$SU(N) \times U(1)$ゲージ対称性の破れにおける embedded stringの安定性

based on arXiv:2303.09517[hep-ph]

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Introduction①

Probing high energy physics with gravitational wave observations

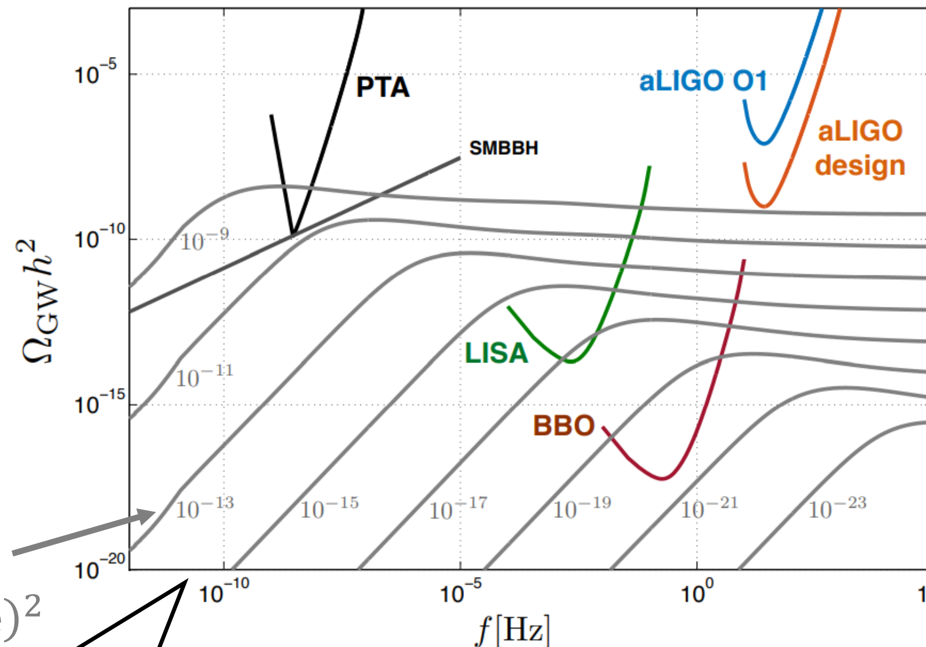
1st order phase transition,
Inflation, **Cosmic strings**, ...

One of classical
solutions in SSB

Gravitational waves

Corresponds to the string tension
$\sim (\text{breaking scale})^2$

[Blanco-Pillado, Olum, Siemens (2018)]



GW spectrum from cosmic strings $\Rightarrow$ Energy scale of SSB ⇐ We want to know!

Which models beyond the SM predict cosmic strings?

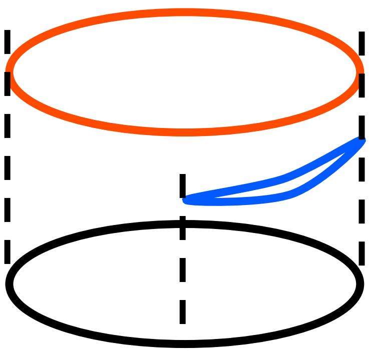
Introduction②

Non-contractible loops on the moduli space $\mathcal{V}$

→ Cosmic strings as **topological defects** [Kibble (1976)]

Ex. $U(1) \to \times$

Non-contractible



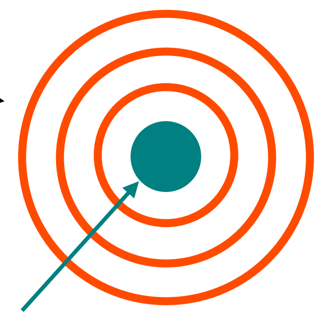
Contractible

$\mathcal{V}(\simeq S^1)$

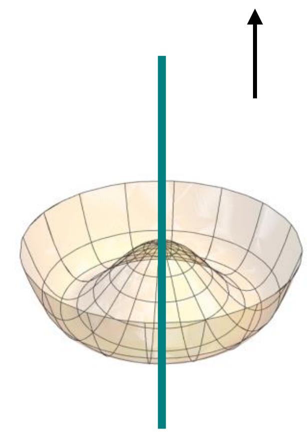
VEV does not change at all the points

$S^1 \to \mathcal{V}$

Boundary condition on the (r, θ) plane



Cannot be on $\mathcal{V}$



z

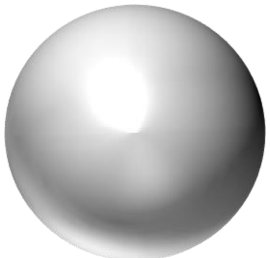
"Nielsen-Olesen string"

[Nielsen, Olesen (1973)]

Is no cosmic string produced when all loops on $\mathcal{V}$ are contractible? ➡ No!!

Introduction③

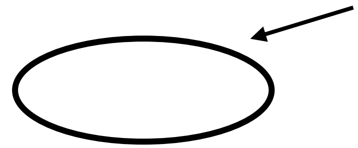
$\mathcal{V}$



No non-contractible loop...

$\supset$

$\mathcal{V}_{sub} \simeq S^1$

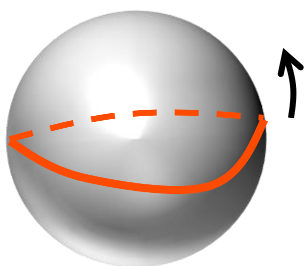


Non contractible loop on $\mathcal{V}_{sub}$

Embedded string

[Vachaspati, Barriola (1992)]
[Vachaspati, Barriola, Bucher (1994)]

Classical solution of a subsystem



$\delta E > 0 \Rightarrow$ classically stable $\Rightarrow$ produced in SSB

$\delta E < 0 \Rightarrow$ classically unstable $\Rightarrow$ not produced in SSB

Well-studied for $SU(2) \times U(1) \rightarrow U(1)$ [James, Perivolaropoulos, Vachaspati (1993)]

But not other symmetry breaking

Our work

The embedded string in $SU(N) \times U(1) \rightarrow SU(N-1) \times U(1)$

1.Introduction

2.Z-string and its stability

3.Embedded strings in $SU(N) \times U(1)$

4.Applications

Z-string

[Nambu (1977)], [Vachaspati (1992)]

$$SU(2) \times U(1) \xrightarrow{H:\ (\mathbf{2}, 1/2)} U(1) \quad \text{with } V(H) = \lambda\left(H^\dagger H - v^2\right)^2$$

$$\cup \qquad\qquad \cup$$

$$U(1) \longrightarrow \times$$

related to the Z boson

$\mathcal{V} \simeq SU(2) \times U(1)/U(1) \simeq S^3$

No non-contractible loops on S^3

Consider Nielsen-Olesen string

Z-string

$$H = \begin{pmatrix} 0 \\ f(r)e^{i\theta} \end{pmatrix}, \qquad \vec{Z} = -\frac{z(r)}{r}\,\vec{e}_\theta, \qquad (\text{others}) = 0$$

in cylindrical coordinate

$$(f(0) = z(0) = 0, f(\infty) = v, z(\infty) = 2/\alpha \quad (\alpha^2 = g_1^2 + g_2^2))$$

Shape of the Z-string

EoM for $f(r), z(r)$

$$f'' + \frac{f'}{r} - \left(1 - \frac{\alpha}{2}z\right)^2 \frac{f}{r^2} + 2\lambda(v^2 - f^2)f = 0$$

$$z'' - \frac{z'}{r} + \alpha\left(1 - \frac{\alpha}{2}z\right)f^2 = 0$$

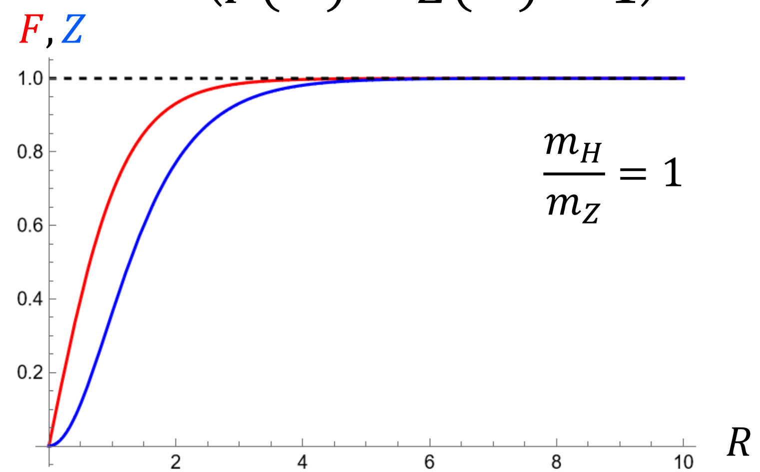
$$\begin{pmatrix} f(0) = z(0) = 0, \\ f(\infty) = v, z(\infty) = \dfrac{2}{\alpha} \end{pmatrix}$$

Normalize as $R \equiv \frac{\alpha v}{2} r, F(R) \equiv \frac{f(r)}{v}, Z(R) \equiv \frac{\alpha}{2} z(r)$

$$F'' + \frac{F'}{R} - (1 - Z)^2 \frac{F}{R^2} + \frac{8\lambda}{\alpha^2}(1 - F^2)F = 0$$

$$Z'' - \frac{Z'}{R} + 2(1 - Z)F^2 = 0$$

$$\begin{pmatrix} F(0) = Z(0) = 0, \\ F(\infty) = Z(\infty) = 1 \end{pmatrix}$$



$F(R)$ and $Z(R)$ are determined

by $\frac{8\lambda}{\alpha^2} = \frac{m_H}{m_Z}$ (m_H: the mass of Higgs; m_Z: the mass of Z boson)

Classical stability of the Z-string

[James, Perivolaropoulos, Vachaspati (1993)]

Consider perturbations from the Z-string

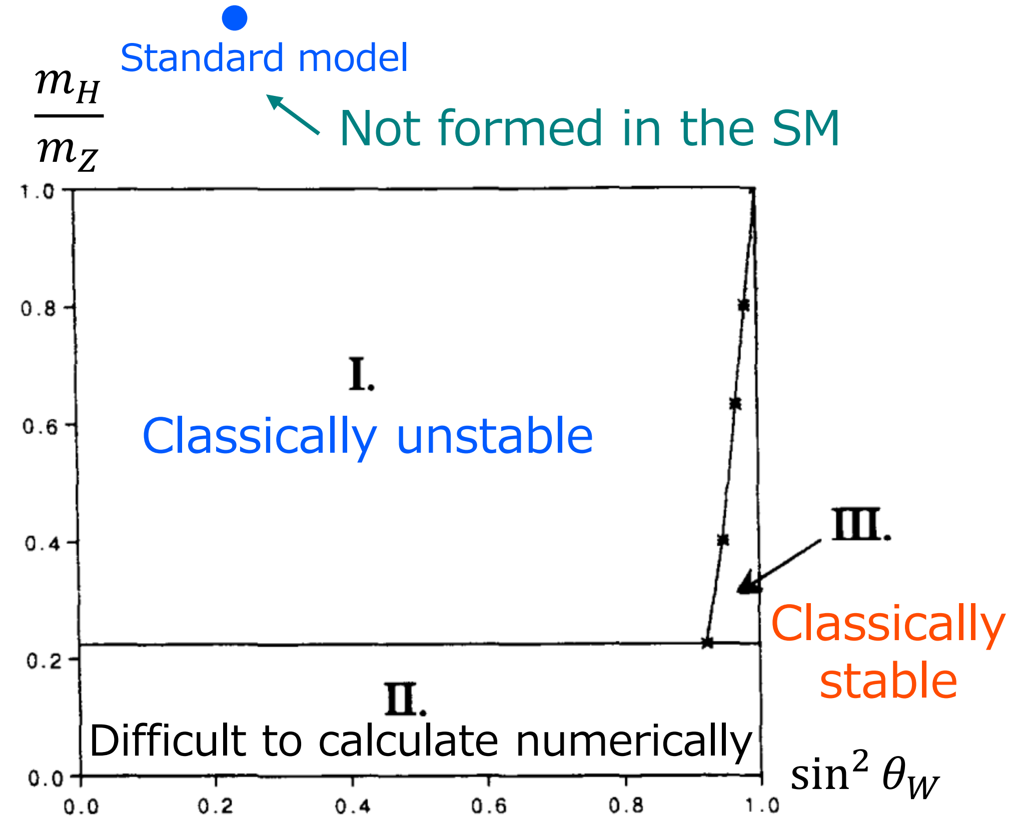
$$H = \begin{pmatrix} \phi(x) \\ f(r) v e^{i\theta} + \delta h(x) \end{pmatrix}, \vec{Z} = -\frac{z(r)}{\alpha r}\, \vec{e}_\theta + \delta\vec{Z}(x), \vec{W}^{\pm}(x), \vec{A}(x)$$

Calculate the variations of the energy and find modes decreasing it

Taking up to the quadratic terms, they are divided into two parts

$\phi(x), \vec{W}^{\pm}(x)$	$\delta h(x), \delta\vec{Z}(x), \vec{A}(x)$
Charged	Neutral (Non-negative)

Some calculations



1. Introduction

2. Z-string and its stability

3. Embedded strings in $SU(N) \times U(1)$

4. Applications

Embedded string in $SU(N) \times U(1)$

We consider $SU(N) \times U(1) \xrightarrow{\phi: \left(\boldsymbol{N}, \frac{1}{2}\right)} SU(N-1) \times U(1)$

Higgs potential: $V(\phi) = \lambda(|\phi|^2 - v^2)^2$ ⟵ $\mathcal{V} \simeq S^{2N-1}$

➡ There is a neutral massive gauge boson $\tilde{Z}_\mu$

$$\tilde{Z}_\mu \equiv \sqrt{\frac{2(N-1)}{N}} \frac{g_N}{\alpha_N} G_\mu^{N^2-1} - \frac{g_1}{\alpha_N} B_\mu$$

(G_μ^a, B_μ: $SU(N), U(1)$ gauge bosons

$T^{N^2-1} = \frac{1}{\sqrt{2N(N-1)}} \mathrm{diag}(1, \ldots, 1, 1-N)$

$\alpha_N^2 \equiv \frac{2(N-1)}{N} g_N^2 + g_1^2$)

⬇ Make an embedded string

Generalized Z-string

$f(0) = z(0) = 0, f(\infty) = v, z(\infty) = 2/\alpha_N$

$$\phi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f(r)e^{i\theta} \end{pmatrix}, \quad \vec{\tilde{Z}} = -\frac{z(r)}{r} \vec{e}_\theta, \quad \text{(others)}=0$$

Note that it is the Z-string when $N = 2$

Classical stability

Up to the quadratic terms in the variation of the energy linear density, they are divided into 3 parts from the perspective of $SU(N-1)$ rep.

Higgs: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_{N-1}(x) \\ f(r)e^{in\theta} + \delta\phi(x) \end{pmatrix}$, Gauge boson: $\begin{pmatrix} \vec{G}^a(x) & \vec{G}^+(x) \\ \vec{G}^-(x) & \end{pmatrix}$,

Diagonal part:

$$\vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\vec{e}_\theta + \delta\vec{\tilde{Z}}(x),$$

$$\vec{\tilde{A}}(x) \equiv \frac{g_1}{\alpha_N}\vec{G}^{N^2-1} + \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}\vec{B}$$

- :$SU(N-1)$ adjoint
- :$SU(N-1)$ fundamental
- :$SU(N-1)$ singlet

The variation made by $SU(N-1)$ adjoint modes and singlet modes do not become negative

$$\delta\mu_{ad} \propto \sum_a \left(\nabla \times \vec{G}^a\right)^2, \delta\mu_s = (\text{perturbation from the N}-0\text{ string in } U(1) \text{ Higgs model})$$

Classical stability

The variation made by $SU(N-1)$ fundamental modes are divided into $N-1$ parts

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_k(x) \\ \vdots \\ \phi_{N-1}(x) \\ f(r)e^{in\theta} + \delta\phi(x) \end{pmatrix}, \quad \vec{G}^{\pm}(x) = \begin{pmatrix} \vec{G}_1(x) \\ \vdots \\ \vec{G}_k(x) \\ \vdots \\ \vec{G}_{N-1}(x) \end{pmatrix}, \quad \vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\vec{e}_\theta$$

After some calculations and the normalization of $r, f(r), z(r)$, we can see that the classical stability depends on (g_1, g_N, λ, N)

$$g_N \to m_G/v, \quad g_1 \to \sqrt{m_{\tilde{Z}}^2 - 2(N-1)m_G^2/N}/v, \quad \lambda \to m_\phi^2/(8v^2)$$

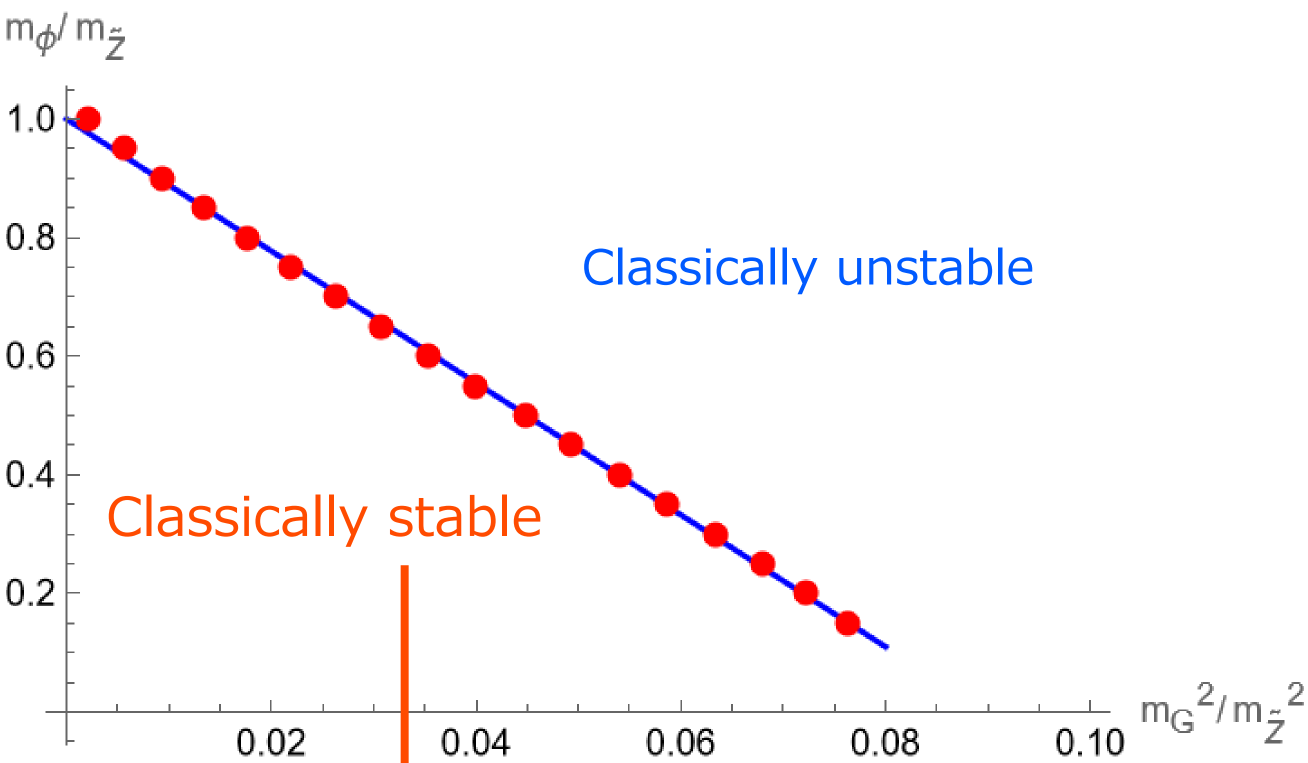
We can remove N-dependence by using $(m_\phi/m_{\tilde{Z}}, m_G/m_{\tilde{Z}})$

The result for the Z-string can be applied!

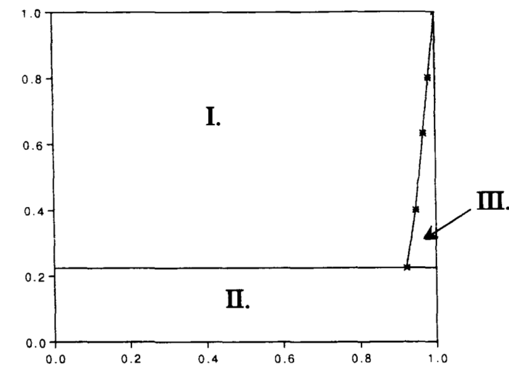
$$\left(\frac{m_H}{m_Z}, \cos\theta_W = \frac{m_W}{m_Z}\right)$$

($m_\phi, m_{\tilde{Z}}, m_G$: the mass of Higgs, neutral gauge boson, charged gauge boson)

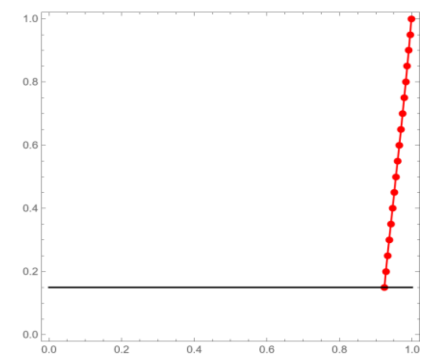
Classical stability



[James, Perivolaropoulos, Vachaspati (1993)]



Ours result



approximately evaluation

$$\frac{m_\phi}{m_{\tilde{Z}}} \leq 1 - 11\frac{m_G^2}{m_{\tilde{Z}}^2} \quad \Leftrightarrow \quad g_1 \geq \sqrt{\frac{11}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N$$

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Unification into $SU(N+1)$

We consider the case that $SU(N)$ and $U(1)$ have the same origin

Ex. Unified into $SU(N+1)$

$$\phi = (\boldsymbol{N}, q)\Big|_{g_1'} = (\boldsymbol{N}, 1/2)\Big|_{g_1}$$

$$SU(N+1) \to SU(N) \times U(1) \to SU(N-1) \times U(1)$$

$$g_{N+1} = g_N = g_1' \qquad g_N = g_1' = \frac{1}{2q} g_1$$

(Cf. $SU(5)$ GUT
$g_5 = g_{3C} = g_{2L} = g_1 = \sqrt{5/3}\, g_Y$)

The generalized Z-strings are formed when g_N and g_1 satisfy

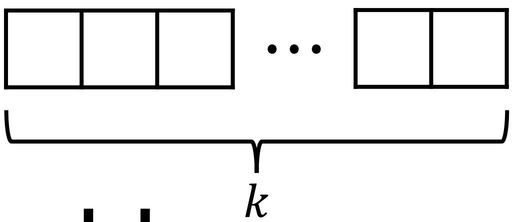
$$g_1 \geq \sqrt{\frac{11}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N \quad \Rightarrow \quad q^2 \geq \frac{2.75}{1 - m_\phi/m_{\tilde{Z}}} - \frac{N-1}{2N}$$

Constraint for $|q|$

$|q|$ depends on what representation of $SU(N+1)$ includes ϕ

Example of $SU(N+1)$

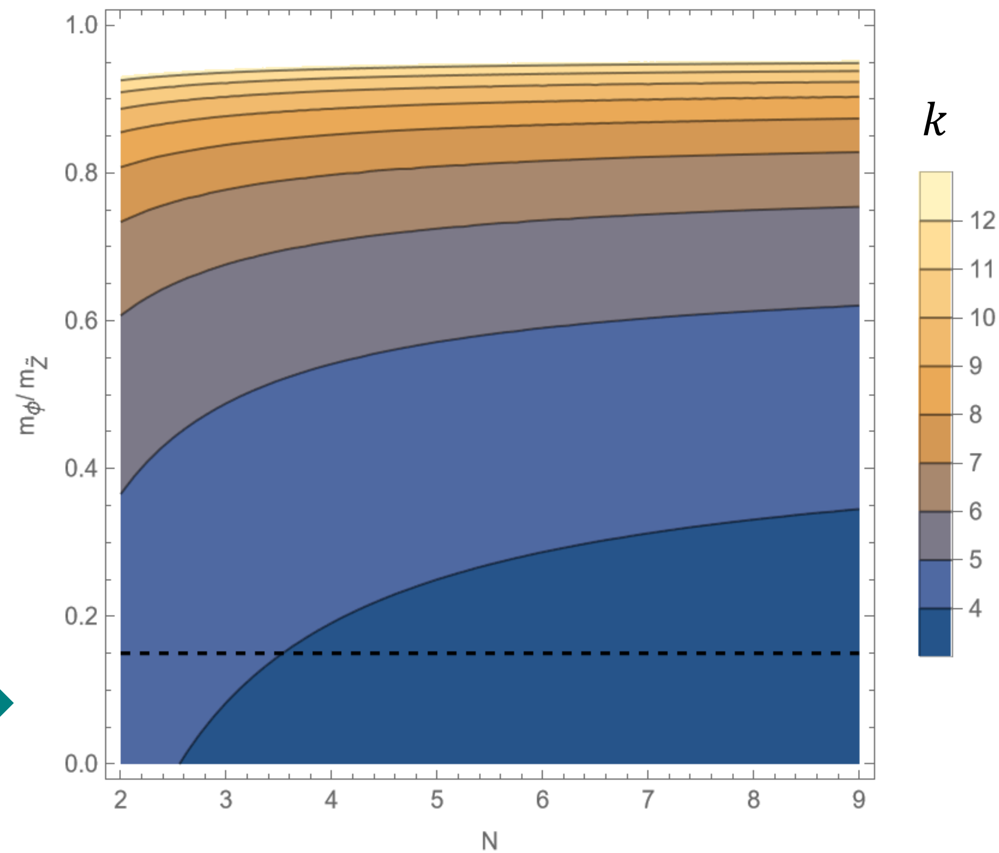
Completely symmetric
k-th rank tensor of $SU(N+1)$



$\cup$

$$\phi = \left(\boldsymbol{N}, \frac{1-(k-1)N}{\sqrt{2N(N+1)}} \right)$$

$$q^2 \geq \frac{2.75}{1 - m_\phi / m_{\tilde{Z}}} - \frac{N-1}{2N}$$



At least, $k \geq 4$ is needed to produce the generalized Z-string

Application for GUT breaking

General unification

$$\phi = (\boldsymbol{N}, q, \boldsymbol{1})\Big|_{g_1{}'} = (\boldsymbol{N}, 1/2, \boldsymbol{1})\Big|_{g_1}$$

$$G \to \cdots \to SU(N) \times U(1) \times H \to SU(N-1) \times U(1) \times H$$

$$g_U = g_N = g_1{}' \xrightarrow{\text{RG running}} g_N = \alpha_{RG}\, g_1{}' = \frac{\alpha_{RG}}{2q} g_1$$

$$\Rightarrow \quad q^2 \geq \alpha_{RG}^2 \left[\frac{2.75}{1 - m_\phi / m_{\tilde{Z}}} - \frac{N-1}{2N}\right]$$

We apply it for

① $\boldsymbol{SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y}$

$\phi = (\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{2}, q)$

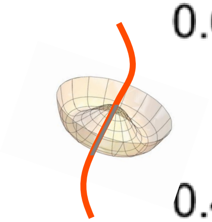
② $\boldsymbol{SO(10) \to SU(4)_C \times SU(2)_L \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y}$

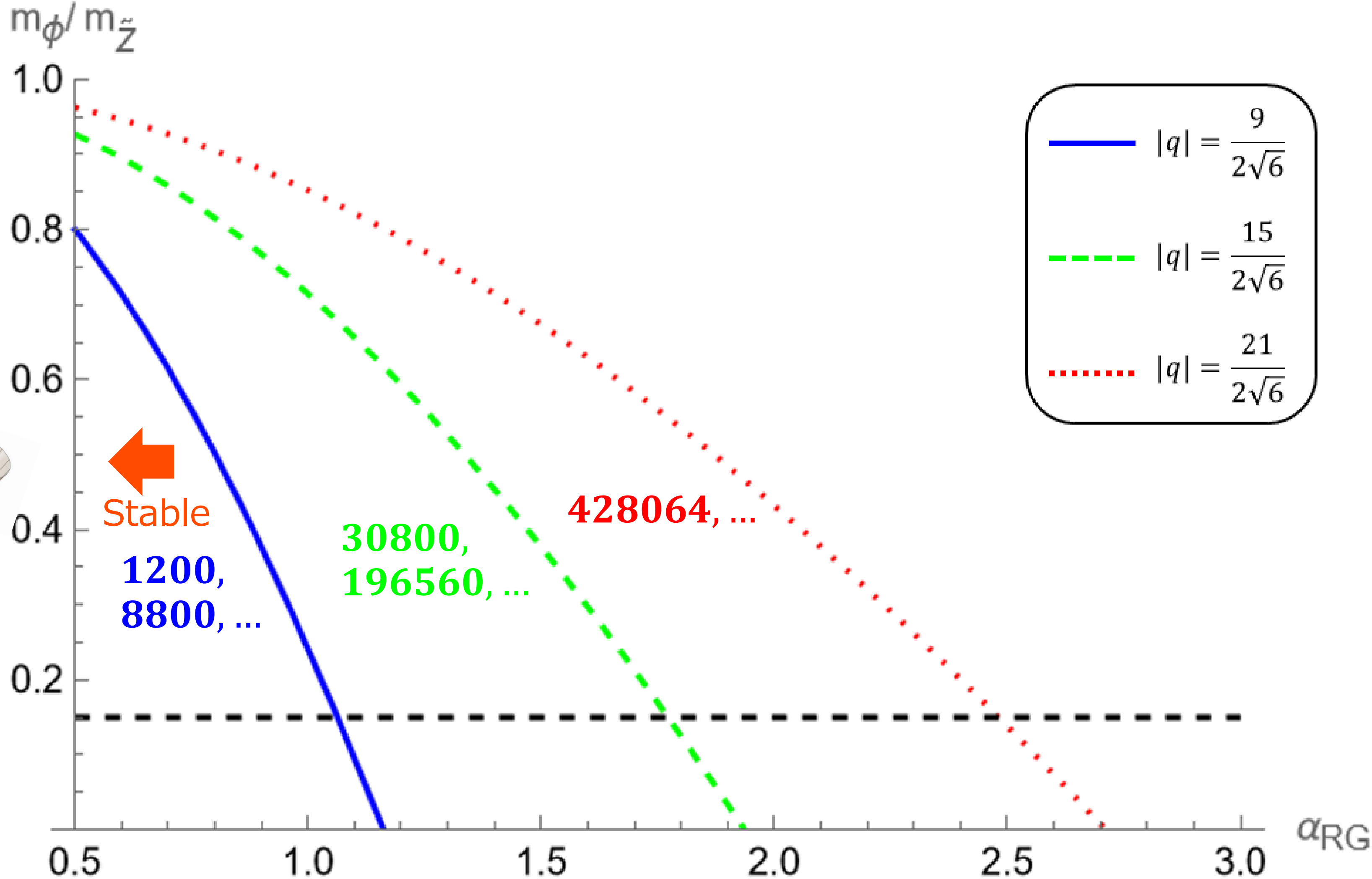
$\phi = (\boldsymbol{4}, \boldsymbol{1}, q)$

① $SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y$

($\alpha_{RG} = g_{2R}/g_{1X}$ at the breaking scale)

$\phi = (1, 1, 2, q)$

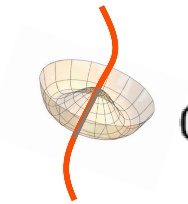


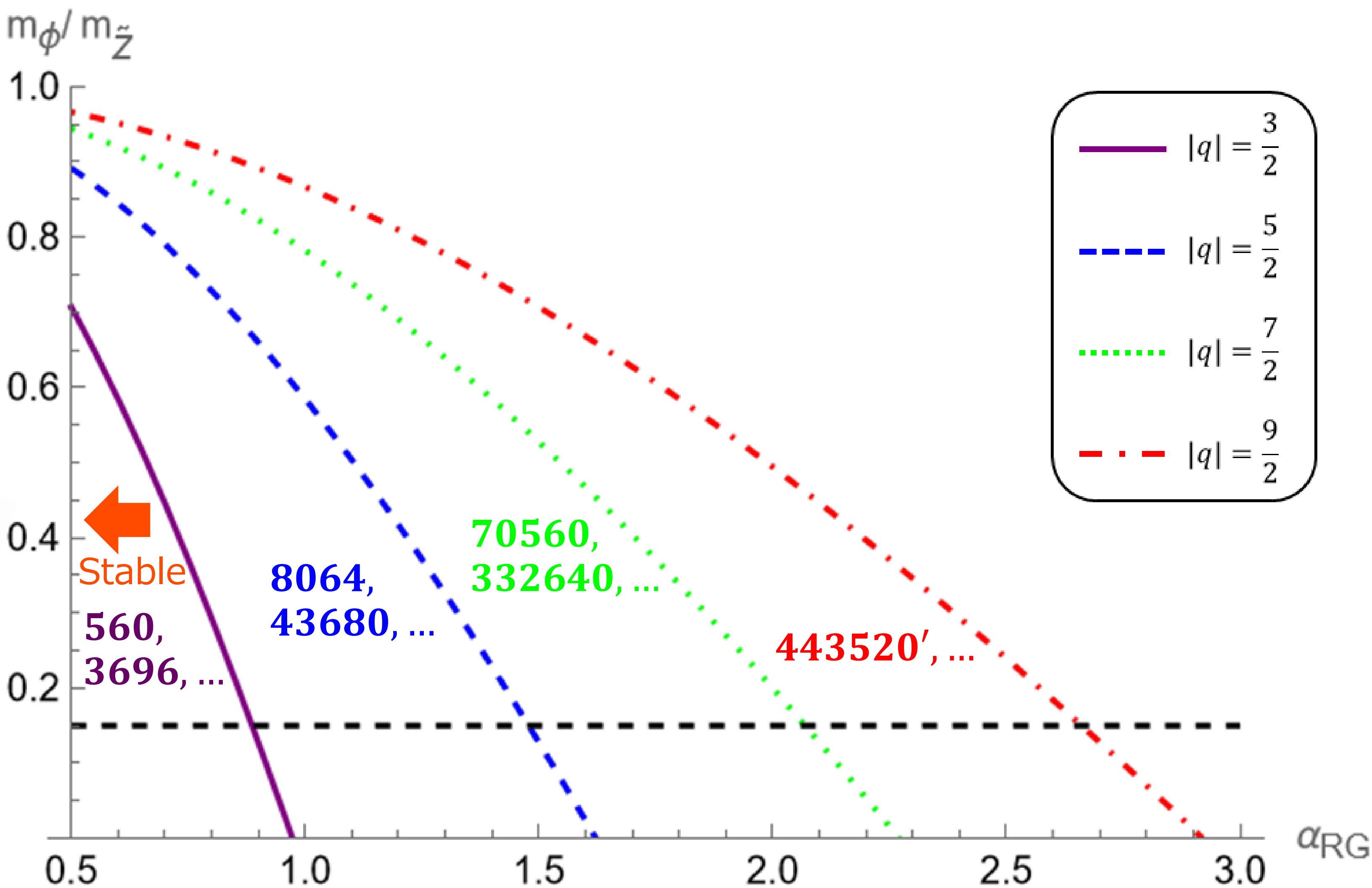


② $SO(10) \to SU(4)_C \times SU(2)_L \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y$

($\alpha_{RG} = g_{4C}/g_{1X}$ at the breaking scale)

$\phi \supset (\mathbf{4}, \mathbf{1}, q)$





Summary and outlook

- The embedded string solutions exist even if there are no non-contractible loop on $\mathcal{V}$.
- The classical stability of the embedded string in $SU(N) \times U(1) \to SU(N-1) \times U(1)$ is determined by the mass ratios of Higgs and massive gauge bosons.
- If $SU(N)$ and $U(1)$ are unified into a simple group, the large representation scalar is needed to produce the generalized Z-string.
- For GUT, it is difficult to unify matter fermion with large representation Higgs.
- We want to know how the embedded string will be observed by GW observation or other cosmological observation.