Interpolation and Exponentially Suppressed Cosmological Constant in Non-Supersymmetric Heterotic Strings with General Z_2 Twists

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Introduction

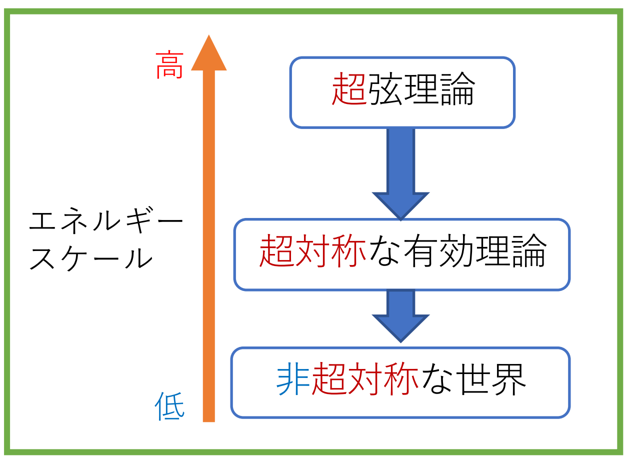
- 加速器実験

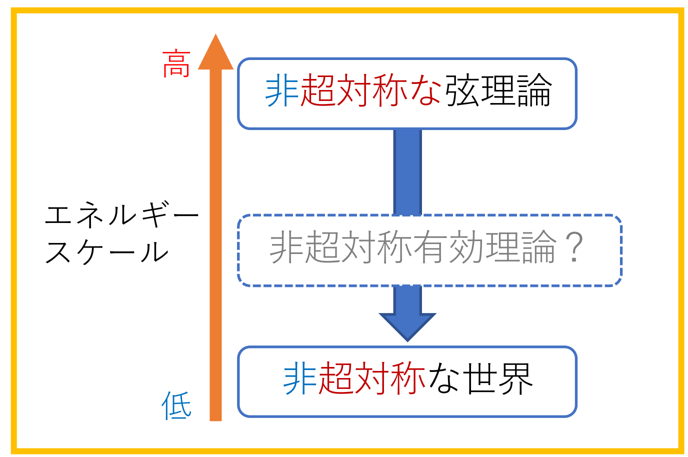
⟹ 到達可能なエネルギースケールではSUSYの証拠なし

- String/Planck scale 程度の高エネルギー領域で
  既にSUSYが破れている可能性

➡ 非超対称な弦理論 (Non-SUSY string) に注目

（概略図）





Introduction

- 10 次元の弦理論 (with modular invariance)：

- Type IIB
- Type IIA
- Type I
- Heterotic $SO(32)$
- Heterotic $E_8 \times E_8$

例外群

- Type 0B
- Type 0A
- Heterotic $SO(32)$
- Heterotic $SO(16) \times E_8$
- Heterotic $SO(16) \times SO(16)$
- Heterotic $E_7^2 \times SU(2)^2$
- Heterotic $SO(24) \times SO(8)$
- …

tachyon free

(Strings with SUSY) < # (Strings without SUSY)

- Non-SUSY "Heterotic" string theoryに焦点を当てる

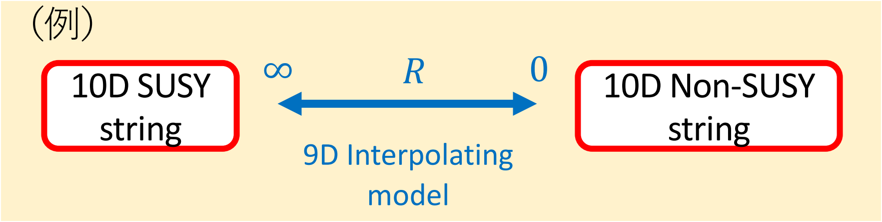
（後述）

Introduction

➢Non-SUSY stringの問題点：宇宙定数が極めて大きくなる

Our approach：

Interpolating model



SUSYが漸近的に回復する $R \approx \infty$ の領域での宇宙定数：　Itoyama, Taylor (1986)

$$\Lambda^{(9)} = (n_F - n_B)\xi / R^9 + \mathcal{O}\left(e^{-R/\sqrt{\alpha'}}\right)$$

(ξ : positive const.
n_B , n_F : # of massless B, F)

- $n_F = n_B \Rightarrow$ 指数関数的に抑圧された宇宙定数をもつ
- しかしモジュライ不安定性の問題（最小値negative等）があった
- 構成法：1方向に対してのみ $\boldsymbol{Z}_2$ Scherk-Schwarz twists

任意の次元数に対して $\boldsymbol{Z}_2$ twistsして得られる模型を調べた

Outline

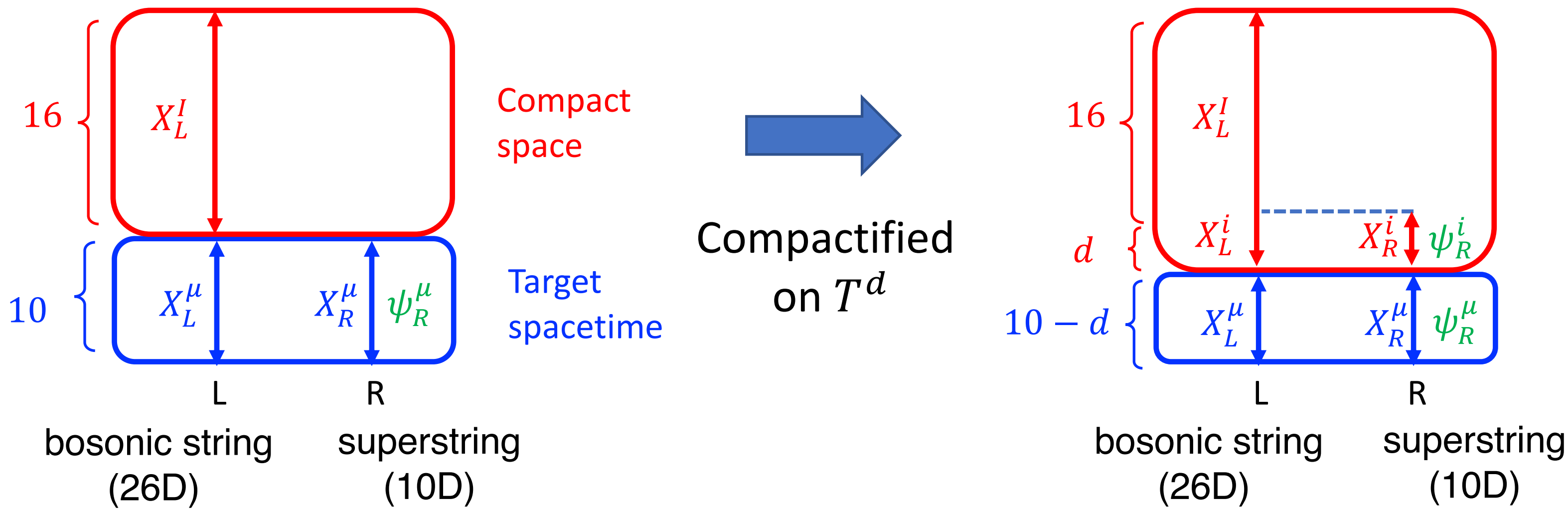
1. Introduction
2. Non-SUSY Hetero with general Z_2 twists
3. Endpoint limits & Interpolations
4. Cosmological Constant
5. Summary

Outline

Heterotic Superstring

Gross, Hervey, Martinec, Rohm (1985)

- Hybrid("heterotic") theory including only closed strings:
  - Left: bosonic string (26D)
  - Right: superstring (10D)
- Compactified on T^d (with maximal SUSY)



- $X_{L,R}^{\mu}, X_{L,R}^{I,i}$ / $\psi_R^{\mu,i}$: bosonic / fermionic coordinates
  $(\mu = 0, \dots 9-d\ ,\ i = 10-d, \dots, 9\ , I = 1, \dots, 16)$

Heterotic Superstring

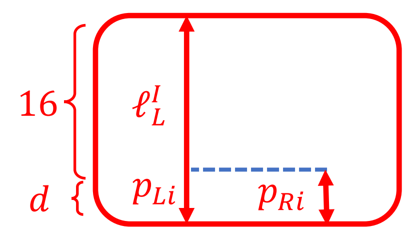
➢ The internal momenta $P = (\ell_L, p_L; p_R) \in \Gamma^{16+d,d}$

- Narain lattice: even self-dual lattice w/ Lorentz. sign. $(16+d, d)$
- Labeled by an integer vector $Z = (q^I, m^i, n_i) \in \boldsymbol{Z}^{16} \times \boldsymbol{Z}^d \times \boldsymbol{Z}^d$

  winding numbers (m^i)   KK momenta (n_i)
- Turn on full moduli: $d(d+16) = d^2 + 16d \quad \Rightarrow (G_{ij} + B_{ij}) + A_i^I$
- Consider a rectangular d-torus: $G_{ij} = R_i^2 \delta_{ij}$

Narain, Sarmadi, Witten, (1986)

$$
\begin{cases}
\ell_L^I = \pi^I - m^i A_i^I, \\
p_{Li} = \dfrac{1}{\sqrt{2}R_i}\left(\pi \cdot A_i + n_i + m^j \left(G_{ij} + B_{ij} - \dfrac{1}{2} A_i \cdot A_j\right)\right) \\
p_{Ri} = \dfrac{1}{\sqrt{2}R_i}\left(\pi \cdot A_i + n_i - m^j \left(G_{ij} - B_{ij} + \dfrac{1}{2} A_i \cdot A_j\right)\right)
\end{cases}
$$



$\pi^I \equiv q^I \alpha_{16} \in \Gamma^{16} \Leftarrow Spin(32)/\boldsymbol{Z}_2$ or $E_8 \times E_8$ lattice

Construction of Non-SUSY Hetero

Dixon, Harvey (1986)
Ginsparg, Vafa (1987)

- Z_2 freely acting orbifold (stringy Scherk-Schwarz comp.)
  - Project out SUSY hetero on T^d by $\frac{1+(-)^F\alpha}{2}$ (+ twisted sec. added)

    Z_2 generator : $(-)^F\alpha$ $\begin{cases} F\text{: spacetime fermion \#} \\ \alpha\text{: shift of order 2 such as } \alpha\,|P\rangle = e^{2\pi i P\cdot\delta}\,|P\rangle \end{cases}$

  - δ is called a shift vector : $2\delta \in \Gamma^{16+d,d}$
    - labeled by a vector $\hat{Z} = (\hat{q}^I, \hat{m}^i, \hat{n}_i)$ whose components are 0 or 1

      → Non-SUSY strings depend on $\hat{Z}$

    - Splitting the Narain lattice $\Gamma^{16+d,d}$ into $\Gamma_+^{16+d,d}$ and $\Gamma_-^{16+d,d}$:

$$\Gamma_+^{16+d,d} = \{P \in \Gamma^{16+d,d} \,|\, \delta\cdot P \in \mathbb{Z}\}$$
$$\Gamma_-^{16+d,d} = \{P \in \Gamma^{16+d,d} \,|\, \delta\cdot P \in \mathbb{Z}+1/2\}$$

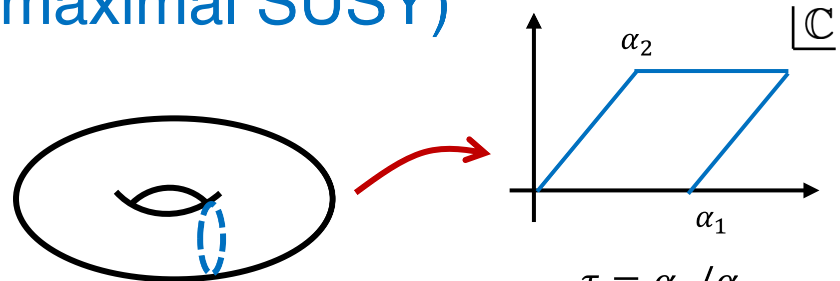
➡ $$\alpha\,|P\rangle = \begin{cases} +\,|P\rangle & \text{for} \quad P \in \Gamma_+^{16+d,d} \\ -\,|P\rangle & \text{for} \quad P \in \Gamma_-^{16+d,d} \end{cases}$$

Boson/Fermion lives in $\Gamma_+^{16+d,d}$ / $\Gamma_-^{16+d,d}$ respectively ➡ SUSY breaking

1-loop Partition Function

- Heterotic strings on T^d (with maximal SUSY)

$$Z^{T^d} = \underbrace{Z_B^{(8-d)}}_{X_L^\mu,\, X_R^\mu} \underbrace{(\bar{V}_8 - \bar{S}_8)}_{\psi_R^\mu,\, \psi_R^i} \underbrace{Z_{\Gamma^{16+d,d}}}_{X_L^I,\, X_L^i,\, X_R^i}$$



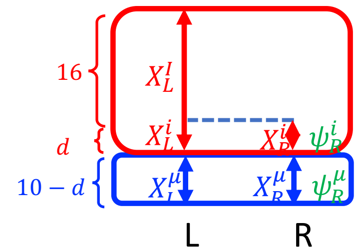
$\tau = \alpha_2/\alpha_1$

orbifolding by $(-)^F \alpha$

$$Z_B^{(8-d)} = \tau_2^{-\frac{8-d}{2}} (\eta\bar{\eta})^{-(8-d)}$$

$$Z_{\Gamma^{16+d,d}} = \eta^{-(16+d)} \bar{\eta}^{-d} \sum_{p \in \Gamma^{16+d,d}} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2}$$

$q = e^{2\pi i \tau}$, V_8, S_8, O_8, C_8: $SO(8)$ characters



- Non-SUSY Heterotic strings

$$Z^{\cancel{SUSY}}_{(\hat{Z})} = Z_B^{(8-d)} \left\{ \underbrace{\bar{V}_8 Z_{\Gamma_+^{16+d,d}}}_{\text{vector}} - \underbrace{\bar{S}_8 Z_{\Gamma_-^{16+d,d}}}_{\text{spinor}} + \underbrace{\bar{O}_8 Z_{\Gamma_\pm^{16+d,d}+\delta}}_{\text{scalar}} - \underbrace{\bar{C}_8 Z_{\Gamma_\mp^{16+d,d}+\delta}}_{\text{co-spinor}} \right\}$$

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Endpoint limits &Interpolations

- Consider $d = 1,2$ cases with $A = B = 0$
- 1-loop partition function:

$$Z^{\cancel{SUSY}}_{(\hat{Z})} = Z_B^{(8-d)} \left\{ \bar{V}_8 \underline{Z_{\Gamma_+^{16+d,d}}} - \bar{S}_8 \underline{Z_{\Gamma_-^{16+d,d}}} + \bar{O}_8 \underline{Z_{\Gamma_\pm^{16+d,d}+\delta}} - \bar{C}_8 \underline{Z_{\Gamma_\mp^{16+d,d}+\delta}} \right\}$$

- The behavior of $Z_{\Gamma_\pm^{16+d,d}(+\delta)}$ in the limits of $R_i \to 0, \infty$
  - Take $R_i \to \infty \Rightarrow$ only $m^i = 0$ contributes $(i = 1,2)$
  - Take $R_i \to 0 \Rightarrow$ only $n_i = 0$ contributes

Example in $d = 1$ with $(\widehat{m}^1, \hat{n}_1) = (1,0)$:

$$Z_{\Gamma_\pm^{17,1}} \xrightarrow{R_1 \to \infty} \frac{R_1}{\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma^{16}}, \quad Z_{\Gamma_\pm^{17,1}+\delta} \xrightarrow{R_1 \to \infty} 0, \qquad \text{SUSY}$$

$$Z_{\Gamma_\pm^{17,1}} \xrightarrow{R_1 \to 0} \frac{1}{R_1\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma_\pm^{16}}, \quad Z_{\Gamma_\pm^{17,1}+\delta} \xrightarrow{R_1 \to 0} \frac{1}{R_1\sqrt{\tau_2}} (\eta\bar{\eta})^{-1} Z_{\Gamma_\pm^{16}+\frac{\pi}{2}}. \qquad \text{Non-SUSY}$$

Endpoint limits &Interpolations

- 9D Non-SUSY heterotic models ($d = 1$)

Itoyama, Koga, Nkajima (2021)

$(\hat{Z} = (\hat{q}, \hat{m}, \hat{n}) \in \boldsymbol{Z}^{16} \times \boldsymbol{Z} \times \boldsymbol{Z})$

$\hookrightarrow \hat{\pi} = \hat{q}\alpha_{16} \in \Gamma^{16}$

- class (1): $|\hat{\pi}|^2 = 0$ (mod 4), $(\hat{m}; \hat{n}) = (0; 0)$

10D non-SUSY $\xleftarrow[0 \qquad\qquad \infty]{R}$ 10D non-SUSY

non-SUSY heterotic strings on a circle

- class (2): $|\hat{\pi}|^2 = 0$ (mod 4), $(\hat{m}, \hat{n}) = (1; 0)$

10D non-SUSY $\xleftarrow[0 \qquad\qquad \infty]{R}$ 10D SUSY

Itoyama, Taylor '86

- class (3): $|\hat{\pi}|^2 = 0$ (mod 4), $(\hat{m}, \hat{n}) = (0; 1)$

10D SUSY $\xleftarrow[0 \qquad\qquad \infty]{R}$ 10D non-SUSY

interpolation between SUSY and non-SUSY vacua (Interpolating model)

- class (4): $|\hat{\pi}|^2 = 2$ (mod 4), $(\hat{m}, \hat{n}) = (1; 1)$

10D SUSY $\xleftarrow[0 \qquad\qquad \infty]{R}$ 10D SUSY

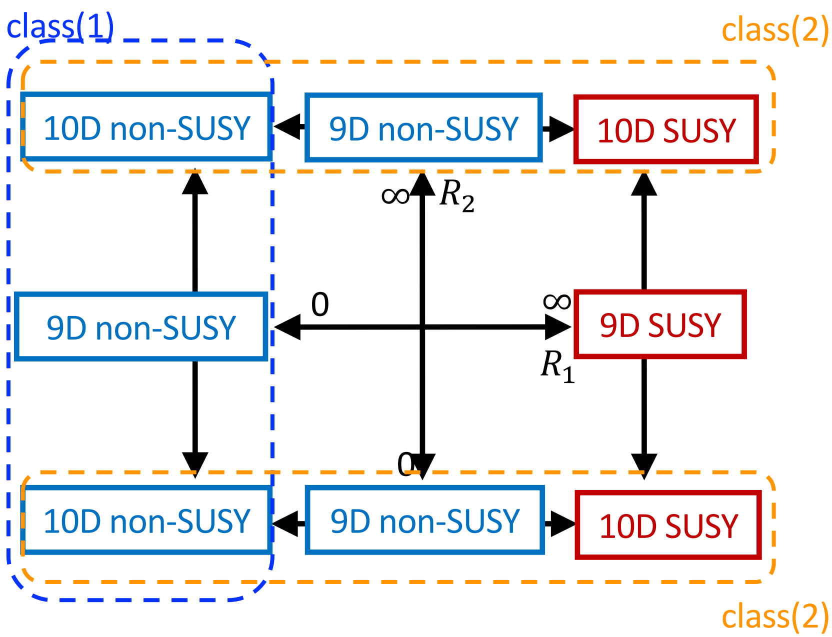
SUSY restored at both of the endpoints

Endpoint limits & Interpolations

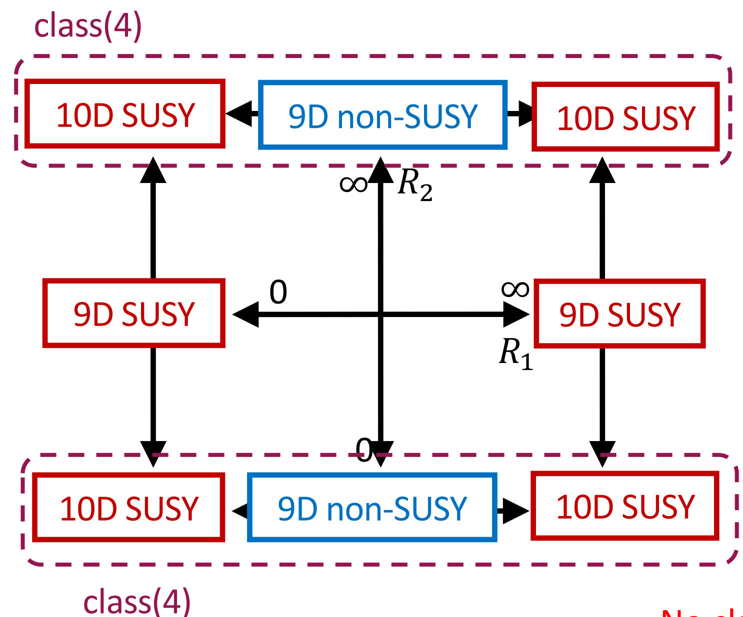
- 8D Non-SUSY heterotic models ($d = 2$)

Koga (2022)

- class (1) & (2) :
  $|\hat{\pi}|^2 = 0 \pmod 4$, $(\hat{m};\hat{n}) = (1,0;0,0)$



- class (1) & (4) :
  $|\hat{\pi}|^2 = 2 \pmod 4$, $(\hat{m};\hat{n}) = (1,0;1,0)$



No class(1)

10D (Non-)SUSY condition

Limits of R_1, R_2	10D SUSY model	10D Non-SUSY model
$(R_1, R_2) \to (\infty, \infty)$	$\hat{m}^1 + \hat{m}^2 > 0$	$\hat{m}^1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (\infty, 0)$	$\hat{m}^1 + \hat{n}_2 > 0$	$\hat{m}^1 + \hat{n}_2 = 0$
$(R_1, R_2) \to (0, \infty)$	$\hat{n}_1 + \hat{m}^2 > 0$	$\hat{n}_1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (0, 0)$	$\hat{n}_1 + \hat{n}_2 > 0$	$\hat{n}_1 + \hat{n}_2 = 0$

Outline

Cosmological Constant

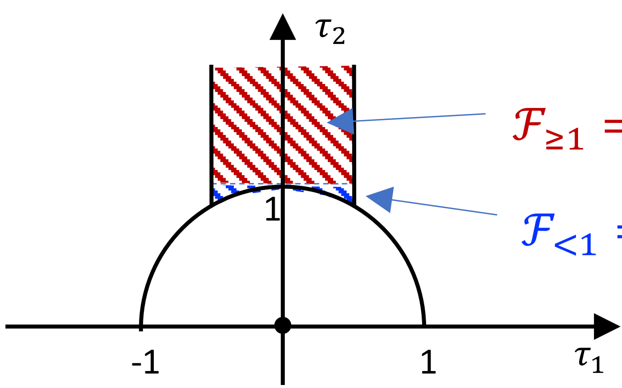
- 1-loop cosmological constant (vacuum energy density) :

$$\Lambda^{(10-d)} = -\frac{1}{2}(2\pi\sqrt{\alpha'})^{-(10-d)} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z^{\cancel{SUSY}}_{(\hat{Z})}$$

Fundamental Region :

$$\mathcal{F} = \left\{ \tau = \tau_1 + i\tau_2 \in \mathbb{C} \,\middle|\, -\frac{1}{2} \le \tau_1 \le \frac{1}{2},\ |\tau| \ge 1 \right\}$$

- decompose $\mathcal{F}$ into $\mathcal{F}_{\ge 1} = \{\tau \in \mathcal{F} | \tau_2 \ge 1\}$ and $\mathcal{F}_{<1} = \{\tau \in \mathcal{F} | \tau_2 < 1\}$



$\mathcal{F}_{\ge 1} = \{\tau \in \mathcal{F} | \tau_2 \ge 1\}$ The integral can be evaluated

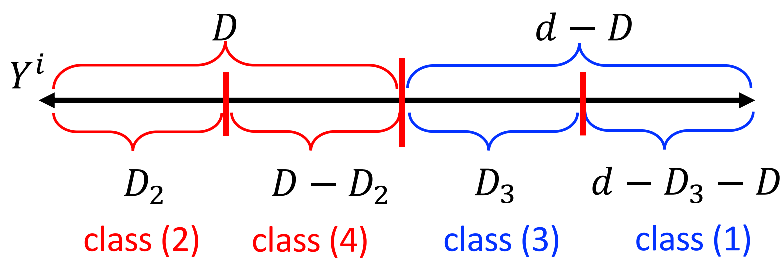
$\mathcal{F}_{<1} = \{\tau \in \mathcal{F} | \tau_2 < 1\}$ exp. suppressed

(10–d)D non-SUSY Heterotic models

- Consider compact coordinates Y^i $(i = 1, \dots, d)$ below :

$$i = a_{(2)} + a_{(4)} + b_{(3)} + b_{(1)}$$

class (#) in 9D



- Assignment of $(\hat{m}, \hat{n})$:

$$(\hat{m}^{a_{(2)}}, \hat{n}_{a_{(2)}}) = (1, 0) \quad \text{for} \quad a_{(2)} = 1, \dots, D_2$$

$$(\hat{m}^{a_{(4)}}, \hat{n}_{a_{(4)}}) = (1, 1) \quad \text{for} \quad a_{(4)} = D_2 + 1, \dots, D$$

SUSY at $R_a \to \infty$

$$(\hat{m}^{b_{(3)}}, \hat{n}_{b_{(3)}}) = (0, 1) \quad \text{for} \quad b_{(3)} = D + 1, \dots, D + D_3$$

$$(\hat{m}^{b_{(1)}}, \hat{n}_{b_{(1)}}) = (0, 0) \quad \text{for} \quad b_{(1)} = D + D_3 + 1, \dots, d$$

Non-SUSY at $R_b \to \infty$

Formula for CC

- Consider $D \geq 1$: SUSY is restored at all $R_i \approx \infty$
  - Up to exponentially suppressed terms,

$$\Lambda^{(10-d)} \sim -\frac{4! \cdot 2^{d-1}}{\pi^{15-d}(\sqrt{\alpha'})^{10-d}} \left(\prod_{i=1}^{d} R_i\right) \sum_{\boldsymbol{n}} \left\{ \sum_{a=1}^{D} (2n_a - 1)^2 R_a^2 + \sum_{b=D+1}^{d} (2n_b)^2 R_b^2 \right\}^{-5}$$
$$\times 8 \left(24 + \sum_{\pi \in \Delta_g} \exp\left[2\pi i \left\{ \sum_{a=1}^{D} (2n_a - 1)(\pi \cdot A_a) + \sum_{b=D+1}^{d} n_b (\pi \cdot A_b) \right\} \right] \right)$$

Δ_g: nonzero roots of $SO(32)$ or $E_8 \times E_8$

$\Rightarrow$ CC does not depend on all the other endpoint models

massless condition

$2\pi \cdot A_a \in \mathbb{Z}$

$\pi \cdot A_b \in \mathbb{Z}$

$$\Lambda^{(10-d)} \sim \frac{4! \cdot 2^{d-1}}{\pi^{15-d}} \left(\prod_{i=1}^{d} R_i\right) \sum_{\vec{n}} \left\{ \sum_{a} (2n_a - 1)^2 R_a^2 + \sum_{b} (2n_b)^2 R_b^2 \right\}^{-5} (n_F - n_B)$$

Solutions of $n_F = n_B$

- SUSY $SO(32)$ endpoint models:

$$\Delta_{SO(32)} = \left\{\left(\underline{\pm, \pm, 0^{14}}\right)\right\}$$

- Simplest configurations:
  - A_a^I $(a = 1, \ldots, D)$ are the same configuration
  - $A_b^I (b = D + 1, \ldots, d)$ is taken to be 0

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \quad (p + q = 16)\,, \quad A_b = (0^{16})$$

  - $D \in 2\boldsymbol{Z}$: $\quad n_F - n_B = -504 \neq 0$
  - $D \in 2\boldsymbol{Z} + 1$: $\quad n_F - n_B = 4pq - \{2p(p-1) + 2q(q-1)\} - 24$

$$n_F = n_B \Rightarrow (p, q) = (9,7), (7,9)$$

CC is exp. supp. when gauge group is $SO(18) \times SO(14)$

Wilson-line Moduli Stability (1)

- SUSY $SO(32)$ endpoint models:

$$\Lambda^{(10-d)} \sim -\sum_{\boldsymbol{n}} C_{\boldsymbol{n}} \left(24 + 4 \sum_{1 \le I < J \le 16} \cos\left[2\pi\theta^I\right] \cos\left[2\pi\theta^J\right] \right)$$

$$\begin{cases} \theta^I = \sum_{a=1}^{D} (2n_a - 1) \underline{A_a^I} + \sum_{b=D+1}^{d} n_b \underline{A_b^I} \\ C_{\boldsymbol{n}} = \frac{4! \cdot 2^{d+2}}{\pi^{15-d} (\sqrt{\alpha'})^{10-d}} \left(\prod_{i=1}^{d} R_i \right) \left\{ \sum_{a=1}^{D} (2n_a - 1)^2 R_a^2 + \sum_{b=D+1}^{d} (2n_b)^2 R_b^2 \right\}^{-5} \end{cases}$$

sum of WLs

- Simplest configurations are critical points:

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q \right) \quad (p + q = 16)\,, \quad A_b = (0^{16})$$

$$\Rightarrow \quad \frac{\partial \Lambda^{(10-d)}}{\partial A_i^I} \sim 0 \quad (I = 1, \ldots, 16, \; i = 1, \ldots, d)$$

Wilson-line Moduli Stability (2)

- Hessian matrix:
  - Simplest configurations

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \quad (p+q=16)\,, \quad A_b = \left(0^{16}\right)$$

  - $D \in 2\boldsymbol{Z}$:

$$\frac{\partial^2 \Lambda^{(10-d)}}{\partial A_i^I \partial A_j^J} \sim \xi \delta_{IJ} \delta_{ij} \qquad (I, J = 1, \ldots, 16,\ i, j = 1, \ldots, d) \quad \xi > 0$$

$\Rightarrow$ Hessian is positive definite

- A global minimum when the gauge group is $SO(32)$ and no massless fermions exist ($\Lambda < 0$)

Wilson-line Moduli Stability (3)

- Hessian matrix:
  - Simplest configurations

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \quad (p+q=16)\,, \quad A_b = \left(0^{16}\right)$$

  - $D \in 2\boldsymbol{Z}+1$:

$$\frac{\partial^2 \Lambda^{(10-d)}}{\partial A_i^I \partial A_j^J} \sim \begin{cases} (2p-17)\,\xi' \delta_{IJ}\delta_{ij} & (I=1,\ldots,p), \\ (-2p+15)\,\xi' \delta_{IJ}\delta_{ij} & (I=p+1,\ldots,16) \end{cases} \qquad \xi' > 0$$

⇒ Hessian is positive/negative definite for $p = 0{,}16 / \, p = 8$

- A global minimum when the gauge group is $SO(32)$ while a local maximum when the gauge group is $SO(16) \times SO(16)$

- $p = 7{,}9 \ (n_F = n_B) \Rightarrow$ saddle points

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Summary

- $(10-d)$D Non-SUSY models are constructed by orbifolding by $(-)^F\alpha$

  (α : shift of order 2 in Narain lattice)

- Various interpolation are shown in $d=2$ case
- Cosmological constant of $(10-d)$D Non-SUSY models in $R_i \approx \infty$ is

$$\Lambda^{(10-d)} \sim \frac{4! \cdot 2^{d-1}}{\pi^{15-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{\vec{n}} \left\{ \sum_a (2n_a - 1)^2 R_a^2 + \sum_b (2n_b)^2 R_b^2 \right\}^{-5} (n_F - n_B) + \mathcal{O}(e^{-R/\sqrt{\alpha'}})$$

- Find the configurations of WLs which gives exp. supp. CC
- Analyze WL-moduli stability: $n_F = n_B \leftrightarrow$ saddle points

Out look

Higher-loop correction, (meta)stable vacua, cosmology