

New approaches to/from dark matter

Koji Ishiwata

Kanazawa University

- **PRB** 106 (2022) 19, 195157, **PRD** 104 (2021) 1, 016004
- **PRD** 106 (2022) 10, 103014 (with S. Ando, N. Hiroshima)

素粒子現象論研究会2022

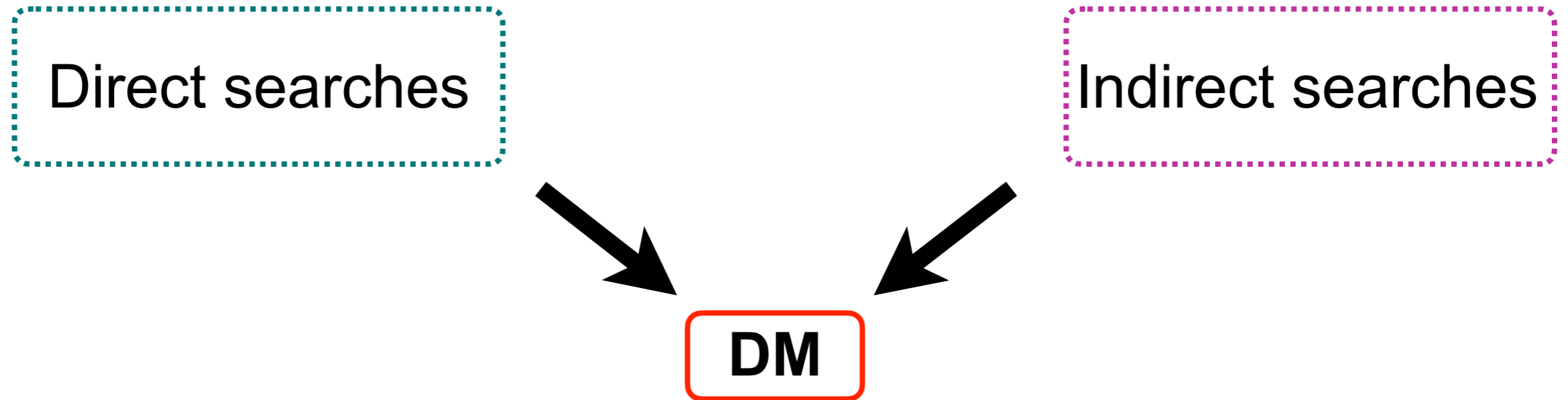
Osaka, March 16, 2023

1. Introduction

Dark matter (DM)

- Electrically neutral
- Non-baryonic
- Stable or sufficiently long-lived
- Non-relativistic
- $\Omega_{\text{DM}} \simeq 0.26$
- $10^{-31} \text{ GeV} < m_{\text{dm}} \lesssim M_{\text{Pl}}$ or $10^{-14} < m_{\text{dm}}/M_{\odot} \lesssim 10^{-12}$

Approaches from astro-particle physics and cosmology



Approaches from astro-particle physics and cosmology

Direct searches

$$\text{DM SM} \rightarrow \text{DM}^{(\prime)} \text{SM}^{(\prime)}$$

Indirect searches

$$\text{DM (DM)} \rightarrow \text{SMs}$$

Motivated and
intensively searched



GeV

TeV



DM mass

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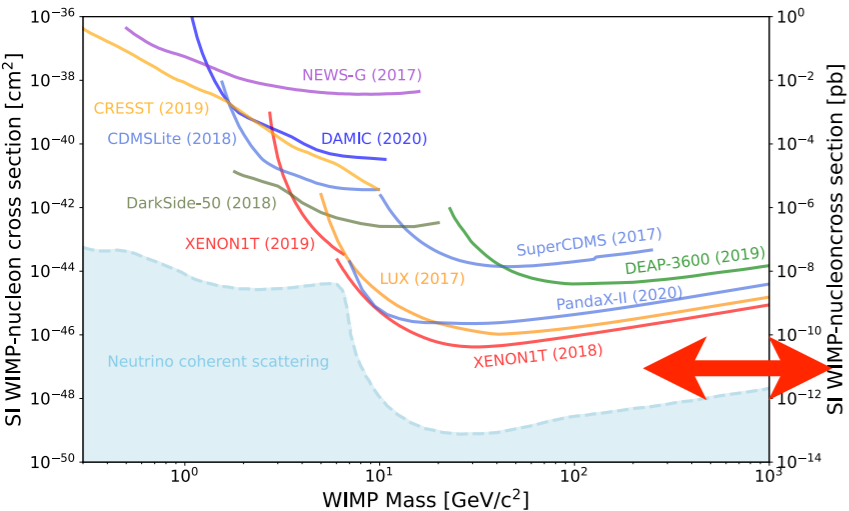
GeV

TeV



DM mass

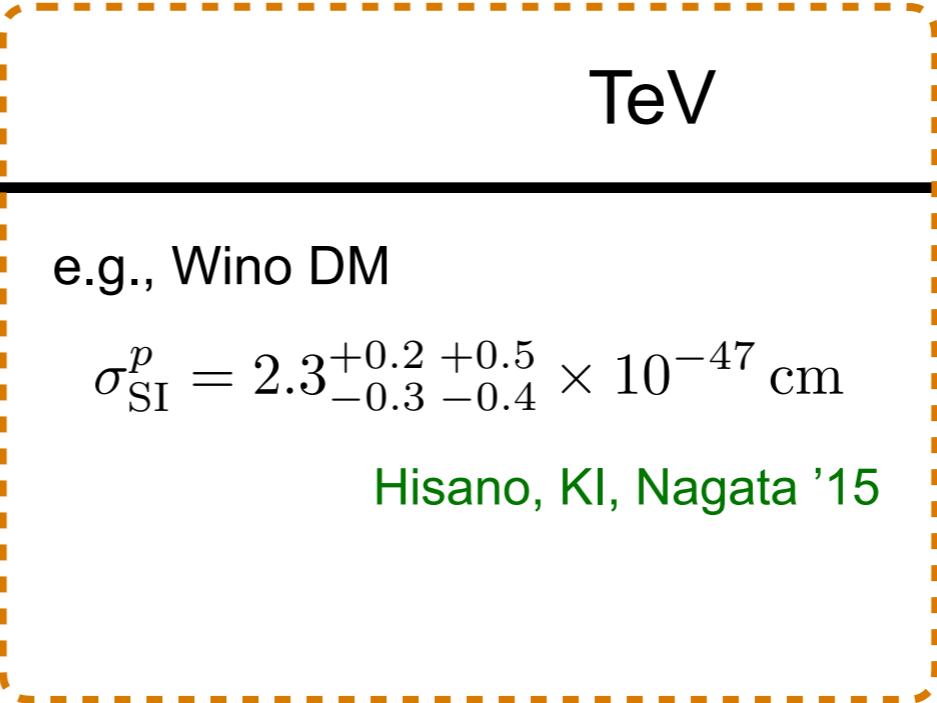
PDG '22



e.g., Wino DM

$$\sigma_{\text{SI}}^p = 2.3_{-0.3}^{+0.2} {}_{-0.4}^{+0.5} \times 10^{-47} \text{ cm}^2$$

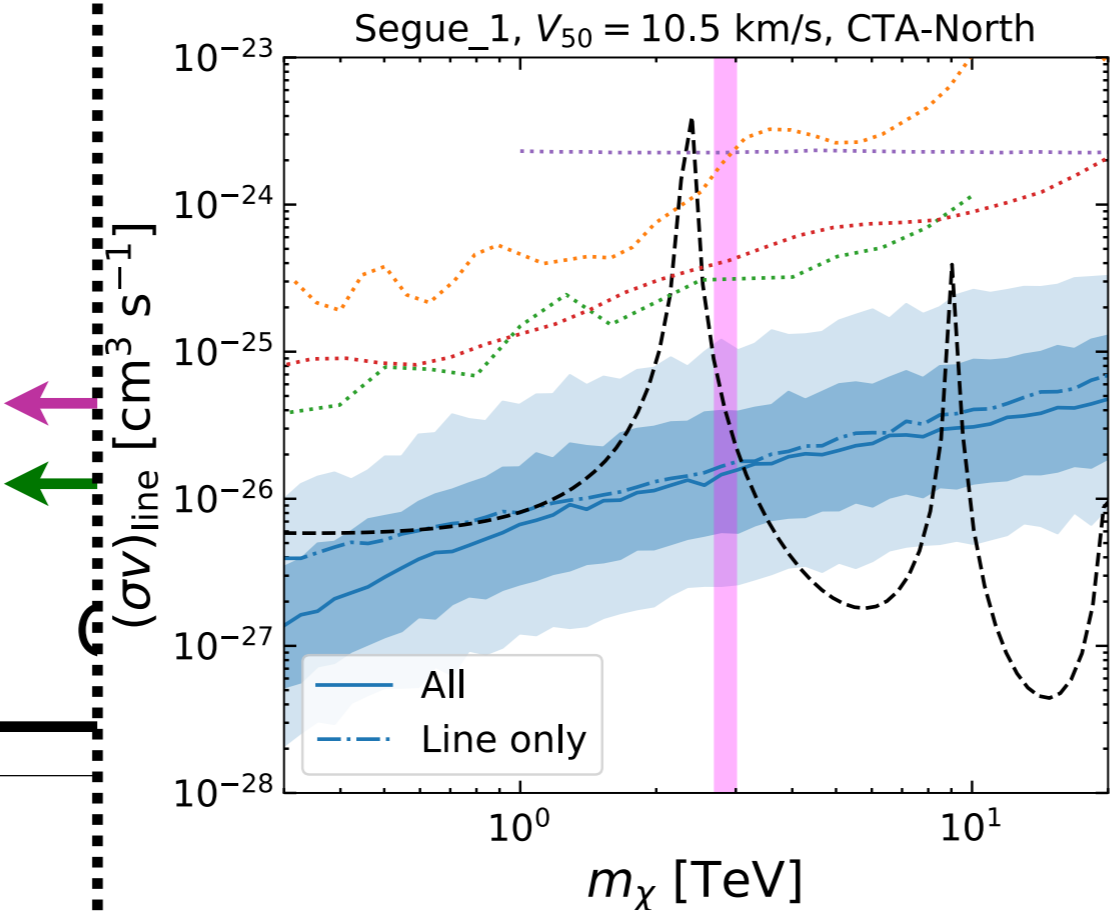
Hisano, KI, Nagata '15



Approaches from astro-particle physics and cosmology

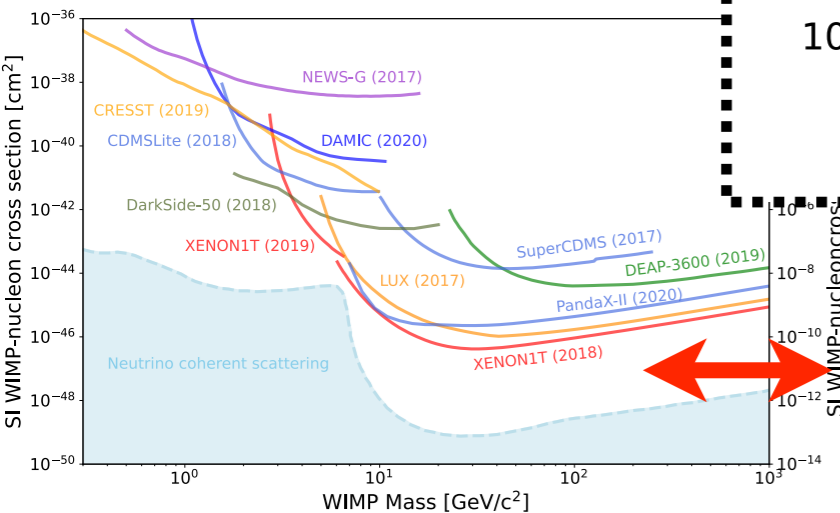
Direct search **We'll find thermal Wino DM from ultrafaint dSphs** indirect searches

DM SM \rightarrow DM SM \rightarrow DM SM (DM) \rightarrow SMs



Ando, KI '21

PDG '22



Hisano, KI, Nagata '15

DM mass

Approaches from astro-particle physics and cosmology

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intensively searched



GeV

TeV



DM mass

e.g.,

- Migdal effect

Ibe et al. '17

- Upscattering

Bringmann et al. '19

Ema et al. '19

Approaches from astro-particle physics and cosmology

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DM mass

Approaches from astro-particle physics and cosmology

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10^{-31} GeV



M_{Pl}

DM mass

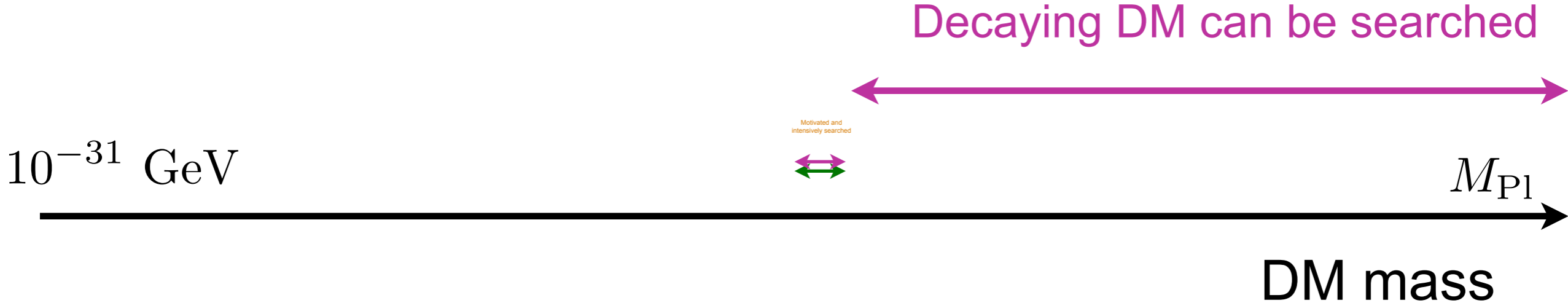
Approaches from astro-particle physics and cosmology

Direct searches

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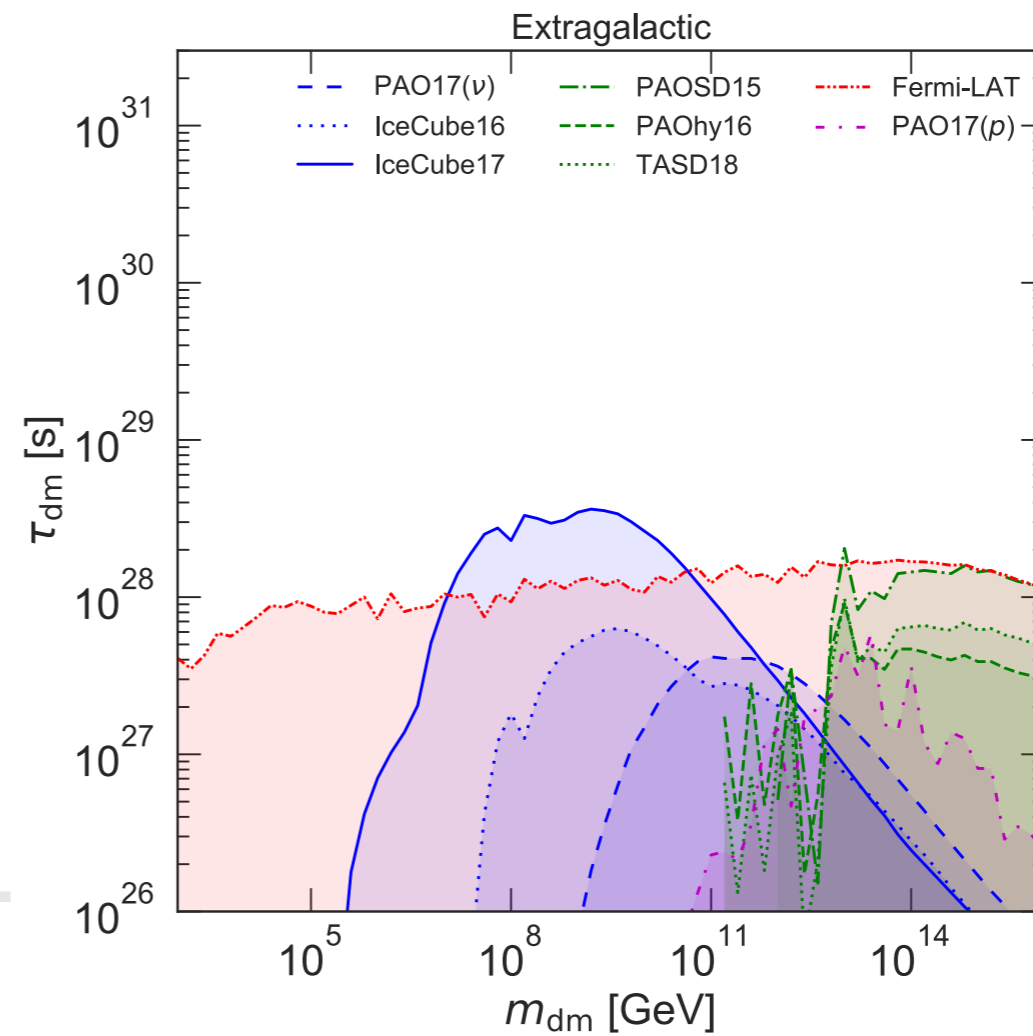
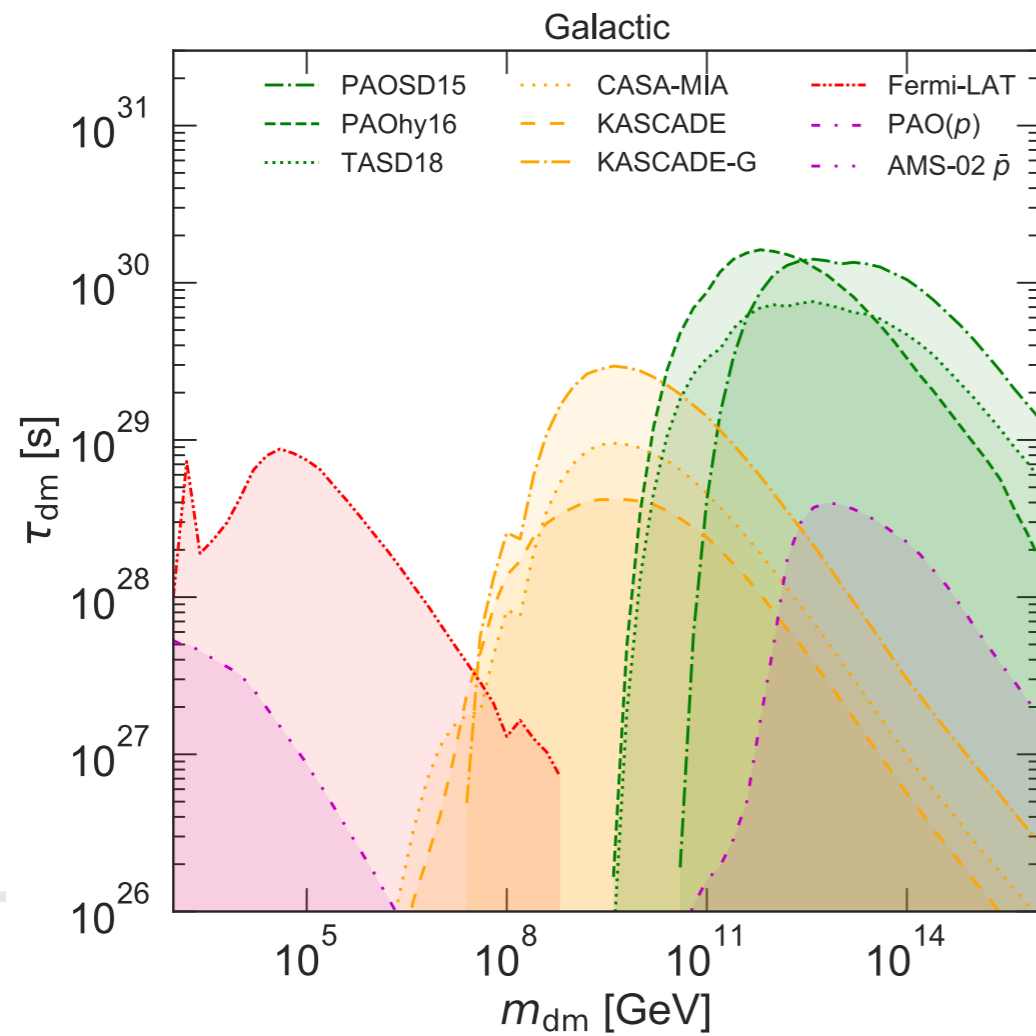


Approaches from astro-particle physics and cosmology

Direct searches

Indirect searches

Ando, Arimoto, KI, Macias '20



SMs

arched

M_{Pl}

ass

Past works:

Esmaili, Ibarra, Peres '12

Murase, Beacom '12

Ahlers, Murase '14

Murase, Laha, Ando, Ahlers '15

Aloisio, Matarrese, Olinto '15

Kalashov, Kuznetsov '16

Cohen, Murase, Rodd, Safdi, Soreq '17

Kachelriess, Kalashov, Kuznetsov '18

Sui, Bhupal Dev '18

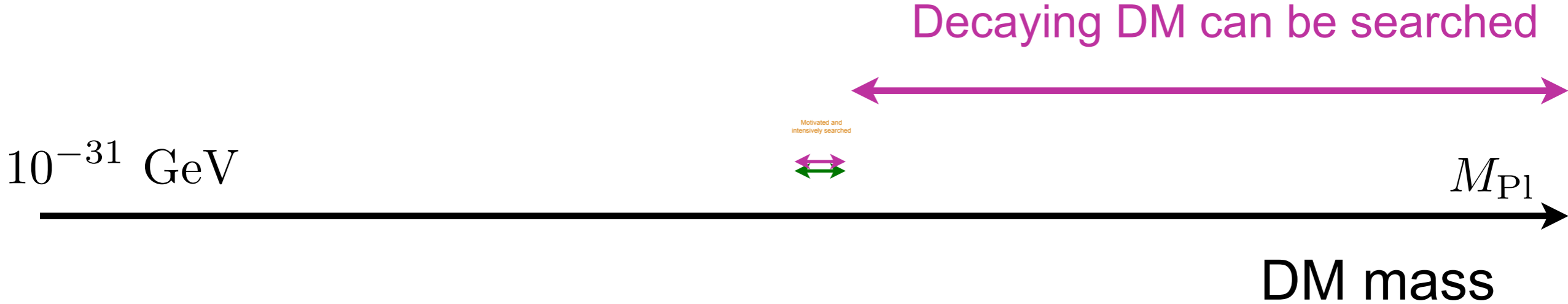
Approaches from astro-particle physics and cosmology

Direct searches

$$\text{DM SM} \rightarrow \text{DM}^{(\prime)} \text{SM}^{(\prime)}$$

Indirect searches

$$\text{DM (DM)} \rightarrow \text{SMs}$$



Approaches from astro-particle physics and cosmology

Direct searches

~~DM SM \rightarrow DM^(l) SM^(l)~~

using **topological insulators**

Axion, Axion-like particles

Indirect searches

DM (DM) \rightarrow SMs

Today's 1st topic

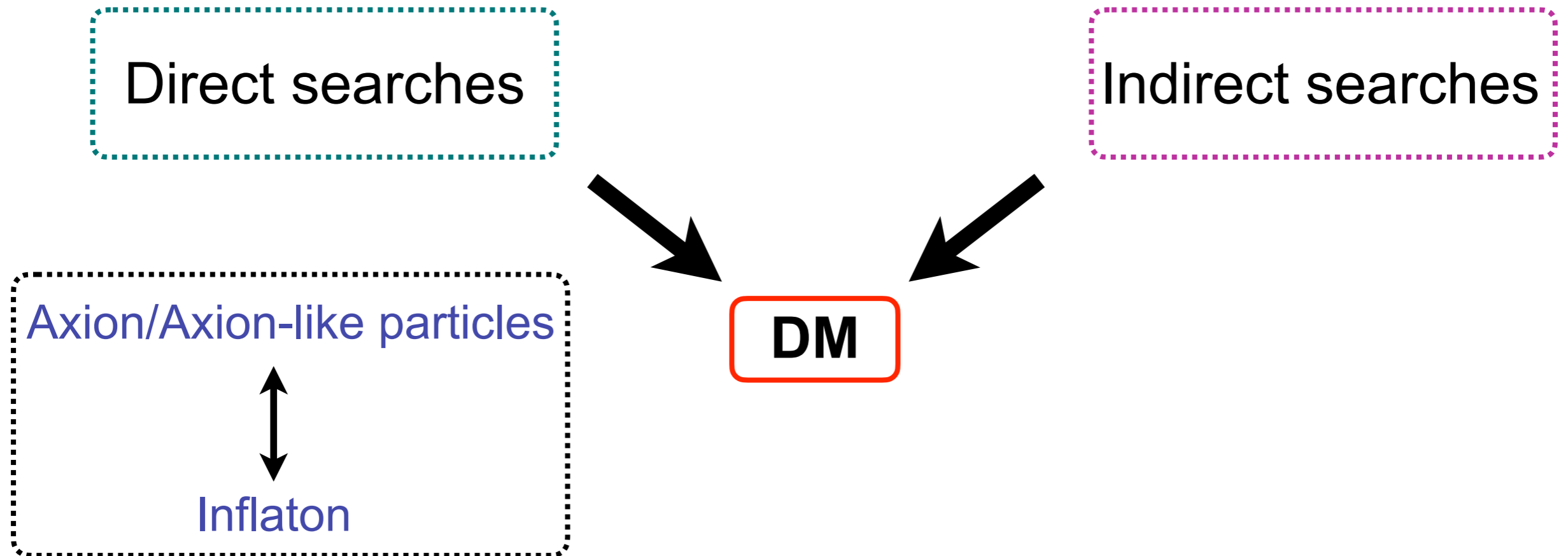
10^{-31} GeV

Motivated and intensively searched

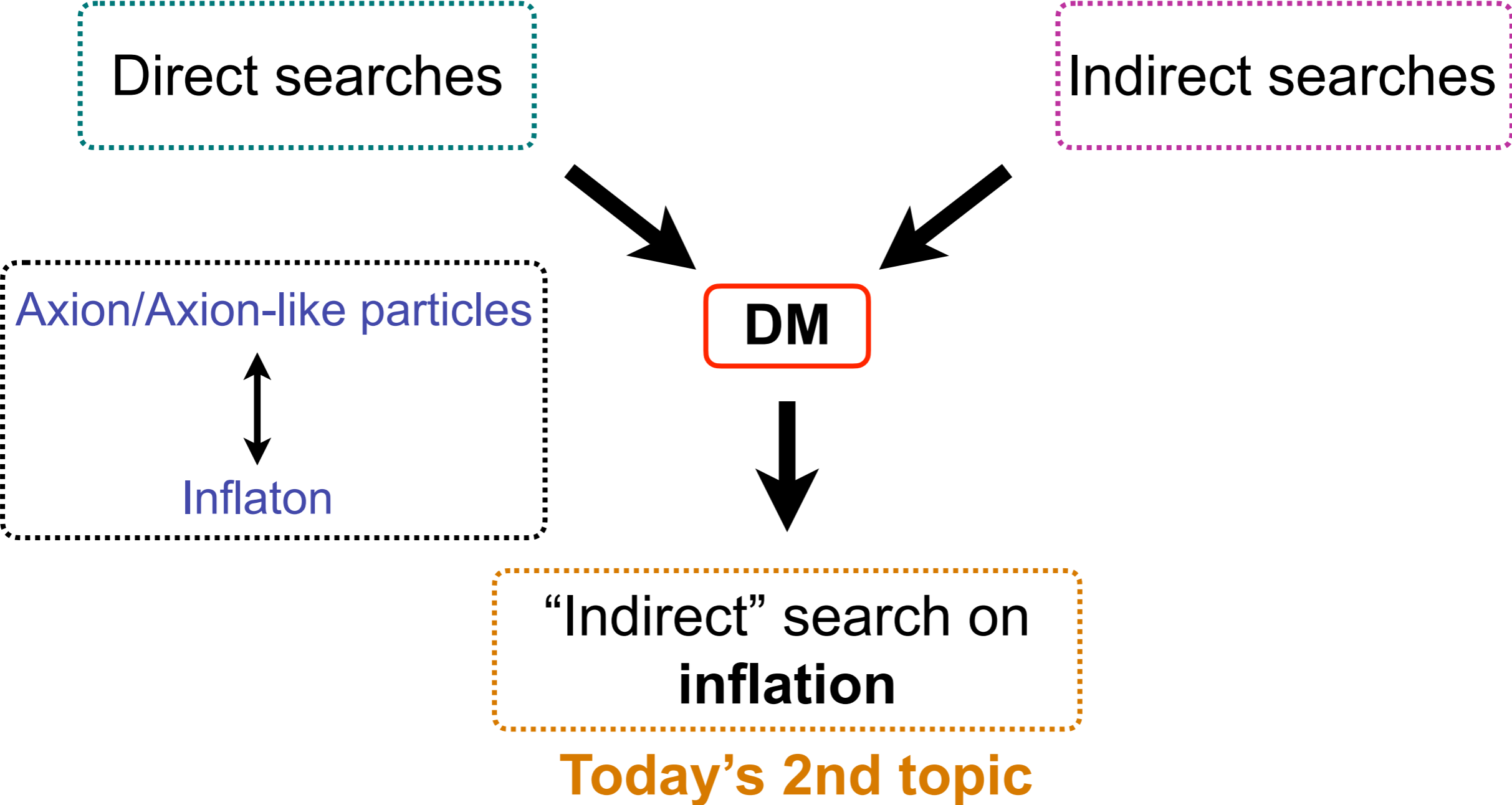
M_{Pl}

DM mass

Approaches from astro-particle physics and cosmology



Approaches from astro-particle physics and cosmology



Outline

1. Introduction

2. Axion in topological insulators

3. Primordial curvature perturbations

4. Conclusion

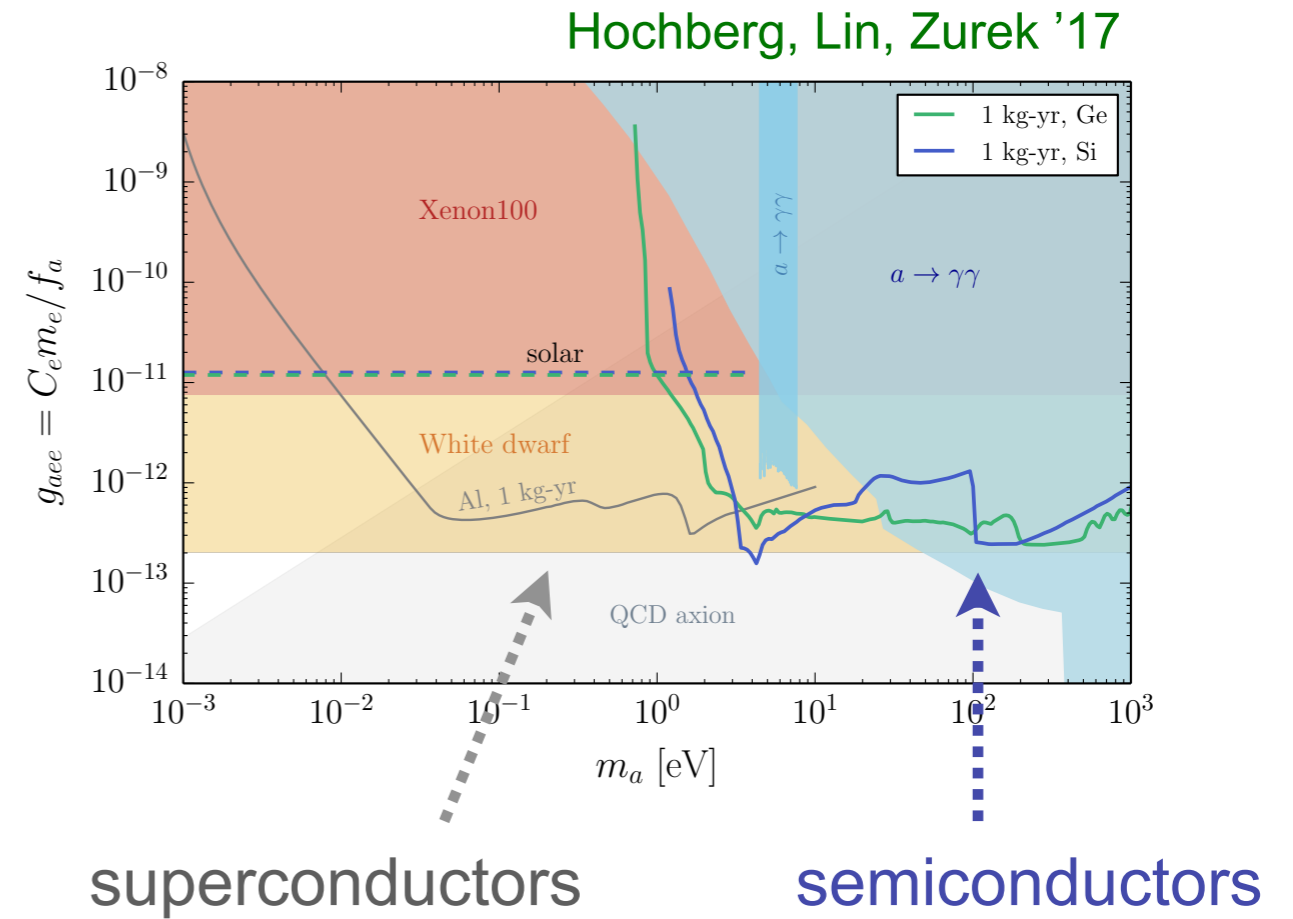
2. Axion in topological insulators

Axion and axion-like particles (ALPs)

- A solution to the strong CP problem (for axion)
- DM candidates
- Inspired by superstring theory
- Impacts on cosmology (axion strings, domain walls, mini-clusters, etc.)
- Lots of searching using various techniques are ongoing

Axion and axion-like particles

- A solution to the strong CP problem
- DM candidates
- Inspired by superstring theory
- Impacts on cosmology (axion etc.)



- Lots of searching using various techniques are ongoing

Axion and axion-like particles (ALPs)

- A solution to the strong CP problem (for axion)
- DM candidates
- Inspired by superstring theory
- Impacts on cosmology (axion strings, domain walls, mini-clusters, etc.)
- Lots of searching using various techniques are ongoing
- 'Axion' is predicted in topological insulators
- 'Axion' in insulators can be used for axion detection

Axion is predicted in topological magnetic insulators

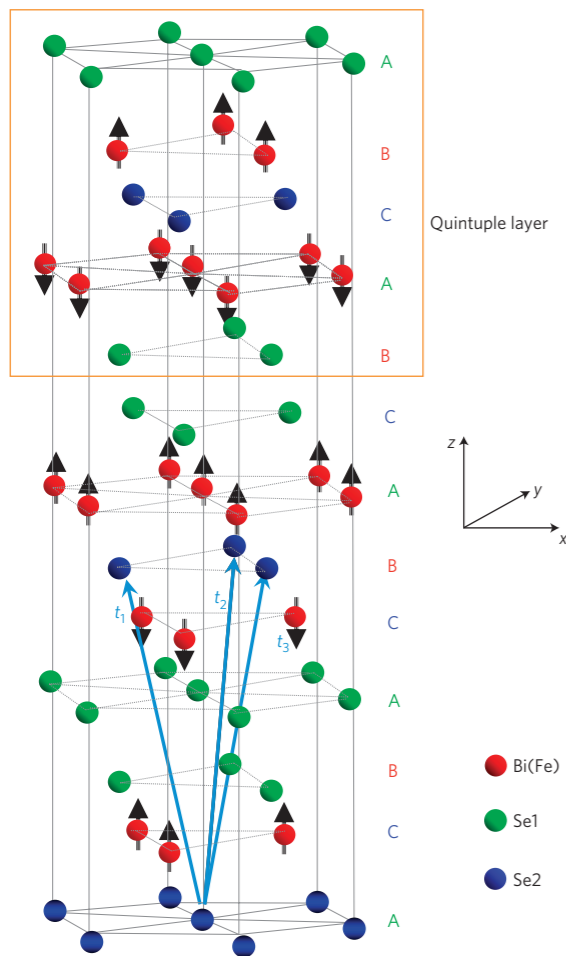
ARTICLES

PUBLISHED ONLINE: 7 MARCH 2010 | DOI: 10.1038/NPHYS1534

nature
physics

Dynamical axion field in topological magnetic insulators

Rundong Li¹, Jing Wang^{1,2}, Xiao-Liang Qi¹ and Shou-Cheng Zhang^{1*}



Bi₂Se₃

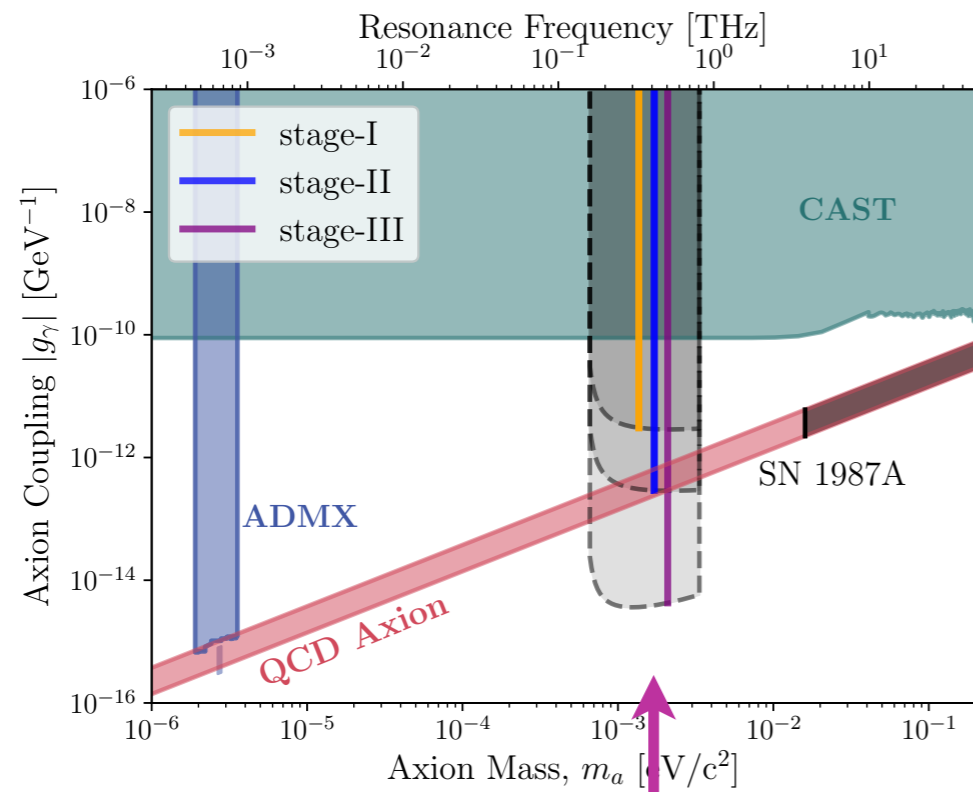
(Topological insulator)

$$\begin{aligned}
 \mathcal{S}_{\text{tot}} &= \mathcal{S}_{\text{Maxwell}} + \mathcal{S}_{\text{topo}} + \mathcal{S}_{\text{axion}} \\
 &= \frac{1}{8\pi} \int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu} \mathbf{B}^2 \right) + \frac{\alpha}{4\pi^2} \int d^3x dt (\theta_0 + \delta\theta) \mathbf{E} \cdot \mathbf{B} \\
 &\quad + g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \quad (4)
 \end{aligned}$$

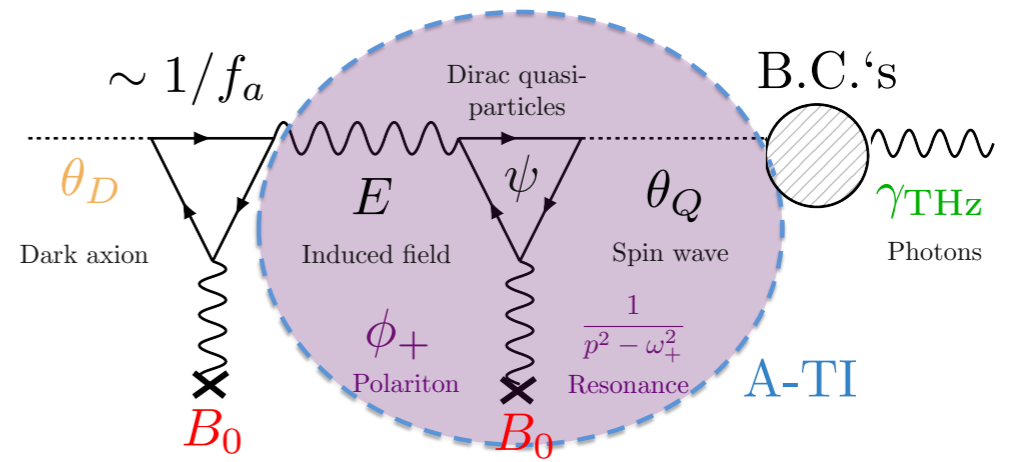
$\theta F_{\mu\nu} \tilde{F}^{\mu\nu}$

Axion mass $\sim \mathcal{O}(\text{meV})$

Proposals for axion/ALPs search using 'axion' in insulators

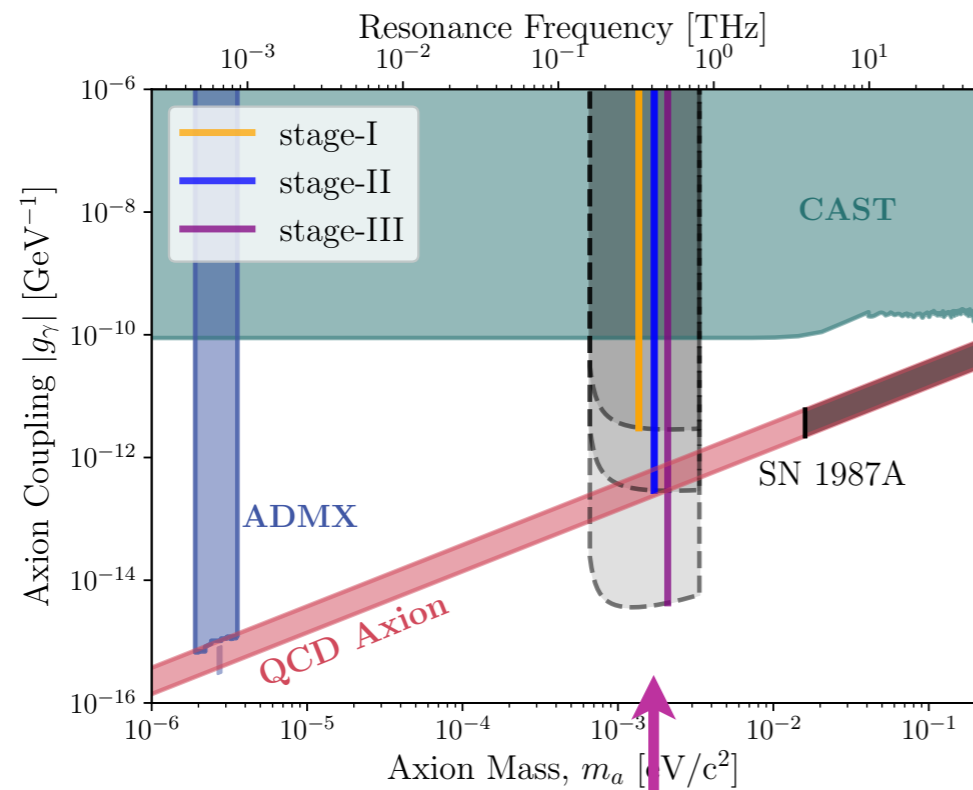


$\mathcal{O}(\text{meV})$

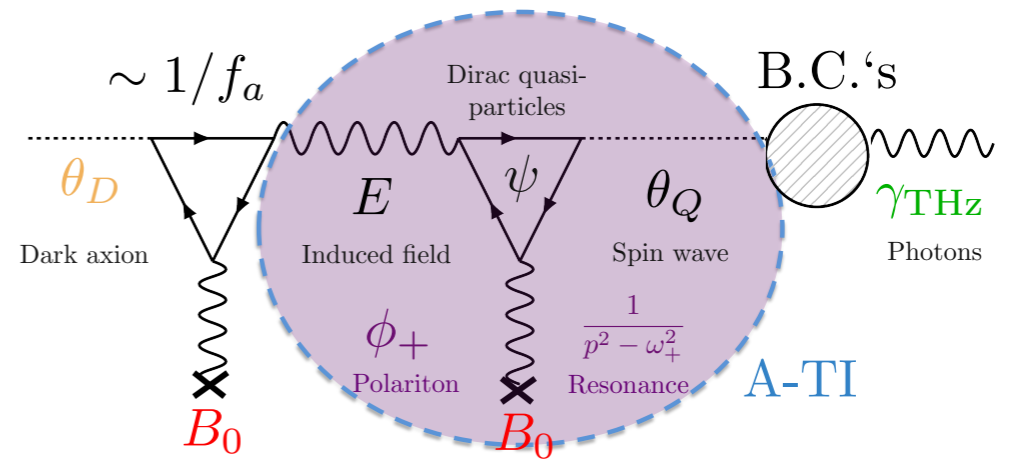


Marsh, Fong, Lentz, Smejkal, Ali '19

Proposals for axion/ALPs search using 'axion' in insulators



$\mathcal{O}(\text{meV})$



Marsh, Fong, Lentz, Smejkal, Ali '19

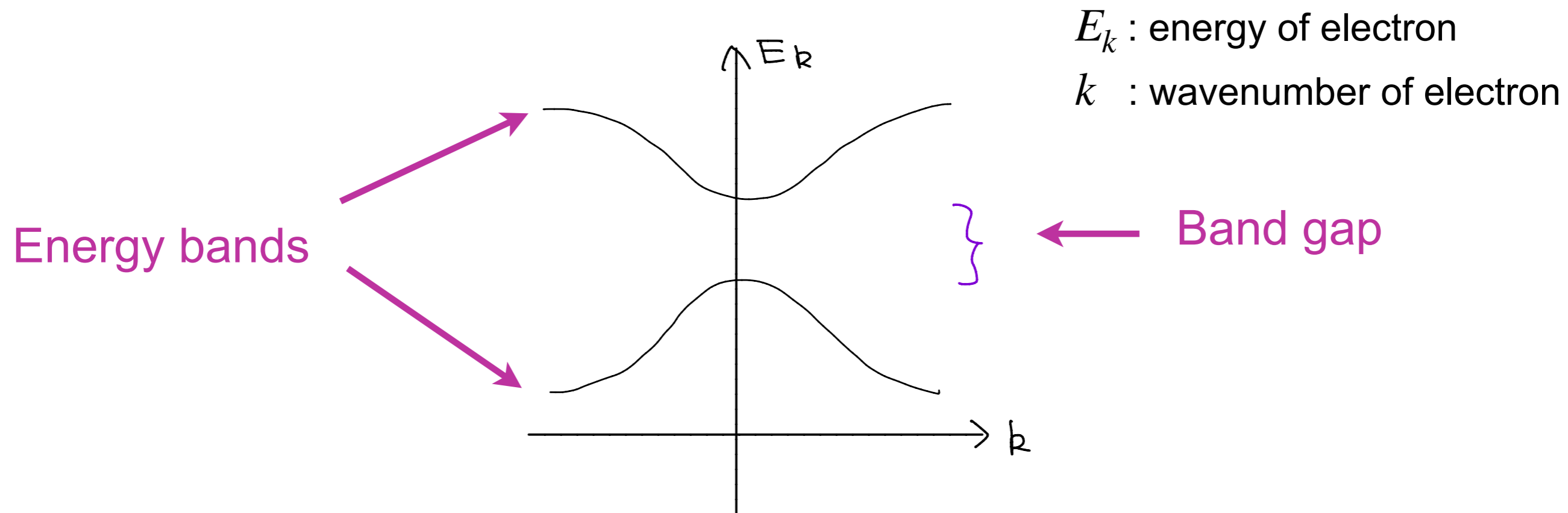
Only meV range?

Basics to axion in insulators

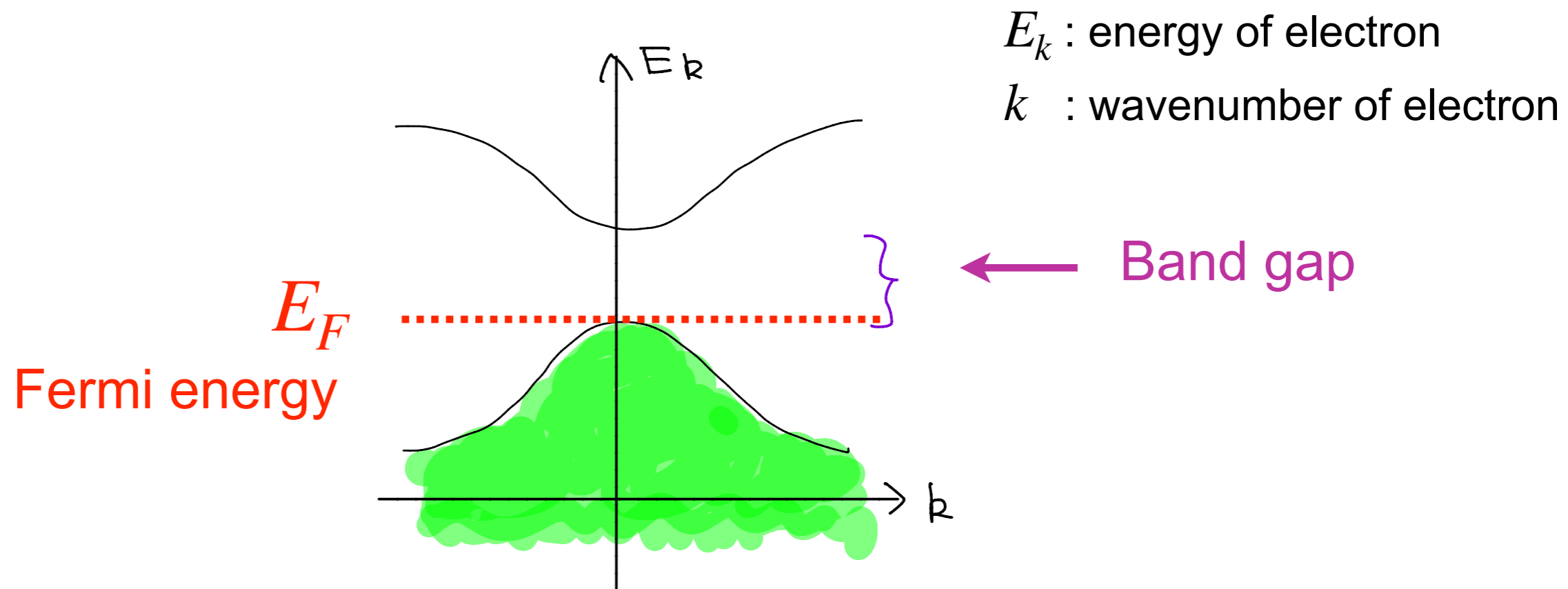
a). Insulators

b). Quantum Hall effect

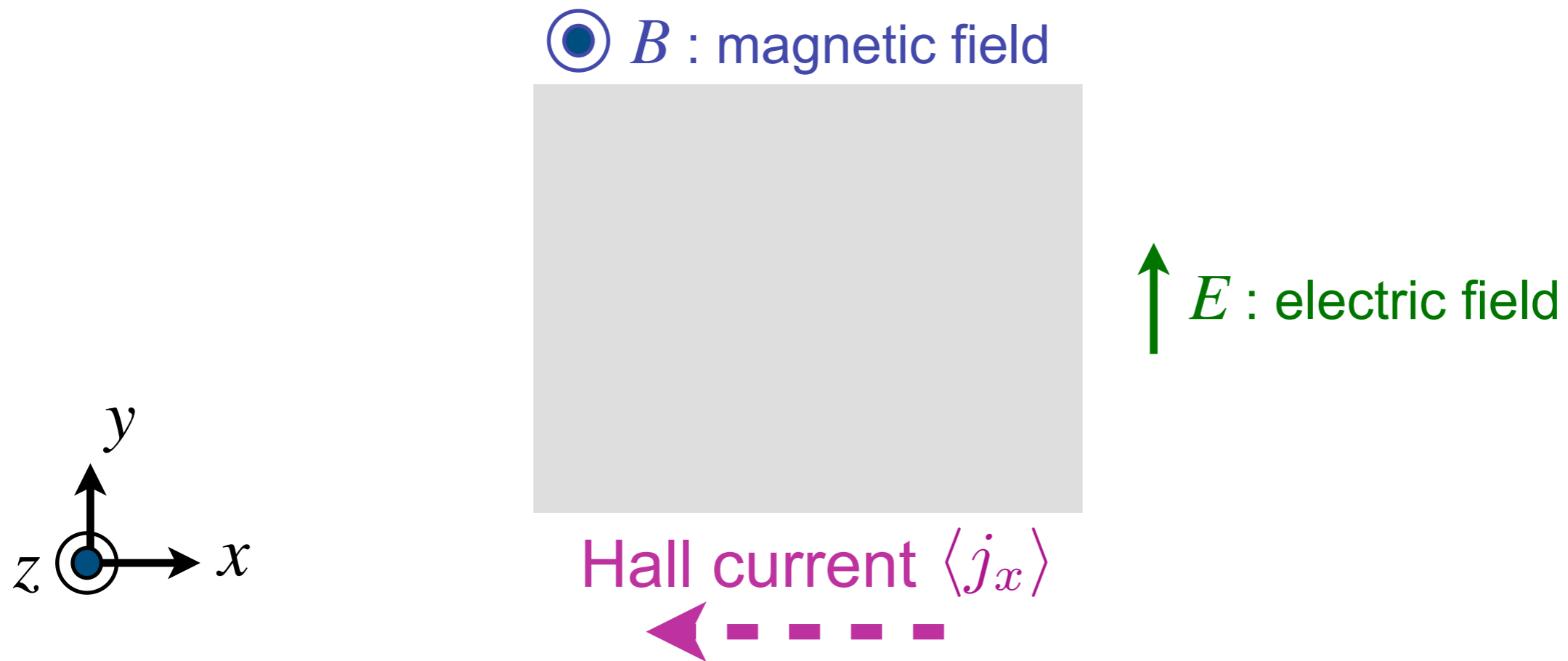
a). Insulators



a). Insulators



e.g., 2D insulator



Quantized electric current is induced in x direction

e.g., a toy model in 2D

$$H = \begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix} = \mathbf{d} \cdot \boldsymbol{\sigma}$$

$$\mathbf{d} = (m, k_x, k_y)$$

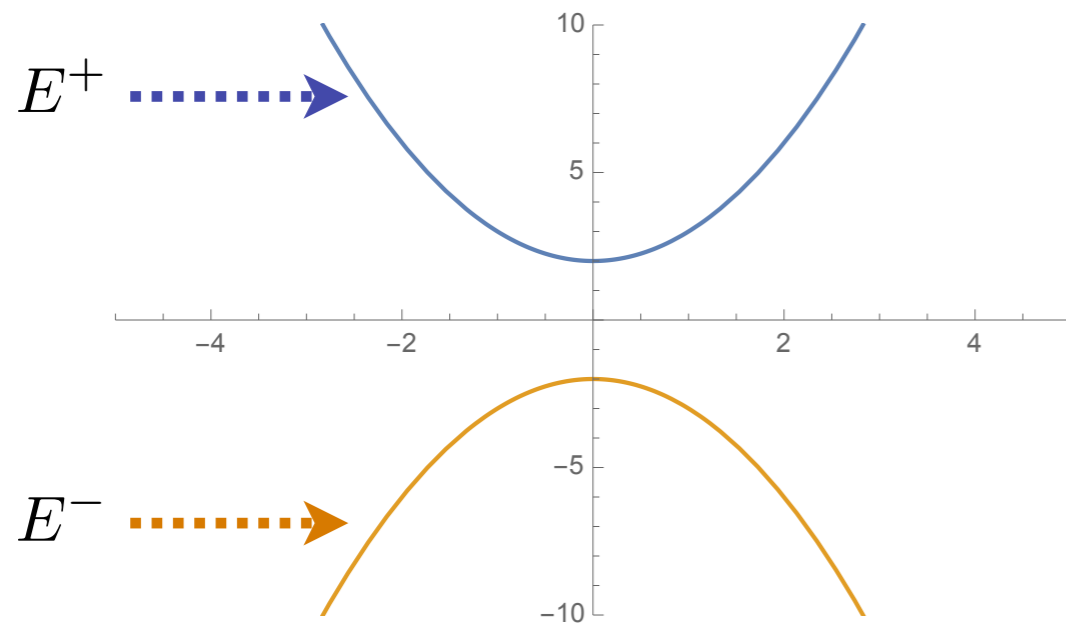


$$\begin{aligned} E^+ &\simeq m + \frac{|\mathbf{k}|^2}{2|m|} \\ E^- &\simeq -m - \frac{|\mathbf{k}|^2}{2|m|} \end{aligned}$$

around $\mathbf{k} = 0$

The band structure

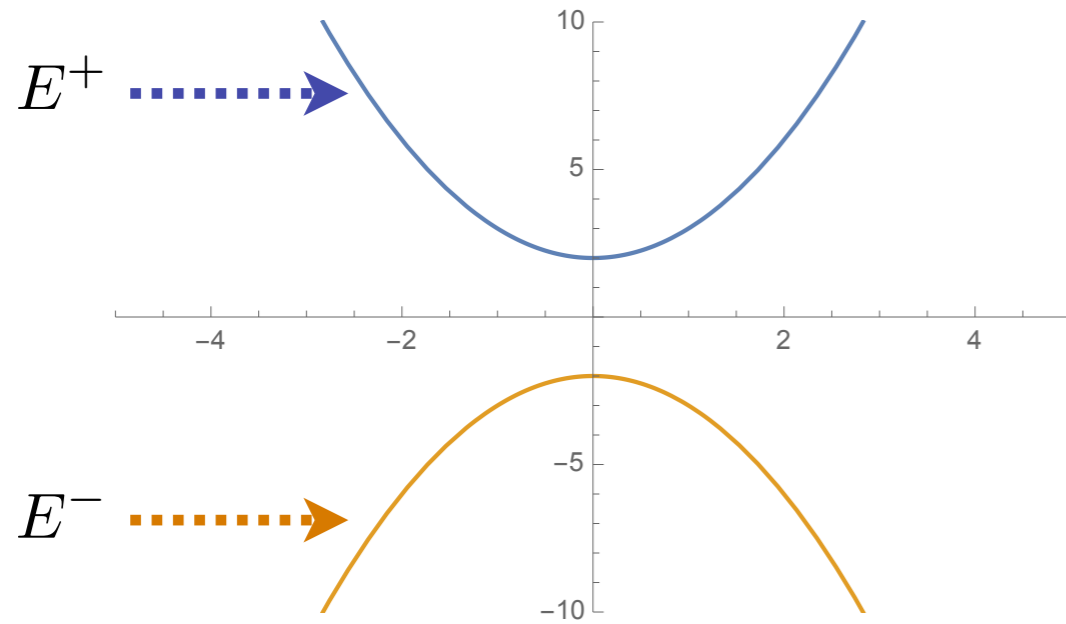
$$m > 0$$



Normal insulator

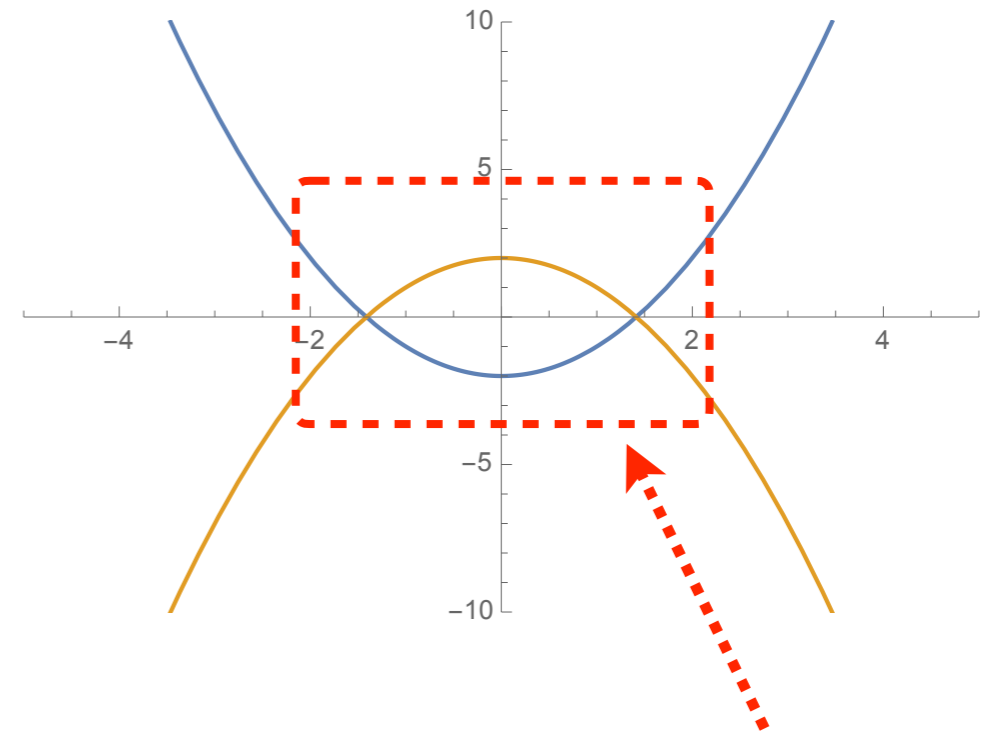
The band structure

$m > 0$



Normal insulator

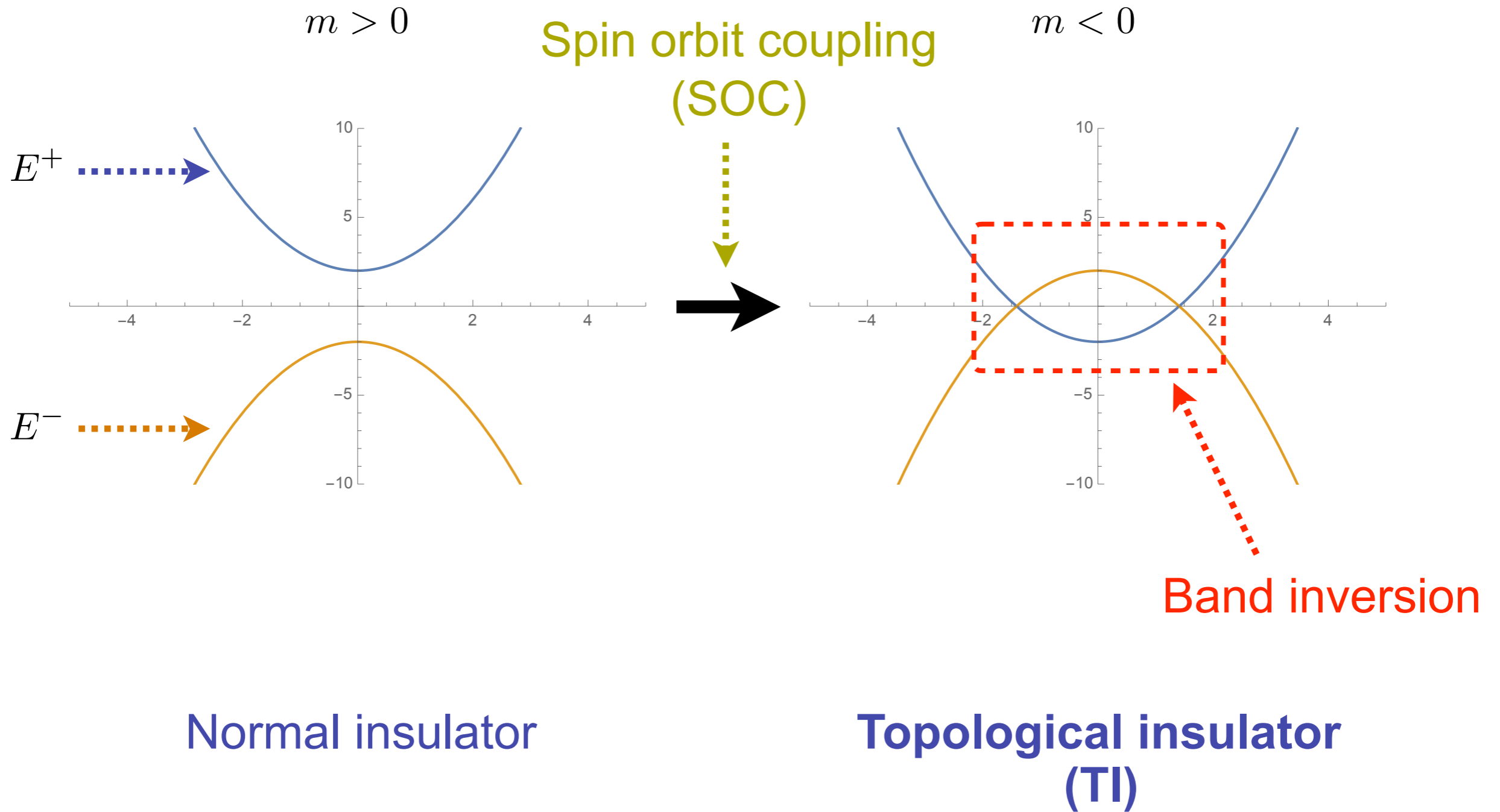
$m < 0$



Band inversion

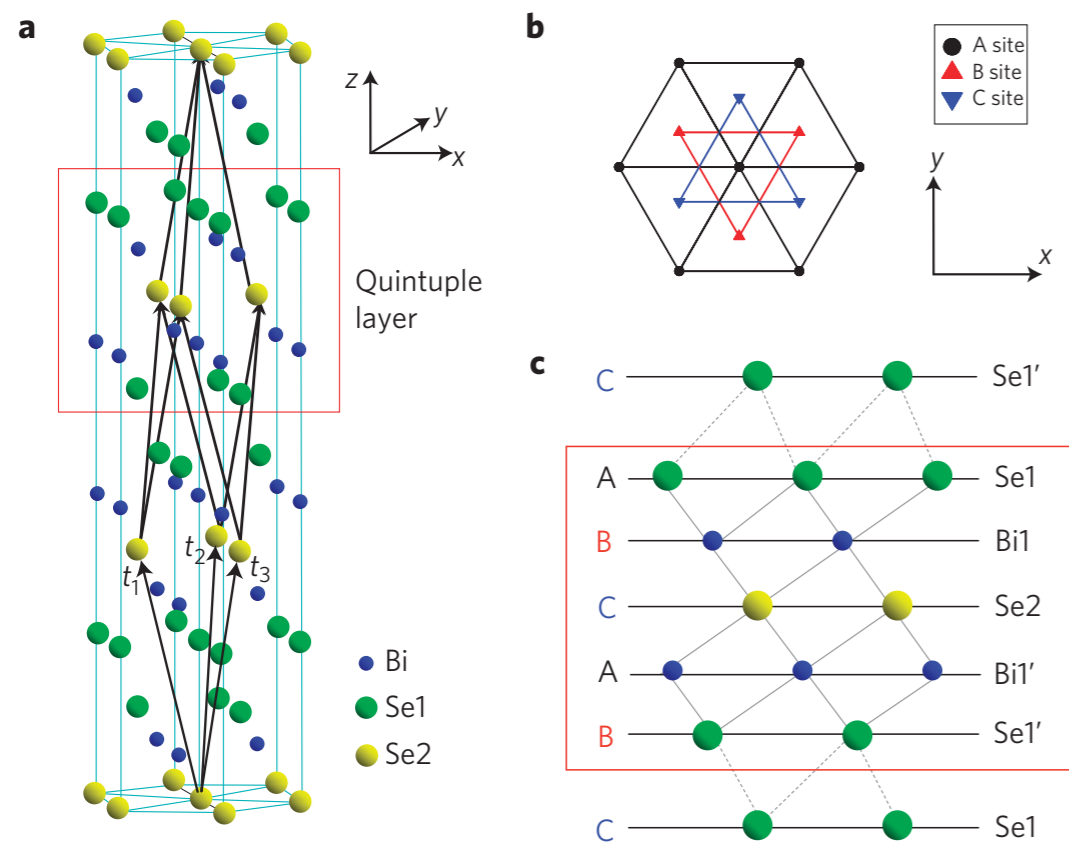
QH insulator

The band structure

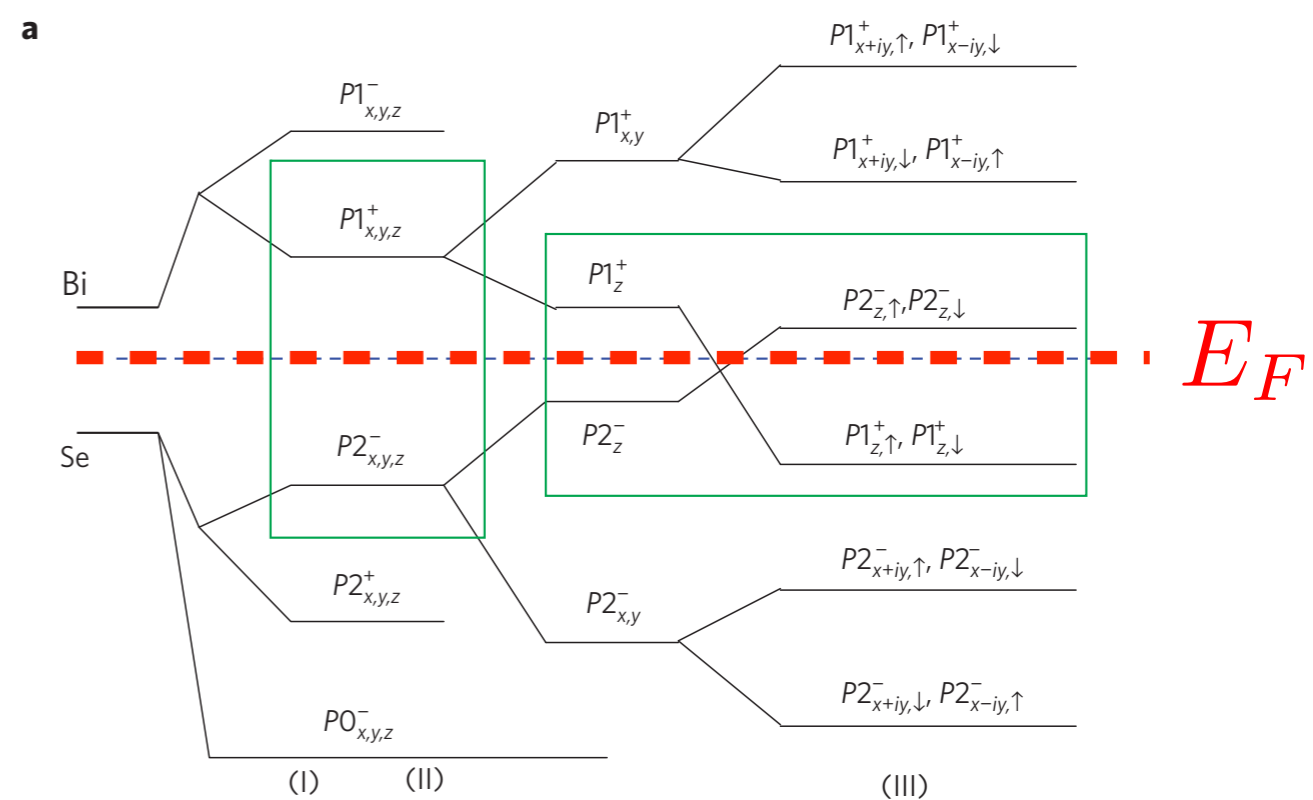


3D TI, Bi_2Se_3

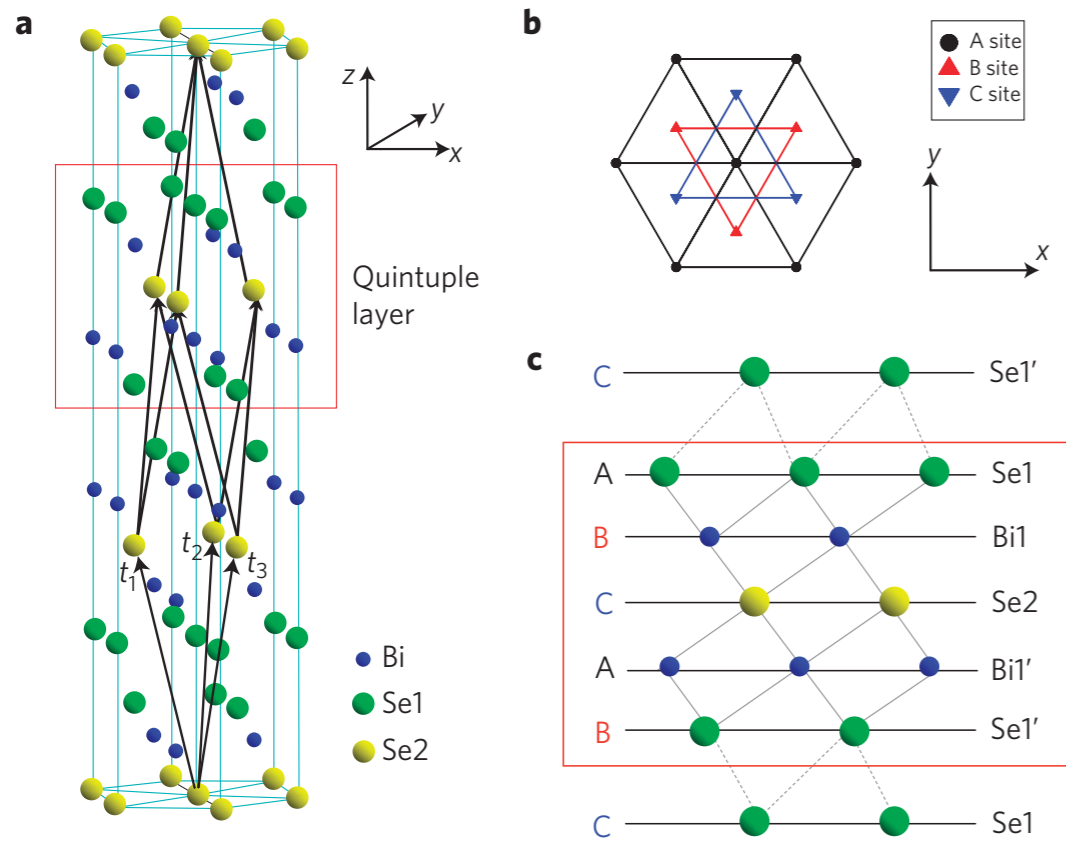
H. Zhang et al. '09



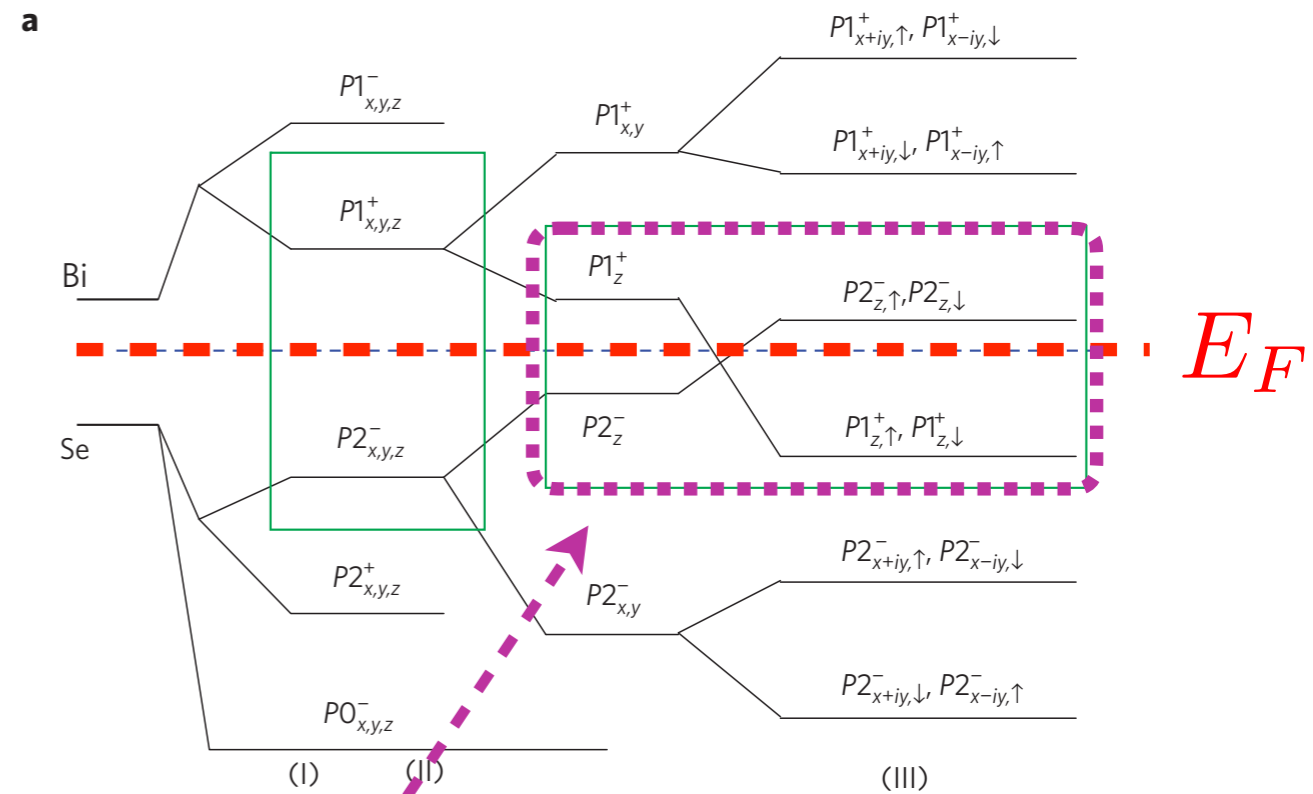
Crystal structure



Energy levels

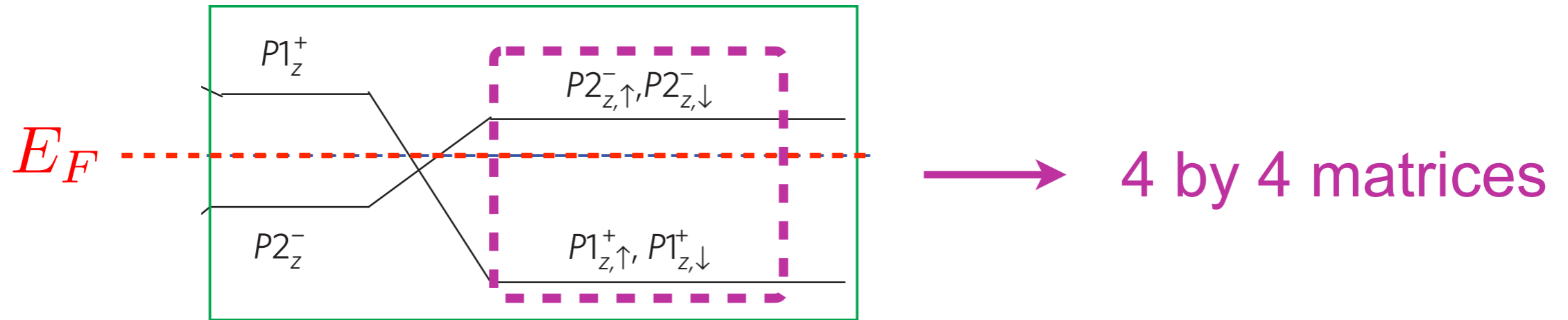


Crystal structure



Energy levels

Band inversion due to strong SOC



$$H_0(\mathbf{k}) = \epsilon_0 \mathbf{1}_{4 \times 4} + \sum_{a=1}^5 d^a \Gamma^a$$

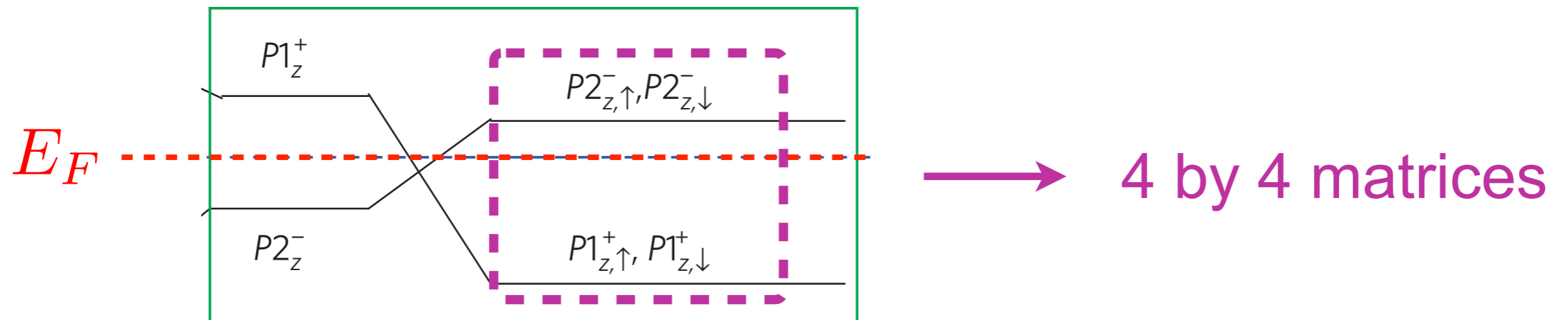
$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x, A_2 \sin k_y, A_1 \sin k_z, \mathcal{M}(\mathbf{k}), 0)$$

$$\mathcal{M}(\mathbf{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \quad \Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \quad \Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

“Effective Hamiltonian for 3D TI”



$$H_0(\mathbf{k}) = \epsilon_0 \mathbf{1}_{4 \times 4} + \sum_{a=1}^5 d^a \Gamma^a$$

$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x, A_2 \sin k_y, A_1 \sin k_z, \mathcal{M}(\mathbf{k}), 0)$$

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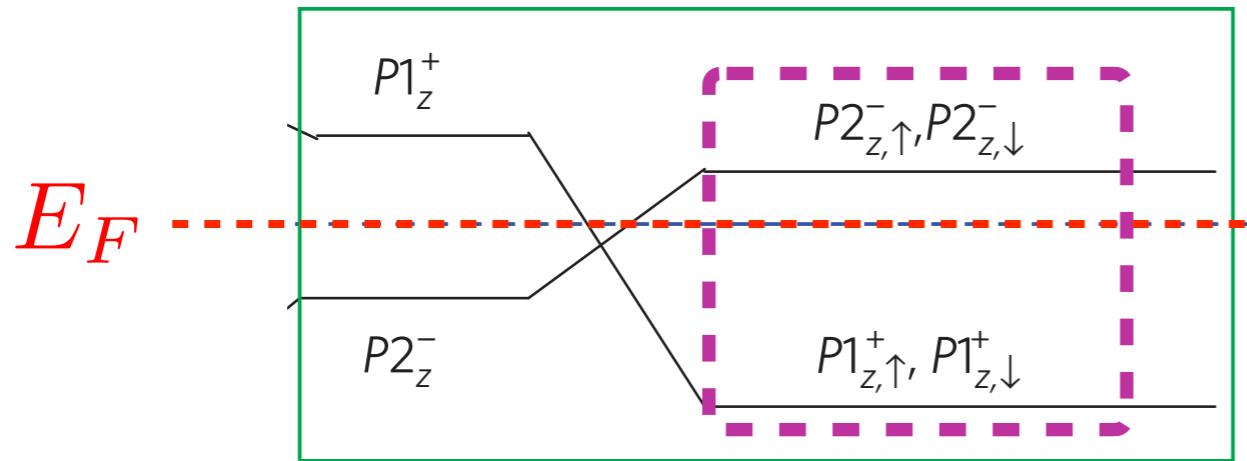
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$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

$$d_5 = 0$$

(see later discussion)

“Effective Hamiltonian for 3D TI”



E_F



4 by 4 matrices

important for topological state

cf. 2D toy model

$$H = \mathbf{d} \cdot \boldsymbol{\sigma}$$

$$\mathbf{d} = (m, k_x, k_y)$$

$$H_0(\mathbf{k}) = \epsilon_0 \mathbf{1}_{4 \times 4} + \sum_{a=1}^5 d^a \Gamma^a$$

$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x, A_2 \sin k_y, A_1 \sin k_z, \mathcal{M}(\mathbf{k}), 0)$$

$$\mathcal{M}(\mathbf{k}) = M + 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \quad \Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \quad \Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

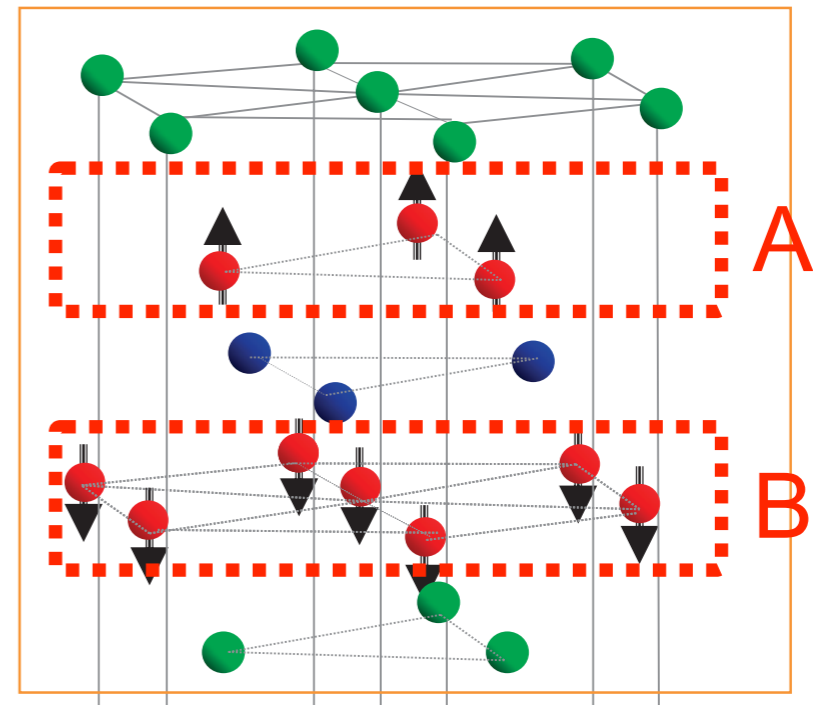
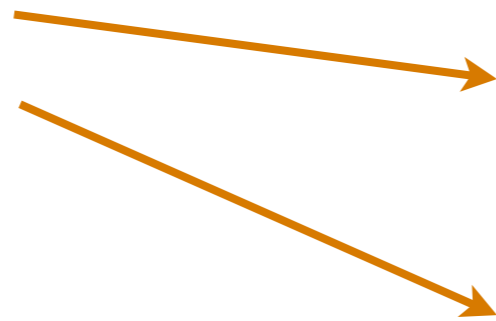
$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

“Effective Hamiltonian for 3D TI”

In addition we consider *antiferromagnetism (AFM)*

R. Li et al. '10

Suppose electrons at Bi
are AFM order



- Bi(Fe)
- Se1
- Se2

In addition we consider *antiferromagnetism (AFM)*

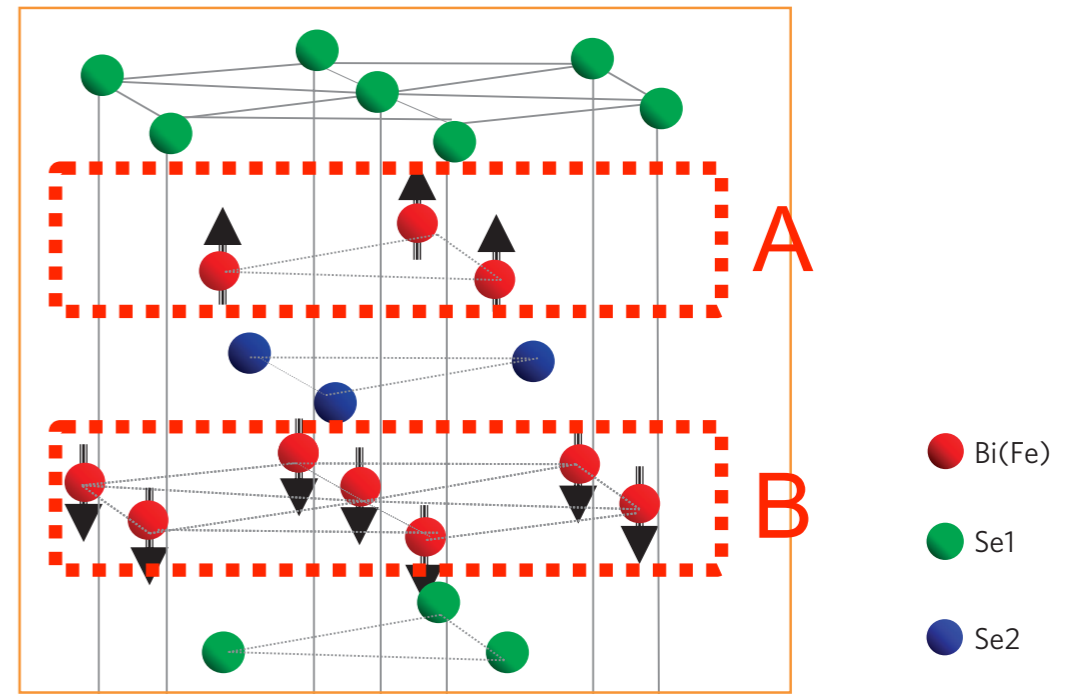
R. Li et al. '10

$$\mathcal{H}_{\text{int}} = \frac{UV}{N} \int d^3x (n_{A\uparrow}n_{A\downarrow} + n_{B\uparrow}n_{B\downarrow})$$

“Hubbard term”

U : parameter to give AFM

Large U \longrightarrow AFM

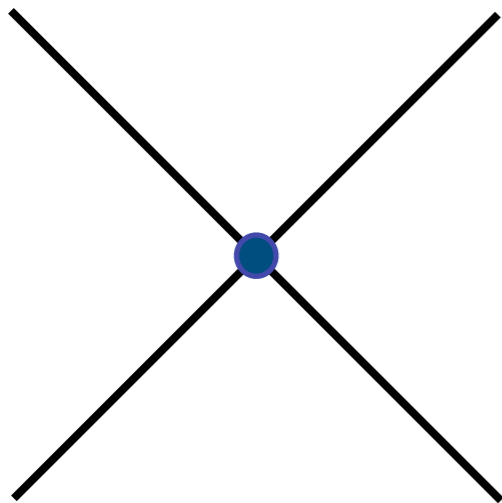


V : volume
 N : number of site
 $n_{A\sigma} = \psi_{A\sigma}^\dagger \psi_{A\sigma}$
 $n_{B\sigma} = \psi_{B\sigma}^\dagger \psi_{B\sigma}$

$$\mathcal{H}_{\text{int}} = \frac{UV}{N} \int d^3x (n_{A\uparrow}n_{A\downarrow} + n_{B\uparrow}n_{B\downarrow})$$



Hubbard-Stratonovich (HS)
transformation



$$n_{A\uparrow}n_{A\downarrow}, n_{B\uparrow}n_{B\downarrow}$$

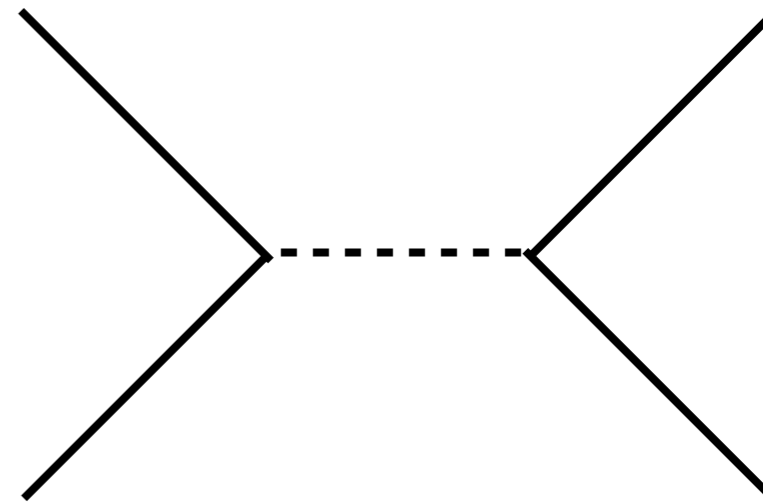
$$(\psi_{\uparrow}^{\dagger}\psi_{\uparrow}\psi_{\downarrow}^{\dagger}\psi_{\downarrow})$$

Four Fermi int.

HS transformation



Integrate out ϕ



$$\phi\psi_{\uparrow}^{\dagger}\psi_{\uparrow}, \phi\psi_{\downarrow}^{\dagger}\psi_{\downarrow}$$

Yukawa int.

$$\mathcal{H}_{\text{int}} = \frac{UV}{N} \int d^3x (n_{A\uparrow}n_{A\downarrow} + n_{B\uparrow}n_{B\downarrow})$$



Hubbard-Stratonovich (HS)
transformation

● A dynamical scalar ϕ that gives $\Gamma^5 d_5$ ($d_5 = \phi$)

● Mass term of ϕ



missed in

Sekine, Nomura '16

Sekine, Nomura '20

Schütte-Engel '21

(confirmed by private communication with
Sekine-san)

$$\mathcal{H}_{\text{int}} = \frac{UV}{N} \int d^3x (n_{A\uparrow}n_{A\downarrow} + n_{B\uparrow}n_{B\downarrow})$$



Hubbard-Stratonovich (HS)
transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ ($d_5 = \phi$)
- Mass term of ϕ
- ϕ relates to the axion field

$$\theta = \frac{1}{4\pi} \int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2 |d|^3} \epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

Derivation as chiral anomaly

$$H(\mathbf{k}) = \sum_{a=1}^5 d^a(\mathbf{k}) \Gamma^a$$

$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x, A_2 \sin k_y, A_1 \sin k_z, \mathcal{M}(\mathbf{k}), \phi)$$

$$\mathcal{M}(\mathbf{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

Derivation as chiral anomaly

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$$\mathcal{M}(\mathbf{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$



- expand around $\mathbf{k} = 0$
- redefine \mathbf{k}

$$H(\mathbf{k}) = k_x \Gamma^1 + k_y \Gamma^2 + k_y \Gamma^3 + M \Gamma^4 + \phi \Gamma^5$$

“Dirac model”

$$H(\mathbf{k}) = k_x \Gamma^1 + k_y \Gamma^2 + k_y \Gamma^3 + M \Gamma^4 + \phi \Gamma^5$$



Unitary transformation of the basis

$$\tilde{U} H(\mathbf{k}) \tilde{U}^\dagger = \beta(\boldsymbol{\gamma} \cdot \mathbf{k} + M + \phi \gamma_5)$$

.....→

$$S = \int d^4x \bar{\psi} [i\gamma^\mu (\partial_\mu - ieA_\mu) - M - i\phi\gamma_5] \psi$$

$\Gamma^5 \phi$ reduces to $i\gamma^5 \phi$

$i\gamma^5\phi$ term can be rotated away, which gives rise to θ term:

$$S_{\Theta} = -\frac{\alpha}{4\pi} \int d^4x \Theta F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\Theta = \frac{\pi}{2} [1 - \text{sgn}(M)] \text{sgn}(\phi) + \tan^{-1} \frac{\phi}{M}$$

it is consistent with

$$\theta = \frac{1}{4\pi} \int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2 |d|^3} \epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi e^{iS + iS_\phi^{\text{mass}}}$$

$$S = \int d^4x \psi^\dagger(x) [i\partial_t - H] \psi(x)$$

$$S_\phi^{\text{mass}} = - \int d^4x M_\phi^2 \phi^2$$

$$H = H_0 + \delta H$$

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi e^{iS + iS_\phi^{\text{mass}}}$$

$$S = \int d^4x \psi^\dagger(x) [i\partial_t - H] \psi(x)$$

$$S_\phi^{\text{mass}} = - \int d^4x M_\phi^2 \phi^2$$

$$H = H_0 + \delta H$$

$\Gamma^5 \phi$
↓
⊞

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi e^{iS + iS_\phi^{\text{mass}}}$$

$$S = \int d^4x \psi^\dagger(x) [i\partial_t - H] \psi(x)$$

$$S_\phi^{\text{mass}} = - \int d^4x M_\phi^2 \phi^2$$

$$H = H_0 + \delta H$$

$\Gamma^5 \phi$
↓

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Summing over ψ, ψ^\dagger

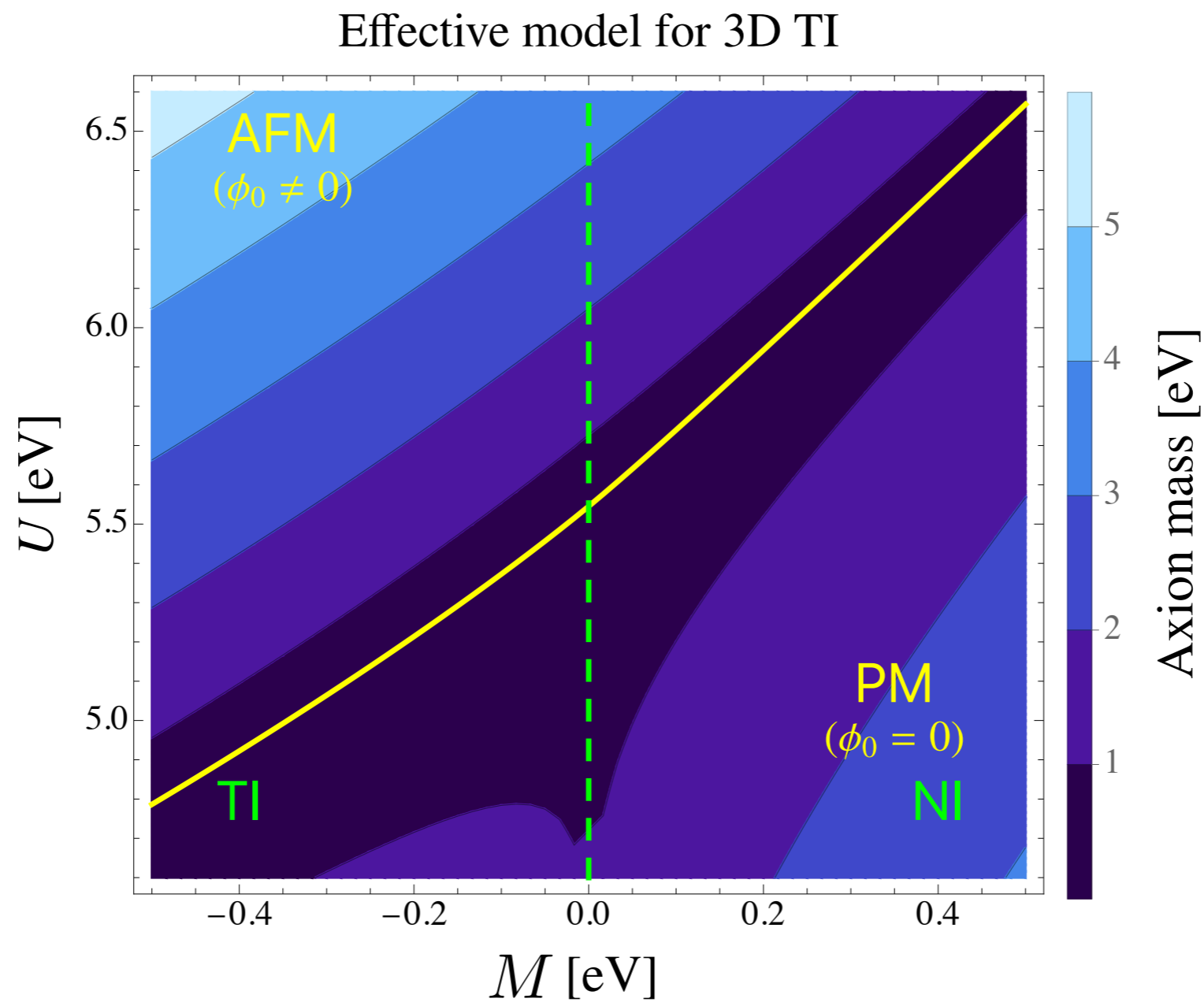
.....➤ Effective potential for ϕ

.....➤ Effective potential for θ

$$\theta = \frac{1}{4\pi} \int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2 |d|^3} \epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

Axion mass

KI '21



Axion mass is $\mathcal{O}(\text{eV})$

Dynamical axion is predicted in topological magnetic insulators

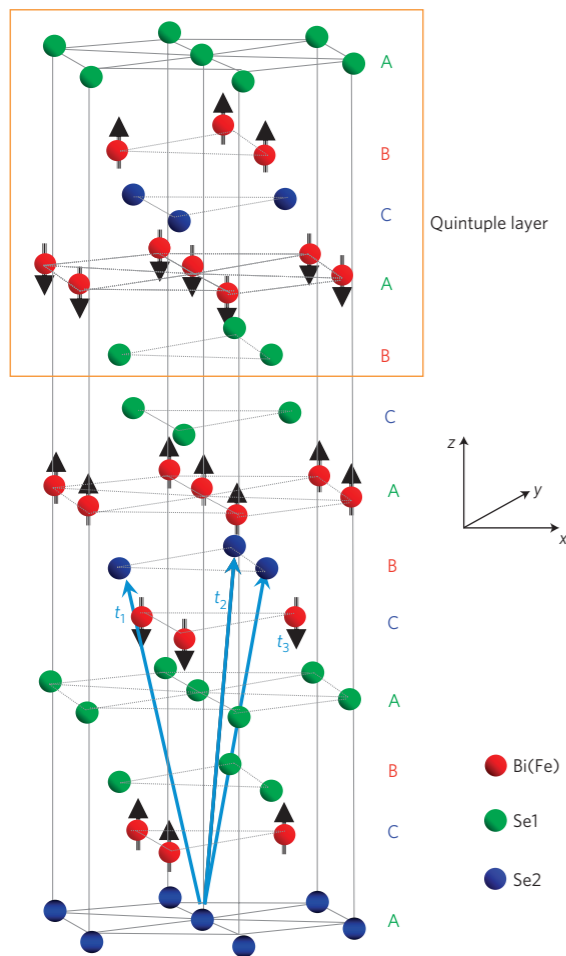
ARTICLES

PUBLISHED ONLINE: 7 MARCH 2010 | DOI: 10.1038/NPHYS1534

nature
physics

Dynamical axion field in topological magnetic insulators

Rundong Li¹, Jing Wang^{1,2}, Xiao-Liang Qi¹ and Shou-Cheng Zhang^{1*}



$$\begin{aligned}
 \mathcal{S}_{\text{tot}} &= \mathcal{S}_{\text{Maxwell}} + \mathcal{S}_{\text{topo}} + \mathcal{S}_{\text{axion}} \\
 &= \frac{1}{8\pi} \int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu} \mathbf{B}^2 \right) + \frac{\alpha}{4\pi^2} \int d^3x dt (\theta_0 + \delta\theta) \mathbf{E} \cdot \mathbf{B} \\
 &\quad + g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \quad (4)
 \end{aligned}$$

$\theta F_{\mu\nu} \tilde{F}^{\mu\nu}$

Axion mass $\sim \mathcal{O}(\text{meV})$

Dynamical axion is predicted in topological magnetic insulators

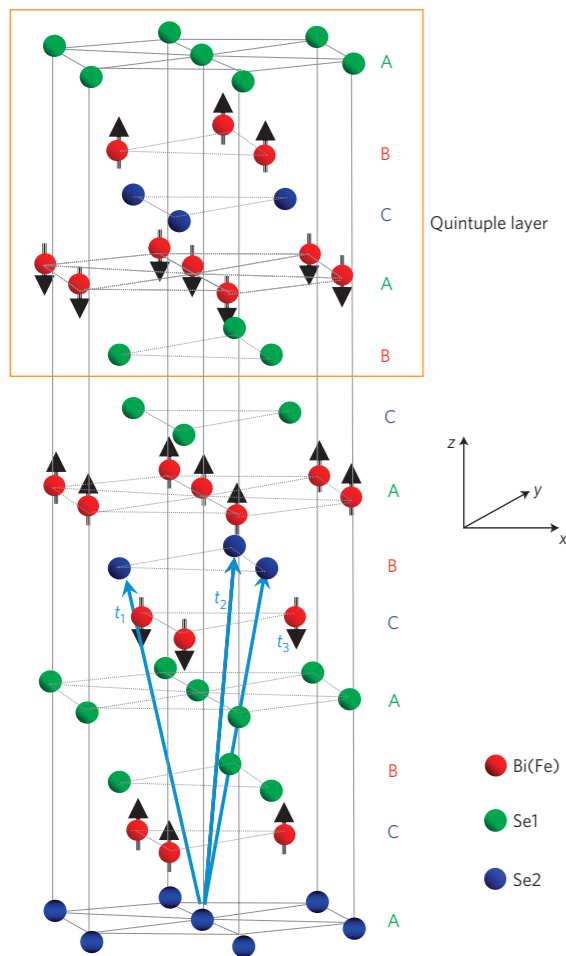
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Dynamical axion field in topological magnetic insulators

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$$\mathcal{S}_{\text{tot}} = \mathcal{S}_{\text{Maxwell}} + \mathcal{S}_{\text{topo}} + \mathcal{S}_{\text{axion}}$$

$$= \frac{1}{8\pi} \int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu} \mathbf{B}^2 \right) + \frac{\alpha}{4\pi^2} \int d^3x dt (\theta_0 + \delta\theta) \mathbf{E} \cdot \mathbf{B}$$

$$+ g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \quad (4)$$

Axion mass $\sim \mathcal{O}(\text{meV})$

?

The thing would be ...

R. Li et al. '10

- $\langle \phi \rangle (= m_5) = 1 \text{ meV}$ is taken
(i.e., $\langle \phi \rangle$ is considered to be a free parameter)

- AFM order is *assumed*

The thing would be ...

R. Li et al. '10

- $\langle \phi \rangle (= m_5) = 1 \text{ meV}$ is taken

(i.e., $\langle \phi \rangle$ is considered to be a free parameter)

→ Axion mass $\sim \mathcal{O}(\text{meV})$ ($\because m_a^2 \propto m_5^2$)

- AFM order is *assumed*

The thing would be ...

R. Li et al. '10

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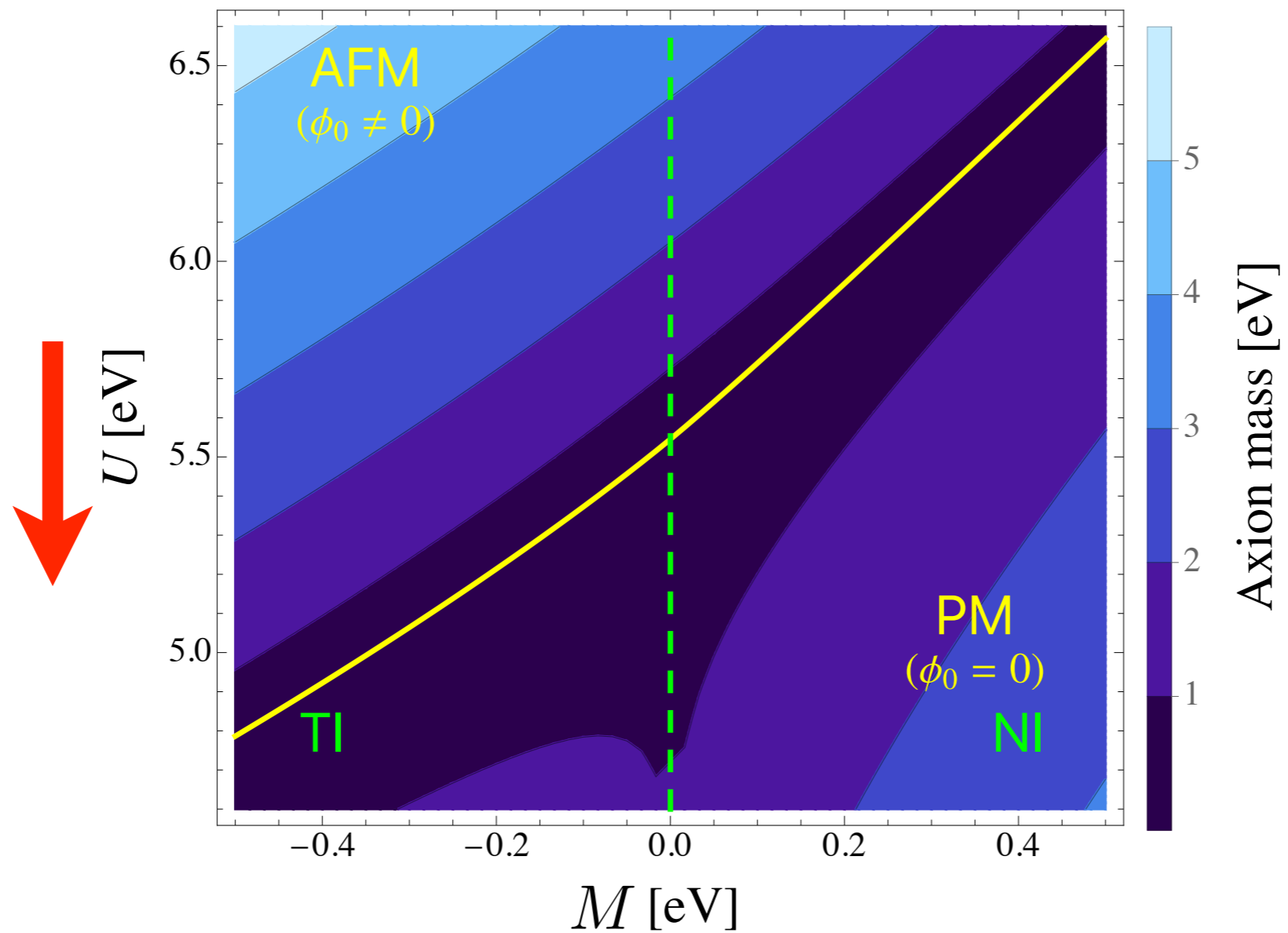
But this is not naively possible since

- AFM order is *assumed*
 $m_5 \sim U \sim \text{eV}$ (in AFM order)

Axion mass

KI '21

Effective model for 3D TI



Suppressed U \longrightarrow No AFM

The thing would be ...

R. Li et al. '10

- $\langle \phi \rangle (= m_5) = 1 \text{ meV}$ is taken

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R. Li et al. '10

- $\langle \phi \rangle (= m_5) = 1 \text{ meV}$ is taken

(i.e., $\langle \phi \rangle$ is considered to be a free parameter)

→ Axion mass $\sim \mathcal{O}(\text{meV})$ ($\because m_a^2 \propto m_5^2$)

- AFM order is *assumed*

No AFM in TI in the first place

→ Fe-doped Bi_2Se_3 is considered

- Fe-doped Bi_2Se_3 , Bi_2Te_3

“likely to be AFM”

(by first-principles calculation)

→ It looks unlikely to be realized ...

J.M. Zhang et al. '13

- $\text{Mn}_2\text{Bi}_2\text{Te}_5$

“rich magnetic topological quantum states”

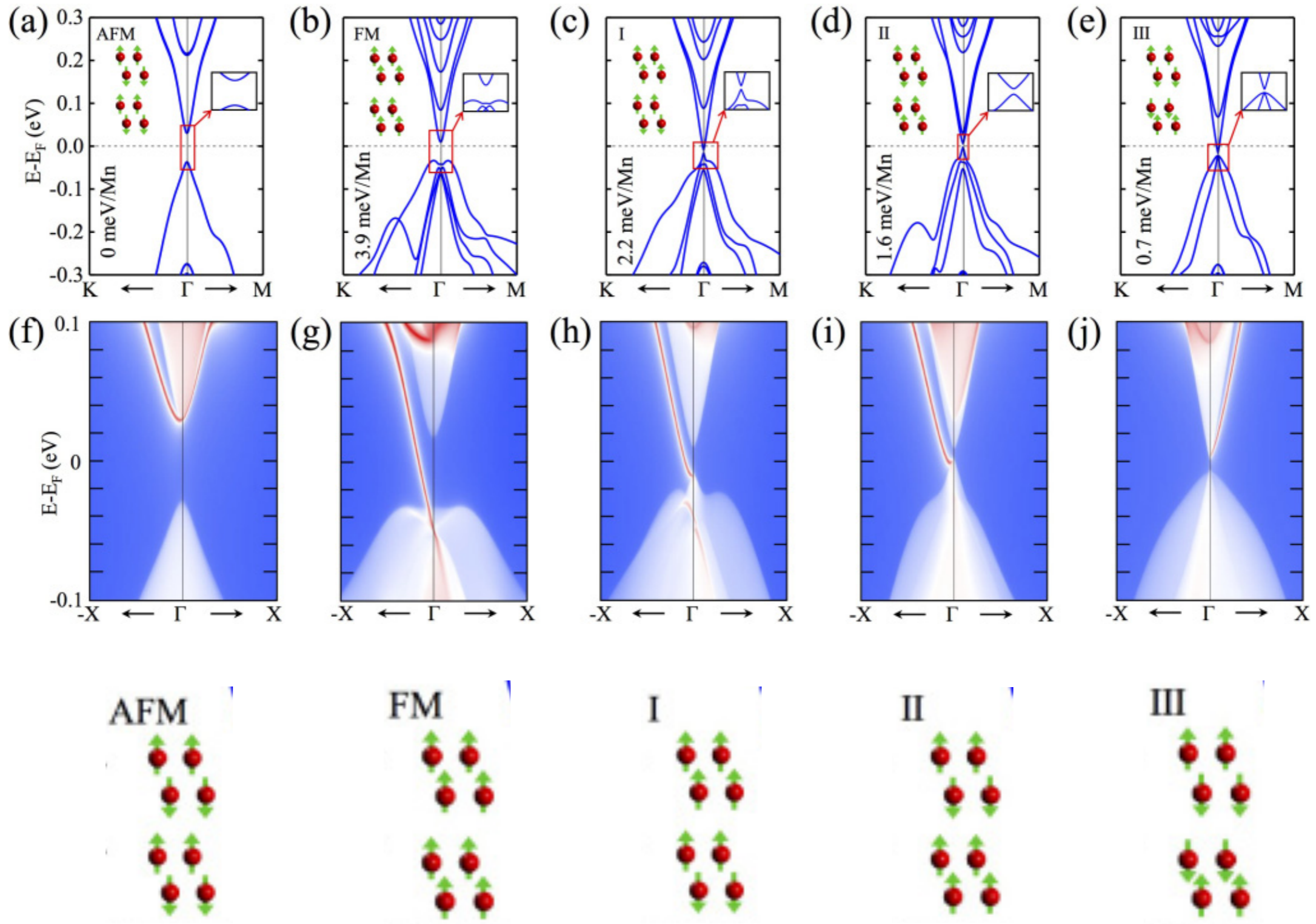
(by first-principles calculation)

J. Zhang et al. '19

Y. Li et al. '20

First-principles calculation

Y. Li et al. '20

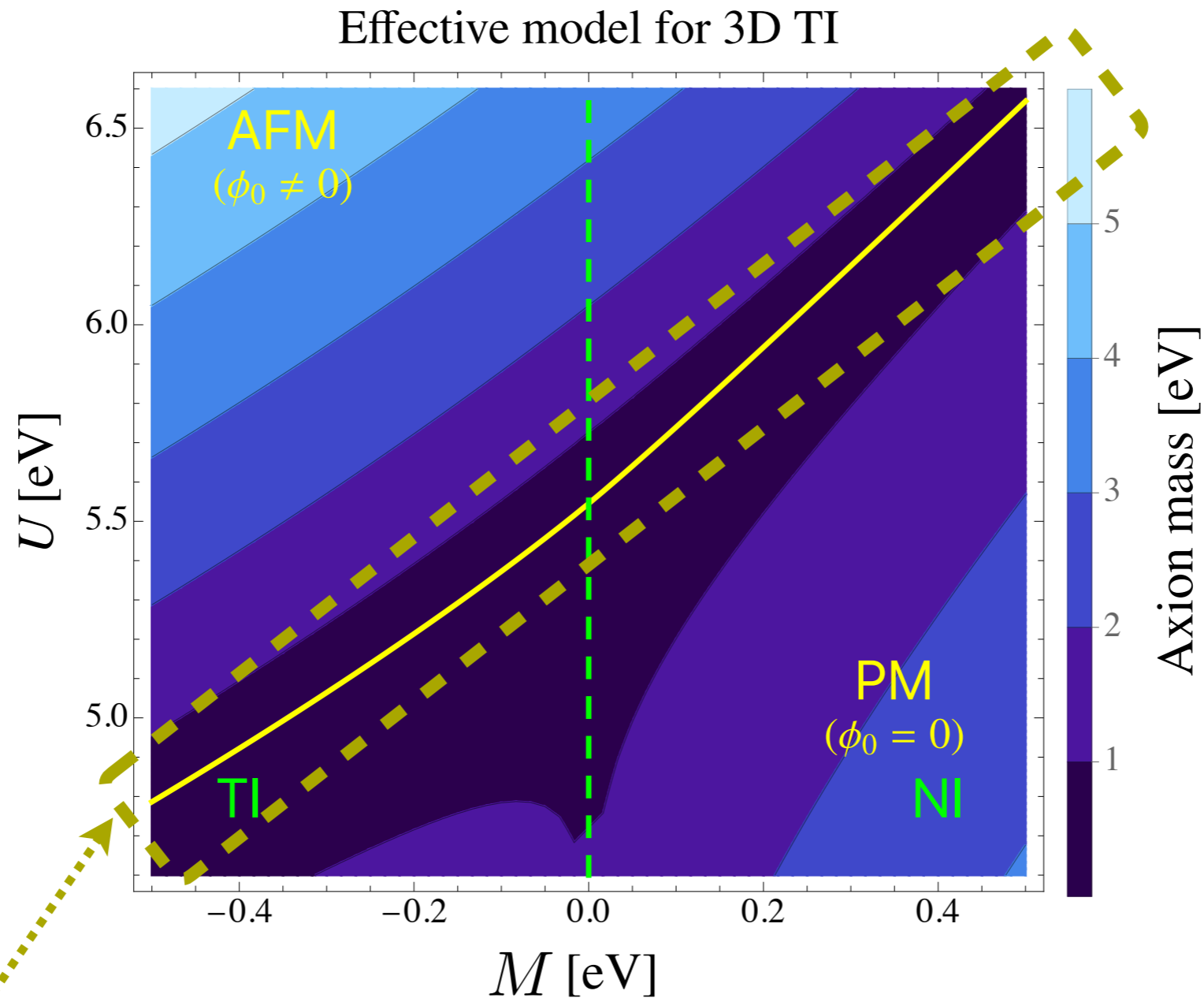


$\text{Mn}_2\text{Bi}_2\text{Te}_5$ is synthesized

L. Cao et al. '21

Axion mass

KI '21



It can be suppressed near the phase boundary

Rich magnetic topological states in that region?

Discussion

- How do we describe axion in $\text{Mn}_2\text{Bi}_2\text{Te}_5$?
- What about axion in NI ?
- Dynamical axion in ferromagnetic state or other magnetic states?

Interaction between impurity and electron

$$H^{\text{TI}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{H}_{\mathbf{k}}^{\text{TI}} c_{\mathbf{k}}$$

$$H_J = \sum_I^{N_s} [J^A \mathbf{S}^A(\mathbf{x}_I) \cdot \mathbf{s}_I^A + J^B \mathbf{S}^B(\mathbf{x}_I) \cdot \mathbf{s}_I^B]$$

Interaction between impurity and electron

$$H^{\text{TI}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{H}_{\mathbf{k}}^{\text{TI}} c_{\mathbf{k}}$$

$$H_J = \sum_I^{N_s} \left[J^A \mathbf{S}^A(\mathbf{x}_I) \cdot \mathbf{s}_I^A + J^B \mathbf{S}^B(\mathbf{x}_I) \cdot \mathbf{s}_I^B \right]$$

Spin of electron

Spin of impurity

Interaction between impurity and electron

$$H^{\text{TI}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{H}_{\mathbf{k}}^{\text{TI}} c_{\mathbf{k}}$$

$$H_J = \sum_I^{N_s} \left[J^A \mathbf{S}^A(\mathbf{x}_I) \cdot \mathbf{s}_I^A + J^B \mathbf{S}^B(\mathbf{x}_I) \cdot \mathbf{s}_I^B \right]$$

Spin of electron

Spin of impurity

Mean field approximation



$$H_e = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{H}_{e\mathbf{k}} c_{\mathbf{k}}$$

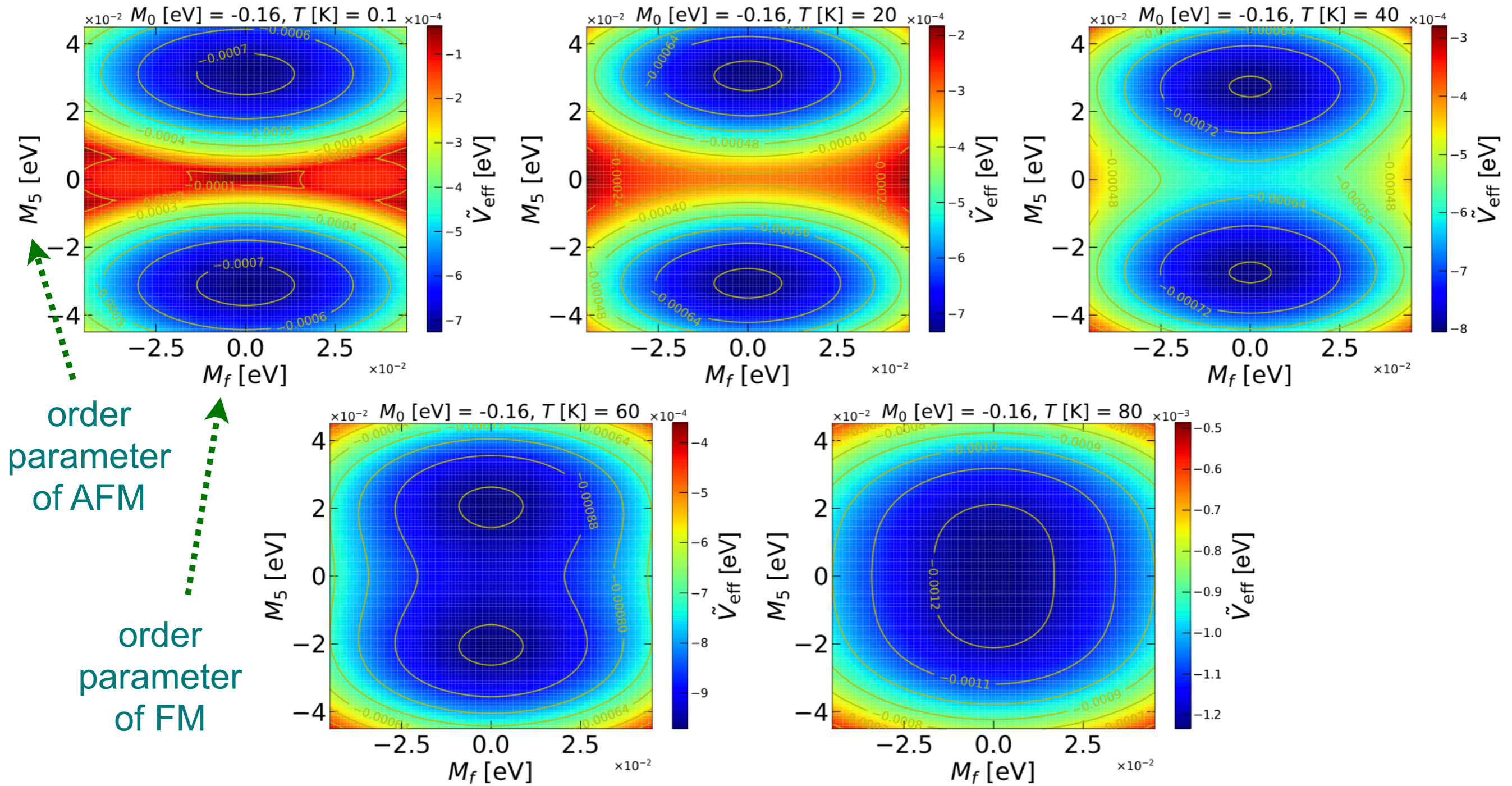
$$\mathcal{H}_{e\mathbf{k}} = \mathcal{H}_{\mathbf{k}}^{\text{TI}} + \mathcal{H}_{\mathbf{k}}^m$$
$$\mathcal{H}_{\mathbf{k}}^m = M_f \Gamma^{12} + M_5 \Gamma^5$$

M_f : order parameter of FM

M_5 : order parameter of AFM

Effective potential

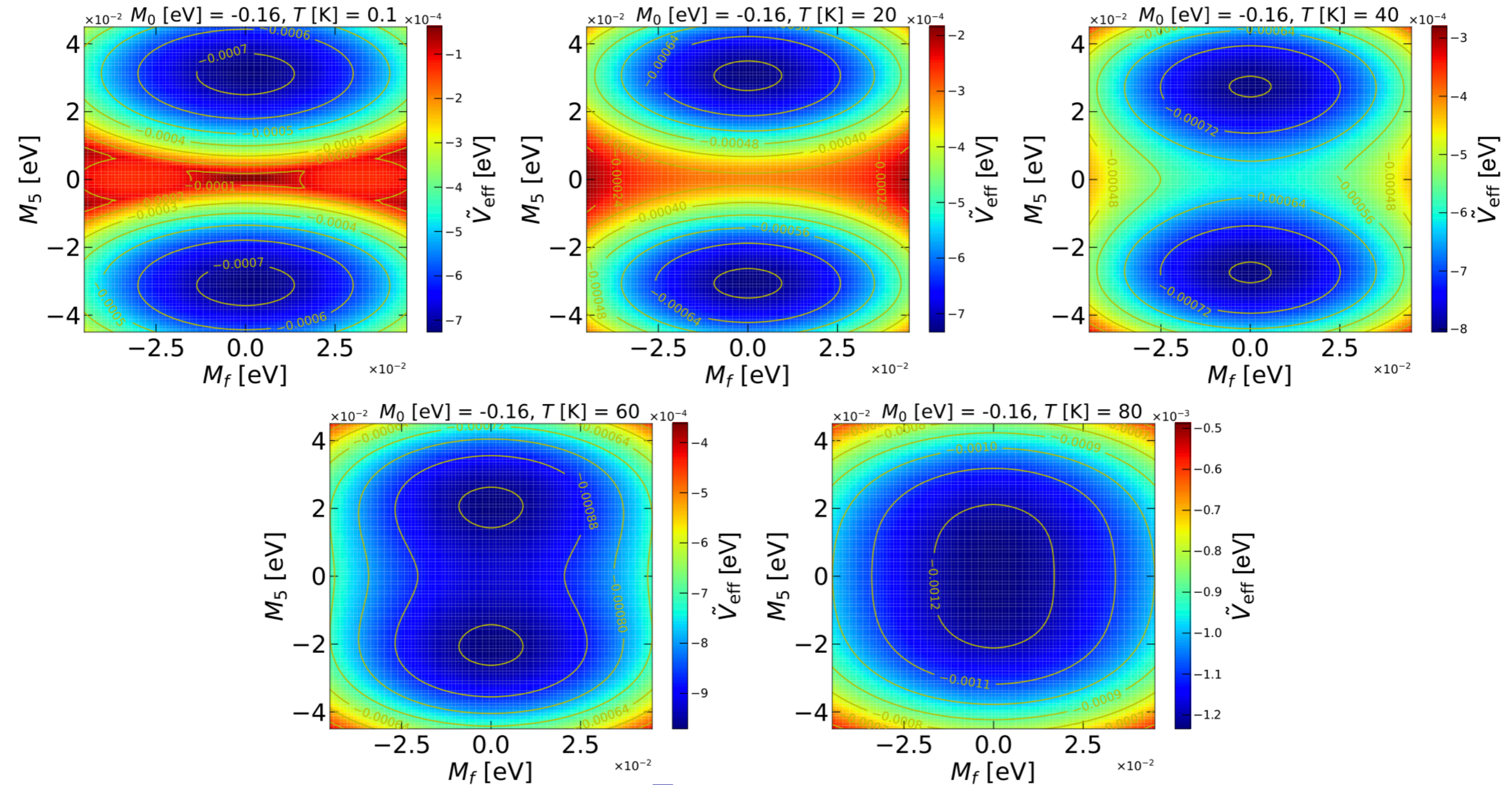
KI '22



Effective potential

Temperature

KI '22



Temperature

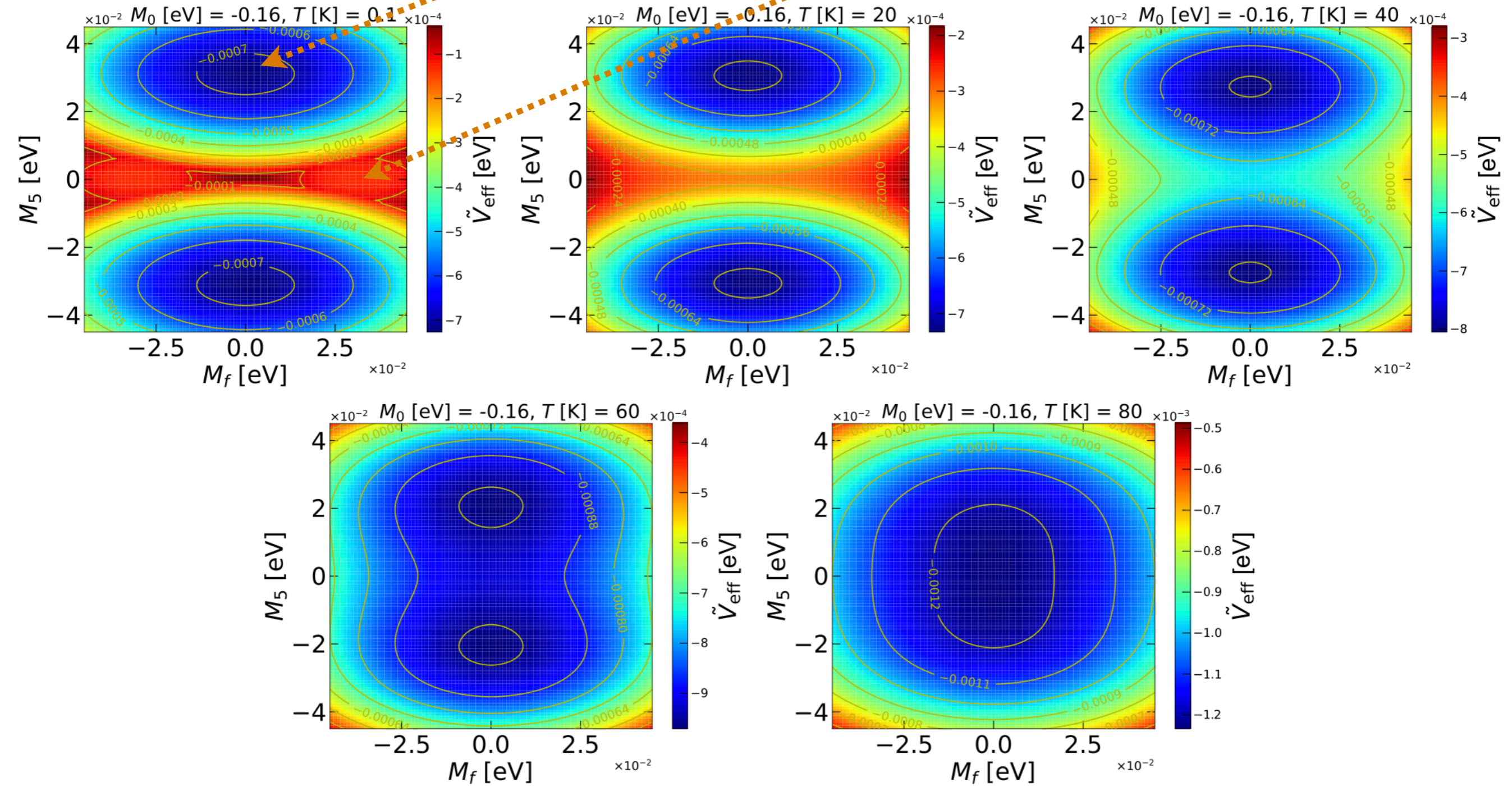


Effective potential

FM state

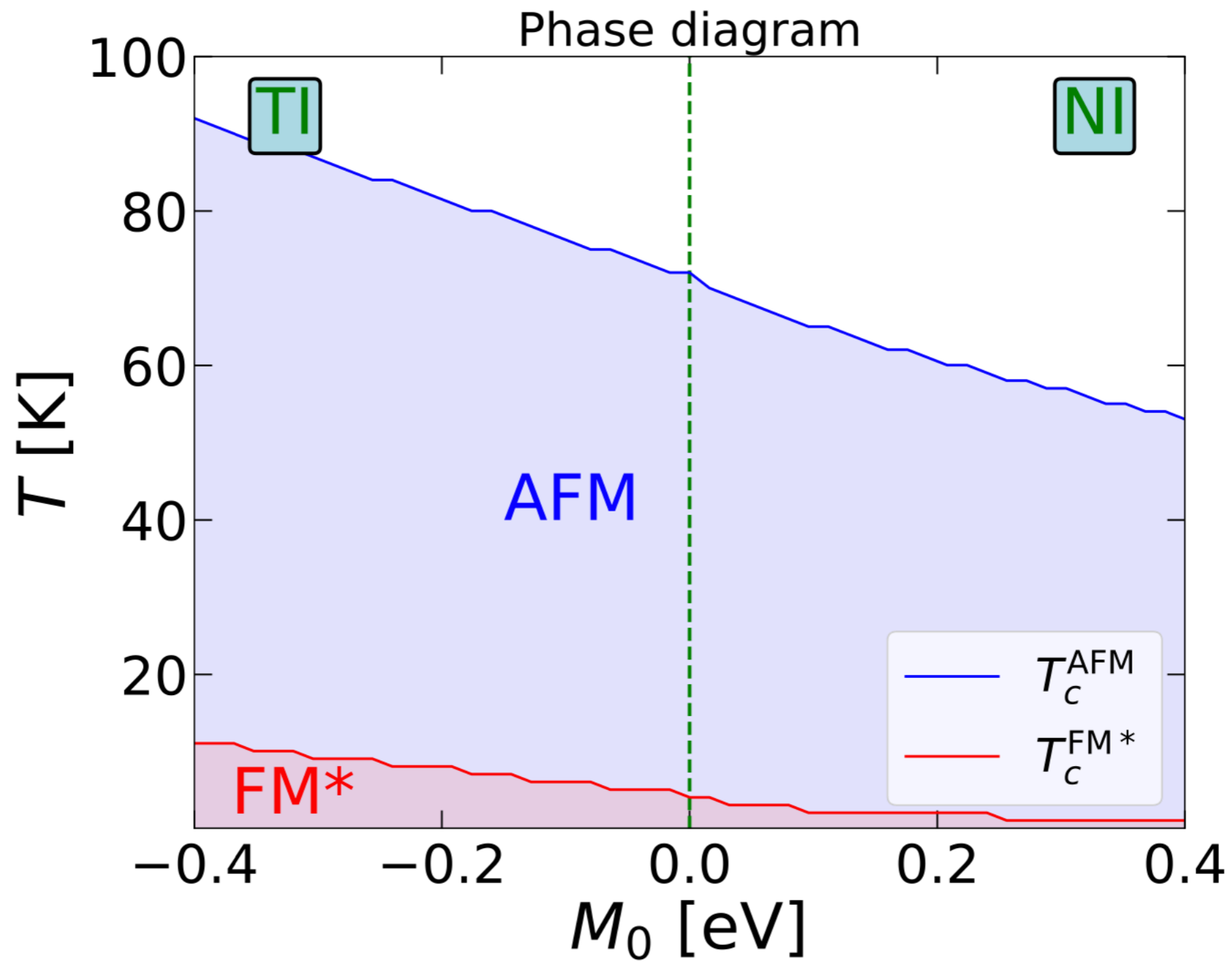
KI '22

AFM state



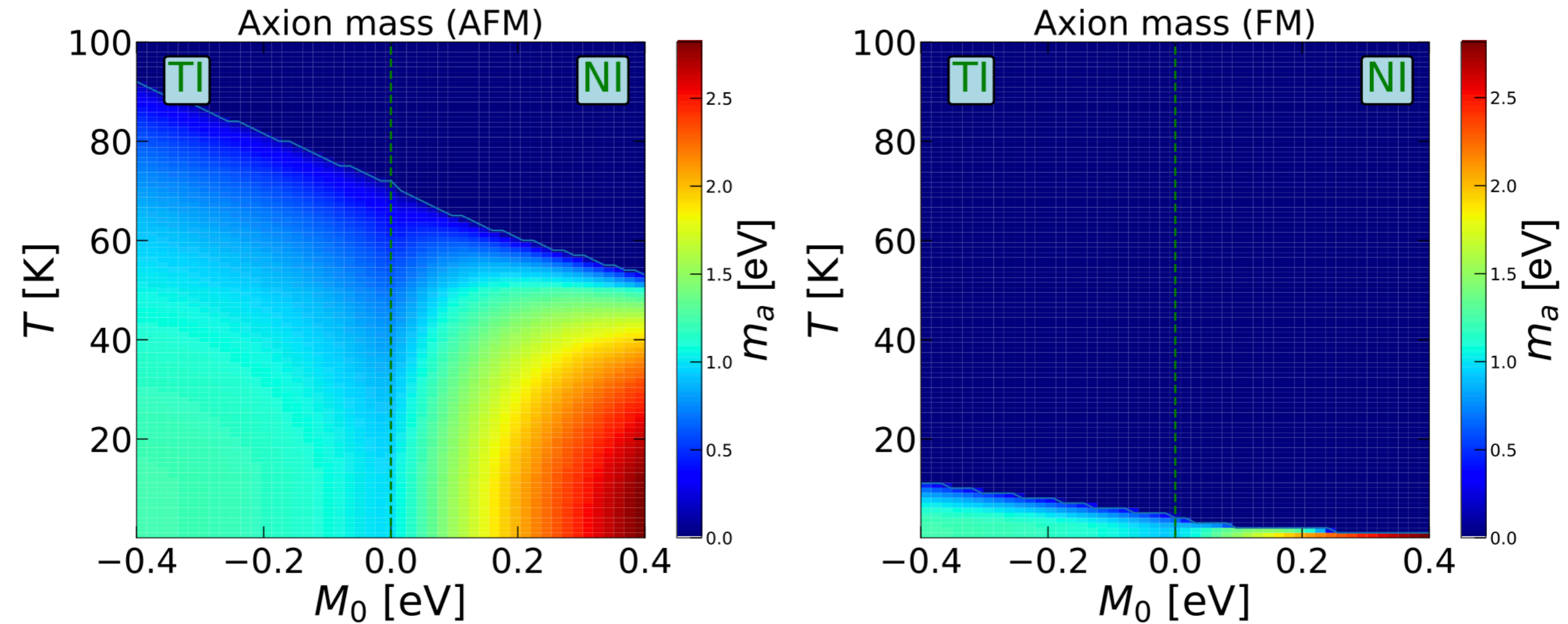
Phase diagram

KI '22



Axion mass

KI '22

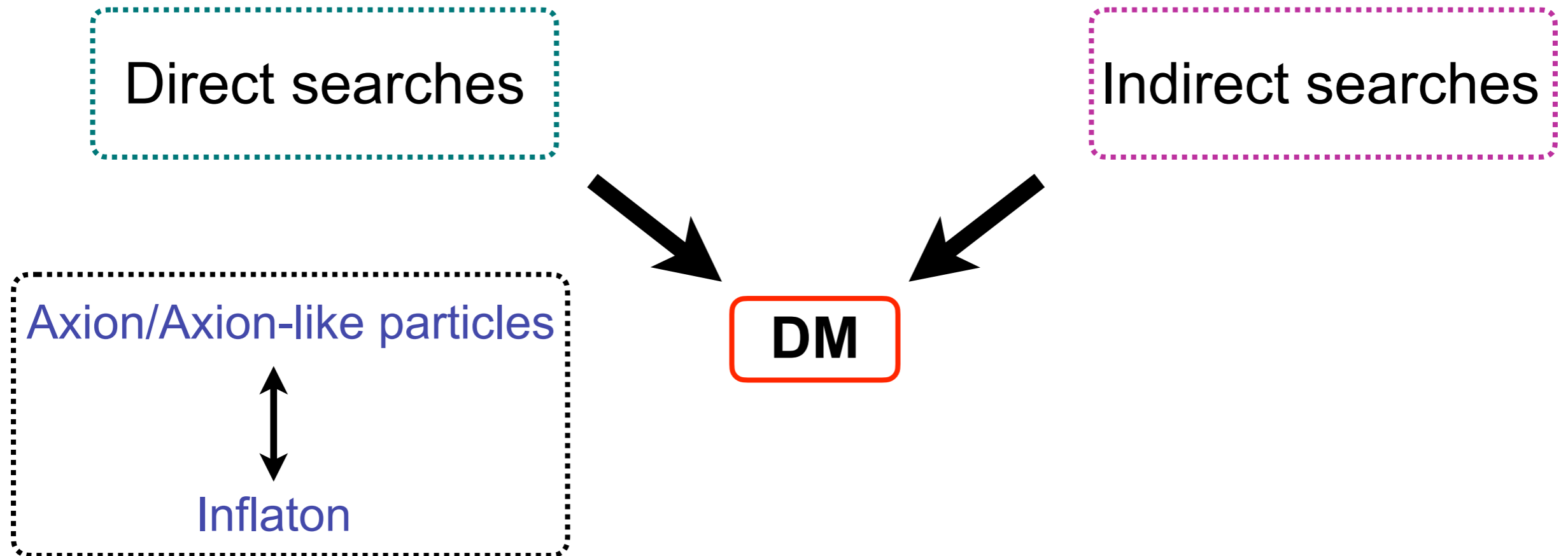


Axion mass is $\mathcal{O}(\text{eV})$ except for the phase boundaries

Quick summary

- Axion mass is $\mathcal{O}(eV)$ while it can be suppressed around the phase boundaries in the magnetic TIs
- Material search is crucial for the particle axion detection

Approaches from astro-particle physics and cosmology



Approaches from astro-particle physics and cosmology

Direct searches

Indirect searches

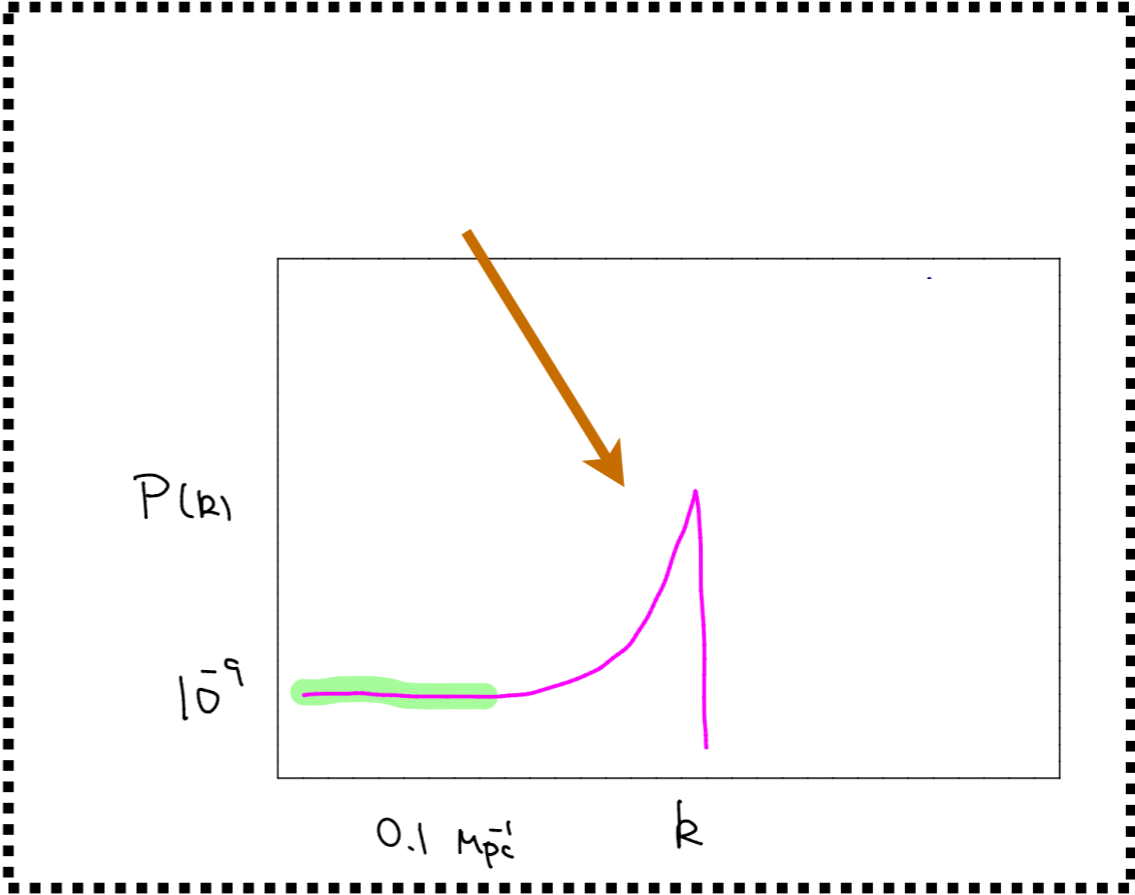


DM

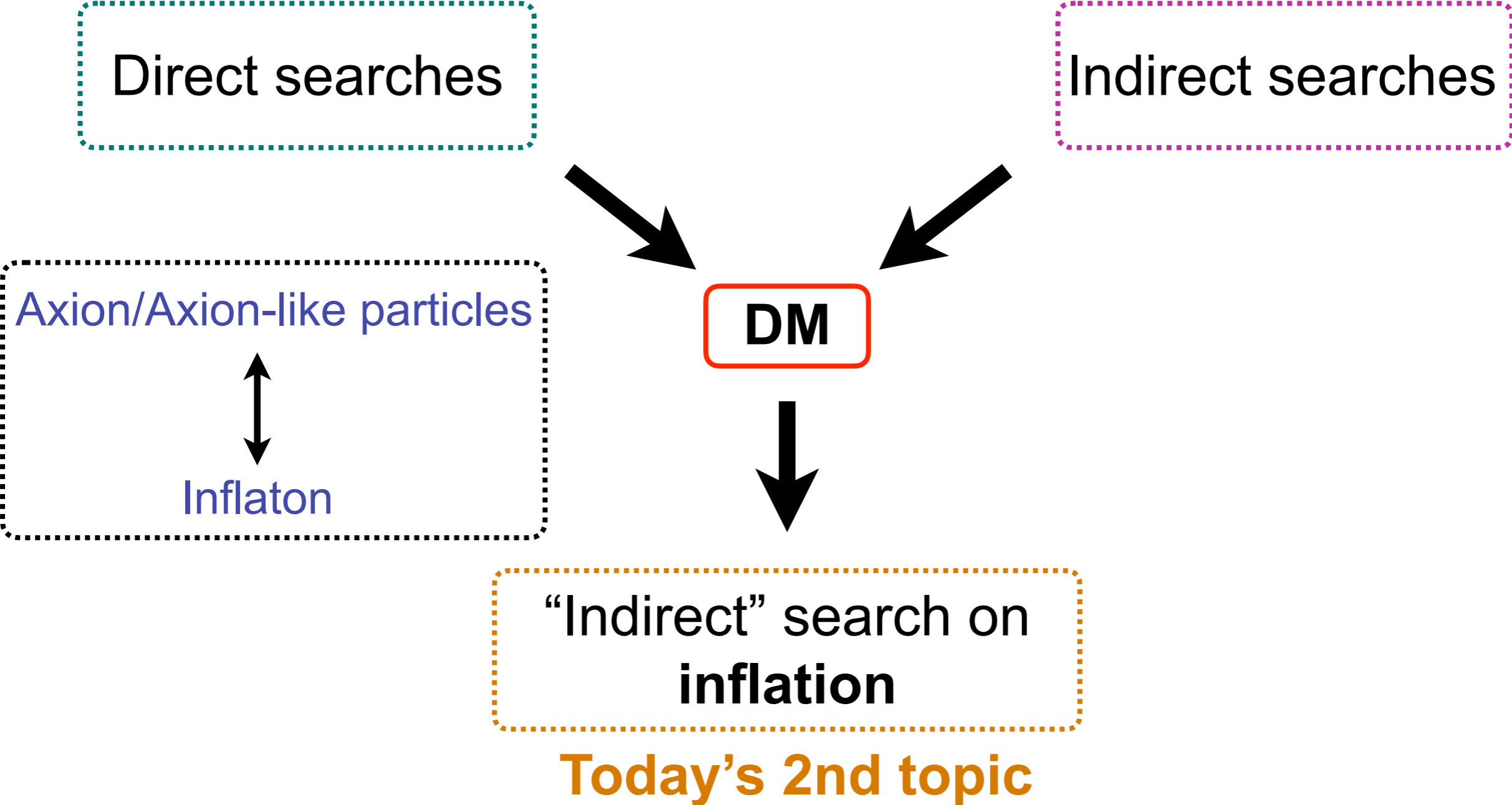
Axion/Axion-like particles
↕
Inflaton

Blue-tilted power spectrum,
e.g., axion-U(1) gauge system

McDonough, Moghaddam, Brandenberger '16

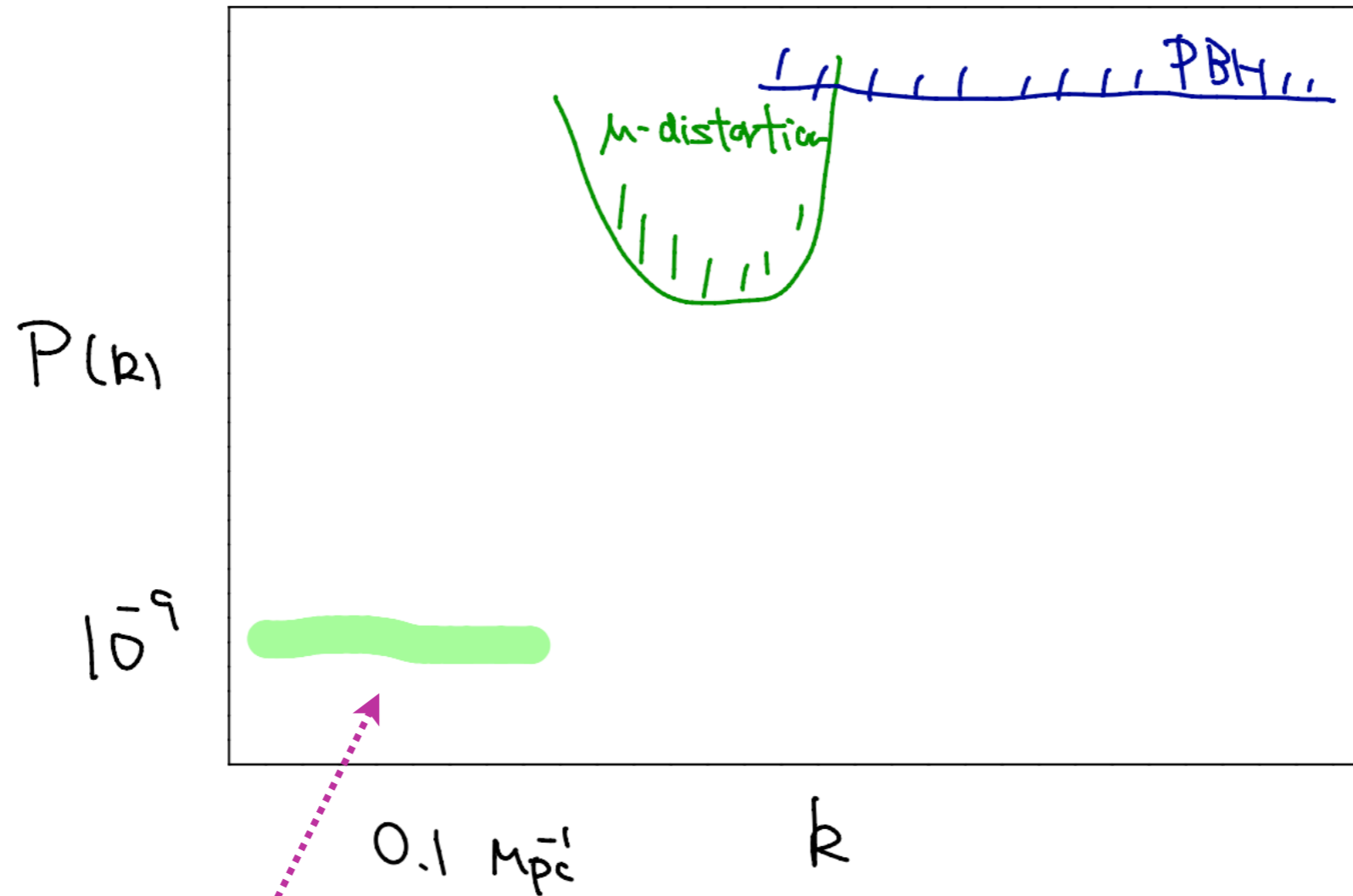


Approaches from astro-particle physics and cosmology



3. Primordial curvature perturbations

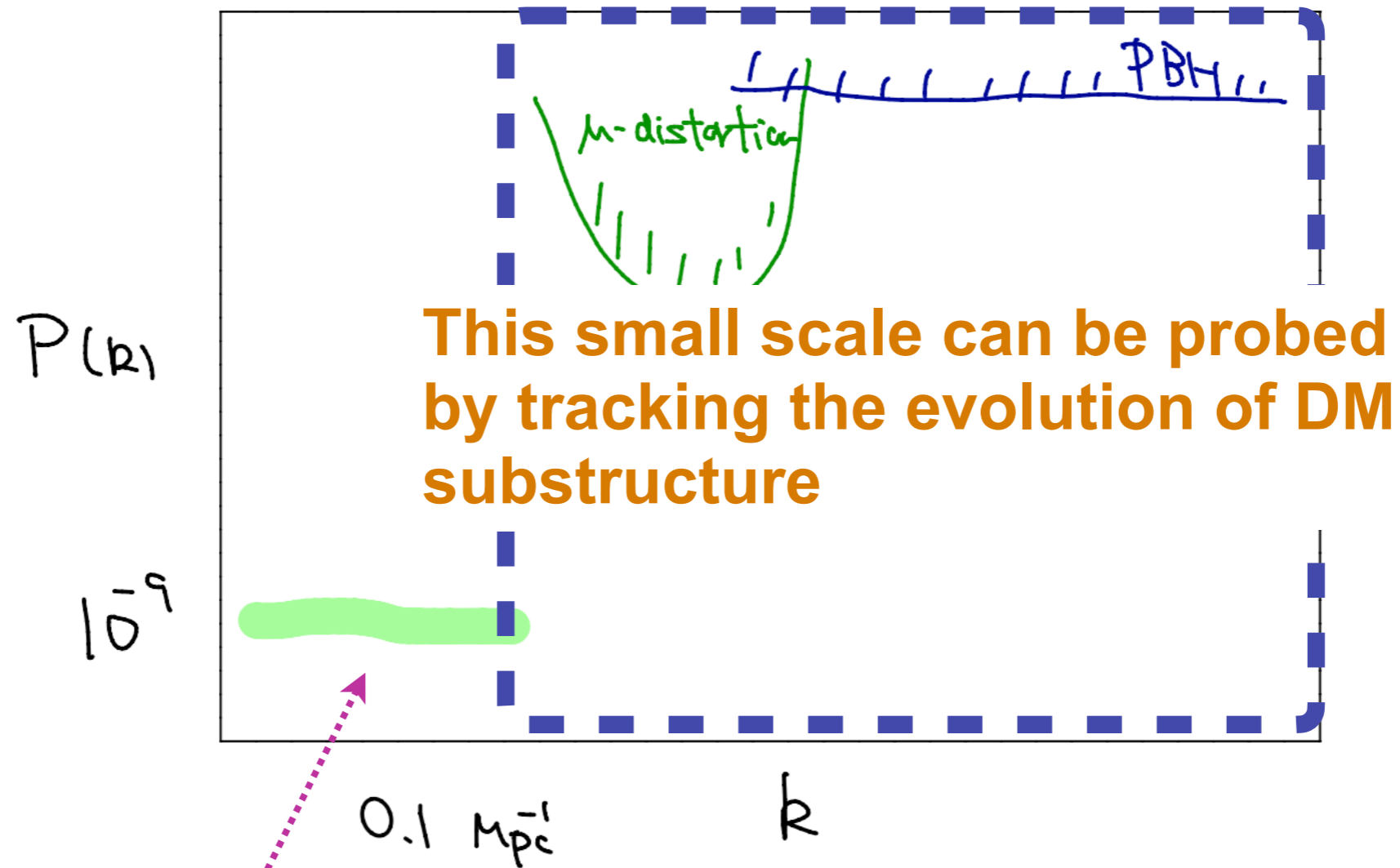
Constraints on primordial curvature power spectrum



$$\mathcal{P}_R = (2.099 \pm 0.029) \times 10^{-9}$$

Planck '18

Constraints on primordial curvature power spectrum



$$\mathcal{P}_R = (2.099 \pm 0.029) \times 10^{-9}$$

Planck '18

Evolution of halo/subhalos

Hiroshima-san's talk in detail

Curvature perturbation



Host halos and subhalos



Subhalos accrete on a host halo



Subhalos or satellite galaxies

Evolution of halo/subhalos

Curvature perturbation



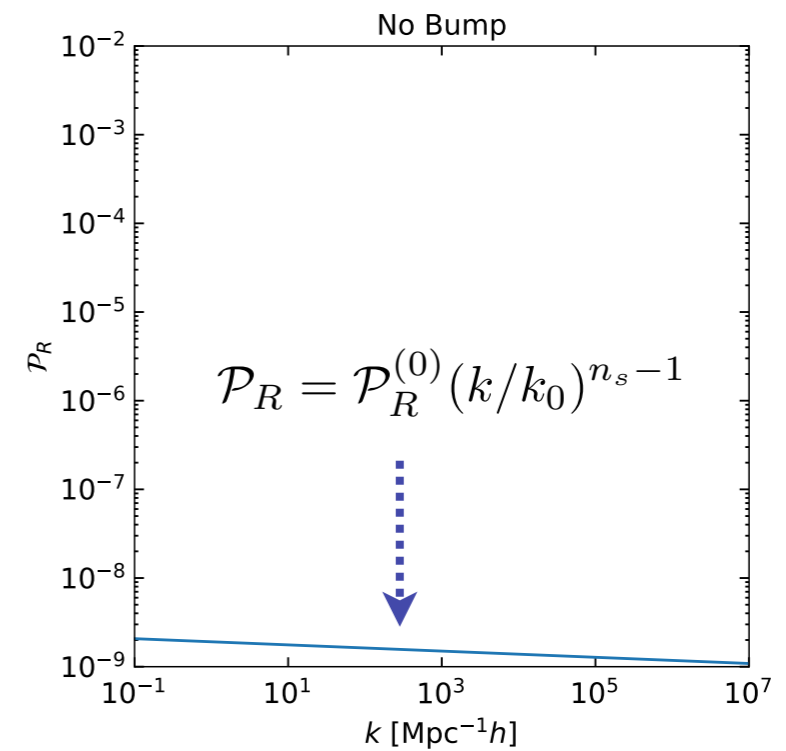
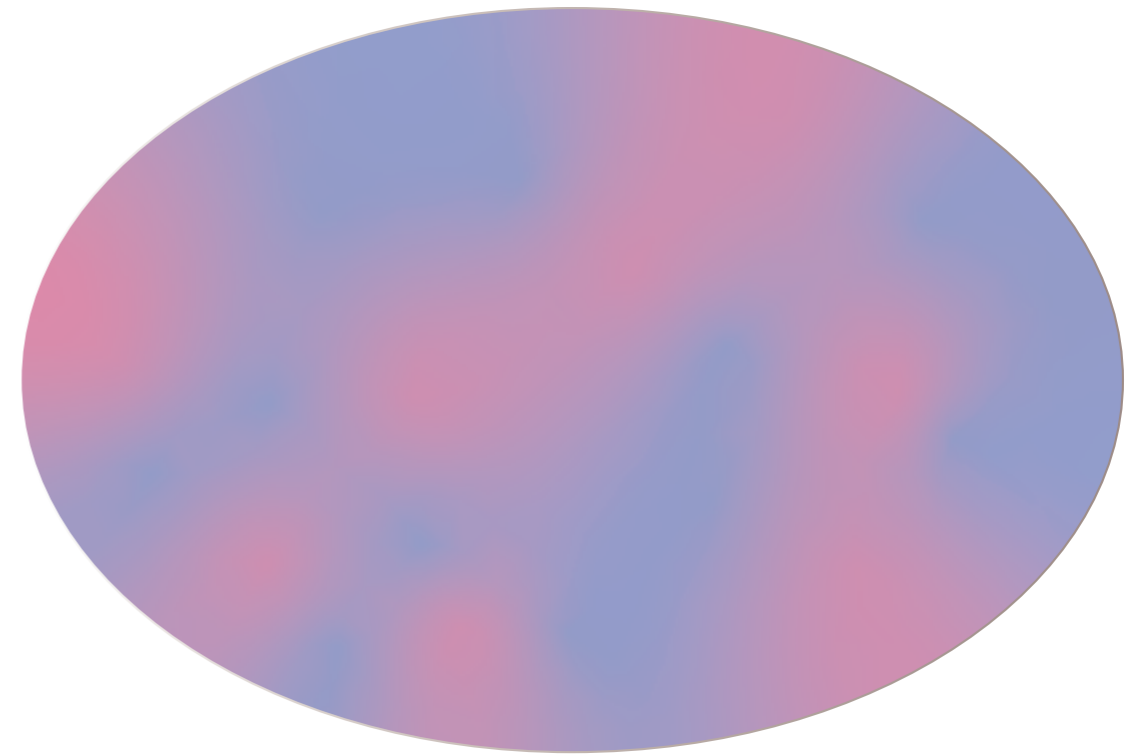
Host halos and subhalos



Subhalos accrete on a host halo



Subhalos or satellite galaxies



Evolution of halo/subhalos

Curvature perturbation



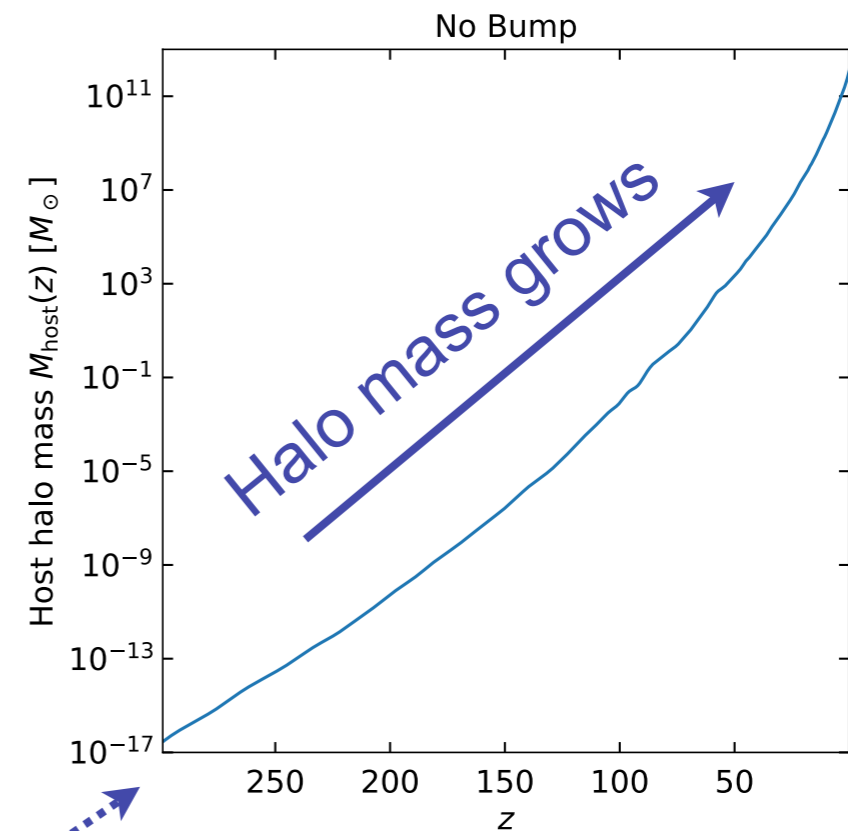
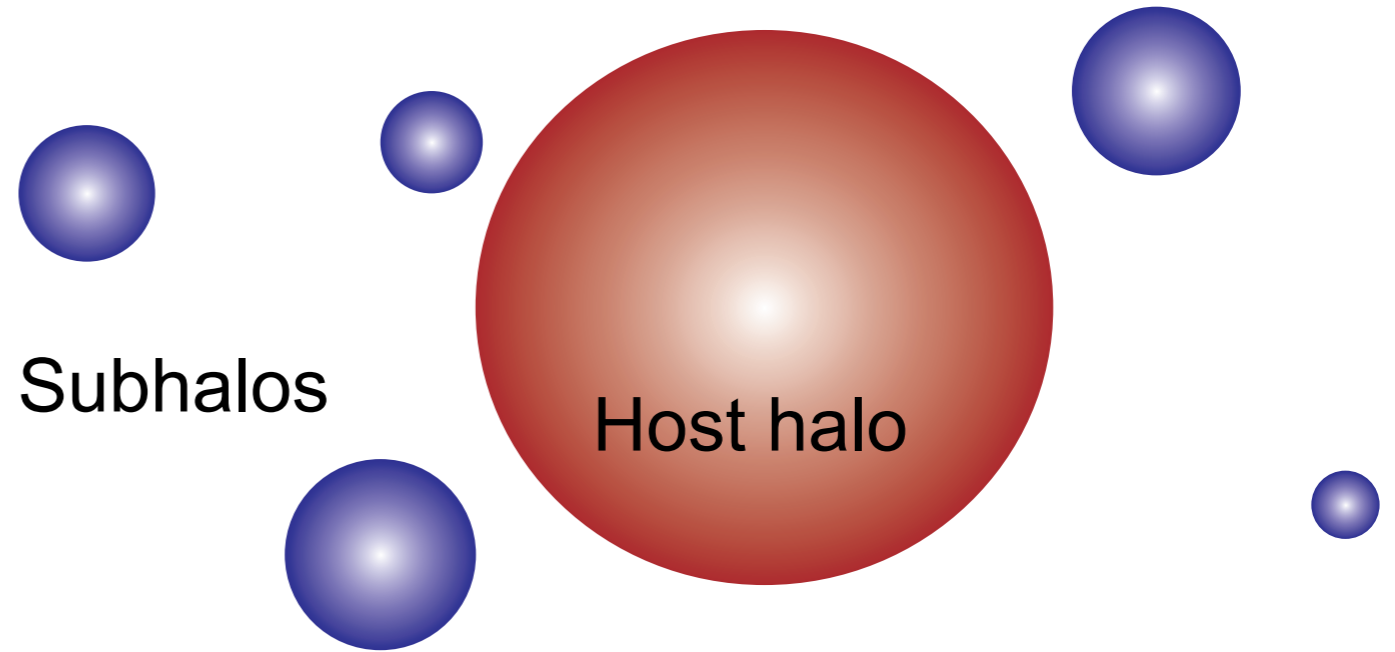
Host halos and subhalos



Subhalos accrete on a host halo



Subhalos or satellite galaxies



Halo mass grows gradually based on the extended Press-Schechter formalism

Bond, Cole, Efstathiou, Kaiser '91

Bower '91

Lacy, Cole '93

Yang, Mo, Zhang, van den Bosch '11

Evolution of halo/subhalos

Curvature perturbation



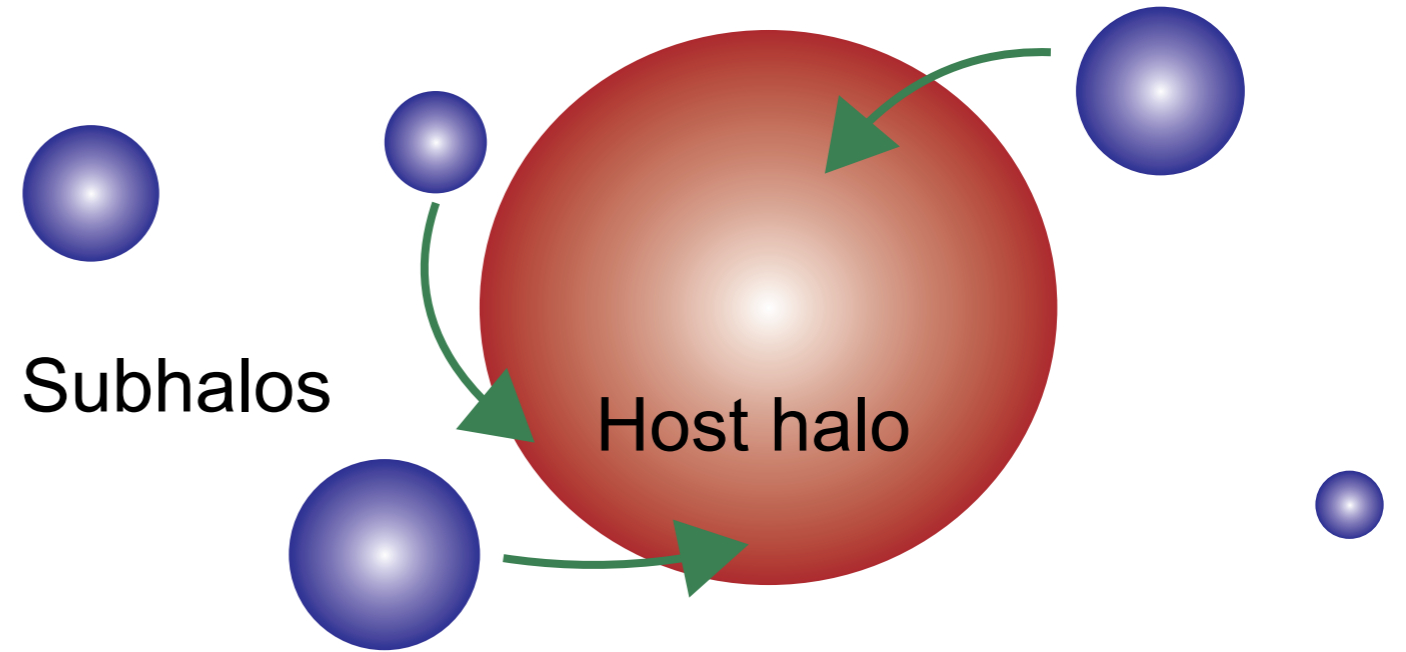
Host halos and subhalos



Subhalos accrete on a host halo



Subhalos or satellite galaxies



Evolution of halo/subhalos

Curvature perturbation



Host halos and subhalos



Subhalos accrete on a host halo

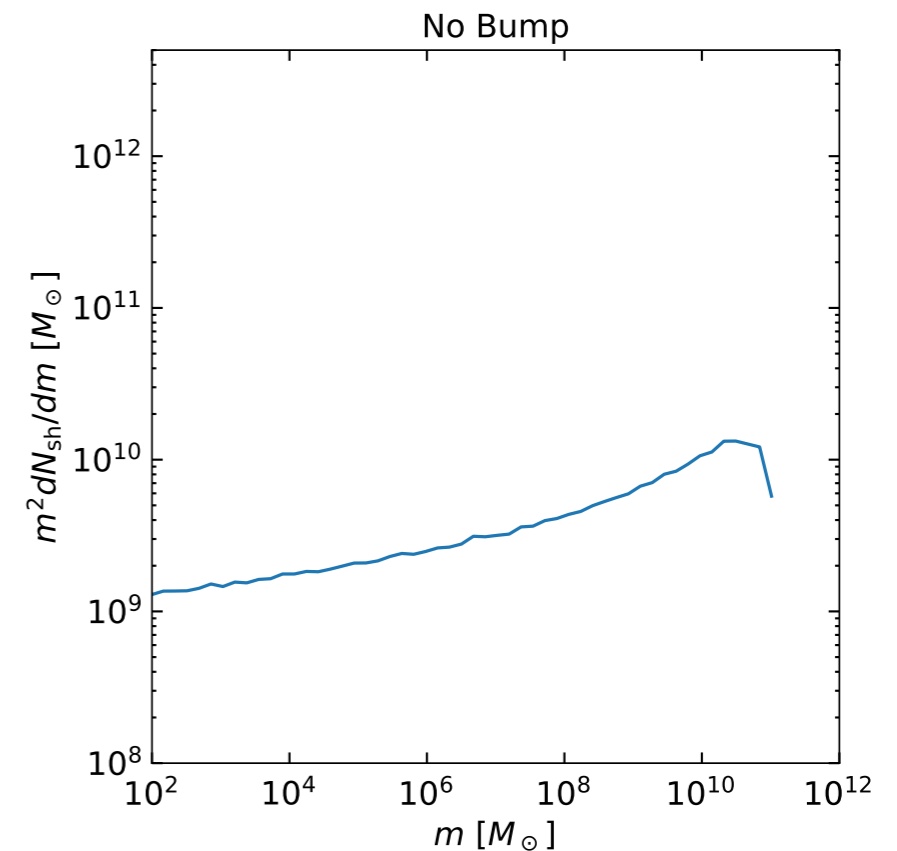
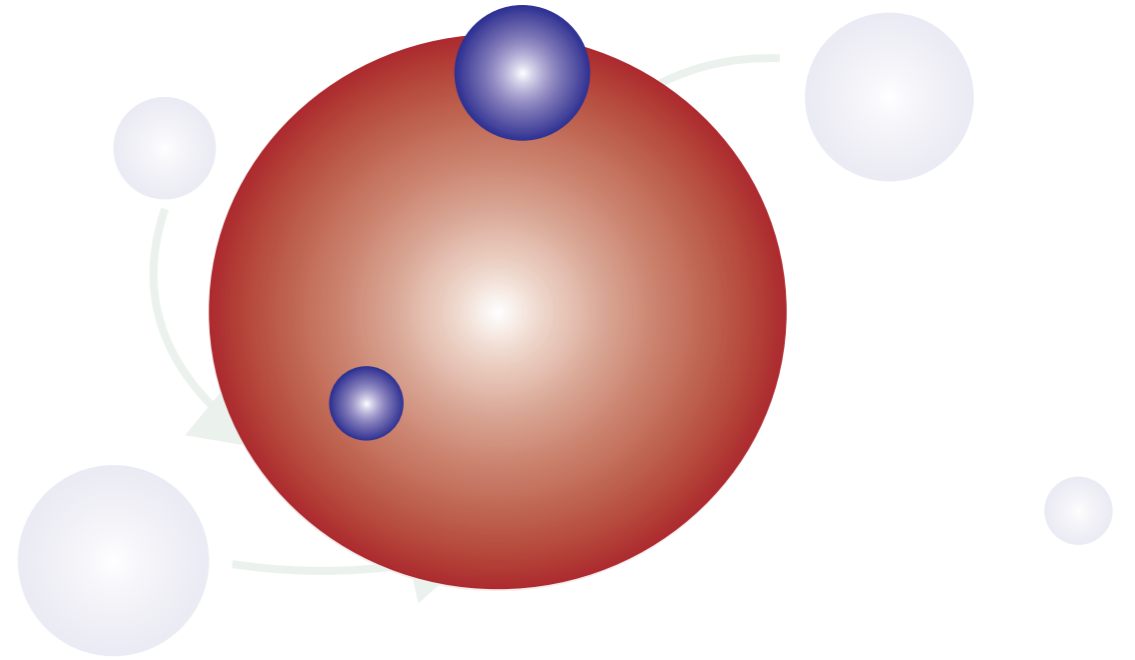


Tidal stripping

Subhalos or satellite galaxies

Studied in semi-analytical way
calibrated by N-body simulation

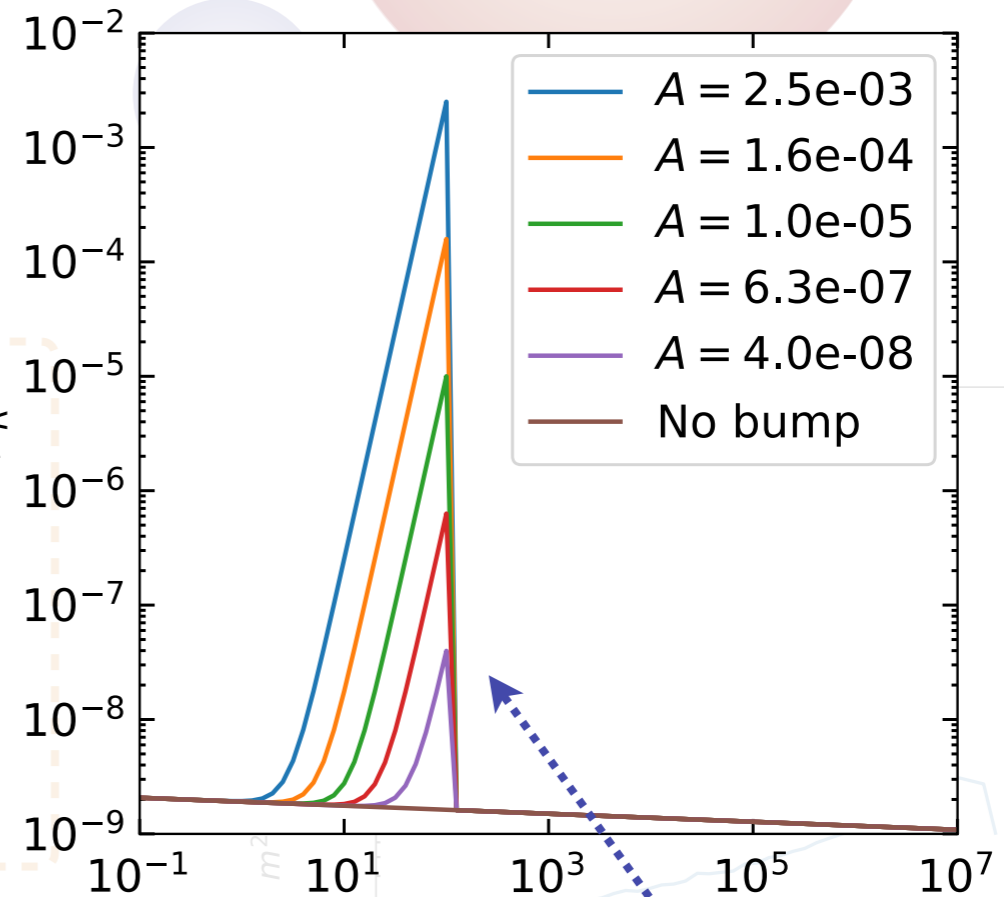
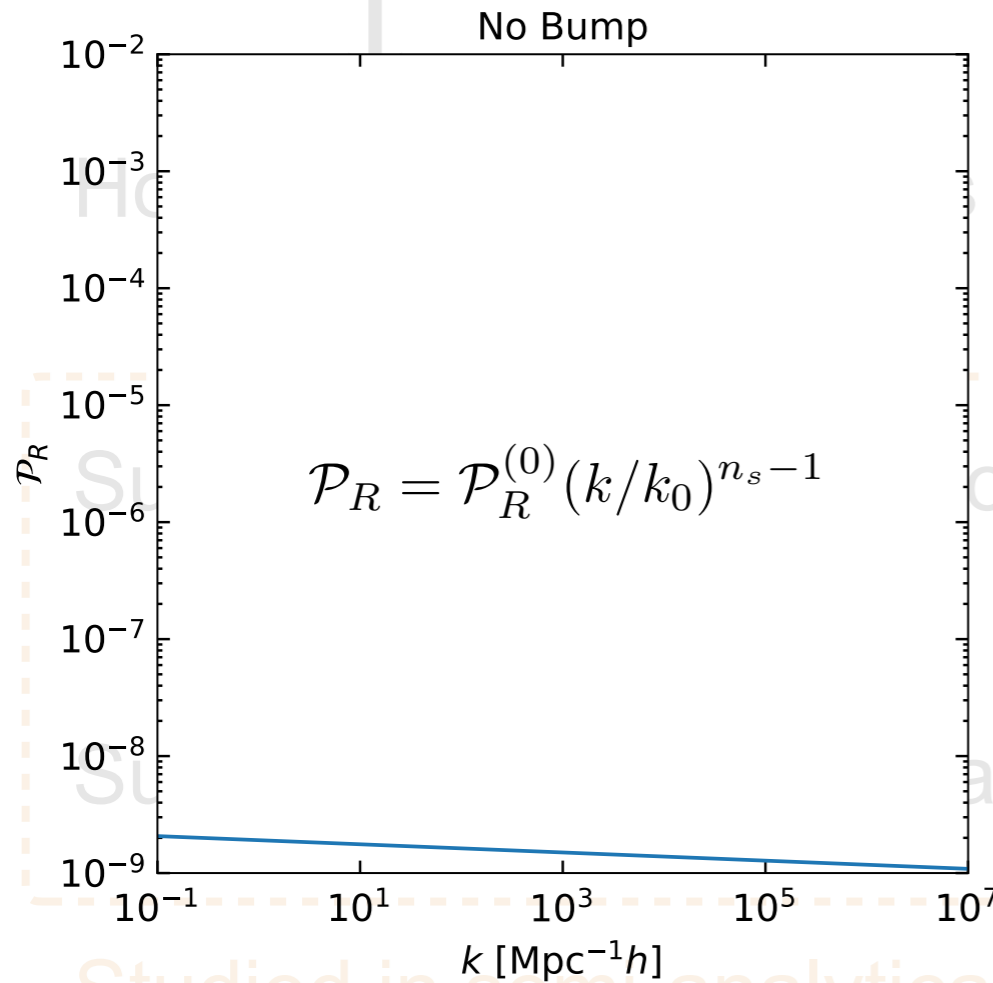
Hiroshima, Ando, Ishiyama '18



Mass distribution of subhalos

Evolution of halo/subhalos

Curvature perturbation



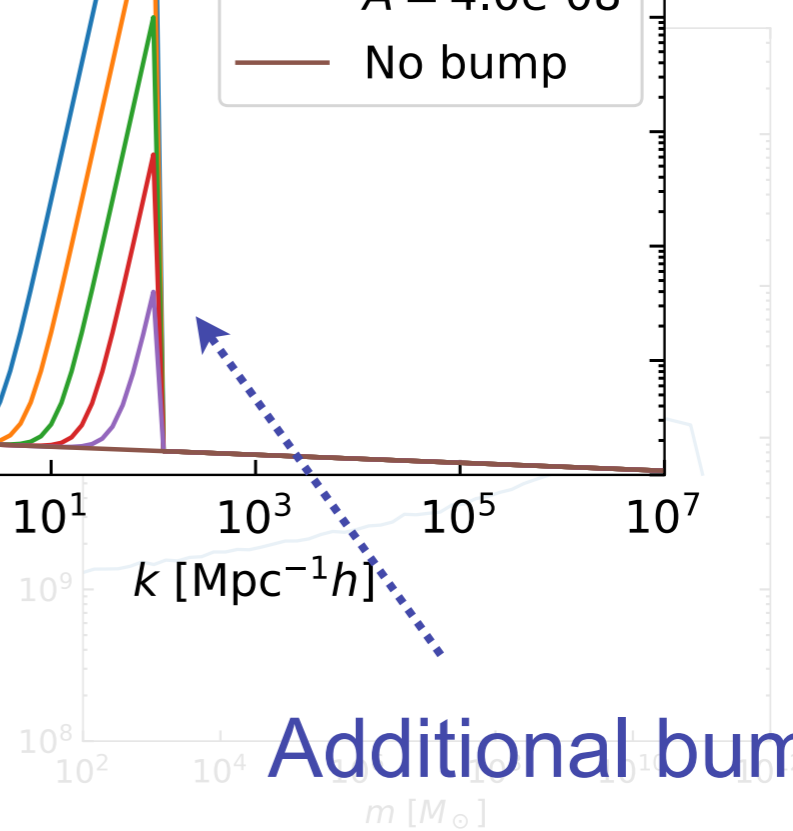
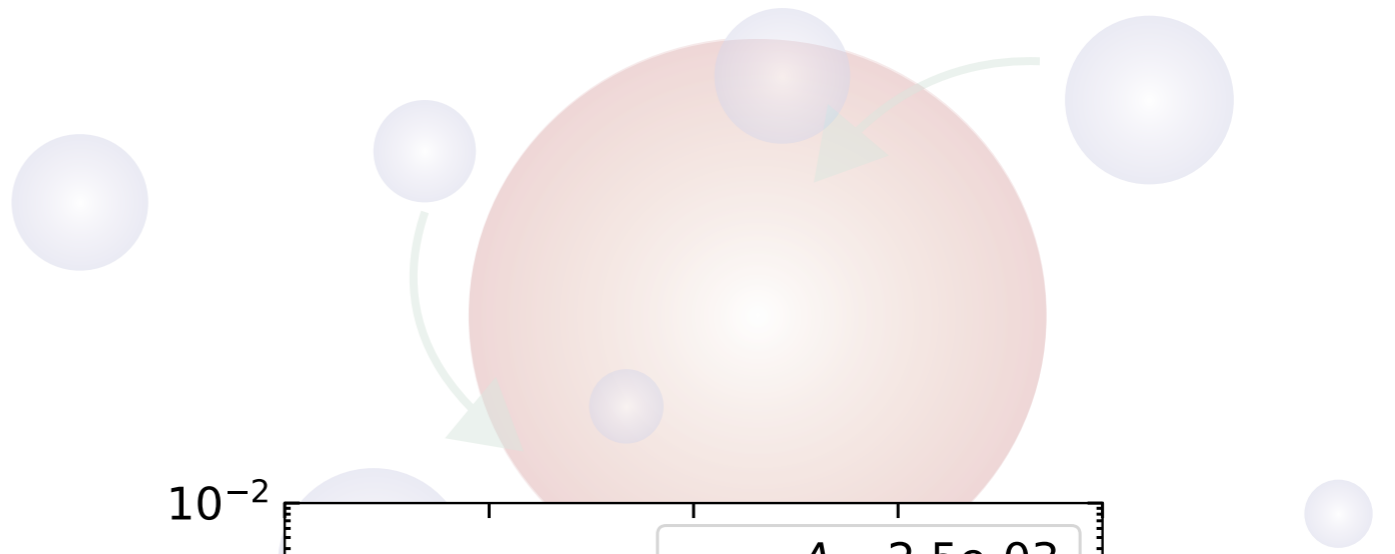
Studied in semi-analytical way
calibrated by N-body simulation

Hiroshima, Ando, Ishiyama '18

What will happen?

Mass distribution of subhalos

Additional bump



Evolution of halo/subhalos

Curvature perturbation



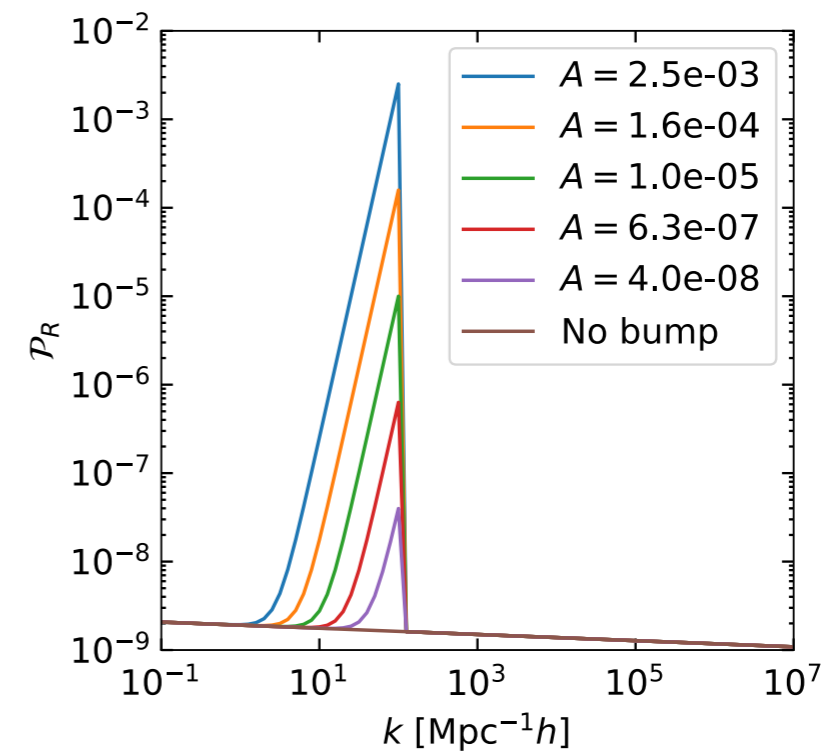
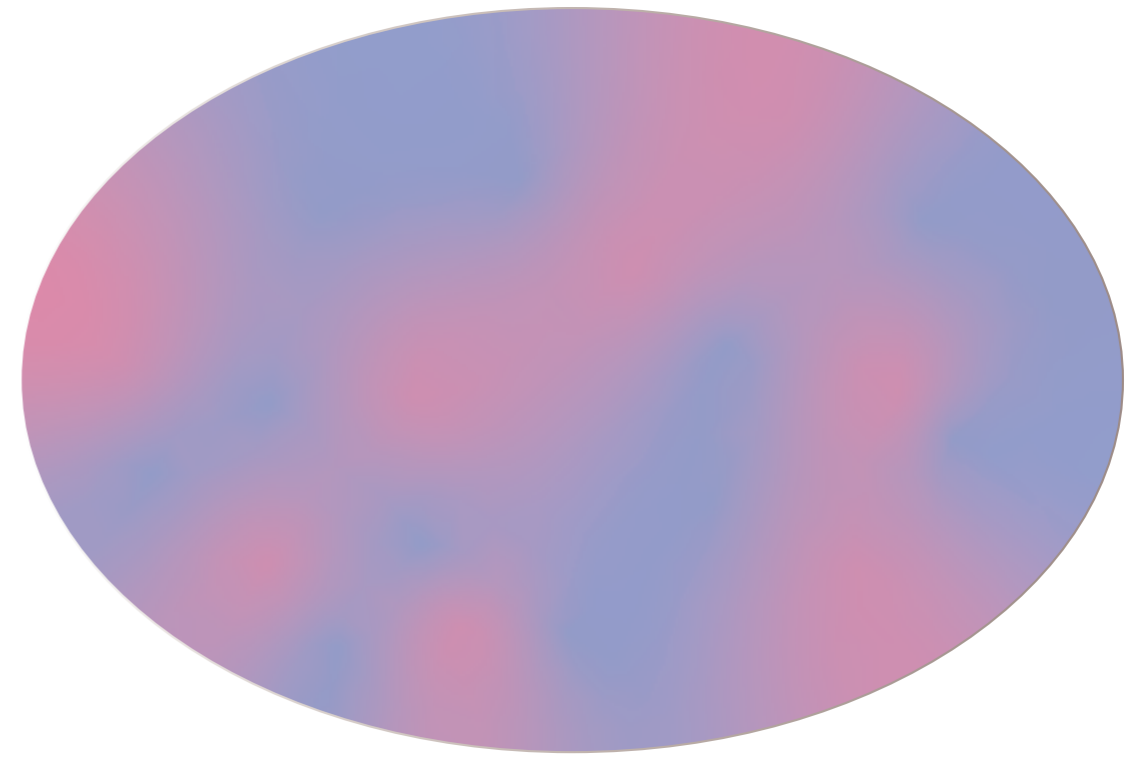
Host halos and subhalos



Subhalos accrete on a host halo



Subhalos or satellite galaxies



Evolution of halo/subhalos

Curvature perturbation



Host halos and subhalos

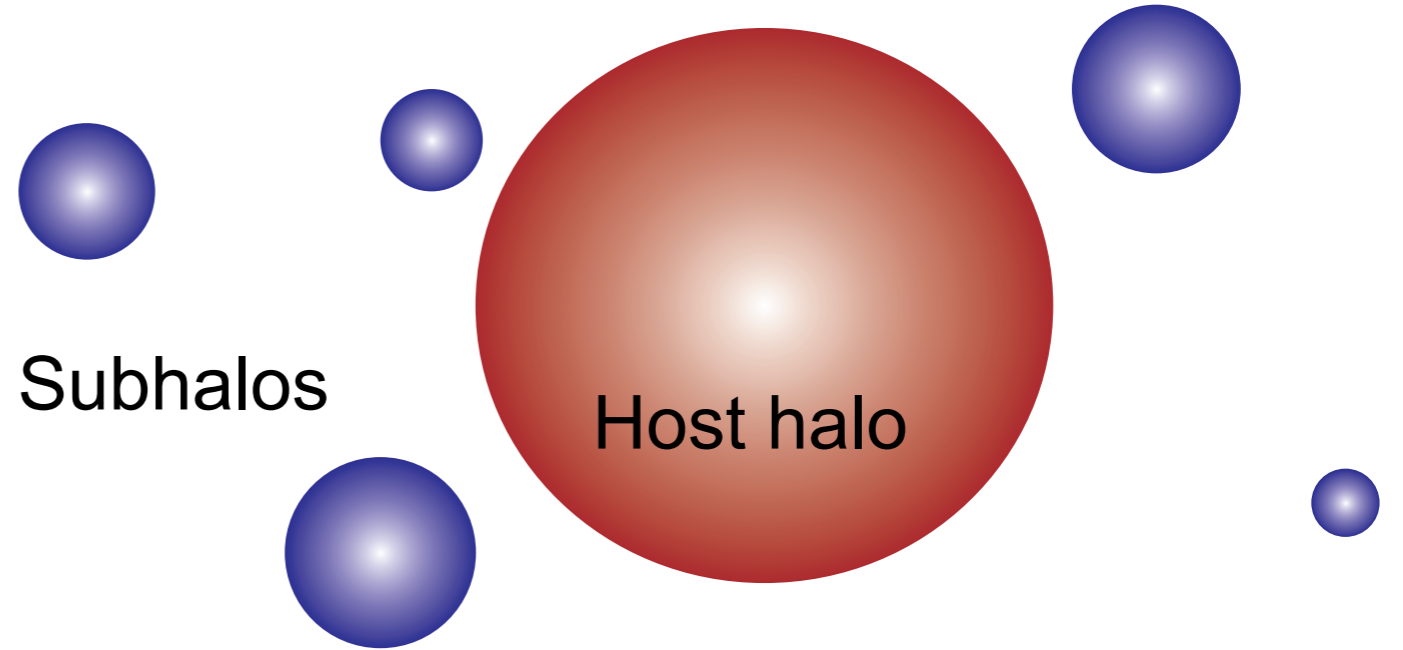


Subhalos accrete on a host halo

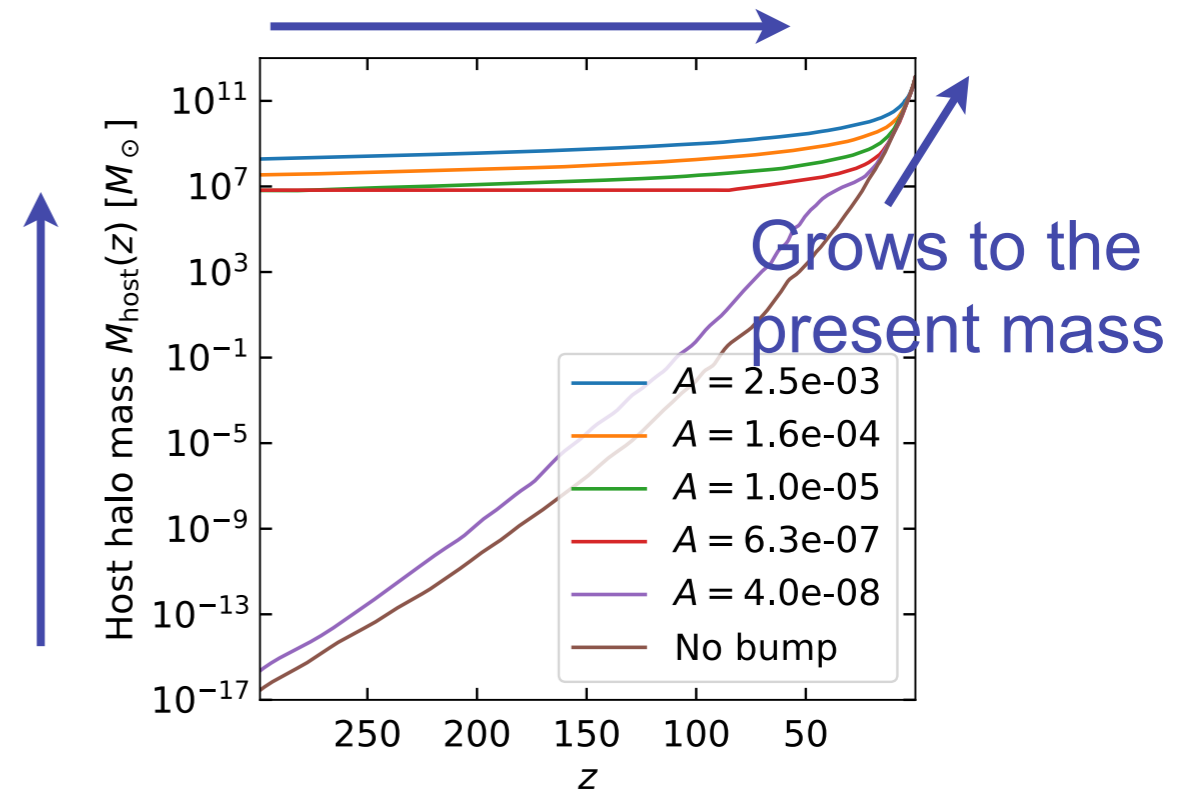


Halo grows to a specific scale *at once*

Subhalos or satellite galaxies



Merely grows for while



Evolution of halo/subhalos

Curvature perturbation



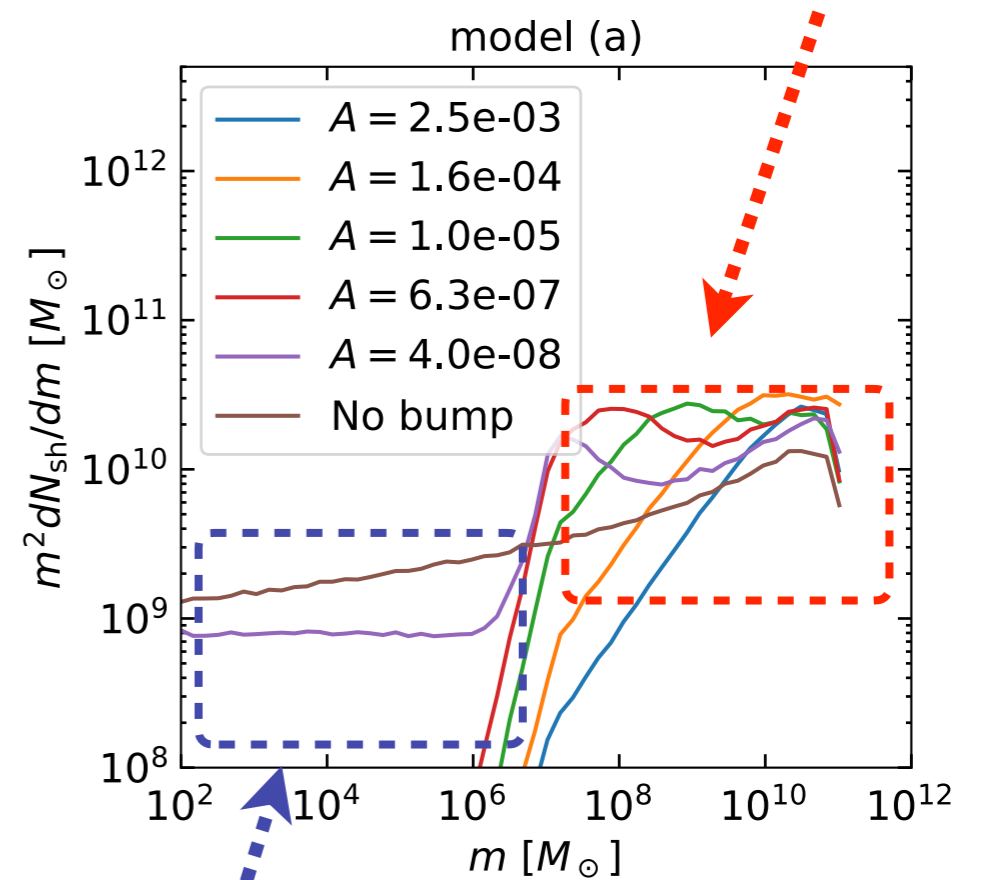
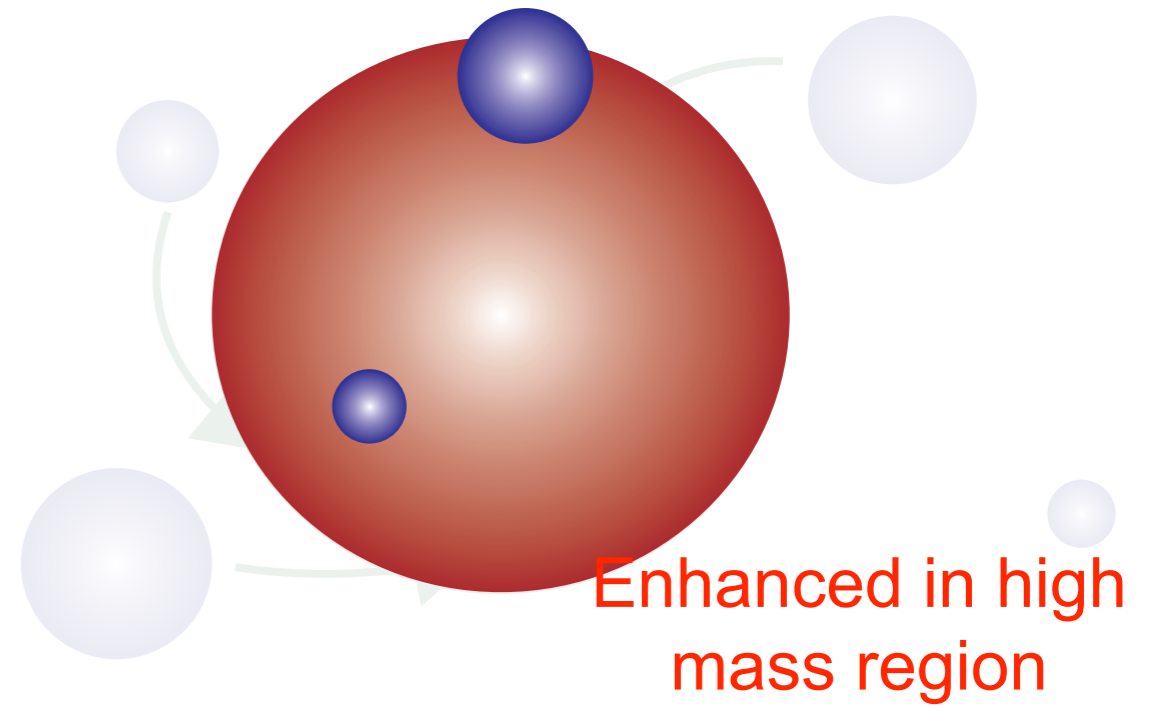
Host halos and subhalos



Subhalos accrete on a host halo

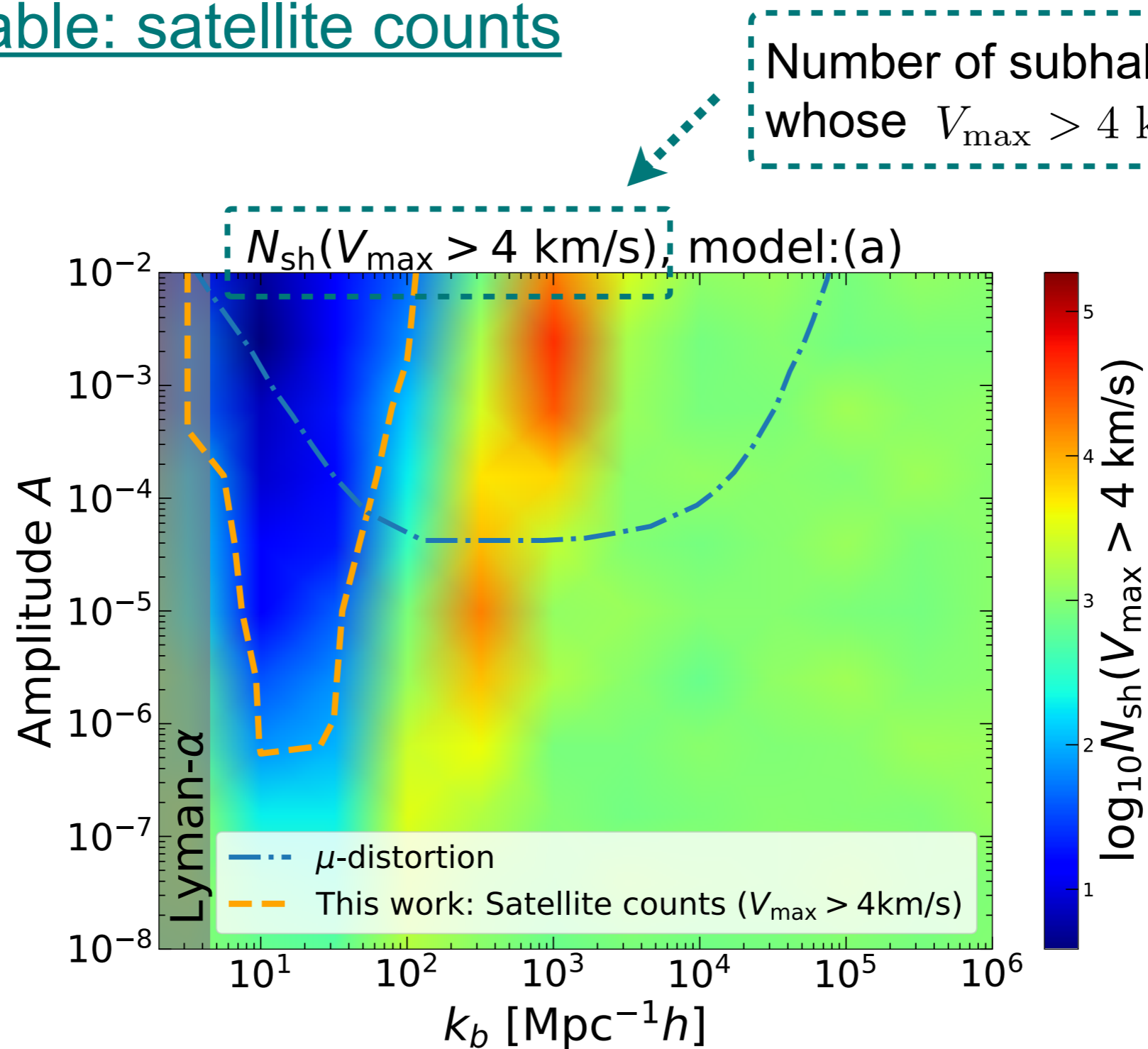


Subhalos or satellite galaxies



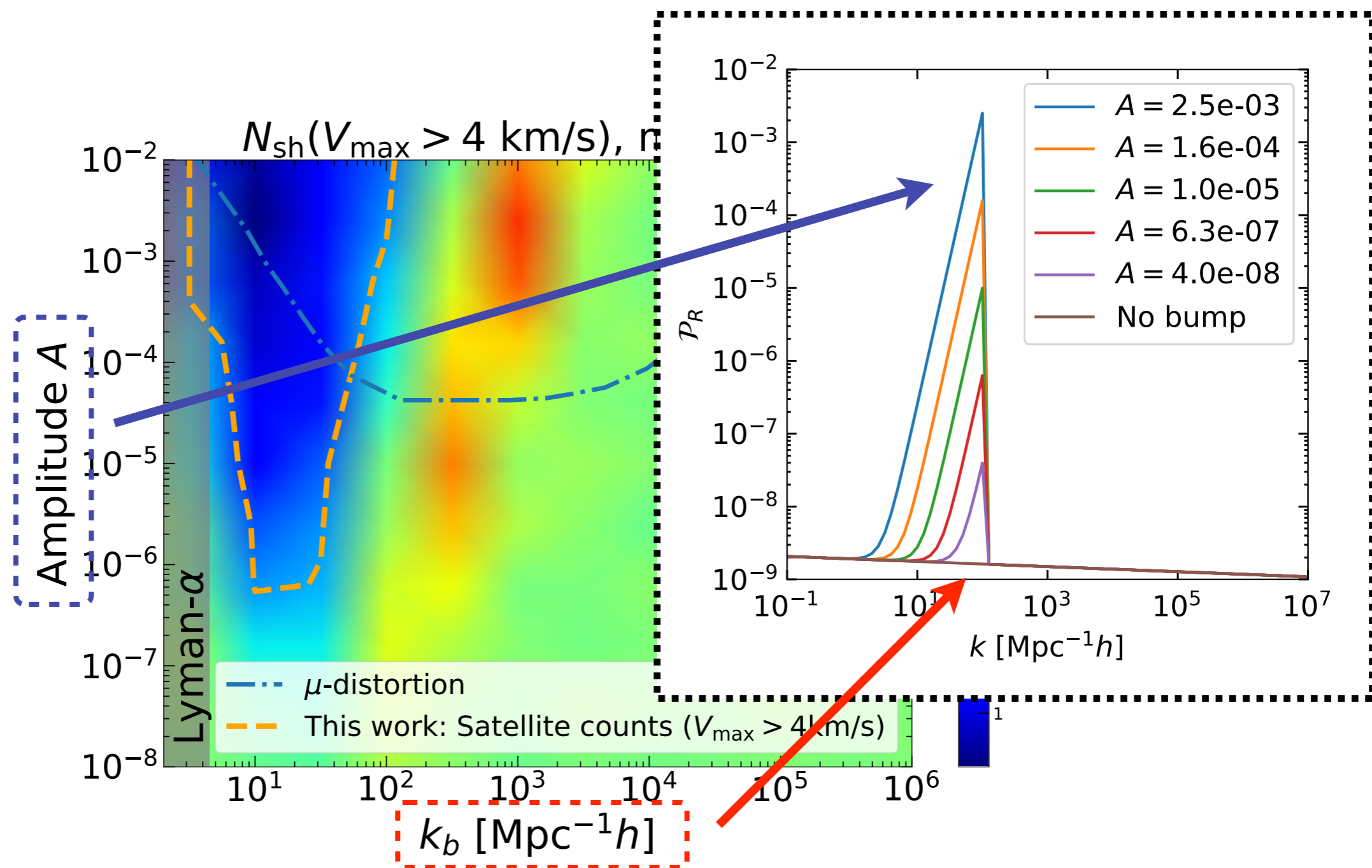
Suppressed in low mass region

The observable: satellite counts



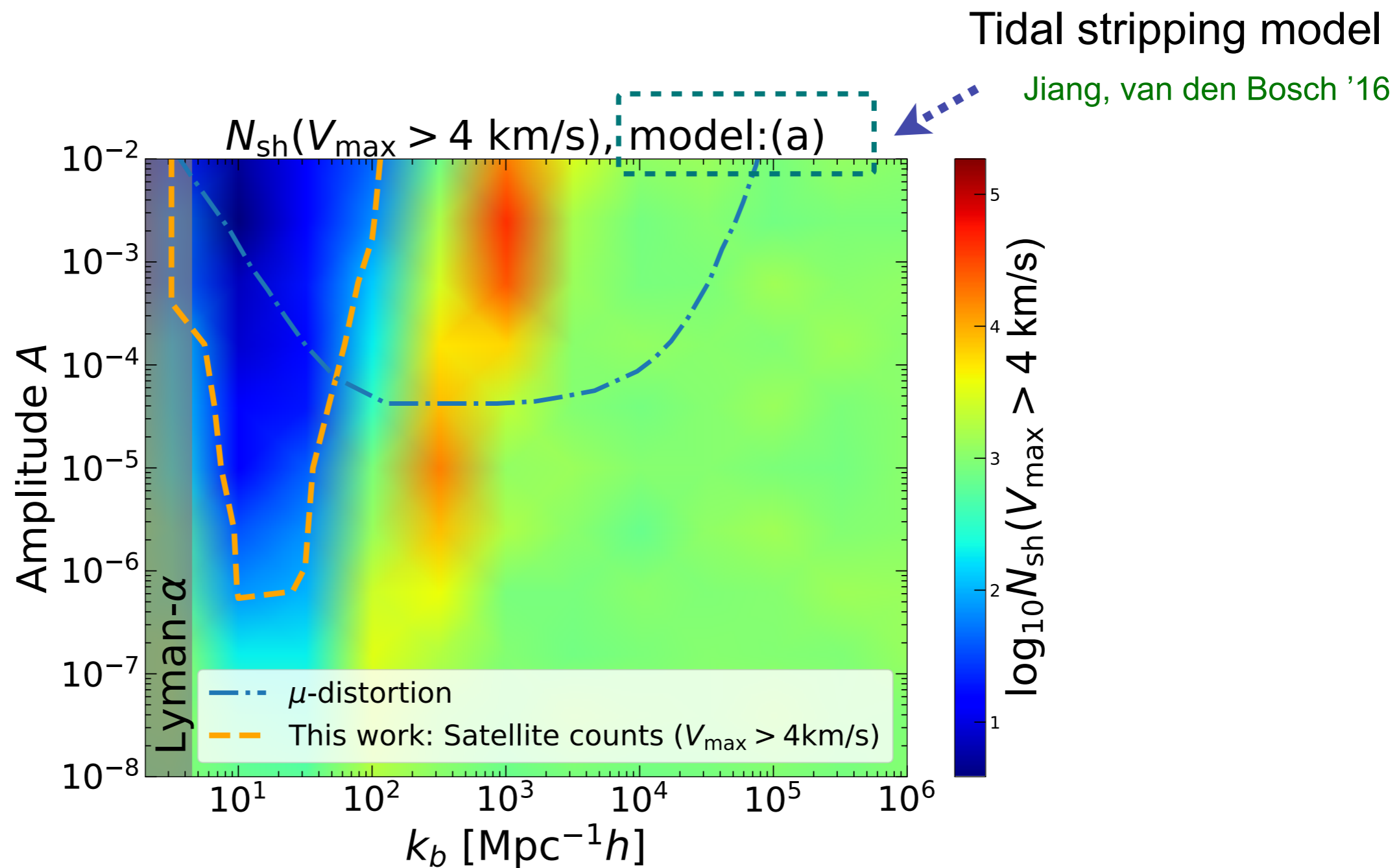
V_{\max} : Maximum circular velocity of subhalos

The observable: satellite counts



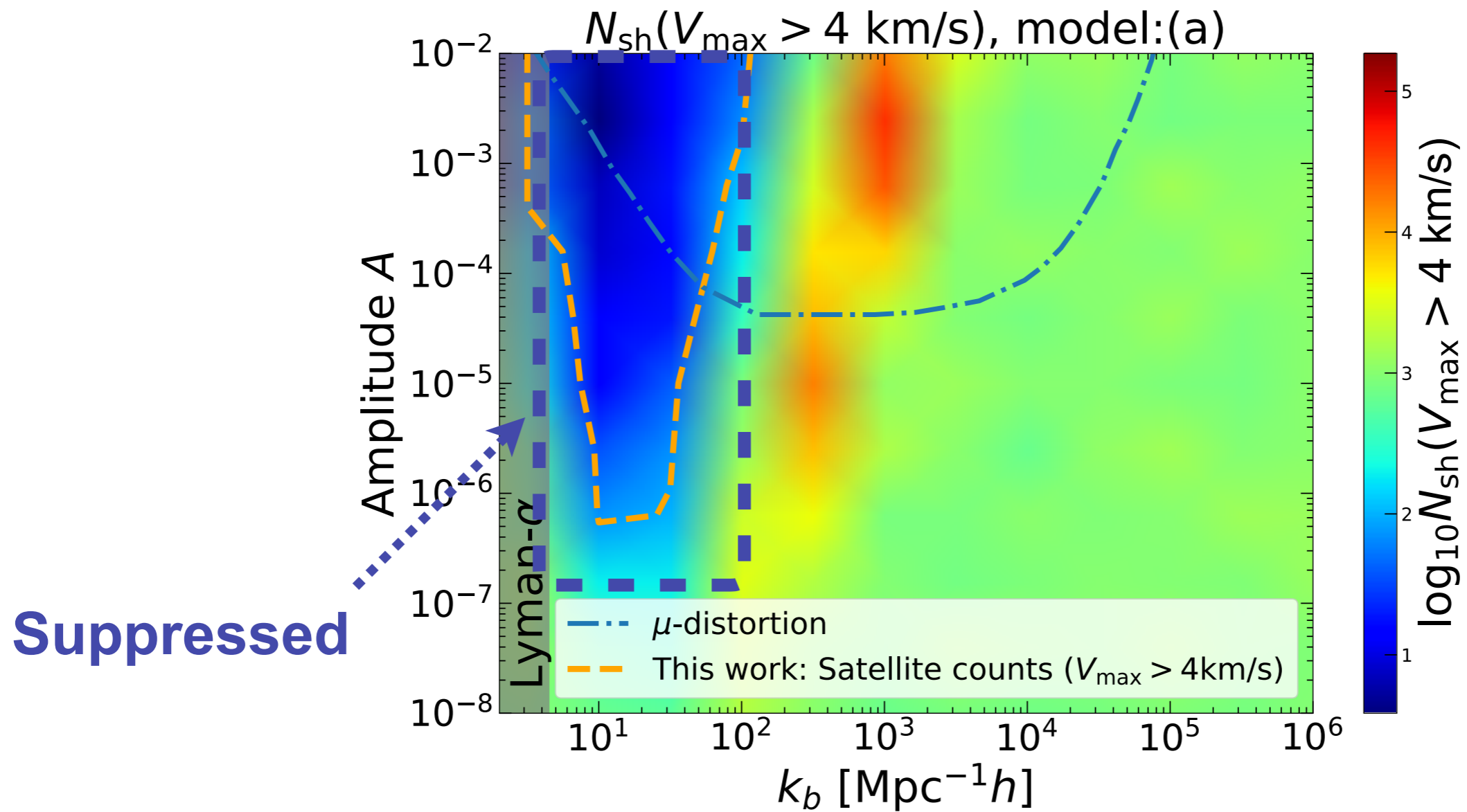
V_{max} : Maximum circular velocity
 of subhalos

The observable: satellite counts



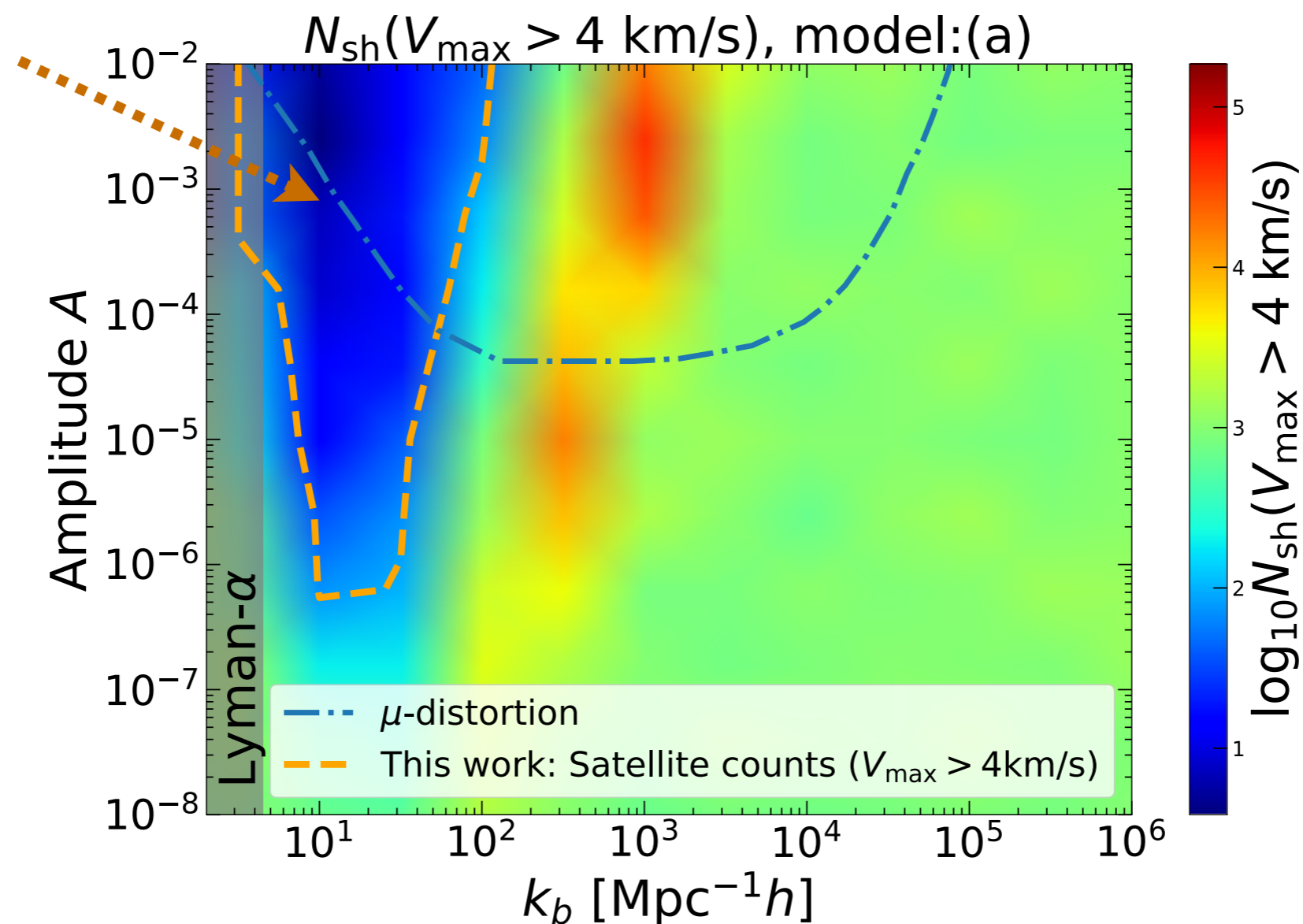
V_{max} : Maximum circular velocity
of subhalos

The observable: satellite counts



The observable: satellite counts

Excluded



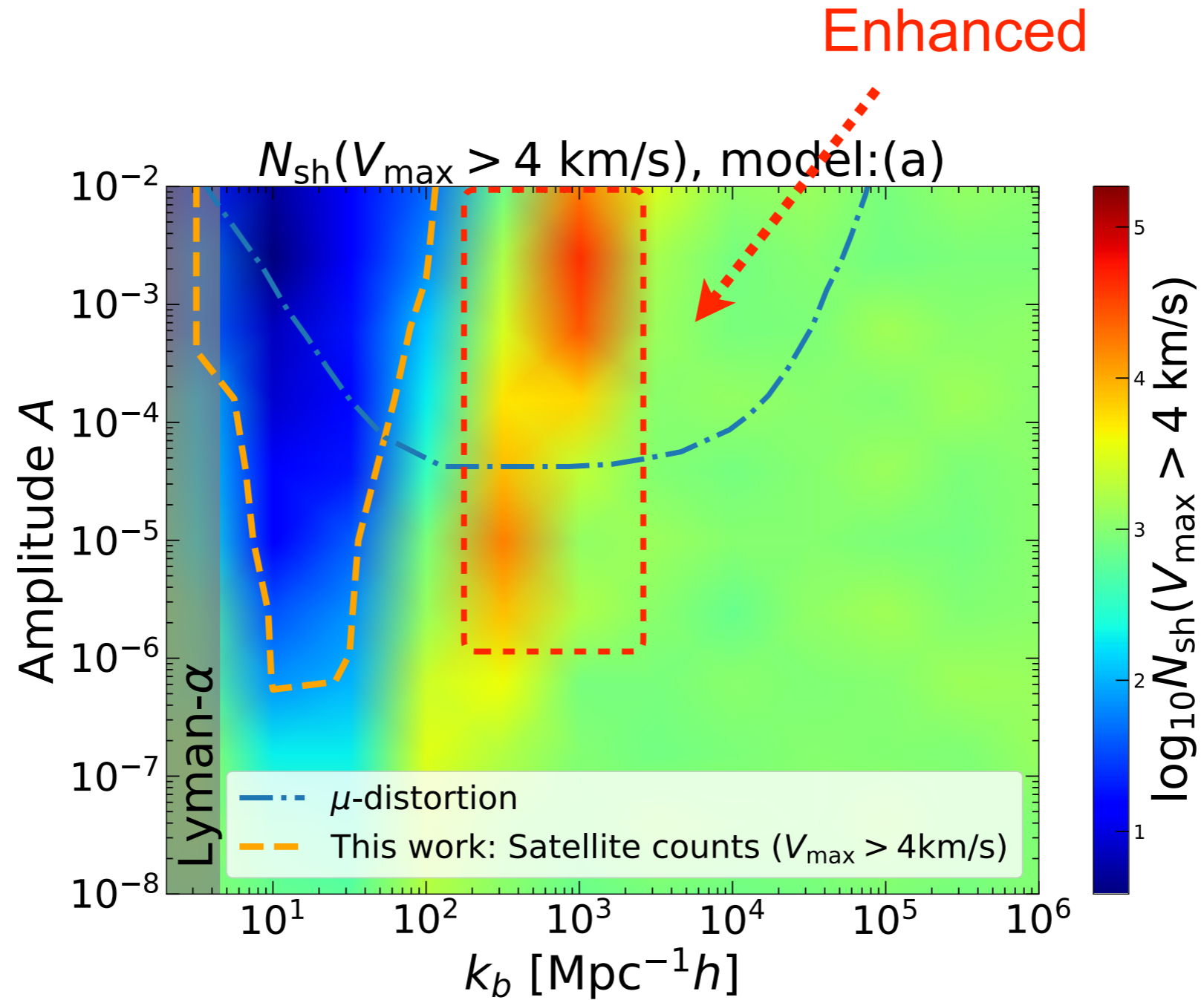
Conservative limit

$$N_{\text{dSph}}^{\text{low}}(V_{\text{max}} > 4 \text{ km/s}) = 94$$

Graus, Bullock, Kelley, Boylan-Kolchin, Garrison-Kimmel, Qi '19

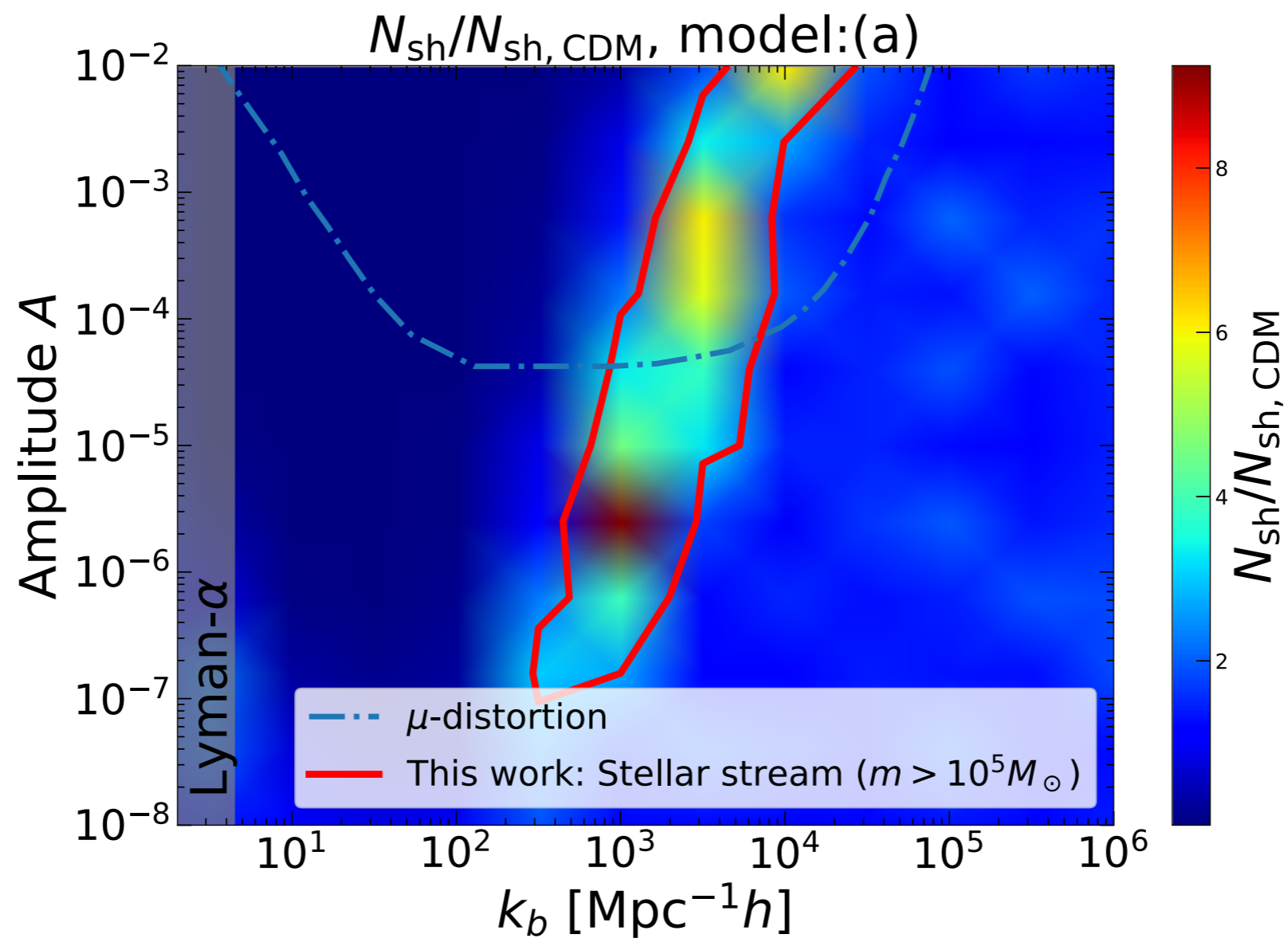
Dekker, Ando, Correa, Ng '21

The observable: satellite counts

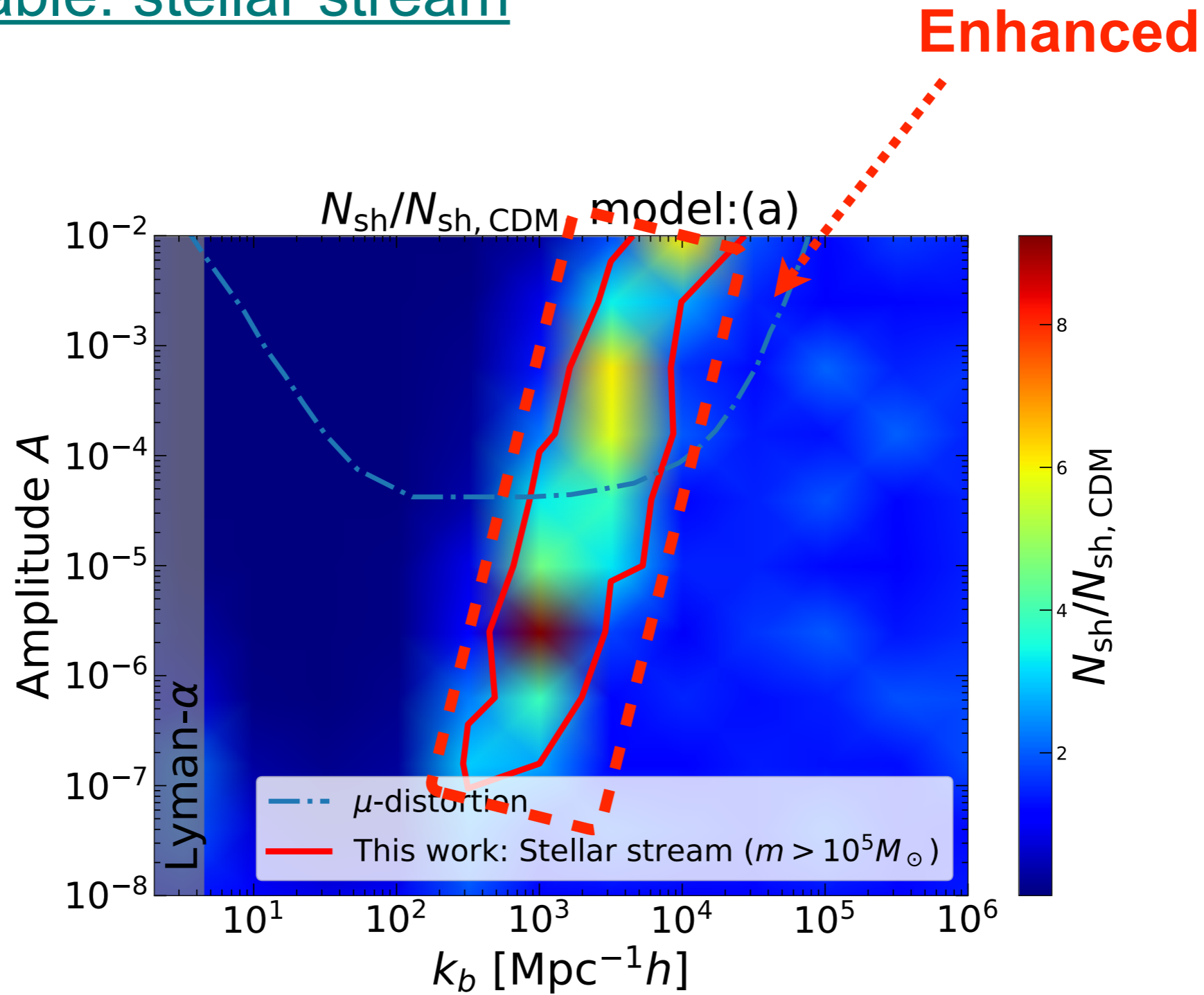


The observable: stellar stream

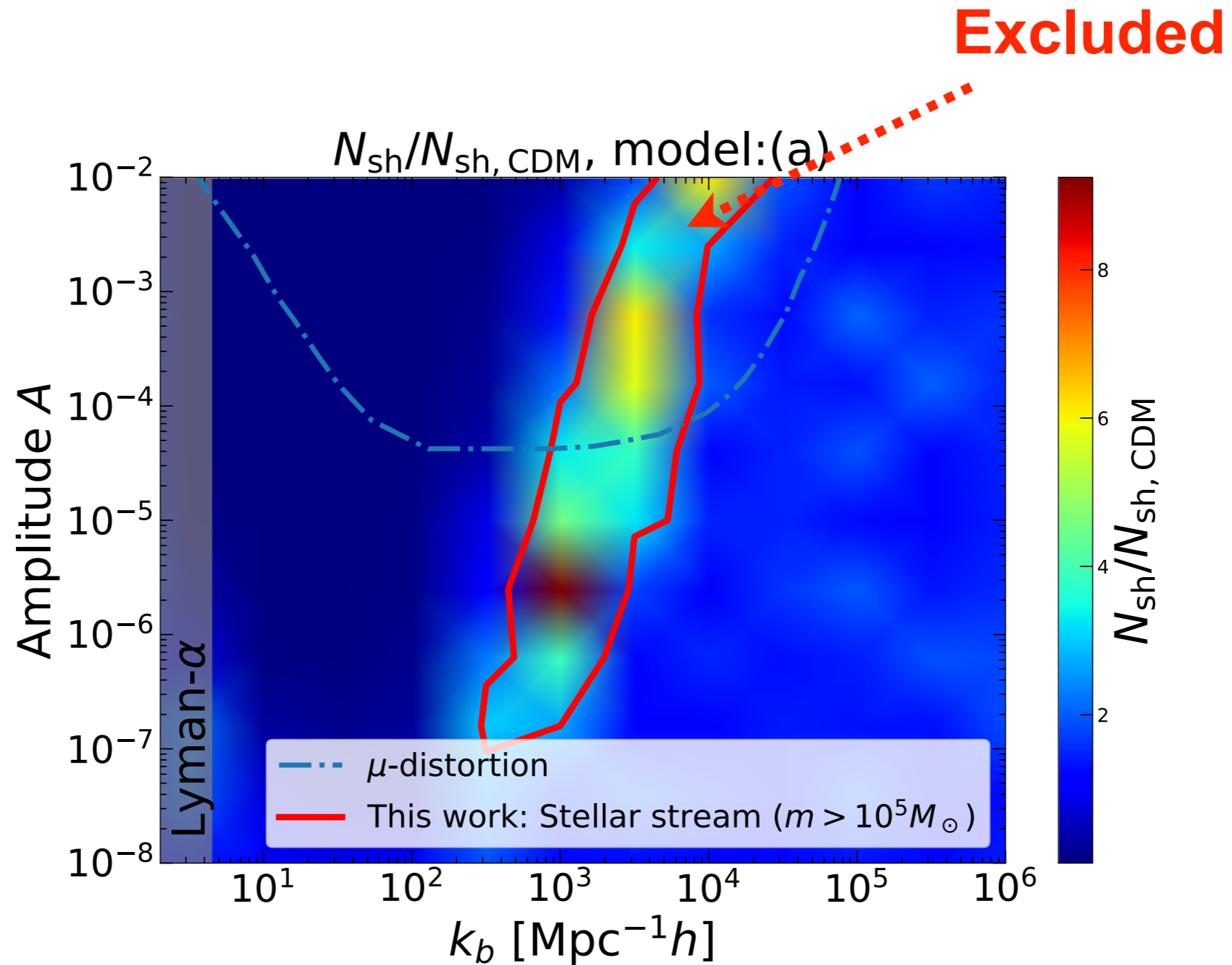
N_{sh} : Number of subhalos
whose mass $> 10^5 M_{\odot}$



The observable: stellar stream



The observable: stellar stream



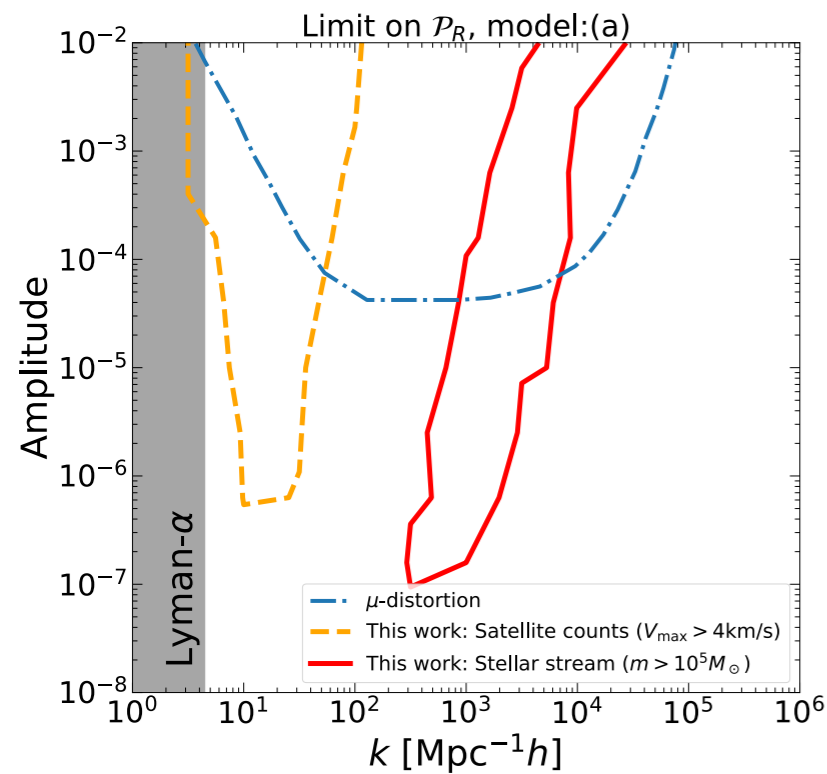
Conservative limit

$$N_{\text{sh}}/N_{\text{sh, CDM}} < 2.7 \text{ (95\% CL)}$$

Banik, Bovy, Bertone, Erkal, de Boer '21

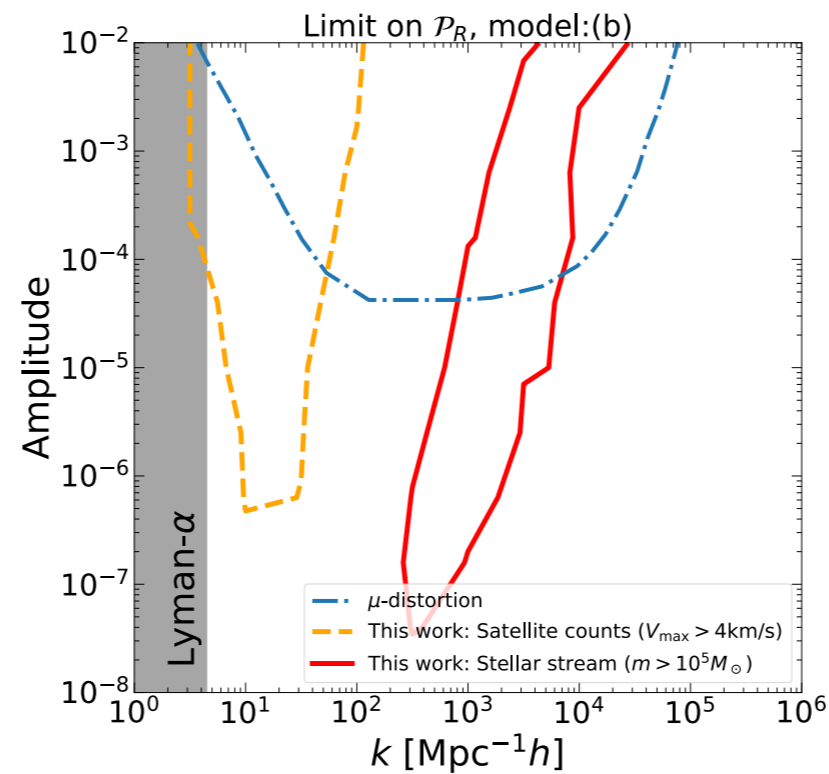
Tidal stripping model

Jiang, van den Bosch '16

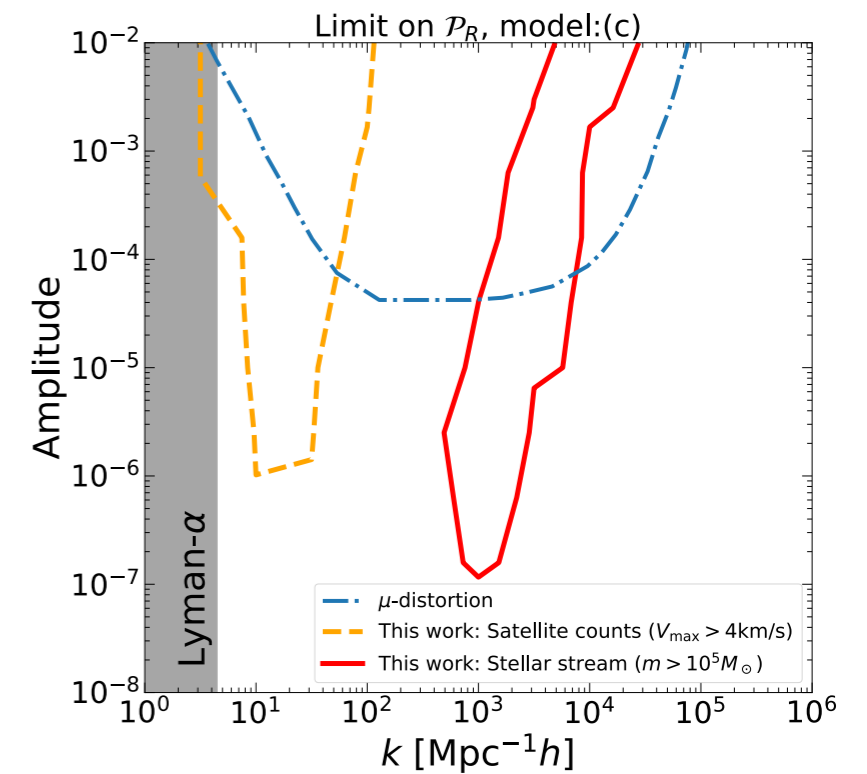


Tidal stripping model

Hiroshima, Ando, Ishiyama '18



No tidal stripping



No tidal model dependence

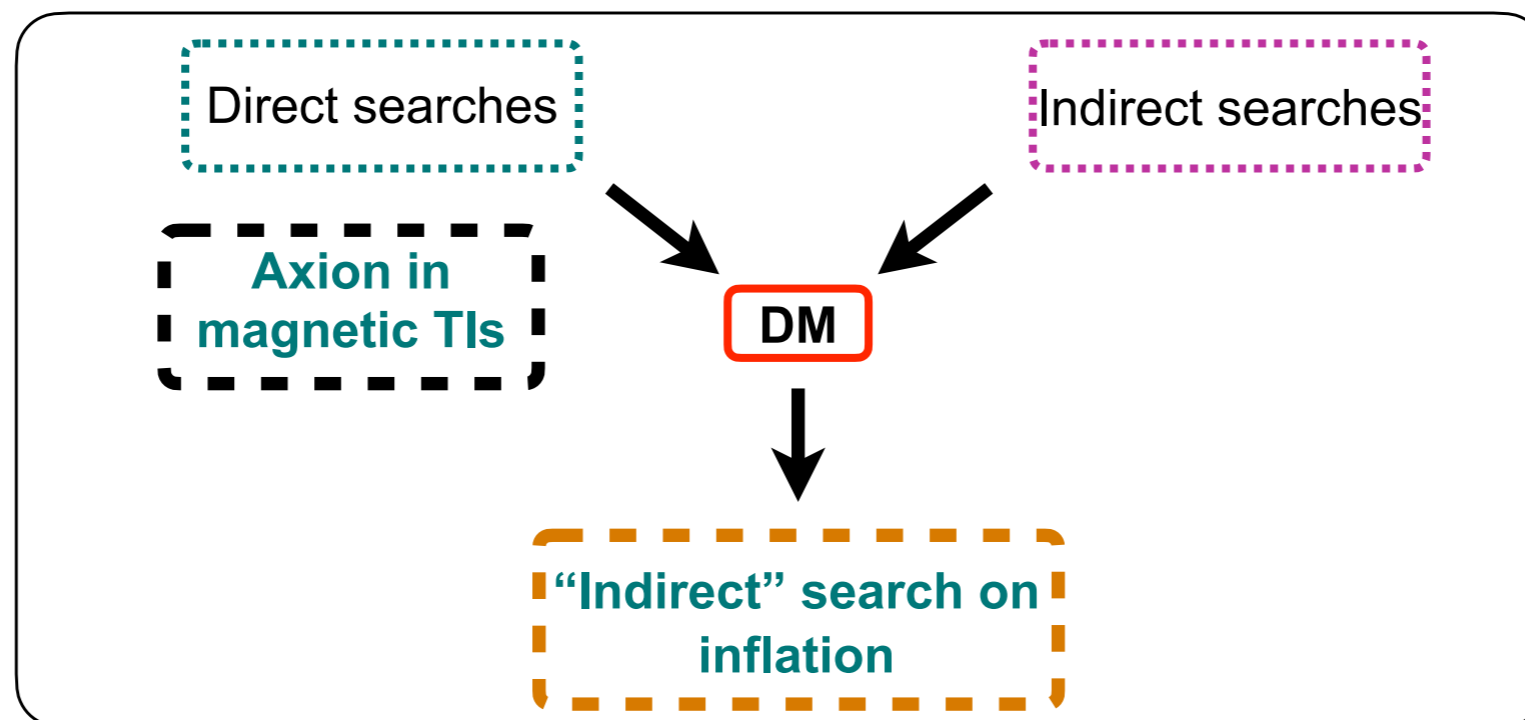
4. Conclusion

Axion in magnetic TIs

- Axion mass is $\mathcal{O}(eV)$ while it can be suppressed around the phase boundary in the magnetic TIs
- Material search is crucial for the particle axion detection

Inflaton sector from DM substructure

- Tracking the evolution of DM substructure is a new technique to probe the primordial curvature perturbation



Backups

Effective potential for ϕ

KI '21

$$V_\phi = -2 \int \frac{d^3 k}{(2\pi)^3} (\sqrt{|d_0|^2 + \phi^2} - |d_0|) + M_\phi^2 \phi^2$$

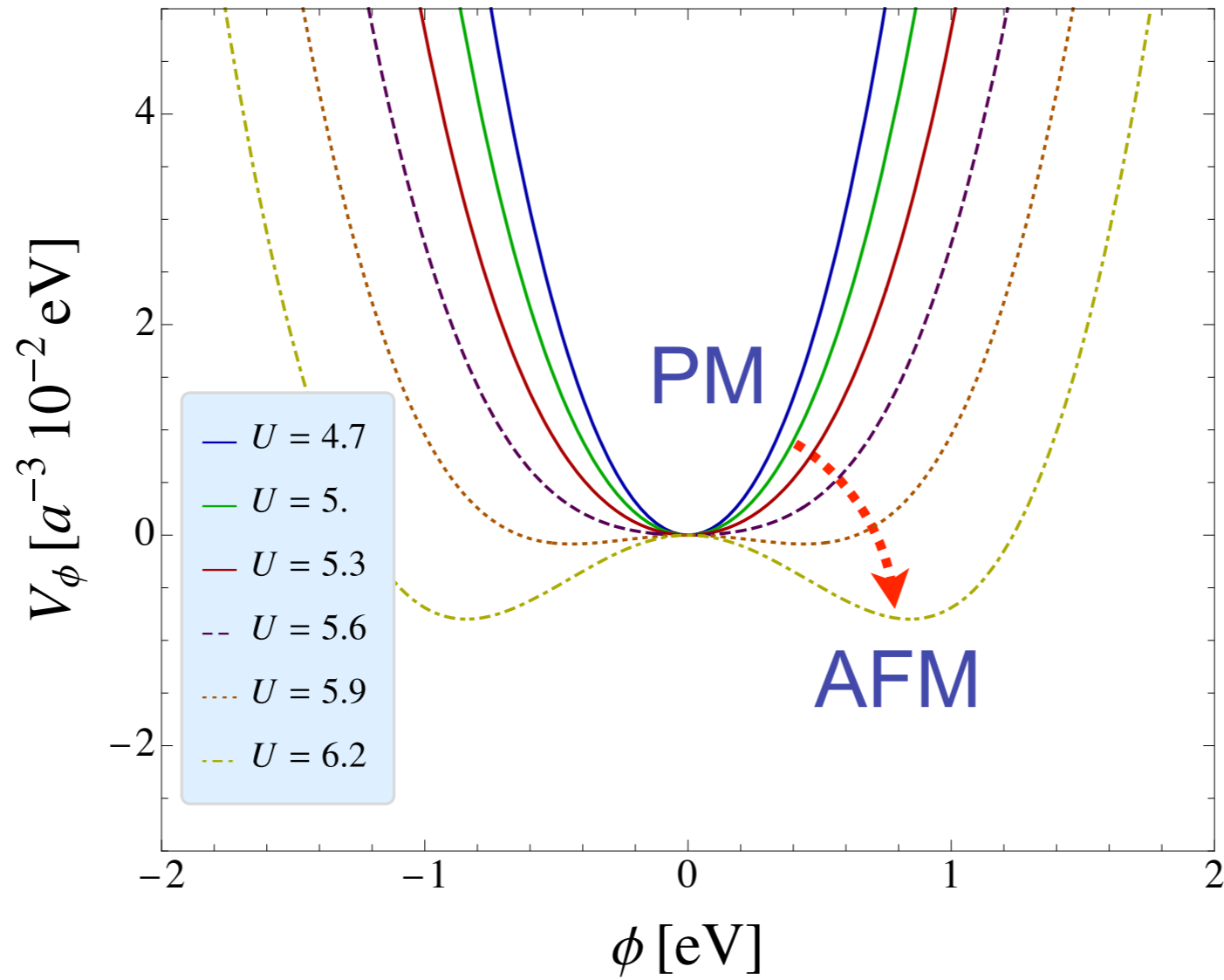
Negative potential



The mass term stabilizes the potential

$$|d_0|^2 = \sum_{a=1}^4 |d^a|^2$$
$$M_\phi^2 = \int \frac{d^3 k}{(2\pi)^3} \frac{2}{U}$$

Effective model for 3D TI, M [eV] = 0.1



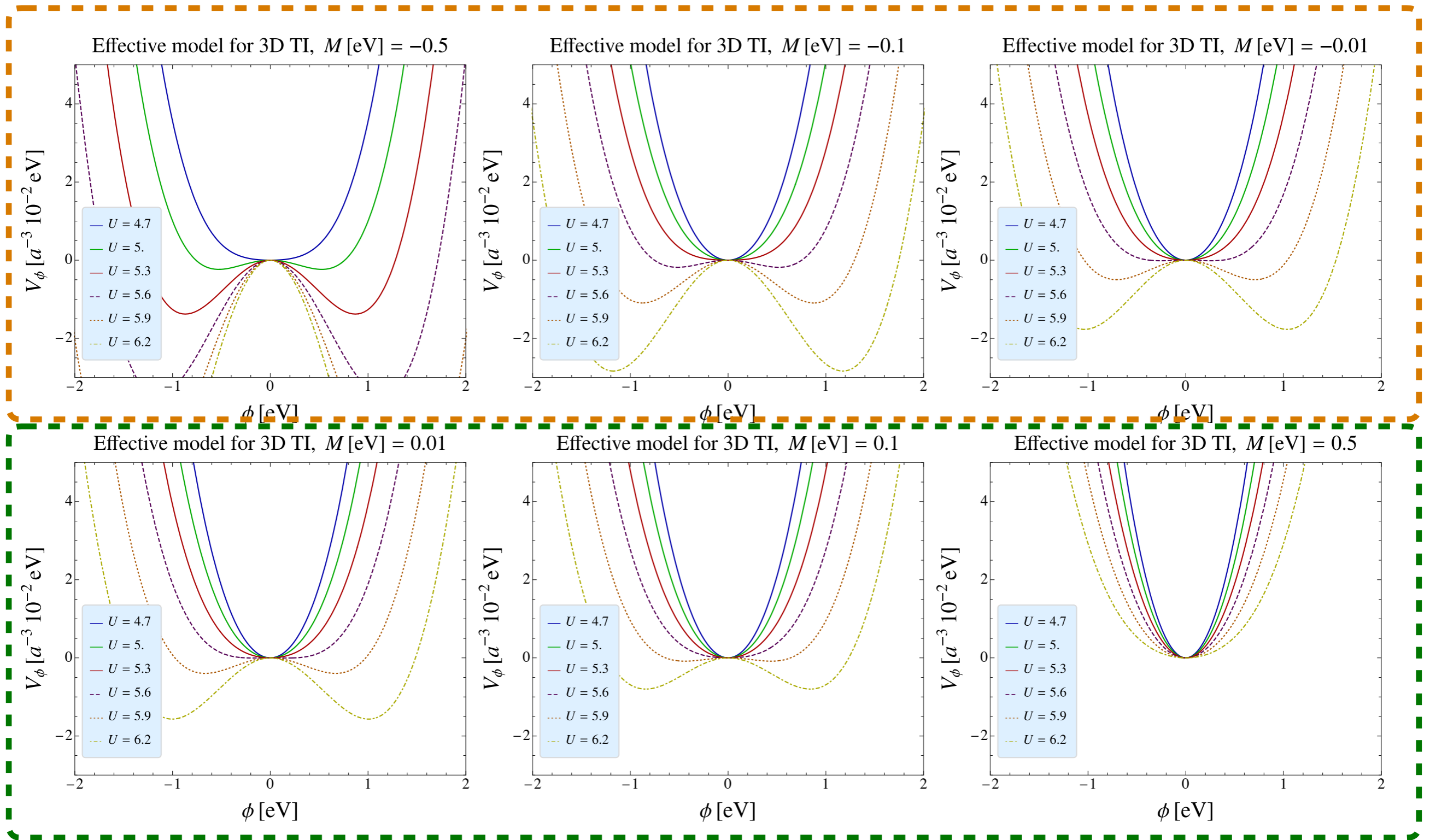
$$A_1 = A_2 = 1$$

$$B_1 = B_2 = -0.5$$

M dependence

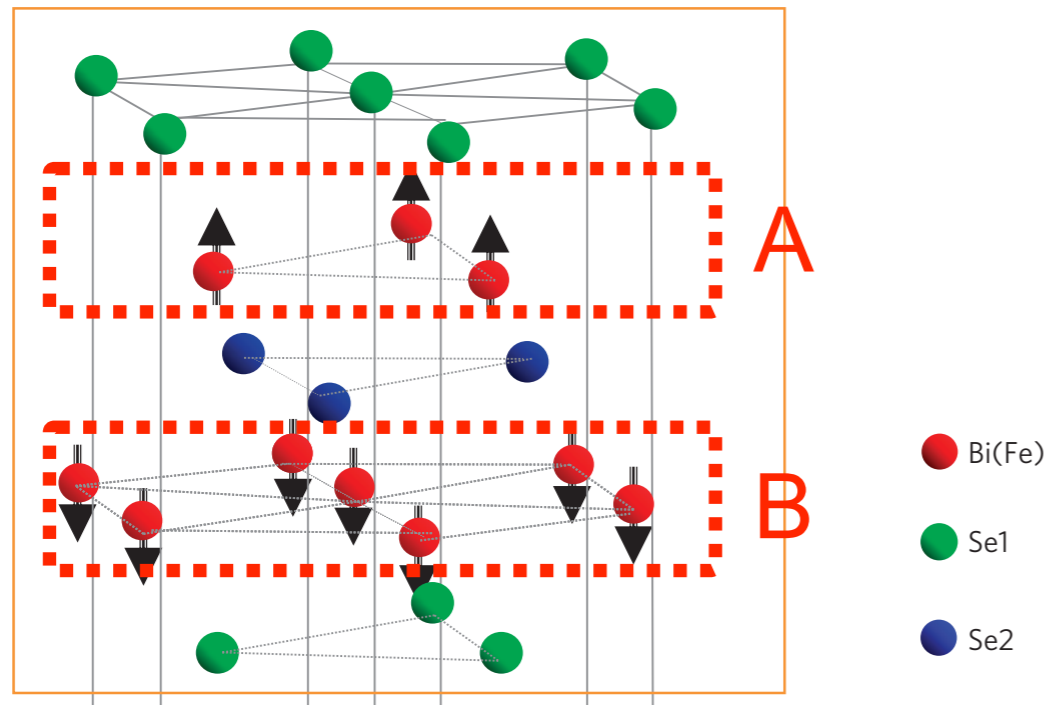
KI '21

TI



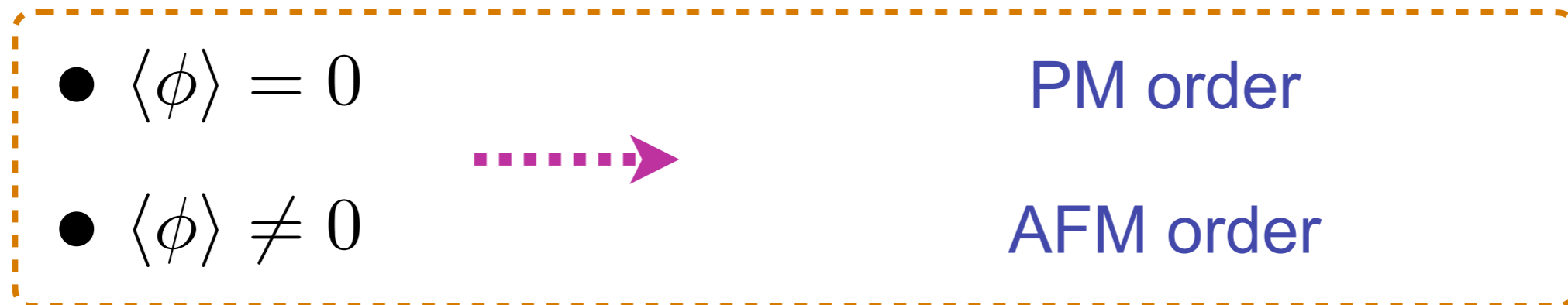
The difference between TI and NI is not clear

NI



$$\langle \phi \rangle \sim \langle S_A \rangle = - \langle S_B \rangle$$

VEV of ϕ is the order parameter of AFM



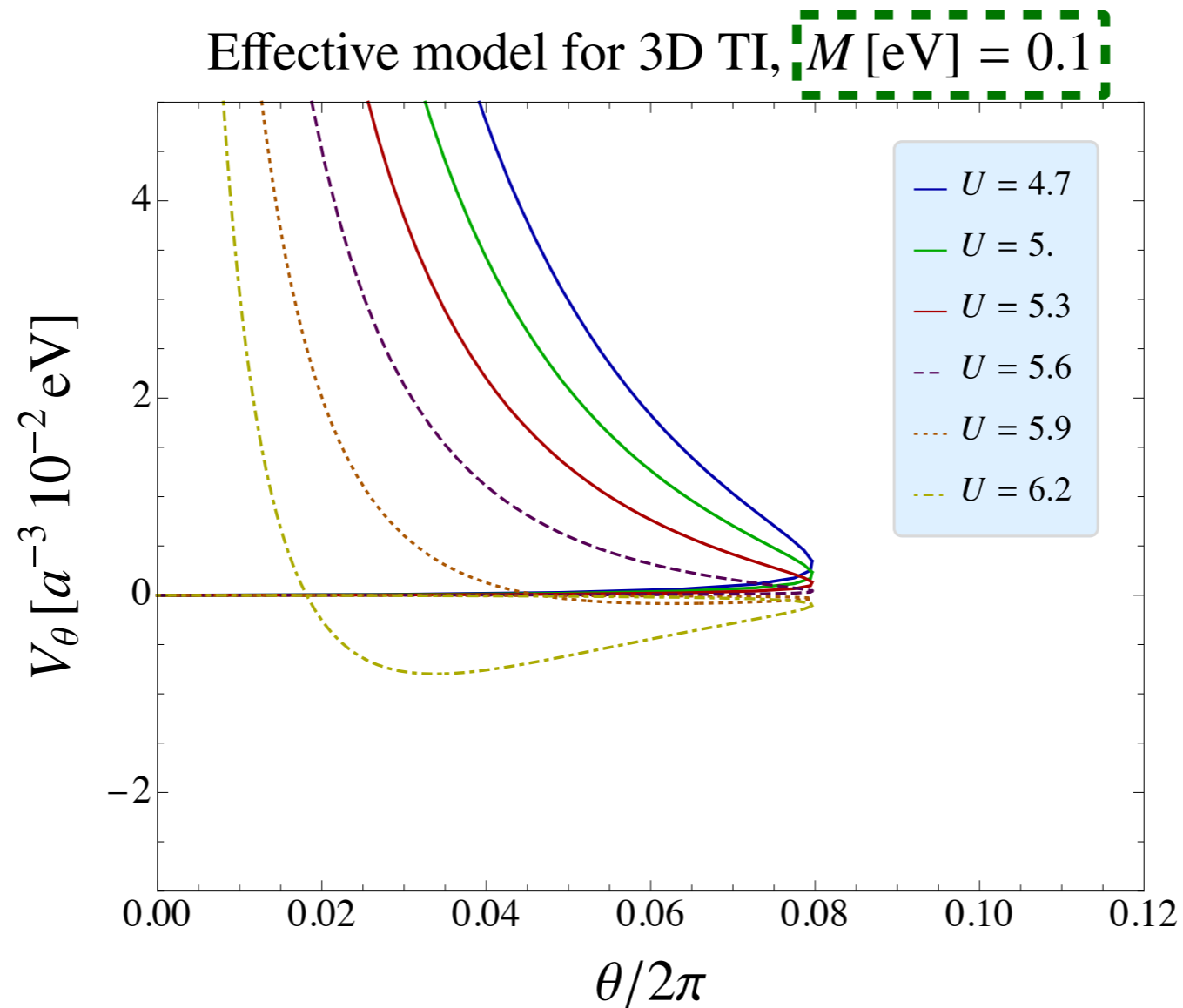
Expected result in the Hubbard model

PM (paramagnetic)

Effective potential in terms of θ

KI '21

NI phase



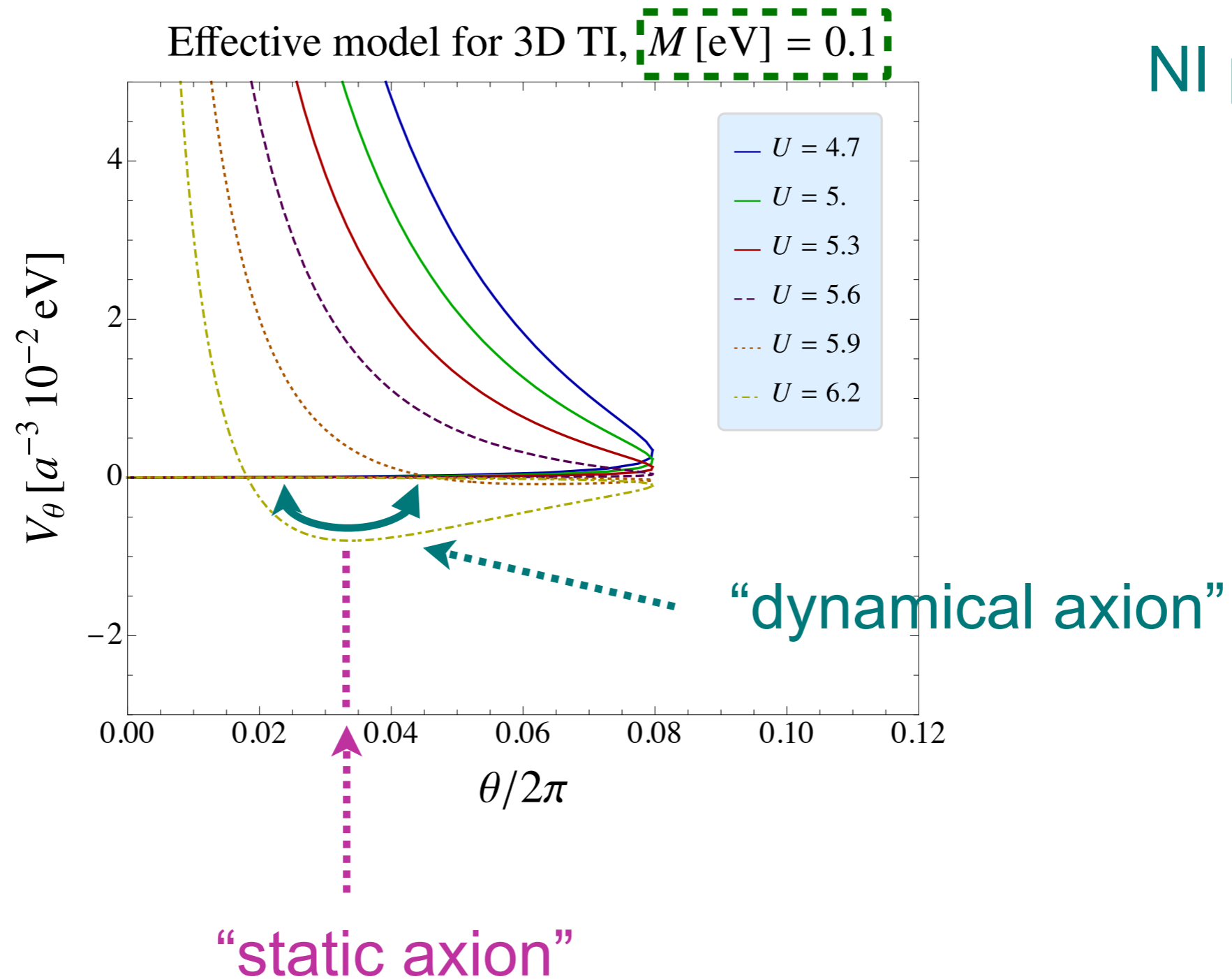
Potential minimum:

- $\theta = 0$ (small U , i.e., PM)
- $\theta \neq 0$ (large U , i.e., AFM)

Effective potential in terms of θ

KI '21

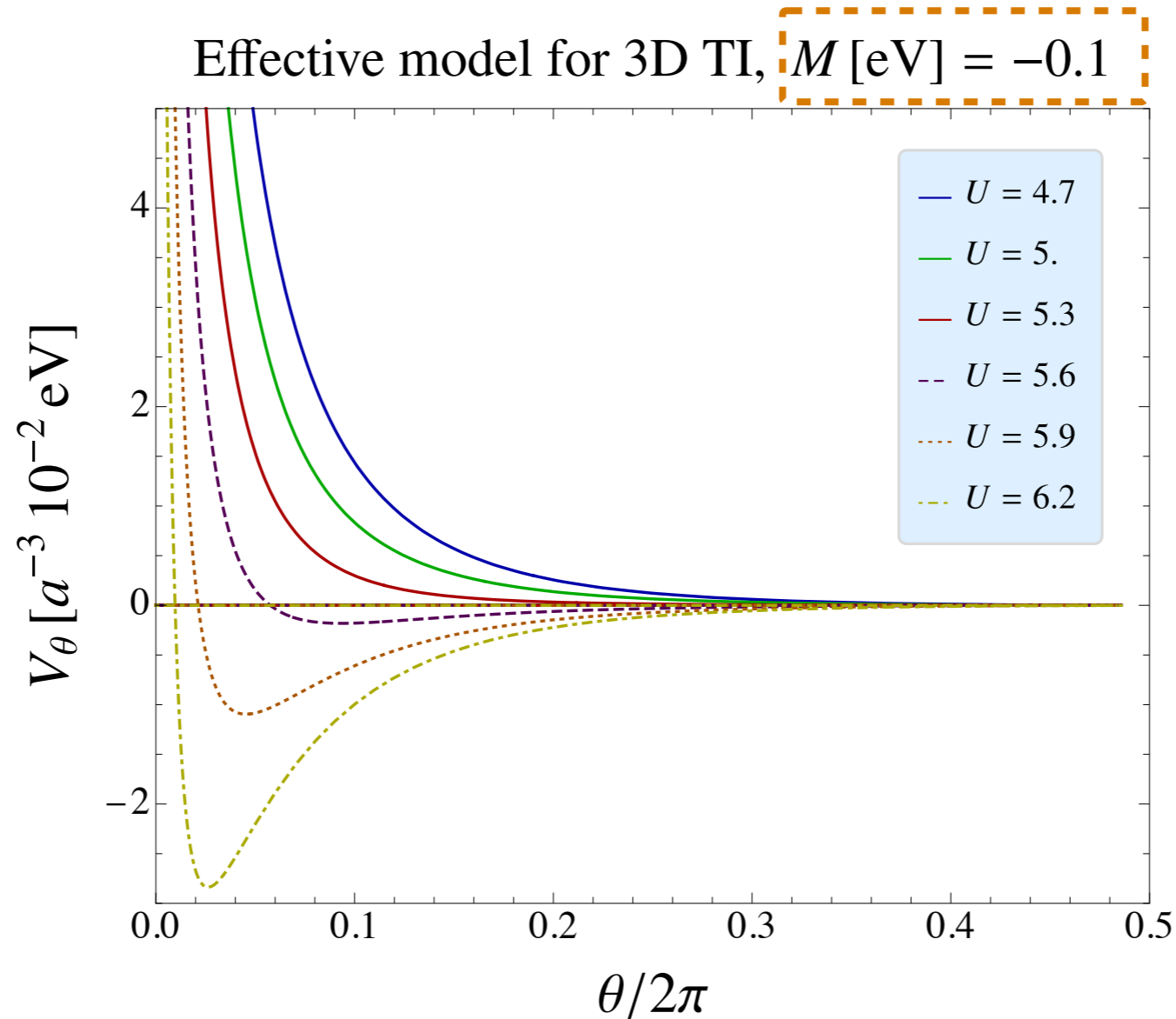
NI phase



Effective potential in terms of θ

KI '21

TI phase



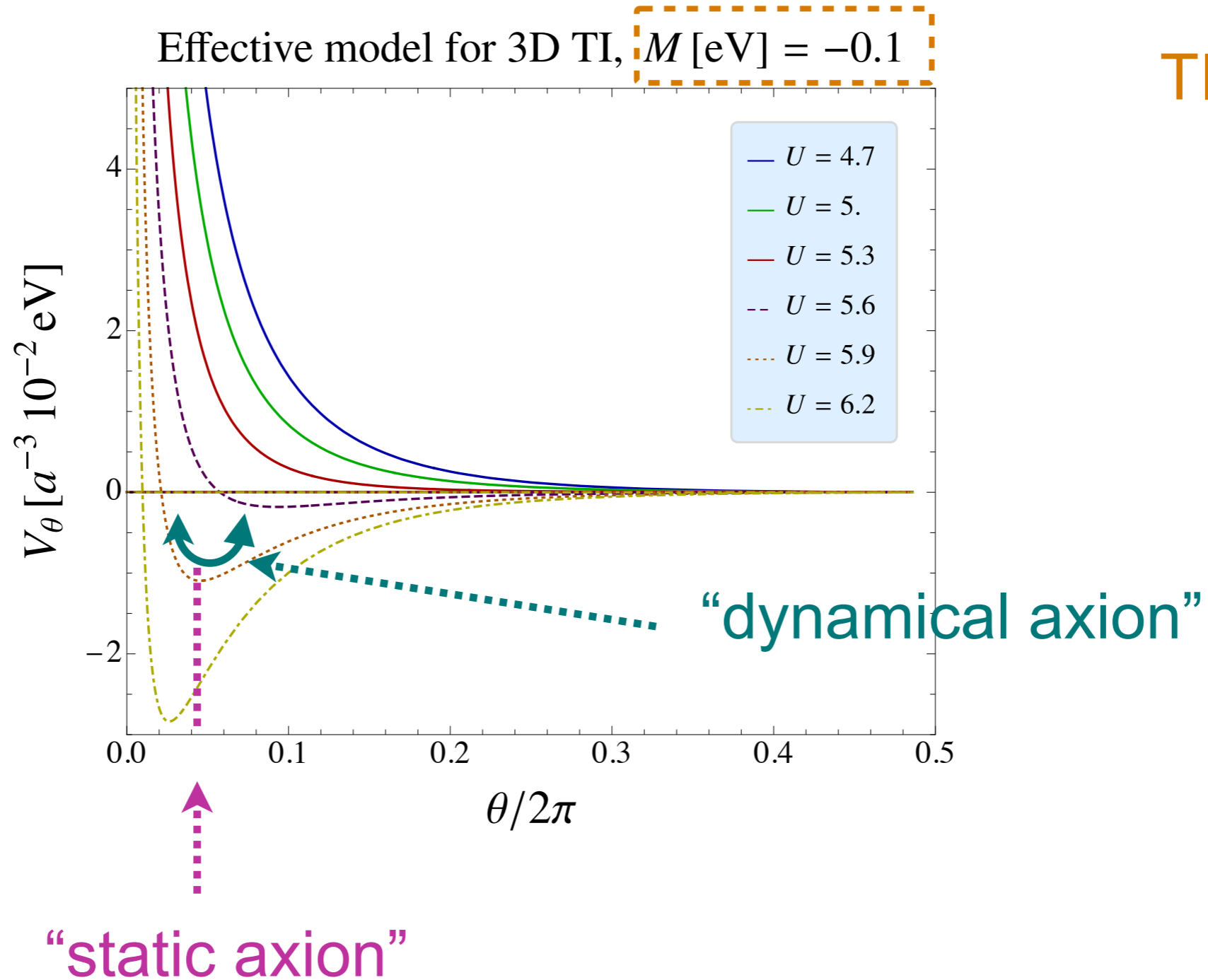
Potential minimum:

- $\theta = \pi$ (small U , i.e., PM)
- $\theta \neq 0$ (large U , i.e., AFM)

Effective potential in terms of θ

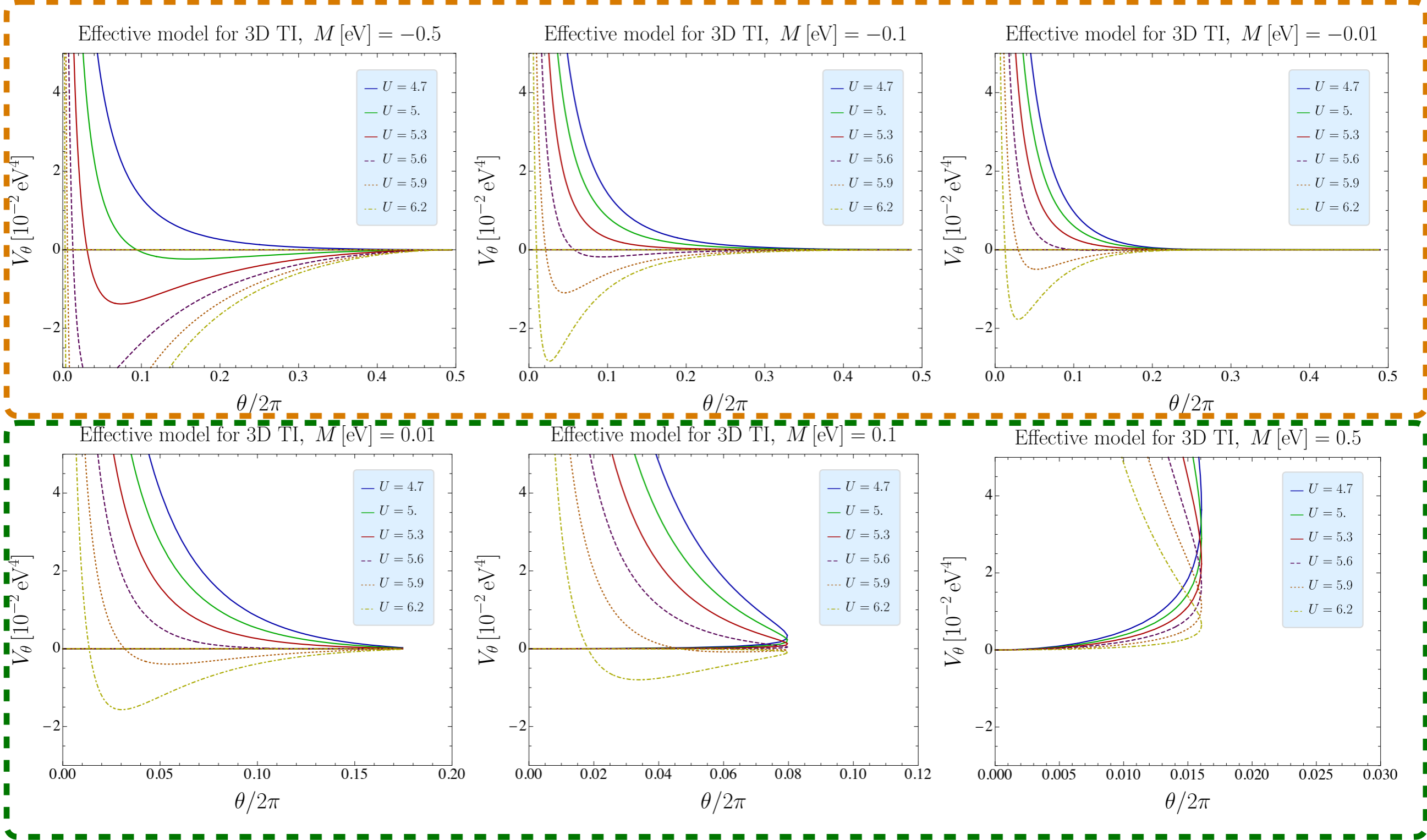
KI '21

TI phase



M dependence

TI

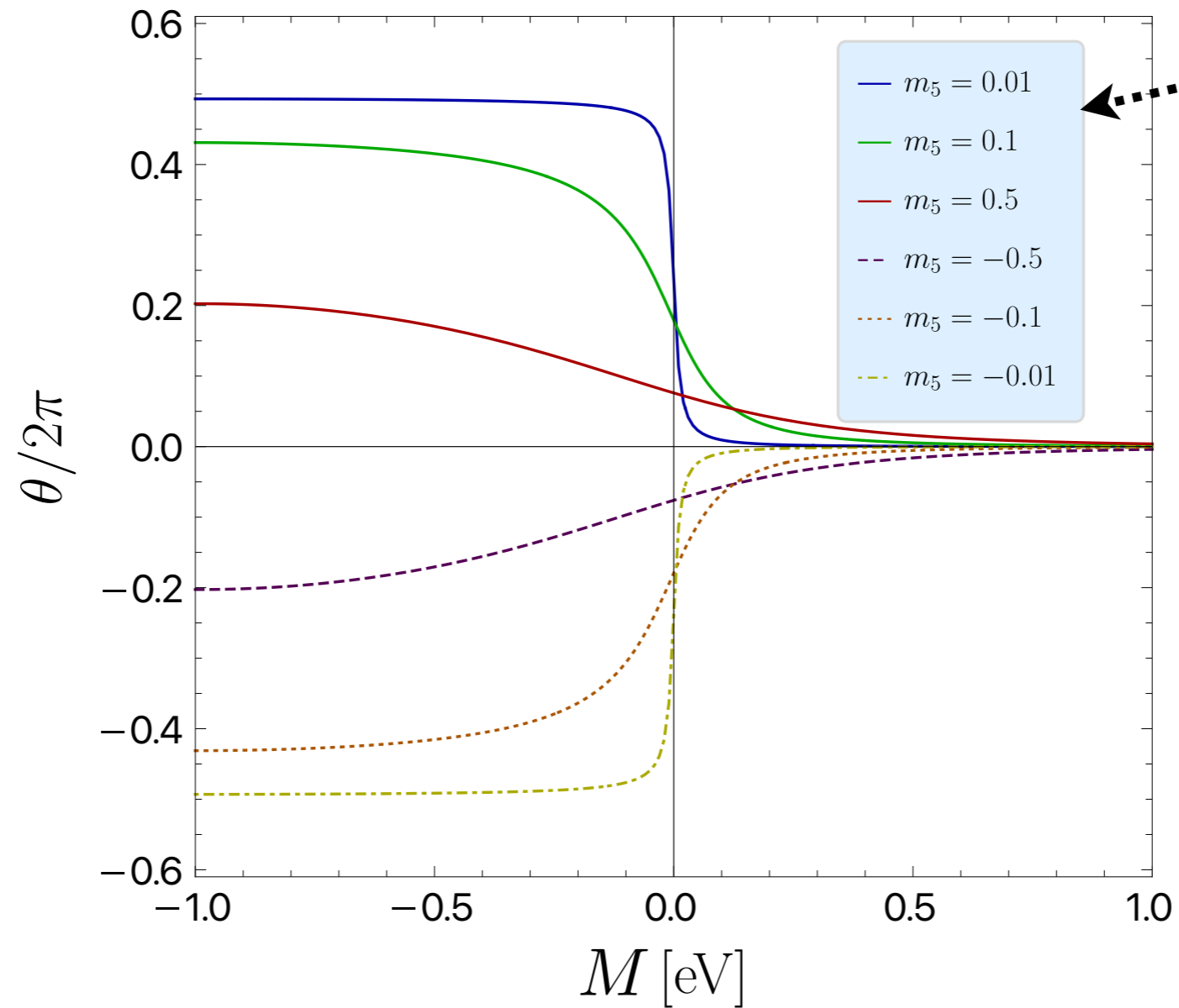


NI

θ as function of M

KI '21

Effective model for 3D TI



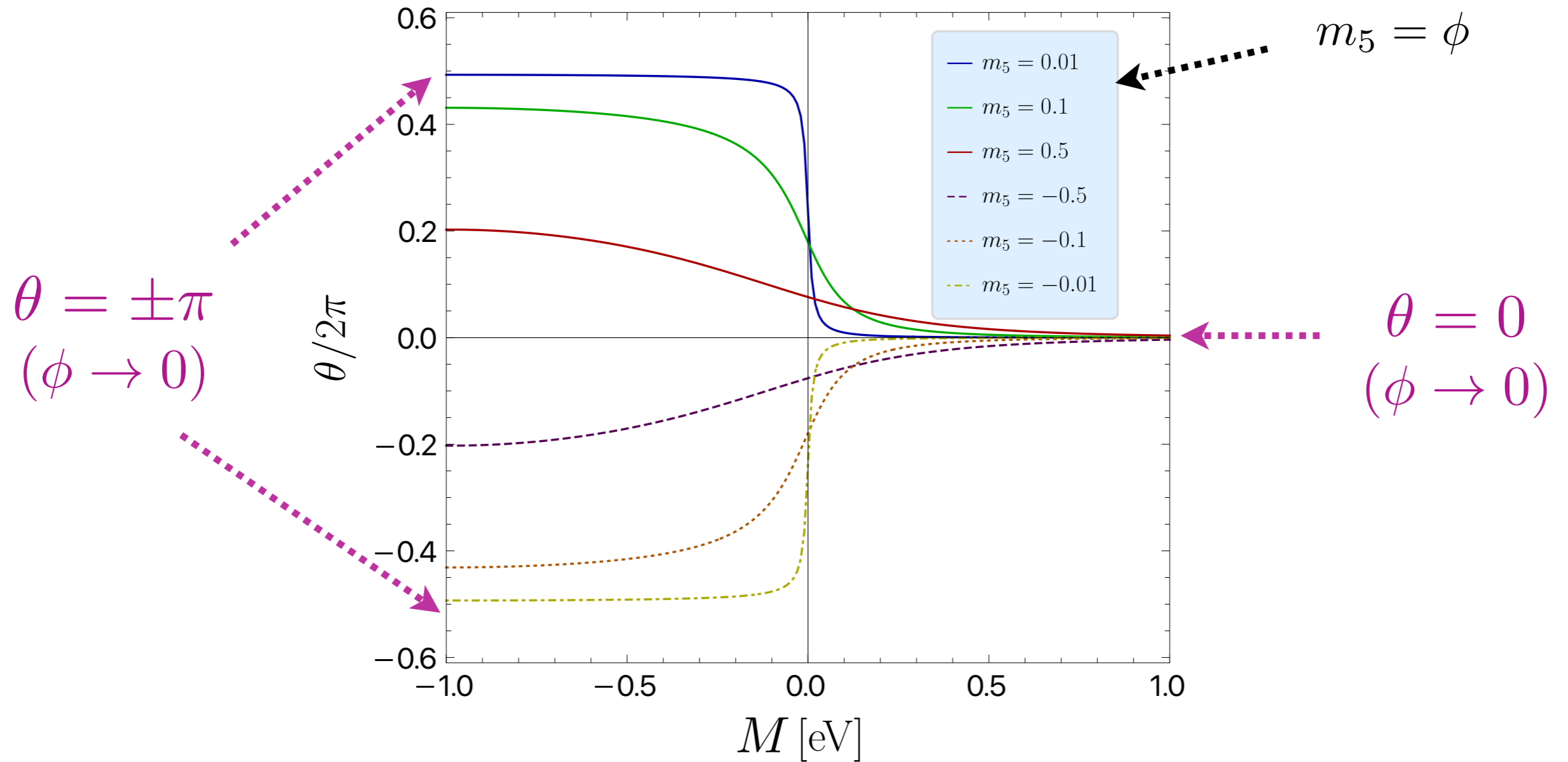
$m_5 = \phi$

calculation for Dirac
model is done by
Zhang '19

θ as function of M

KI '21

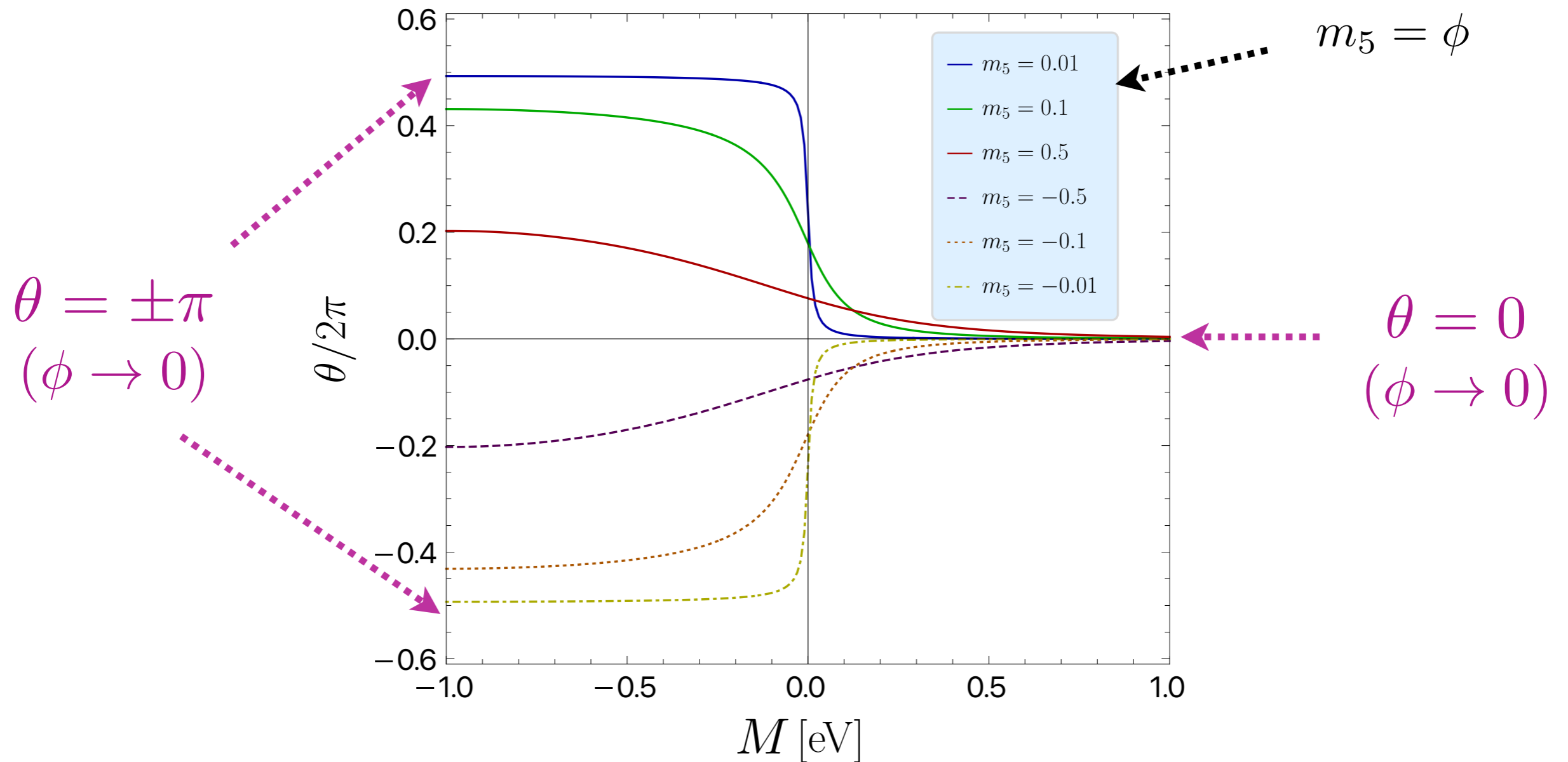
Effective model for 3D TI



θ as function of M

KI '21

Effective model for 3D TI



$\phi = 0$
(\mathcal{T}) \longleftrightarrow PM order

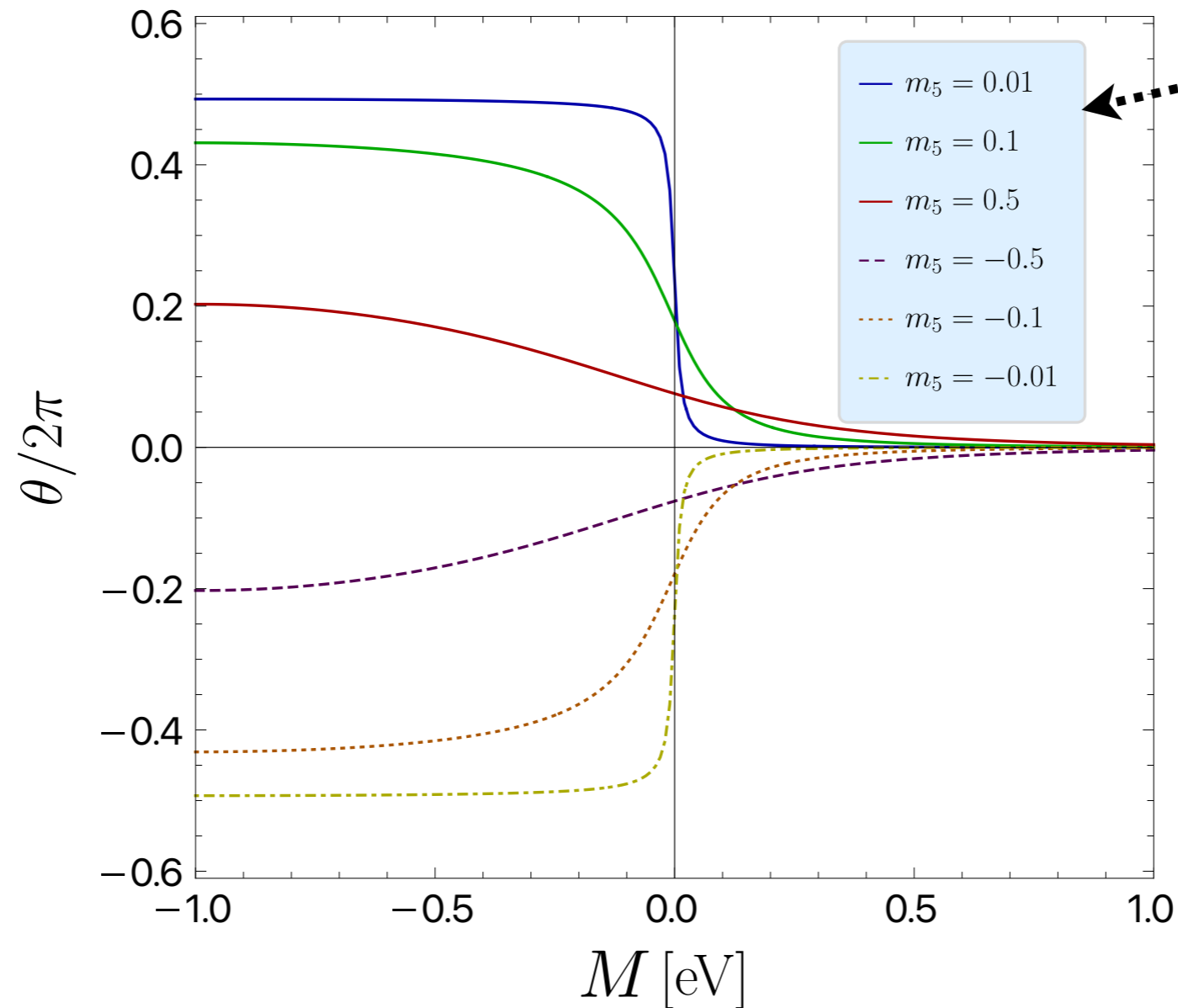


$\theta = \pm\pi$ $M < 0$ (TI)
 $\theta = 0$ $M > 0$ (NI)

θ as function of M

KI '21

Effective model for 3D TI

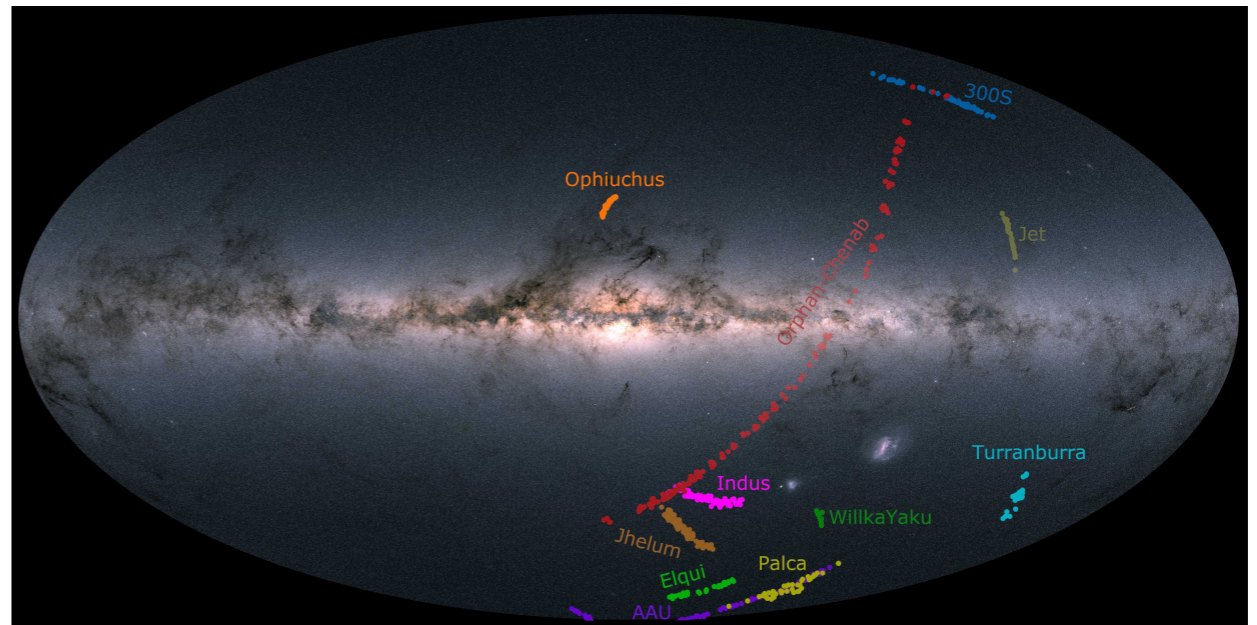
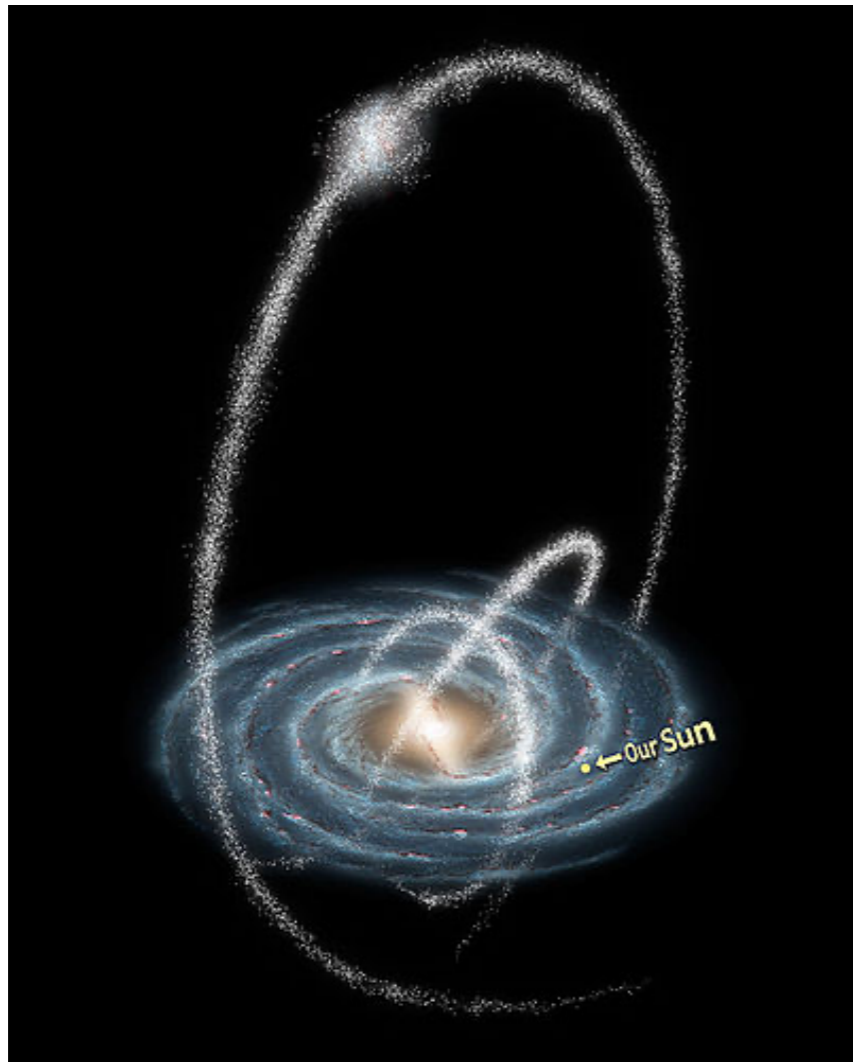


$\phi \neq 0$
(~~\mathcal{F}~~) \longleftrightarrow AFM order



θ takes continuum variable

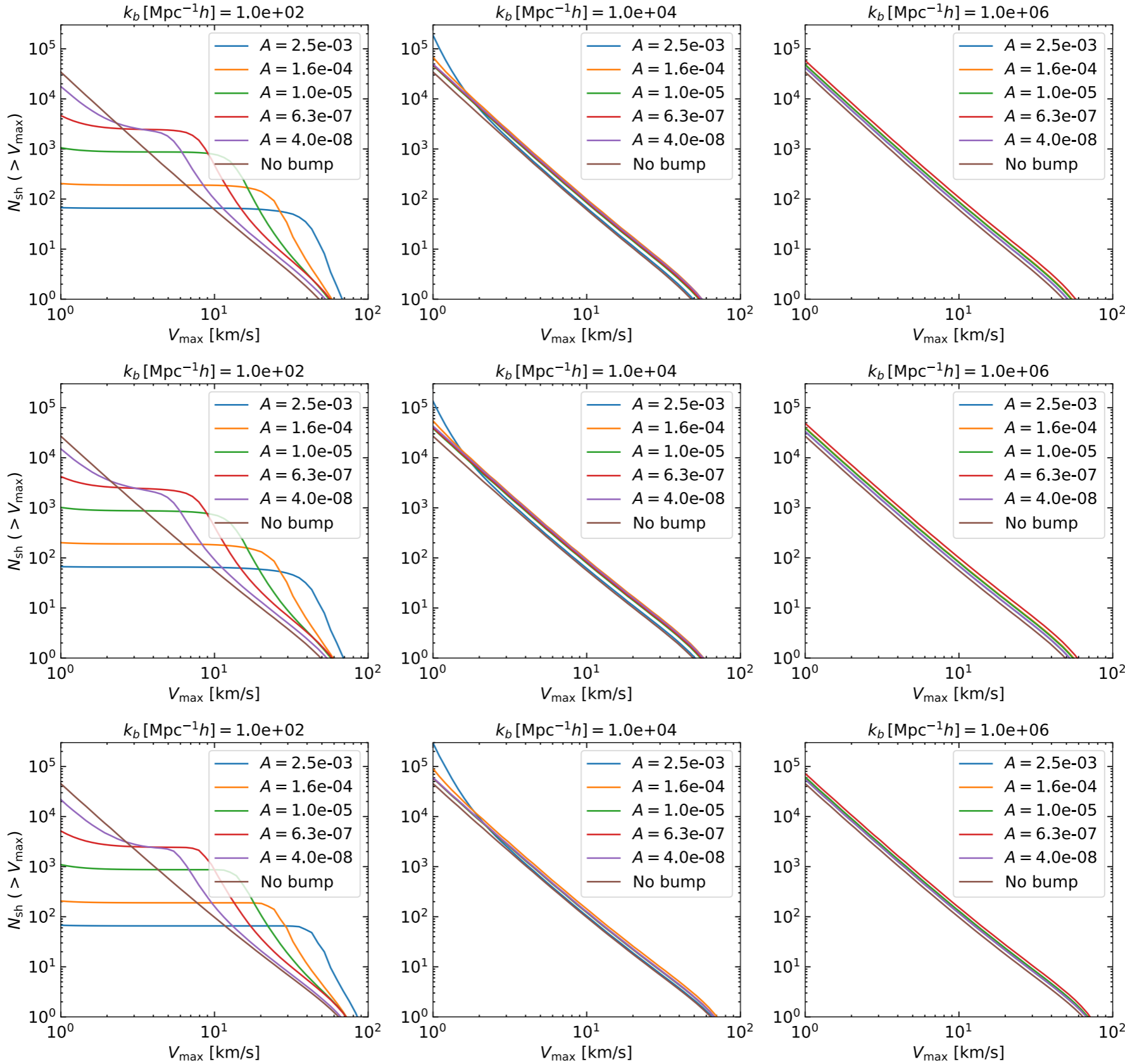
Stellar stream



A passage of subhalos \longrightarrow A gap in the stream

Too small or too large number of subhalos conflict with the observation

Cumulative maximum circular velocity function



Cumulative number of subhalos, maximum circular velocity function, and boost factor

