

Phenomenological aspects of classically conformal extension of the Standard Model

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Talk based on work in collaborations with

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Satomi Okada, Shinsuke Kawai, Taiki Hasegawa,
Hikaru Uchida, Victor Baules, Rabindra Mohapatra

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1. Introduction

Problems of the Standard Model

The Standard Model (SM) is the best theory in describing the nature of elementary particle physics, which is in excellent agreement with almost of all current experimental results (including LHC Run-2 results) as of TODAY

However,

New Physics beyond SM is strongly suggested by both experimental & theoretical points of view

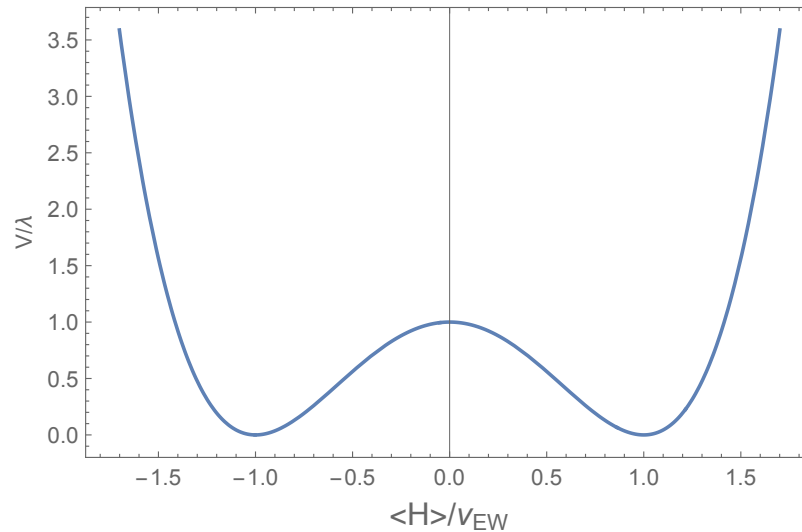
Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?

1. What drives the Electroweak symmetry breaking?

SM Higgs potential with a negative mass squared:

$$V = -m_H^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \text{const}$$



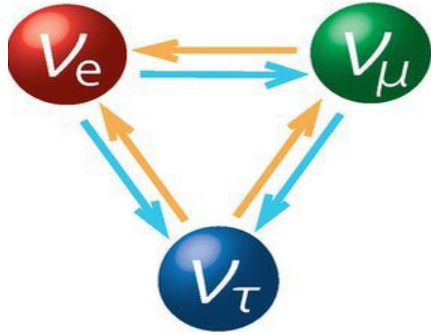
Any “dynamical” reason for $-m_H^2$?

Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?
2. Why are **Neutrino Masses** are non-zero and so tiny?

2. Neutrino Mass problem

Neutrino Oscillation Phenomena



$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2$$

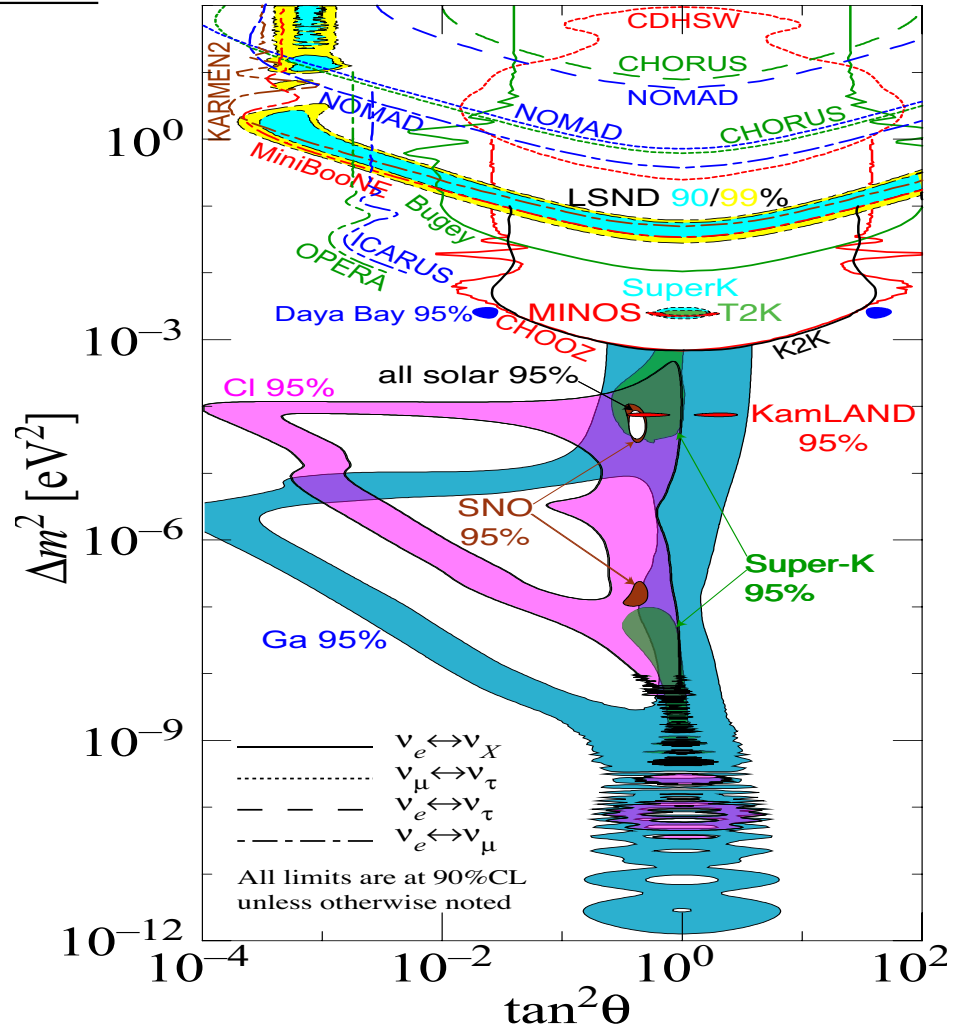
$$\sin^2(2\theta_{12}) = 0.846 \pm 0.021$$

$$\sin^2(2\theta_{23}) = 0.999^{+0.001}_{-0.018}$$

$$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$$

Neutrinos are massless
in the Standard Model

Particle Data Group



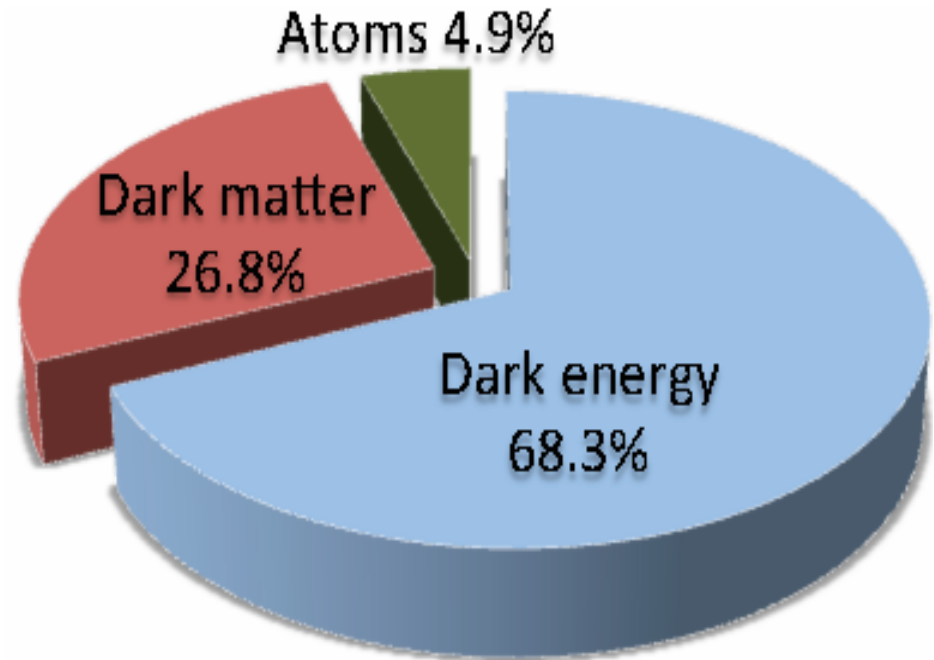
Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?
2. Why are **Neutrino Masses** are non-zero and so tiny?
3. What is the nature of **Dark Matter**?

3. Dark Matter Problem

Existence of Dark Matter has been established!

Energy budget of the Universe is precisely determined by recent CMB anisotropy observations (WMAP & Planck)



Dark Matter particle: non-baryonic
electric charge neutral
(quasi) stable $\tau_{DM} > t_U$

No suitable DM candidate in the Standard Model

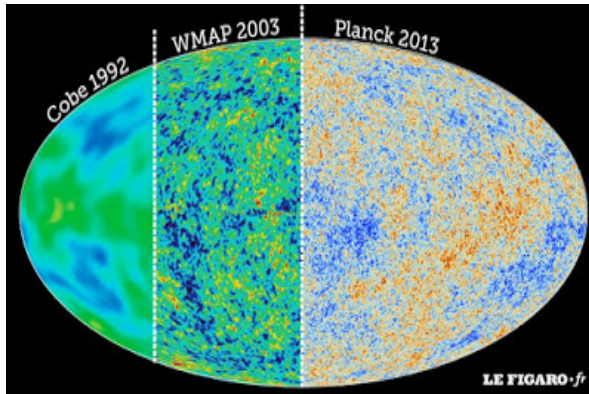
Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?
2. Why are **Neutrino Masses** are non-zero and so tiny?
3. What is the nature of **Dark Matter**?
4. What drives **Cosmic Inflation** before Big Bang?

4, Cosmic Inflation

The problems of Big-Bang Cosmology

- Flatness problem
- Horizon problem
- Need to dilute unwanted topological defects
- Origin of the primordial density fluctuations



$$\frac{\delta T}{T} \simeq 10^{-5}$$

Seeds of the large scale structure

Solution: **Cosmic Inflation** before Big-Bang cosmology, driven by a scalar field (**inflaton**) which has a very flat potential

No suitable inflaton candidate in the SM

Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?
2. Why are **Neutrino Masses** are non-zero and so tiny?
3. What is the nature of **Dark Matter**?
4. What drives **Cosmic Inflation** before Big Bang?
5. What is the origin of **Matter-Antimatter asymmetry** in the Universe?

5. What is the origin of Matter-Antimatter Asymmetry?

Observations: (1) Big asymmetry $n_B \gg n_{\bar{B}}$

(2) Small ratio to entropy

$$\frac{n_B}{s} \simeq \frac{n_B - n_{\bar{B}}}{s} \simeq 10^{-10} \ll 1$$

What is the origin?

*Baryogenesis in the SM context: [Electroweak Baryogenesis](#)
Unfortunately, it doesn't work with the 125 GeV Higgs mass

Questions that the Standard Model cannot answer

1. What derives the **Electroweak Symmetry Breaking**?
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3. What is the nature of **Dark Matter**?
4. What drives **Cosmic Inflation** before Big Bang?
5. What is the origin of **Matter-Antimatter asymmetry** in the Universe?

We will first discuss....

2. Classically conformal extension
of the SM for dynamical/radiative
EW symmetry breaking

U(1) Higgs model and Coleman-Weinberg mechanism

Toy model:

Field	Symbol	U(1)
Higgs Scalar	Φ	+2
Weyl Fermion	Ψ	-1

* some more chiral fermions for anomaly cancellation

By imposing **Classical Conformal symmetry**

$$V_{tree} = \lambda_{\Phi} (\Phi^{\dagger} \Phi)^2$$

*define this theory as "Massless Theory"

Yukawa coupling is allowed:

$$\mathcal{L}_Y = Y \Phi \Psi \Psi + h.c.$$

Coleman-Weinberg mechanism

Coleman & Weinberg,
PRD 7 (1973) 1888

$$V_{CW} = V_{tree} + V_{1-loop}$$
$$= \frac{\lambda_{\Phi}}{4} \phi^4 + \frac{\beta_{\Phi}}{8} \phi^4 \left(\ln \left[\frac{\phi^2}{v_{\phi}^2} \right] - \frac{25}{6} \right),$$

where $\Phi = \frac{1}{\sqrt{2}} (\phi + i\chi)$, $\beta_{\Phi} = \frac{1}{16\pi^2} (96g^4 - Y^4)$

➤ Radiative U(1) symmetry breaking at $\phi = v_{\phi}$

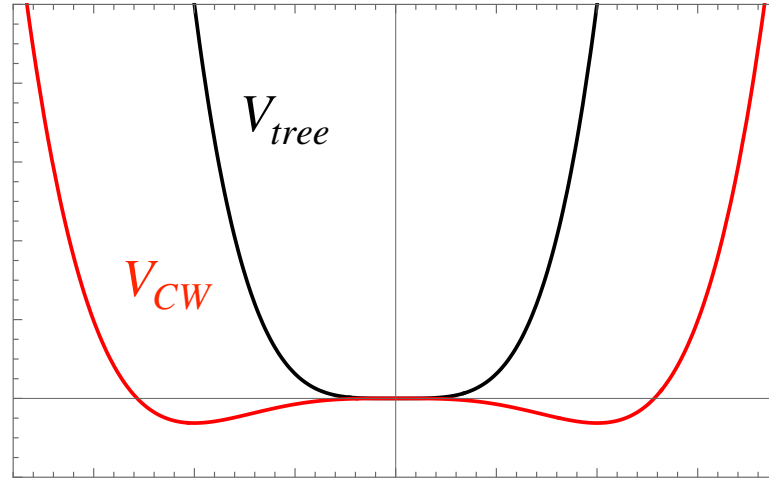
➤ Parameter relations: $\lambda_{\Phi} = \frac{11}{6} \beta_{\Phi}$ $Y \rightarrow 0$

$$m_{\phi}^2 = \left. \frac{d^2 V_{CW}}{d\phi^2} \right|_{\phi \rightarrow v_{\phi}} \rightarrow \frac{3}{2\pi^2} g^2 M_{Z'}^2$$

Interesting properties:

- Origin of gauge symmetry breaking?
quantum corrections (QM system knows where to be)

$$\frac{d^2 V_{CW}}{d\phi^2} \Big|_{\phi \rightarrow 0} = 0$$



- Predictability

Relation between Higgs mass and U(1) gauge boson mass

- Yukawa coupling must be sub-dominant,

$$\beta_{\Phi} = \frac{1}{16\pi^2} (96g^4 - Y^4) > 0,$$

otherwise unstable vacuum

Application to the Standard Model?

- Radiative EW symmetry breaking?
 - **Not working**: top Yukawa dominates 1-loop corrections
 - Even if top Yukawa was not large (80's), $m_H < m_W$

- Induced EW symmetry breaking?

Haba, N. Kitazawa & NO (2005)
Iso, NO & Orikasa (2009)

Classically conformal U(1) extended SM

$$V = \lambda_h (H^\dagger H)^2 - \lambda_{mix} (H^\dagger H) (\Phi^\dagger \Phi) + V_{CW}(\Phi^\dagger \Phi)$$

Negative Higgs mass squared is induced by Φ VEV!

$$m_H^2 = -\lambda_{mix} |\langle \Phi \rangle|^2$$

Symmetry Breaking

1st: Radiative U(1) breaking by Coleman-Weinberg mechanism

$$V(\phi) = \frac{\lambda_{\Phi}}{4} \phi^4 + \frac{12g_X^4}{16\pi^2} \phi^4 \left(\ln \left[\frac{\phi^2}{v_X^2} \right] - \frac{25}{6} \right) \quad \phi = \sqrt{2} \text{Re} [\Phi]$$

$$\langle \Phi \rangle = \frac{v_X}{\sqrt{2}}$$

2nd: Electroweak symmetry breaking is triggered

$$V \supset -\lambda_{\text{mix}} (\Phi^\dagger \Phi) (H^\dagger H) + \lambda_H (H^\dagger H)^2$$
$$\rightarrow -\lambda_{\text{mix}} \langle \Phi^\dagger \Phi \rangle (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

This picture needs **an SM extension with an extra gauge symmetry!**

New Physics beyond the SM: $G_{SM} \rightarrow G_{SM} \times G_X$

3. Classically conformal
U(1) B-L (U(1)_x) Extended SM

Minimal gauged B-L extension of the SM

Davidson (1979);

Mohapatra & Marshak (1980)

B-L (Baryon number minus Lepton number)

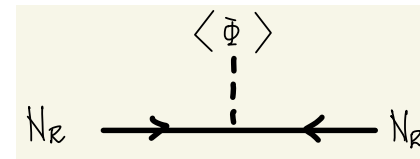
Based on $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

Particle Content

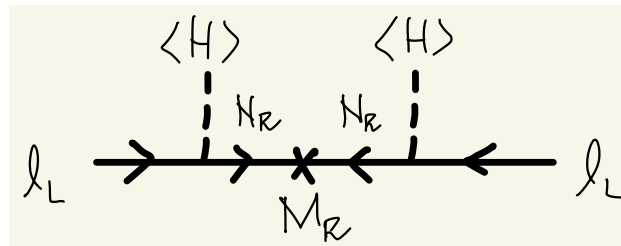
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$	
q_L^i	3	2	+1/6	+1/3	
u_R^i	3	1	+2/3	+1/3	
d_R^i	3	1	-1/3	+1/3	
ℓ_L^i	1	2	-1/2	-1	
N_R^i	1	1	0	-1	3 RHNs
e_R^i	1	1	-1	-1	
H	1	2	-1/2	0	
Φ	1	1	0	+2	B-L Higgs field

Properties of Minimal B-L Model

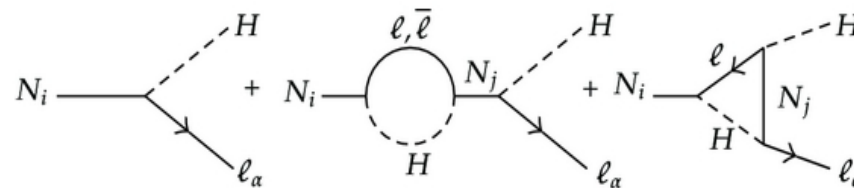
- Anomaly-free global B-L symmetry in the SM is gauged
- Right-handed neutrinos to cancel gauge/gravitational anomaly
- Spontaneous B-L gauge symmetry breaking to generate Majorana mass for RHNs



- Type-I seesaw mechanism after electroweak symmetry breaking



- Leptogenesis via CP-asymmetric out-of-equilibrium NR decay



Comment: History of the SM construction

The Standard Model based is on the gauge symmetry:

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

1960s

QCD

Electroweak

Comment: History of the SM construction

The Standard Model based is on the gauge symmetry:

$$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$

1960s

QCD

Electroweak

1950s and before

global $\text{SU}(3)$: hadron model

global $\text{SU}(2)$: Isospin for particle classification

global hypercharge: Gell-Mann-Nishijima relation

- The gauge groups of the SM were initially introduced as global symmetries ($E < M_{W,Z}, \Lambda_{QCD}$)
- They are now gauge groups \rightarrow gauge bosons

Global $\text{U}(1)_{B-L}$ picture is good since $E < M_{Z'}$?

Generalization of the minimal B-L model

Oda, NO & Takahashi (2015)
 Das, Oda, NO & Takahashi (2016)

	SU(3) _c	SU(2) _L	U(1) _Y	U(1) _X
q_L^i	3	2	1/6	$(1/6)x_H + (1/3)$
u_R^i	3	1	2/3	$(2/3)x_H + (1/3)$
d_R^i	3	1	-1/3	$(-1/3)x_H + (1/3)$
ℓ_L^i	1	2	-1/2	$(-1/2)x_H - 1$
e_R^i	1	1	-1	$-x_H - 1$
H	1	2	-1/2	$(-1/2)x_H$
N_R^i	1	1	0	-1
Φ	1	1	0	2

3 RHNs

U(1)_X Higgs

- ▶ U(1)_X charge: $Q_X = Q_Y x_H + Q_{B-L}$ (x_H=0 is the B-L model)
- ▶ Free from gauge & mixed gauge-gravitational anomalies
- ▶ Seesaw Mechanism is automatically implemented

Classically Conformal extension of Minimal B-L Model

Iso, NO & Orikasa (2009)

$$V = \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\Phi^\dagger \Phi)^2 - \lambda_{\text{mix}} (H^\dagger H) (\Phi^\dagger \Phi)$$

- ▶ No mass terms due to the conformal invariance
- ▶ We set $\lambda_{H,\Phi,\text{mix}} > 0$
- ▶ No symmetry breaking at the tree-level

Assuming a small mixing quartic coupling, the symmetry breaking occurs in the following way.....

Symmetry Breaking

1st: Radiative U(1) breaking by Coleman-Weinberg mechanism

$$V(\phi) = \frac{\lambda_{\Phi}}{4} \phi^4 + \frac{12g_X^4}{16\pi^2} \phi^4 \left(\ln \left[\frac{\phi^2}{v_X^2} \right] - \frac{25}{6} \right) \quad \phi = \sqrt{2} \text{Re} [\Phi]$$

$$\langle \Phi \rangle = \frac{v_X}{\sqrt{2}}$$

2nd: Electroweak symmetry breaking is triggered

$$V \supset -\lambda_{\text{mix}} (\Phi^\dagger \Phi) (H^\dagger H) + \lambda_H (H^\dagger H)^2$$
$$\rightarrow -\lambda_{\text{mix}} \langle \Phi^\dagger \Phi \rangle (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

Negative mass squared generated!

Relations among parameters

CW mechanism: $\lambda_{\Phi} = \frac{11}{\pi^2} g_{BL}^4$

$$m_{\phi} = \sqrt{\frac{3}{2\pi^2}} g_{BL} m_{Z'} = \sqrt{\frac{6}{\pi^2}} g_{BL}^2 v_{BL}$$

Higgs mass relations: $m_h^2 = \lambda_{mix} v_{BL}^2 = 2\lambda_H v_h^2$

Mixing between Higgs bosons:

$$\mathcal{L} \supset -\frac{1}{2} \begin{bmatrix} h & \phi \end{bmatrix} \begin{bmatrix} m_h^2 & \lambda_{mix} v_X v_h \\ \lambda_{mix} v_X v_h & m_{\phi}^2 \end{bmatrix} \begin{bmatrix} h \\ \phi \end{bmatrix}$$

By using $m_h = 125 \text{ GeV}$ & $v_h = 246 \text{ GeV}$, we have

only 2 free parameters: g_X, v_X

Extension of B-L Model with a DM candidate

- Z_2 parity & Z_2 -odd RHN DM

NO & Seto (2009)

$J=1,2$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$	Z_2
N_R^j	1	1	0	-1	+
N_R	1	1	0	-1	-
Φ	1	1	0	+2	+

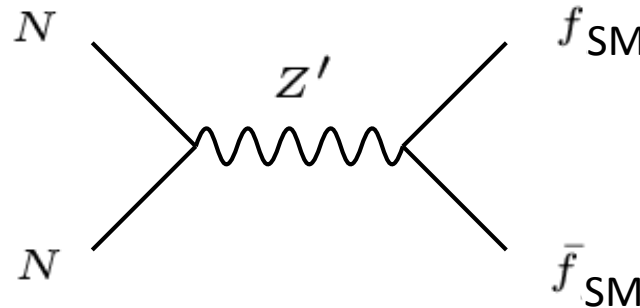
Z_2 -odd RHN is stable \rightarrow DM
The others are even

3 RHNs \rightarrow 2 RHNs for Minimal Seesaw

+

1 B-L Higgs/ Z' -portal WIMP DM

King, NPB 576 (2000) 85;
Frampton, Glashow & Yanagida,
PLB 548 (2002) 119

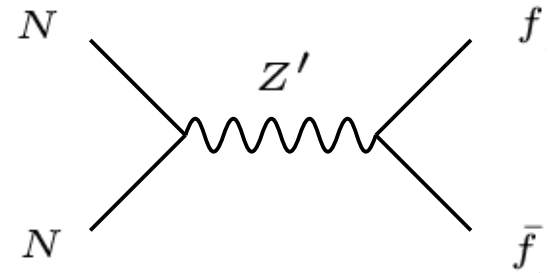


NO & Orikasa (2012);
NO & Burell (2015);
NO & S. Okada (2015)
NO, S. Okada & Raut (2017)
Oda, NO & Takahashi (2017)

Complementarity between DM physics and LHC

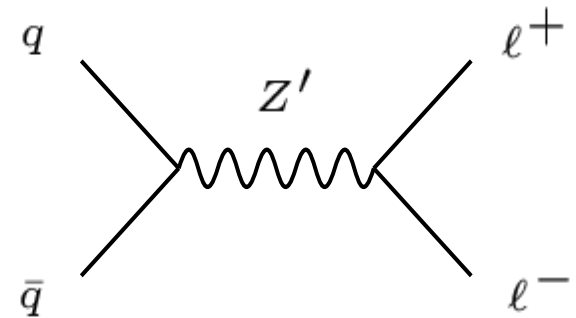
(1) Z' -portal RHN DM

RHN DM communicates with the SM particles through Z' boson mediated processes

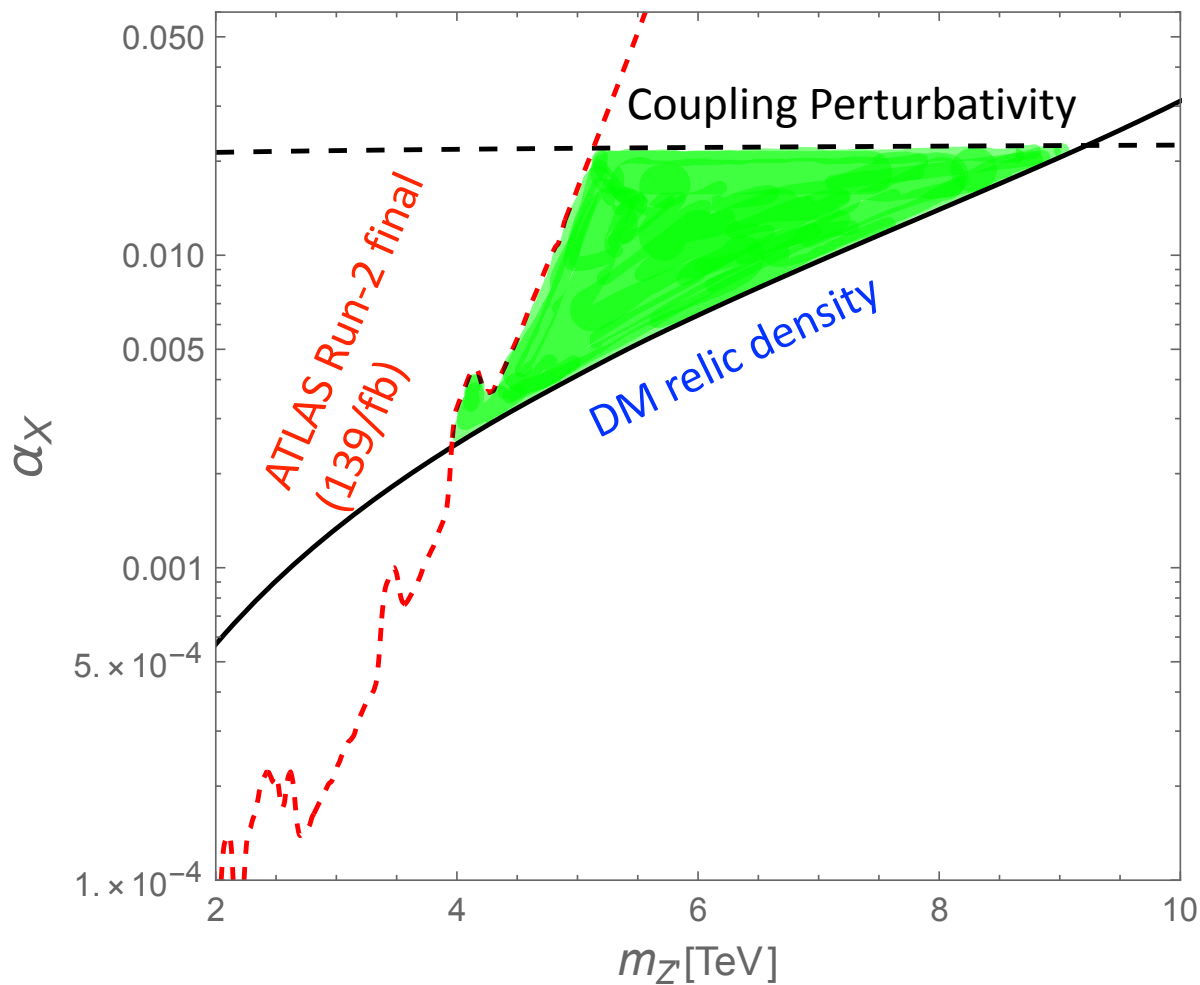


(2) Z' boson search at the LHC Run-2

Search for a narrow resonance with the di-lepton final state at ATLAS and CMS with LHC Run-2



Combining Cosmological & LHC Run-2 Constraints with the gauge coupling perturbativity until Planck



Extension of Minimal B-L Model with inflaton

- B-L Higgs as Inflaton

NO, Rehman & Shafi (2011)

NO & Raut (2015)

Introduce non-minimal gravitational coupling to the B-L Higgs:

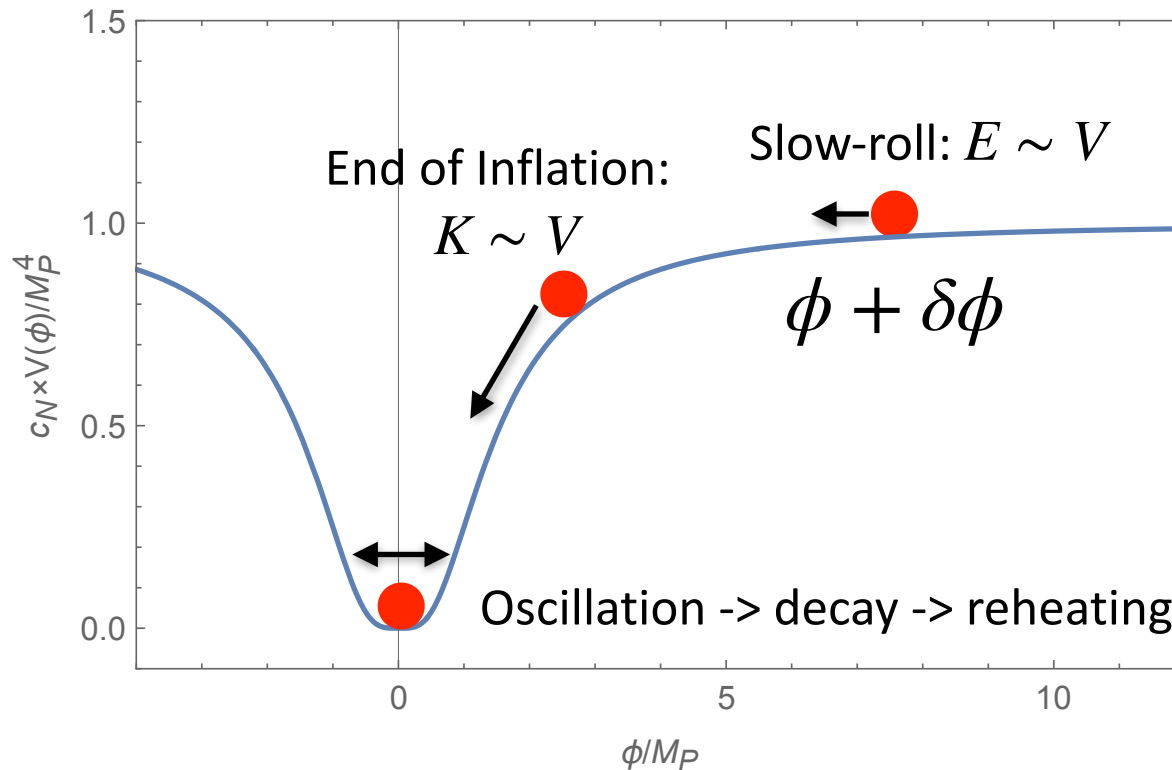
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 f R + \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi) \right]$$

where $f = 1 + 2\xi \frac{\Phi^\dagger \Phi}{M_P^2}$

- $v_{BL} \ll M_P$
- During the inflation, the inflation potential is dominated by $V \sim \lambda_\Phi (\Phi^\dagger \Phi)^2$

“ $\lambda\phi^4$ inflation with non-minimal gravitational coupling”

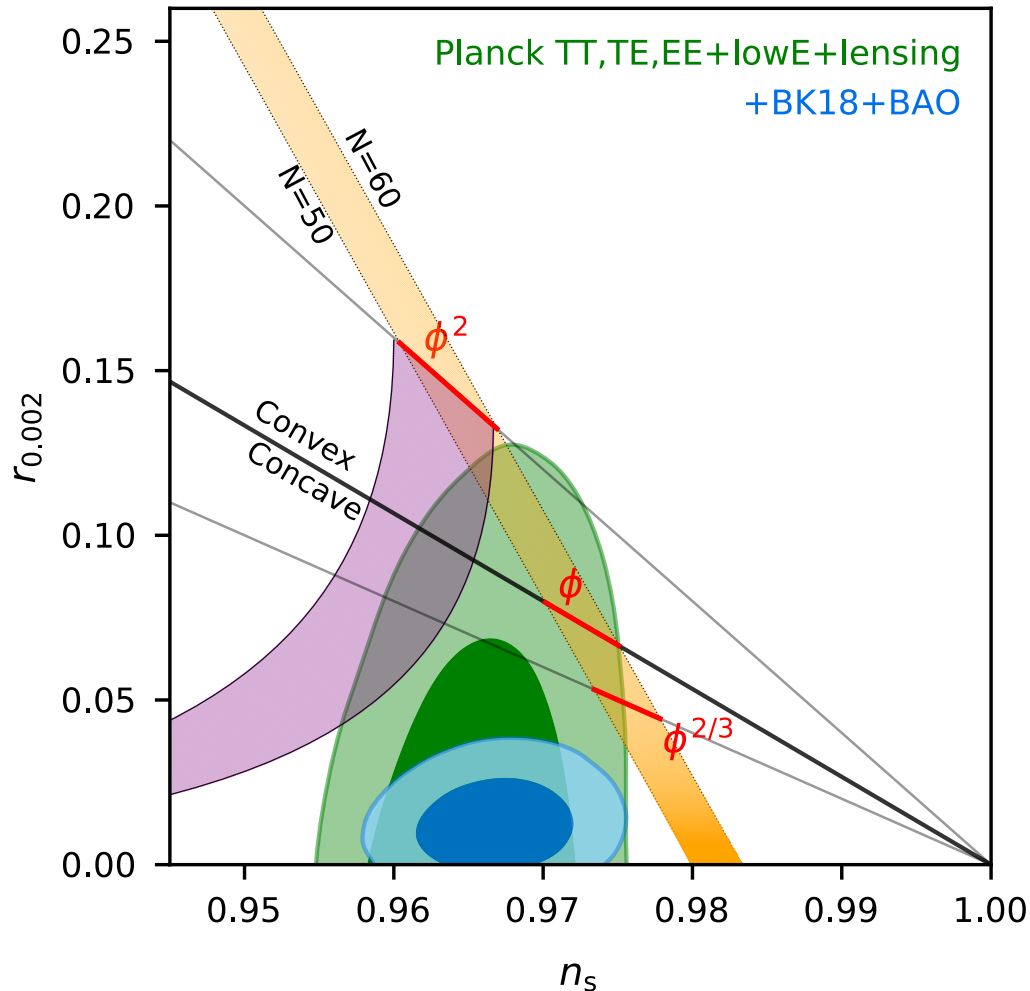
Slow-roll inflation to drive the cosmic inflation



- Inflation takes place during slow-roll: $a(t) \propto e^{H_{inf}t}$
- Quantum fluctuation $\delta\phi$ is magnified to a macroscopic scale \rightarrow primordial density fluctuation

Constraints on inflation scenario from CMB observations

BICEP/Keck 2018
PRL 127 (2021) 151301



Power spectrum of scalar
perturbation:

$$P_S(k_0) = 2.099 \times 10^{-9}$$

$$k_0 = 0.05 \text{ Mpc}^{-1}$$

Spectral index:

$$n_s = 1 + \frac{d \ln P_S}{d \ln k} \simeq 0.965$$

Tensor-to-scalar ratio:

$$\frac{P_T}{P_S} = r \leq 0.036 \text{ (95\%)}$$

Inflationary predictions of a slow-roll inflation

$$\mathcal{L}_{inf} = \frac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) - V(\phi)$$

Defining the slow-roll parameters (in Planck units $M_P = 1$)

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V}$$

Spectral index & tensor-to-scalar ratio:

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon$$

The power spectrum of scalar perturbation: $P_S = \frac{1}{12\pi^2} \frac{V^3}{(V')^2}$

The number of e-folds: $N_e = \int_{\phi_e}^{\phi_0} d\phi \frac{V}{V'}$

Here, $\phi = \phi_0$ at the horizon exit & the end of inflation $\epsilon(\phi_e) = 1$

Inflationary predictions of a slow-roll inflation

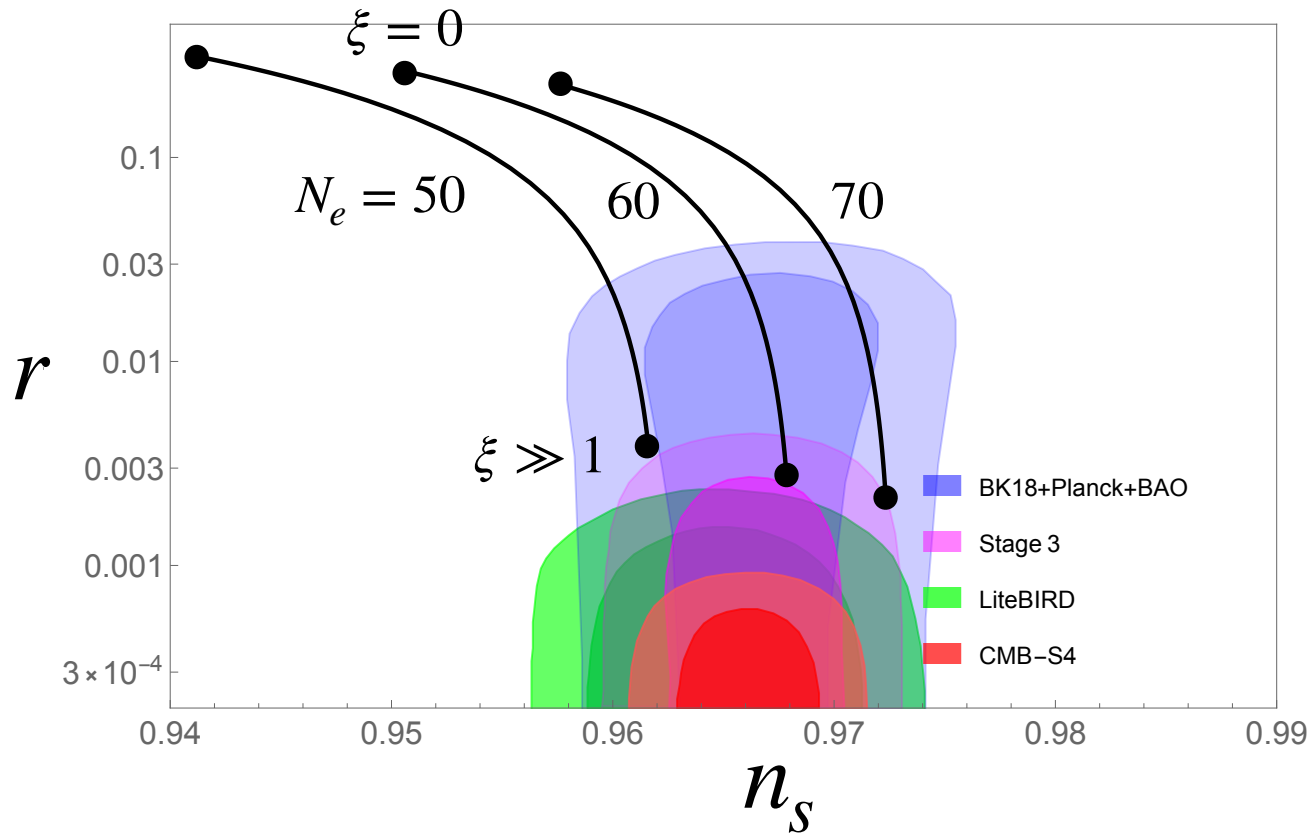
The power spectrum of scalar perturbation:

$$P_S = \frac{1}{12\pi^2} \frac{V^3}{(V')^2} \rightarrow 2.099 \times 10^{-9}$$

The number of e-folds: $N_e = \int_{\phi_e}^{\phi_0} d\phi \frac{V}{V'} \rightarrow \text{Fix (say, 50-60)}$

—————→ $n_s \ \& \ r$
predictions

Inflationary Predictions VS Planck+BK18+BAO results



- Once N_e is fixed, only 1 free parameter (ξ) determines the predictions
- Predicted GWs are $r \gtrsim 0.003$

Future experiments (CMB-S4, LiteBIRD) will cover the region!

Comment on $\lambda\phi^4$ inflation

- Simple 1-field inflation with the introduction of $\xi|\phi|^2 R$
- Consistent with Planck + others with a suitable choice of quartic coupling $\lambda|\phi|^4$
- **Potentially, any scalar can play the role of inflaton**

* SM Higgs is not likely the inflaton since its running quartic coupling runs into negative at high energies

- The classically conformal gauged U(1) B-L extended SM can solve several problems of the SM:

- What drives **Electroweak Symmetry Breaking**?
- Why are **Neutrino Masses** non-zero and so tiny?
- What is the nature of **Dark Matter**?
- What drives **Cosmic Inflation** before Big Bang?
- What is the origin of **Matter-Antimatter asymmetry** in the Universe?

4. Some more phenomenology of
Classically Conformal U(1) Extended SM

High predictability for the parameters ($g_X^2 \gg Y_N^2$)

$$V_{\text{tree}} = \frac{1}{4}\lambda_H h^4 - \frac{1}{4}\lambda_{\text{mix}} h^2 \phi^2 + \frac{1}{4}\lambda_\Phi \phi^4$$

- ▶ No mass term
- ▶ We set $\lambda_{H,\Phi,\text{mix}} > 0$
- ▶ No symmetry breaking at the tree-level

$$V(\phi)_{1\text{-loop}} = \frac{\lambda_\Phi}{4}\phi^4 + \frac{12g_X^4}{16\pi^2}\phi^4 \left(\ln \left[\frac{\phi^2}{v_X^2} \right] - \frac{25}{6} \right)$$

Radiative U(1) symmetry breaking via CW Mechanism, and then induced EW symmetry breaking

Relations among parameters

CW mechanism: $\lambda_\Phi = \frac{11}{\pi^2} g_X^4$

$$m_\phi = \sqrt{\frac{3}{2\pi^2}} g_X m_{Z'} = \sqrt{\frac{6}{\pi^2}} g_X^2 v_X$$

Higgs mass relations: $m_h^2 = \lambda_{\text{mix}} v_X^2 = 2\lambda_H v_h^2$

Mixing between Higgs bosons: $\mathcal{L} \supset -\frac{1}{2} \begin{bmatrix} h & \phi \end{bmatrix} \begin{bmatrix} m_h^2 & \lambda_{\text{mix}} v_X v_h \\ \lambda_{\text{mix}} v_X v_h & m_\phi^2 \end{bmatrix} \begin{bmatrix} h \\ \phi \end{bmatrix}$

By using $m_h = 125 \text{ GeV}$ & $v_h = 246 \text{ GeV}$, we have
only 2 free parameters:

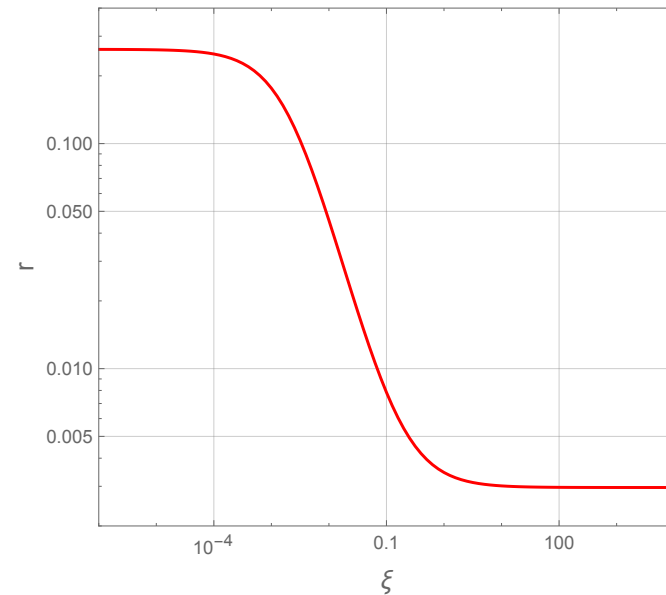
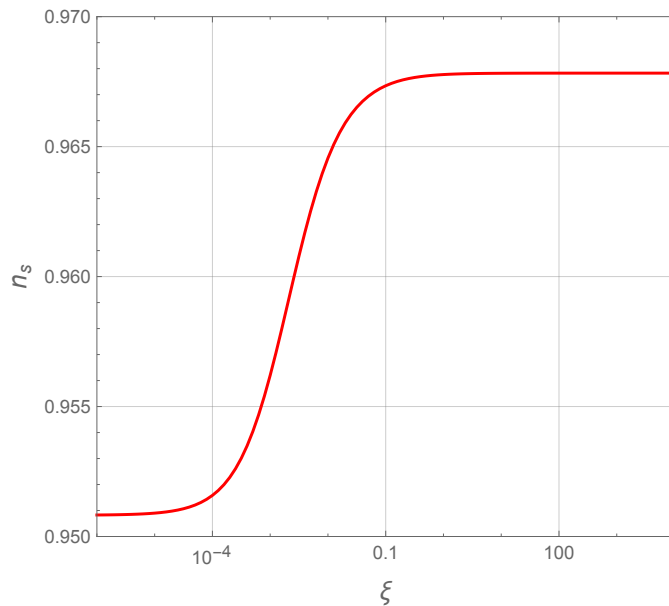
$$g_X, v_X$$

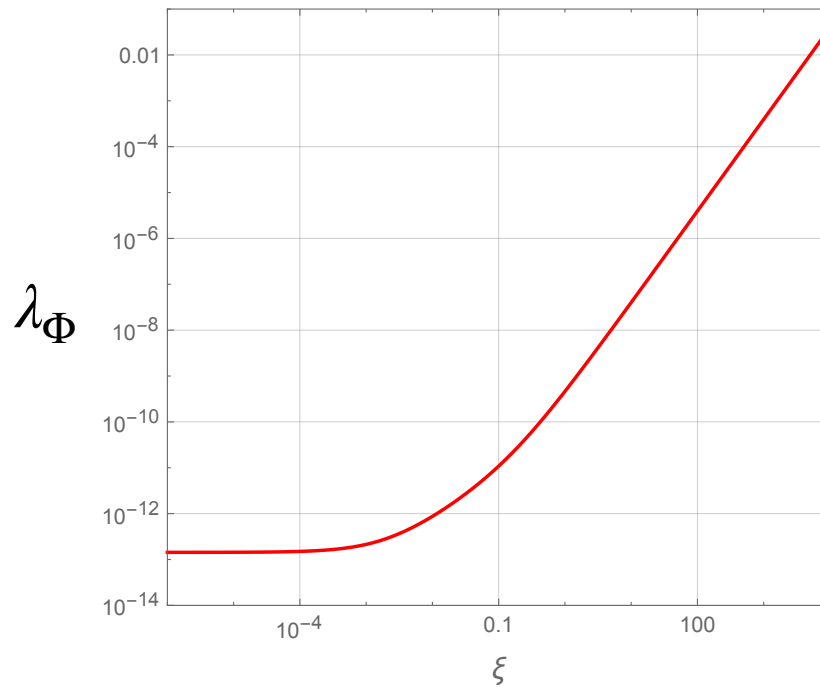
The B-L Higgs inflation scenario (inflaton = B-L Higgs) is more predictive in the the classically conformal B-L model.

Oda, NO, Raut & Takahashi (2017)
NO & Raut (2019)

In non-minimal quartic inflation, once N_e is fixed, the inflationary predictions (n_s, r) and the quartic coupling (λ_Φ) are determined by only ξ .

$$N_e = 60$$





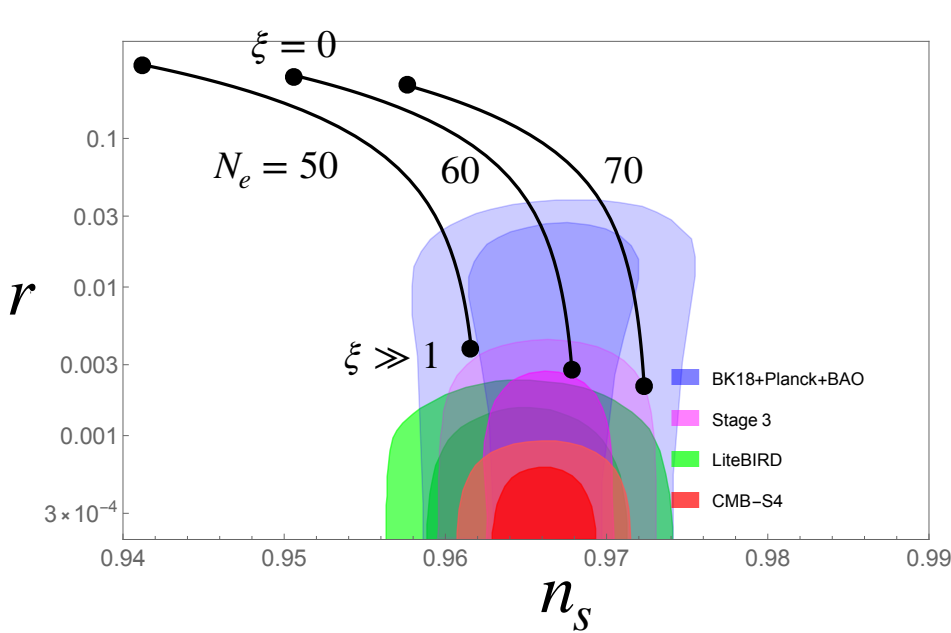
In the classically conformal B-L model, B-L Higgs/Inflaton quartic coupling is determined by the B-L gauge coupling.

$$\lambda_\Phi = \frac{11}{\pi^2} g_{BL}^4$$

Thus, one-to-one correspondence between ξ & g_{BL}

- * The relation is at VEV scale, we take into account RG evolutions to the inflation scale.

Inflationary Predictions VS Planck+BK18+BAO results



$N_e = 55$

ξ	n_s	r	λ_Φ
0.0164	0.962	0.036	1.57×10^{-12}
0.0745	0.964	0.011	8.38×10^{-12}
1	0.965	0.00408	5.23×10^{-10}
10	0.965	0.00356	4.54×10^{-8}
100	0.965	0.00350.	4.47×10^{-6}
1000	0.965	0.00350	4.46×10^{-4}
10^4	0.965	0.00350	4.46×10^{-2}

$$(N_e, \xi) \leftrightarrow (n_s, r)$$

In the classically conformal B-L model,

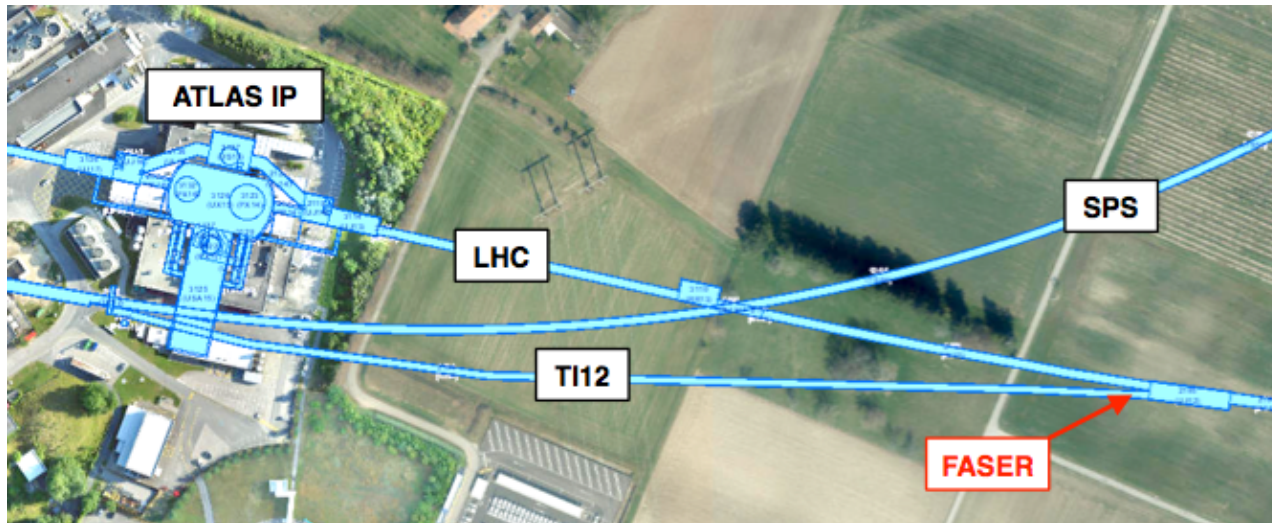
$$(N_e, g_{BL}) \leftrightarrow (n_s, r)$$

4-1.Hunting inflaton at FASER

NO & Raut, PRD 103 (2021) 5, 055022

ForwArd Search ExpeRiment (FASER)

- Recently approved (March 2019) new experiment at CERN to look for **long-lived charge-neutral particles**
- The FASER detector will be installed in a tunnel near the ATLAS detector about 480 m away



FASER Search for Dark Scalar

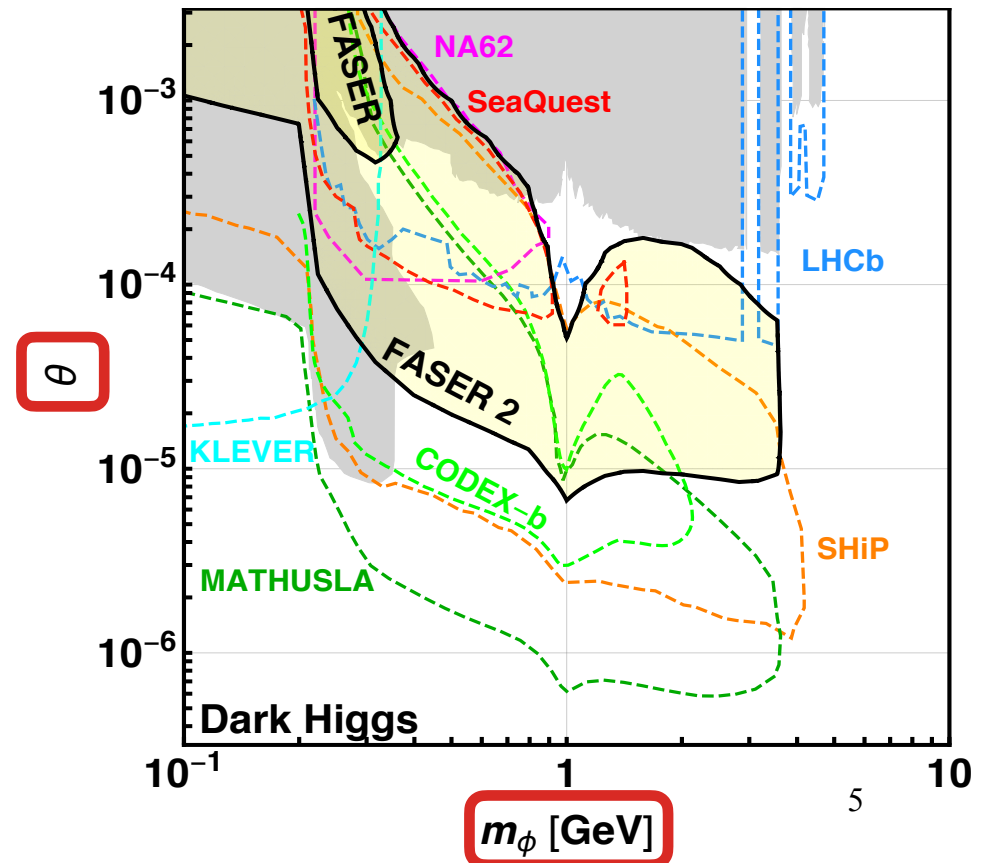
Upcoming FASER experiment will search for a light “Dark Scalar” mainly produced from rare B-meson decays through the mixing with the SM Higgs boson

- FASER at LHC Run-3
- FASER-2 at HL-LHC

$$\begin{bmatrix} h \\ \phi \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \tilde{h} \\ \tilde{\phi} \end{bmatrix}$$

* Gray shaded region is already excluded by CHRAM, Belle & LHCb

arXiv: 1811. 12522



Search for Inflaton at FASER

Let us now identify the U(1)_X Higgs as inflaton in non-minimal Inflation

★ We have a connection among FASER search region, Inflationary predictions & Z'-boson search at LHC

FASER Search: m_ϕ, θ



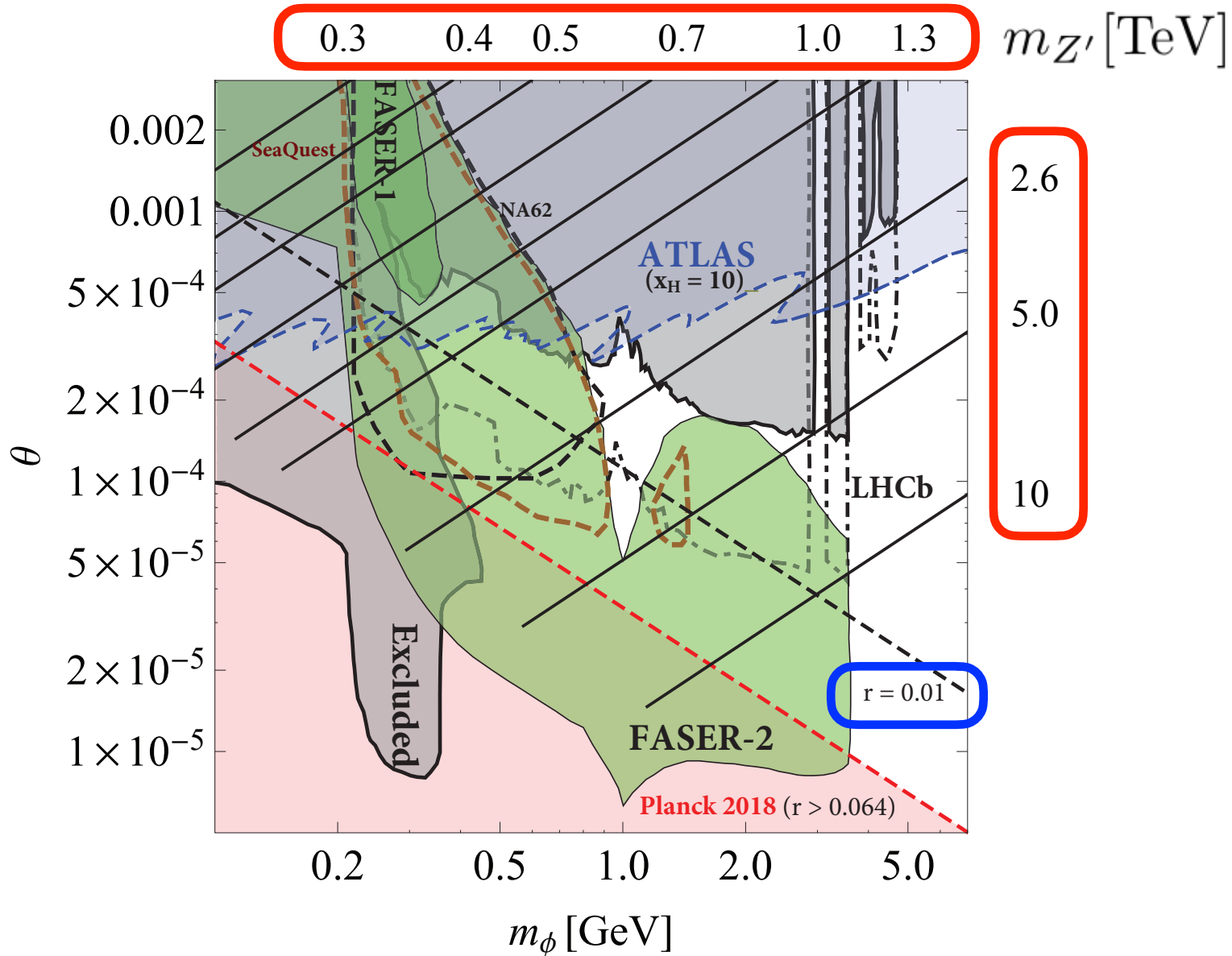
Inflationary predictions: $\xi(m_\phi, \theta)$



Z' boson resonance search: $g_X(m_\phi, \theta), m_{Z'}(m_\phi, \theta)$

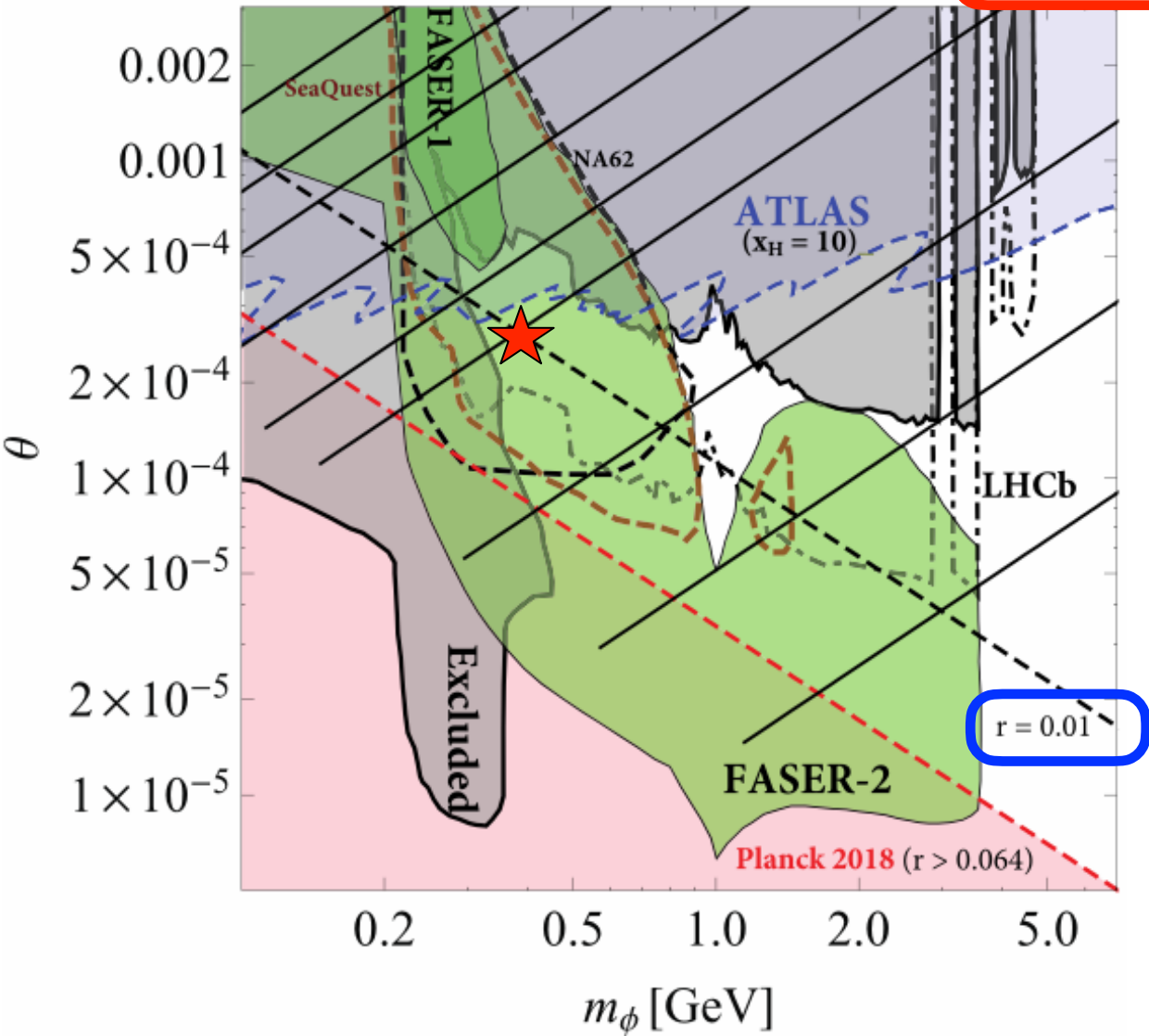
Hunting Inflaton at FASER

NO & Raut, arXiv: 1910.09663



If a Dark Scalar (U(1) Higgs) was discovered by FASER,.....

$m_{Z'} [\text{TeV}] = 1.3$



- Cross checked by
- Future CMB measurements
 - Z' -boson resonance search at HL-LHC

4-2. Reheating consistency condition
on the classically conformal
B-L Higgs inflation model

Kawai & NO, arXiv: 2303.00342

One more important constraint which is not taken seriously

The relation between $N_e(N_k)$ and reheat temperature:

$$N_k \equiv \ln \frac{a_{\text{end}}}{a_k} = 66.5 - \ln h - \ln \frac{k}{a_0 H_0} + \frac{1 - 3w}{12(1 + w)} \ln \frac{\rho_{\text{th}}}{\rho_{\text{end}}} + \frac{1}{4} \ln \frac{V_k}{\rho_{\text{end}}} + \frac{1}{4} \ln \frac{V_k}{M_{\text{P}}^4} + \frac{1}{12} (\ln g_*^{\text{eq}} - \ln g_*^{\text{th}}),$$

- k is the coming wave number of CMB at the horizon exit
- $\rho_{\text{th}} = \frac{\pi^2}{90} g_* T_R^4$
- V_k is the inflaton potential energy at the CMB horizon exit
- ρ_{end} is the inflaton energy density at the end of inflation
- w is the equation of state for the evolving inflaton from the end of inflation to the reheating time

The relation between $N_e(N_k)$ and reheat temperature:

$$N_k \equiv \ln \frac{a_{\text{end}}}{a_k} = 66.5 - \ln h - \ln \frac{k}{a_0 H_0} + \frac{1 - 3w}{12(1 + w)} \ln \frac{\rho_{\text{th}}}{\rho_{\text{end}}} \\ + \frac{1}{4} \ln \frac{V_k}{\rho_{\text{end}}} + \frac{1}{4} \ln \frac{V_k}{M_{\text{P}}^4} + \frac{1}{12} (\ln g_*^{\text{eq}} - \ln g_*^{\text{th}}),$$

Once the inflation potential is determined, **we have a relation between e-folds and reheat temperature.**

However, **this formula is not seriously considered**, since the reheating temperature is **undetermined (free parameter)** in usual inflation scenario

So, for a fixed N_k , we adjust T_R

Inflaton/B-L Higgs decay width

$$m_\phi > 2m_h : \quad \Gamma_{\phi \rightarrow H^\dagger H} = \frac{m_h^4}{8m_\phi v_{BL}^2}$$

$$m_\phi < 2m_h : \quad \Gamma_{\phi \rightarrow SMSM} = \Gamma_{SM}(m_h \rightarrow m_\phi) \times \sin^2 \theta$$

We estimate the reheating temperature by

$$\Gamma_\phi = H(T_R) = \sqrt{\frac{\rho_{\text{th}}}{3M_P}}$$

Therefore, the reheat temperature is not a free parameter, but is determined by g_{BL} & v_{BL}

In the classically conformal B-L model,

$$(N_e, g_{BL}) \leftrightarrow (n_s, r)$$

Imposing the relation between e-folds and the reheating temperature,

$$(g_{BL}, \nu_{BL}) \leftrightarrow (n_s, r)$$

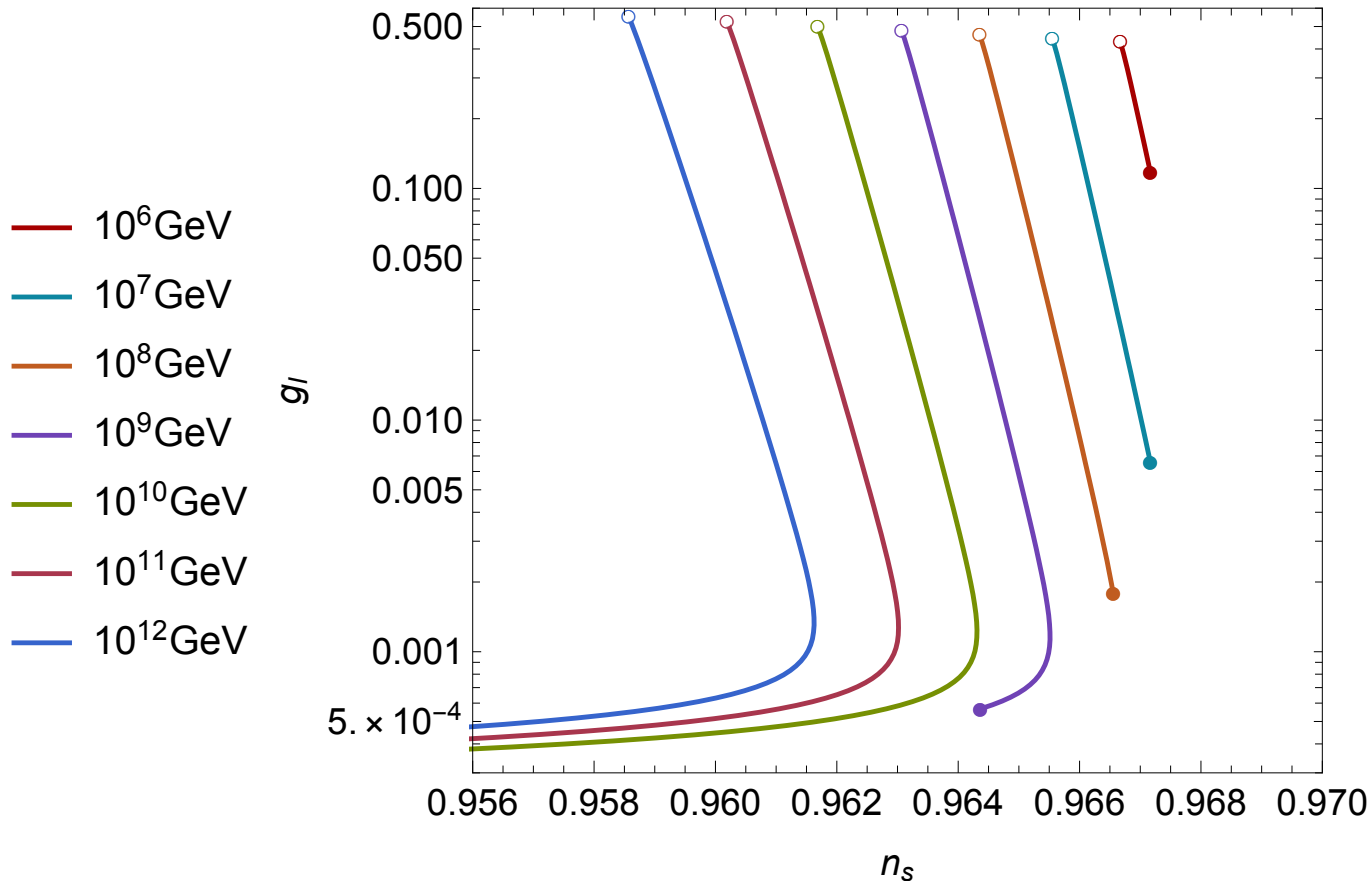
We have one-to-one correspondence between the inflationary predictions (n_s, r) and (g_{BL}, ν_{BL})

* Note that we can not always find a solution for a set of (g_{BL}, ν_{BL})

Results: n_s VS. g_{BL} for various ν_{BL} for $m_\phi > 2m_h$

$$g_{BL}(\mu = M_P) < 1$$

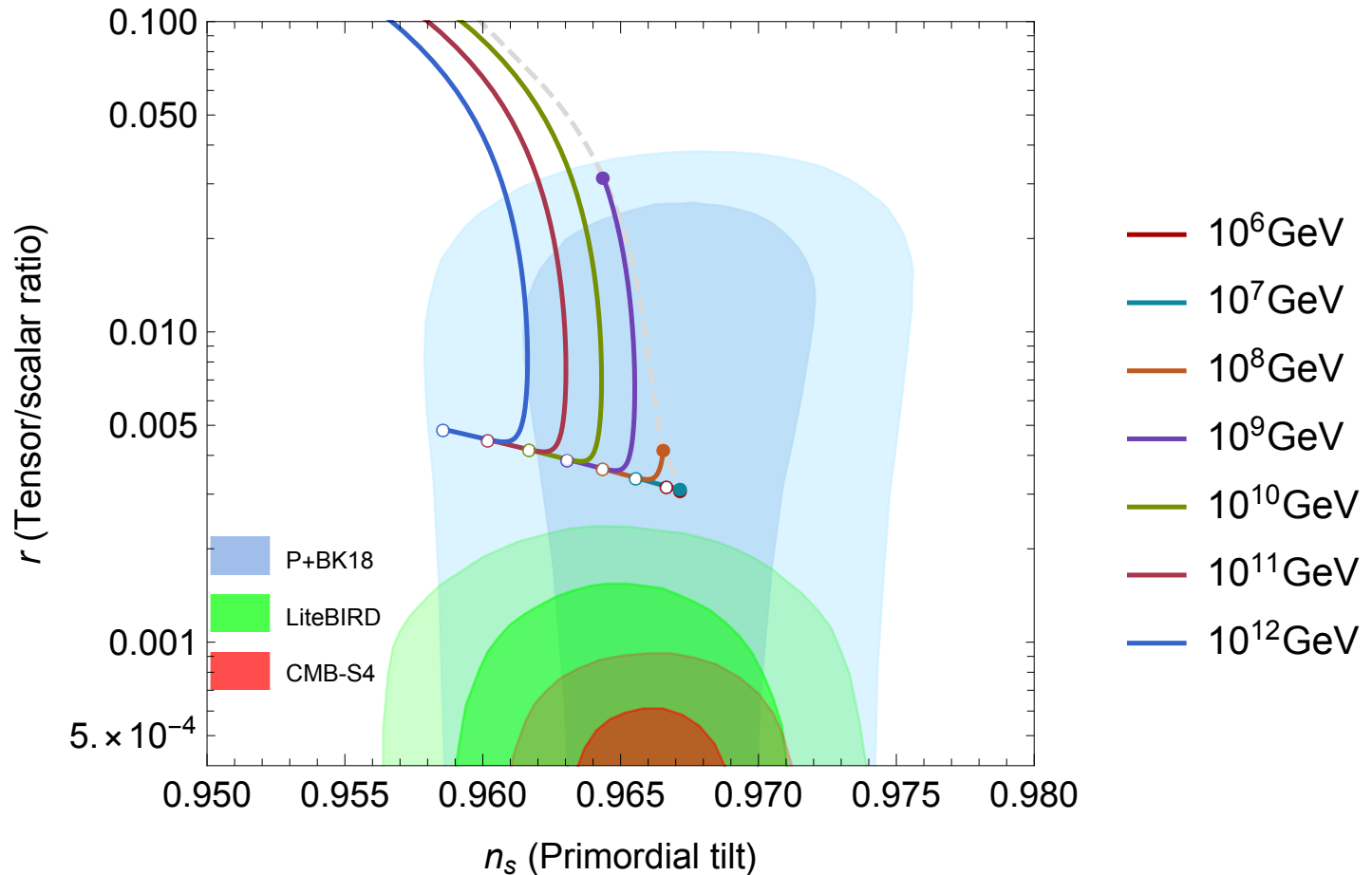
Kawai & NO, arXiv: 2303.00342



* Here, we have considered only the case $m_\phi > 2m_h$, since estimate of the reheating temperature is not easy in the other case.

Results: Inflationary predictions for various v_{BL}

Kawai & NO, arXiv: 2303.00342



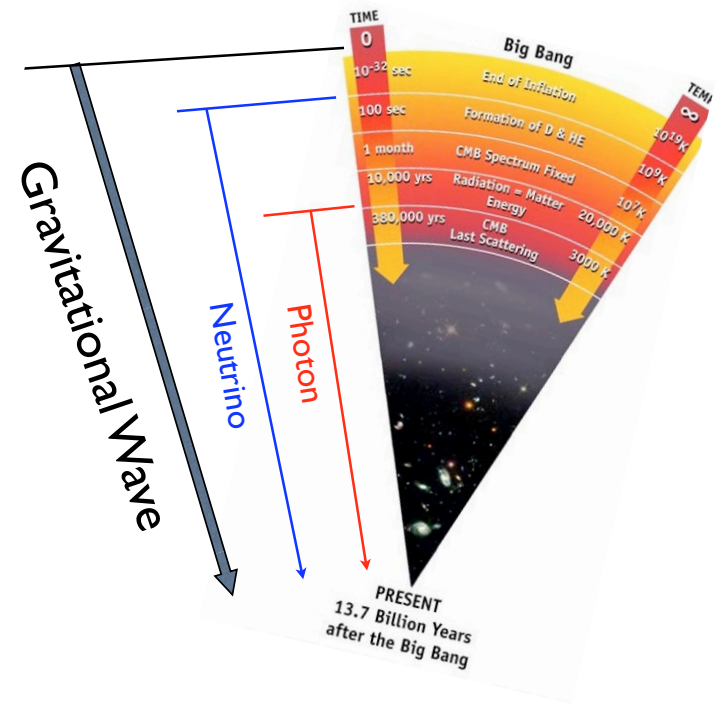
- Theoretically consistent region is very restricted
- $10^6 \lesssim v_{BL}[\text{GeV}] \lesssim 10^{12}$

4-3. Gravitational-Wave Probes of the $U(1)_X$ Extended SM

Exploring Early Universe (Beyond the SM (BSM) in cosmology)

GWs carry the information from the “earliest Universe”!

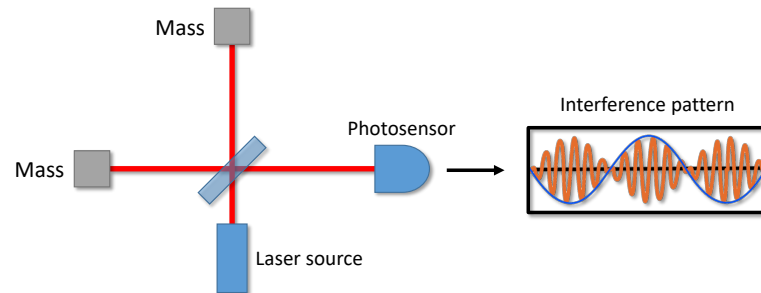
GW detections as a probe of BSM!



Detection of GWs

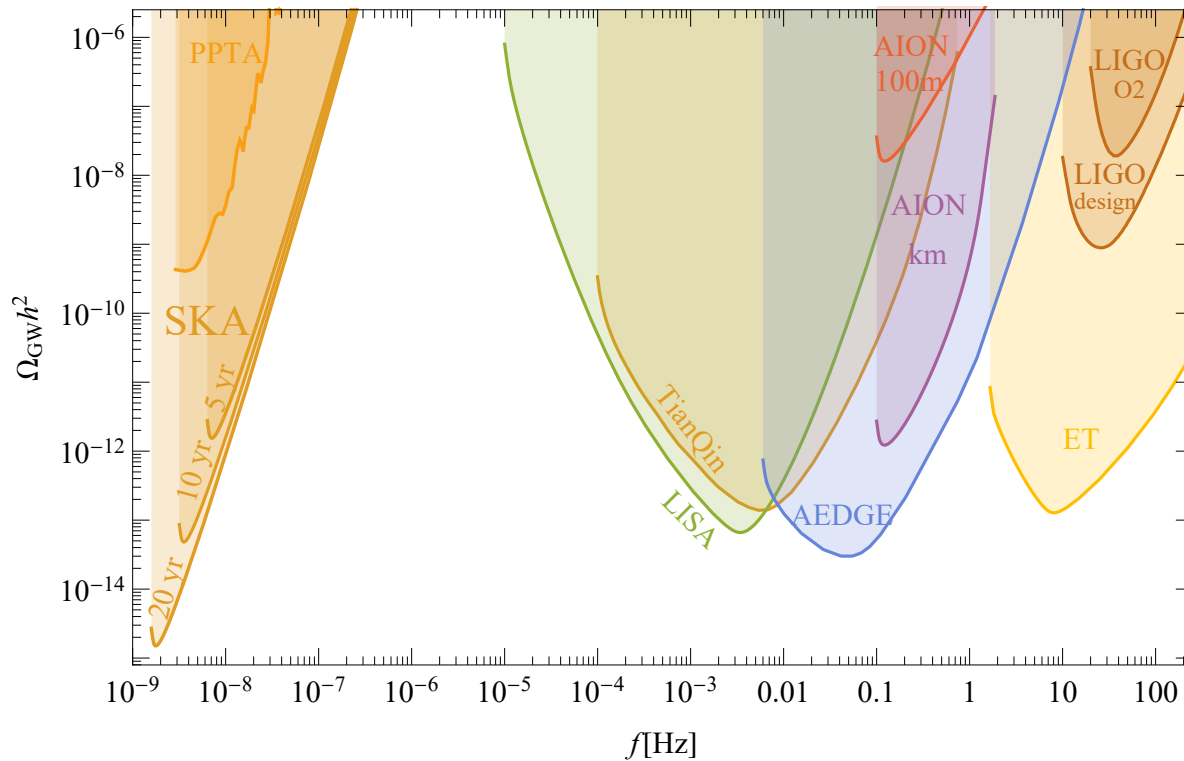
- Indirect: **B-mode polarization** of CMB (**GWs from inflation**)
Pulsar timing arrays: GW effects on pulsar timing

- Direct: Interferometers



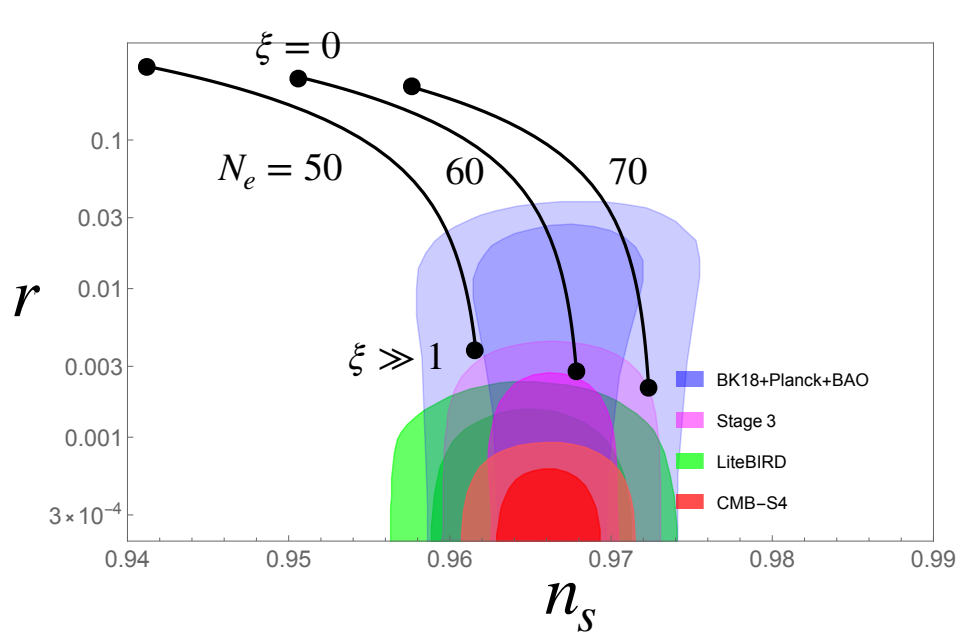
GW150914 detection at LIGO has opened up a possibility to detect GWs in a variety of frequencies.

On-going and planned GW detection experiments



1. Primordial GW from U(1)x Higgs Inflation

Even for $v_X \gg 1 \text{ TeV}$ (beyond the LHC energy), as long as $v_X \ll M_P$, the U(1)x Higgs inflation with non-minimal gravitational coupling is a perfectly conceited with the observations



$N_e = 55$

ξ	n_s	r	λ_Φ
0.0164	0.962	0.036	1.57×10^{-12}
0.0745	0.964	0.011	8.38×10^{-12}
1	0.965	0.00408	5.23×10^{-10}
10	0.965	0.00356	4.54×10^{-8}
100	0.965	0.00350.	4.47×10^{-6}
1000	0.965	0.00350	4.46×10^{-4}
10^4	0.965	0.00350	4.46×10^{-2}

2. GWs from 1st order phase transition

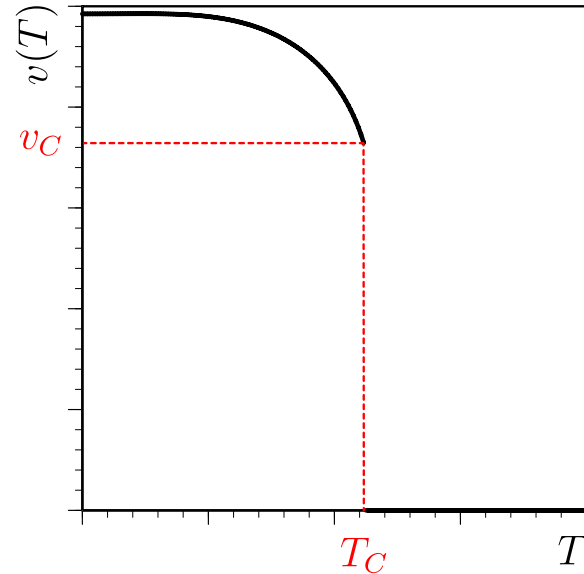
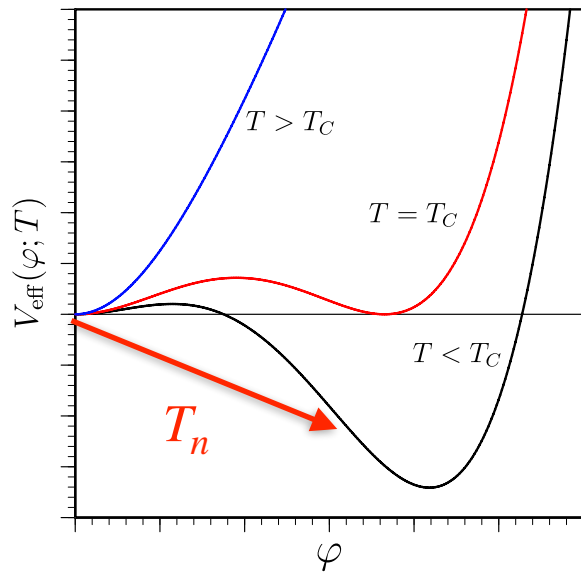
There are many well-motivated models beyond the SM, in which **the SM gauge symmetry is extended**.

We naturally expect that the universe experienced some phase transitions associated with the extended gauge symmetry breaking, in addition to the electroweak & QCD phase transitions in the SM.

If a gauge symmetry breaking exhibits **1st order phase transition**, we may expect a large amplitude of GWs created by **bubble dynamics**.

Our case: GWs from $U(1)_X$ symmetry breaking

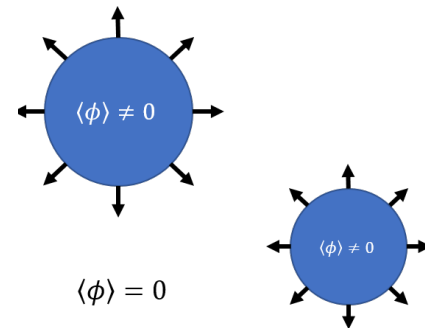
1st order phase transition



Bubble nucleation occurs at T_n (nucleation temp) if the condition is satisfied:

Thermal bubble nucleation rate/vol

$$\Gamma(T_n) \sim T_n^4 e^{-S_3/T_n} \sim H(T_n)^4$$



Theory background: finite-temperature field theory

$$V_{\text{eff}}(\varphi, T) = V_0(\varphi) + \Delta V_{1\text{-loop}}(\varphi) + \Delta V_T(\varphi, T)$$

- Tree-level potential: $V_0(\varphi)$
- 1-loop effective potential:

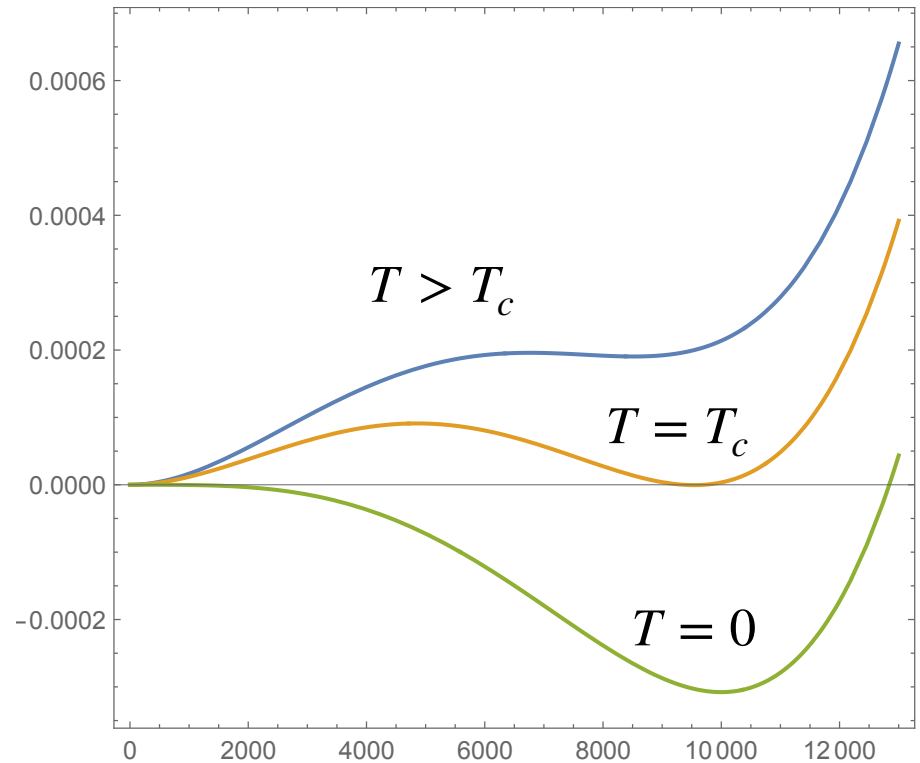
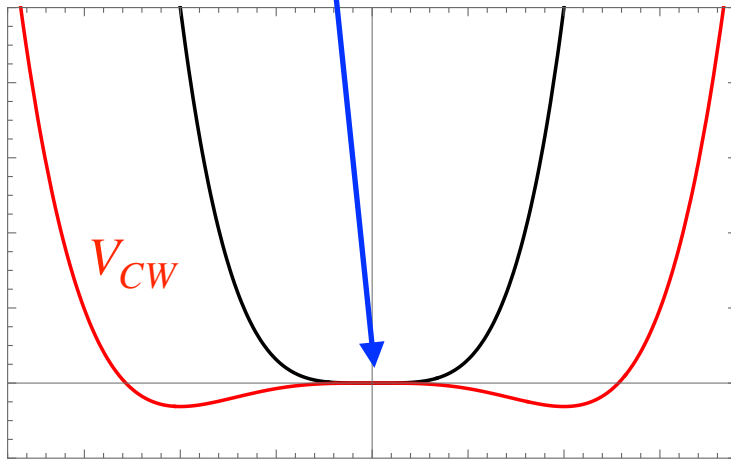
$$\begin{aligned} \Delta V_{1\text{-loop}}(\varphi) = & \sum_s g_s \frac{m_s^4}{64\pi^2} \left(\ln \frac{m_s^2}{Q^2} - c_s \right) - \sum_f g_f \frac{m_f^4}{64\pi^2} \left(\ln \frac{m_f^2}{Q^2} - c_f \right) \\ & + \sum_v g_v \frac{m_v^4}{64\pi^2} \left(\ln \frac{m_v^2}{Q^2} - c_v \right). \end{aligned}$$

- Finite temperature corrections to the effective potential:

$$\begin{aligned} \Delta V_T(\varphi) = & \sum_s g_s \frac{T^4}{2\pi^2} J_B(m_s^2/T^2) - \sum_f g_f \frac{T^4}{2\pi^2} J_F(m_f^2/T^2) + \sum_v g_v \frac{T^4}{2\pi^2} J_B(m_v^2/T^2) \\ J_{B,F}(y^2) = & \int_0^\infty dx x^2 \log \left[1 \mp e^{-\sqrt{x^2 + y^2}} \right] \end{aligned}$$

Classically conformal model is suitable for getting a strong 1st order phase transition

$$\left. \frac{d^2 V_{CW}}{d\phi^2} \right|_{\phi \rightarrow 0} = 0$$



Note $\left. \frac{d^2 V_H}{d\phi^2} \right|_{\phi \rightarrow 0} < 0$ in Higgs potential

Phase transition analysis

- Thermal bubble nucleation rate/vol

$$\Gamma(T) \sim T^4 e^{-S_3/T}$$

- 3-D Euclidean action

$$S_3 = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{d\varphi(r)}{dr} \right)^2 + V(\varphi, T) \right]$$

with a bounce solution of $\frac{d^2\varphi}{dr^2} + \frac{2}{r} \frac{d\varphi}{dr} = V'$

$$\lim_{r \rightarrow \infty} \varphi(r) = 0 \quad \& \quad \lim_{r \rightarrow 0} \frac{d\varphi(r)}{dr} = 0$$

→ We fix T_n by $\Gamma(T_n) \sim T_n^4 e^{-S_3/T_n} \sim H(T_n)^4$

Characterizing the GW spectrum

- Nucleation temperature: T_n
- Phase transition strength: $\alpha = \frac{\Delta\rho(T_n)}{\rho_{\text{rad}}(T_n)}$
- Hubble normalized transition time scale: $\frac{\beta}{H(T_n)} = T \left. \frac{d(S_3/T)}{dT} \right|_{T=T_n}$
- Bubble wall velocity: v_b

GW spectrum

$$\Omega_{GW}(f) = \Omega_{GW}^{\text{coll}}(f) + \Omega_{GW}^{\text{sw}}(f) + \Omega_{GW}^{\text{turb}}(f)$$

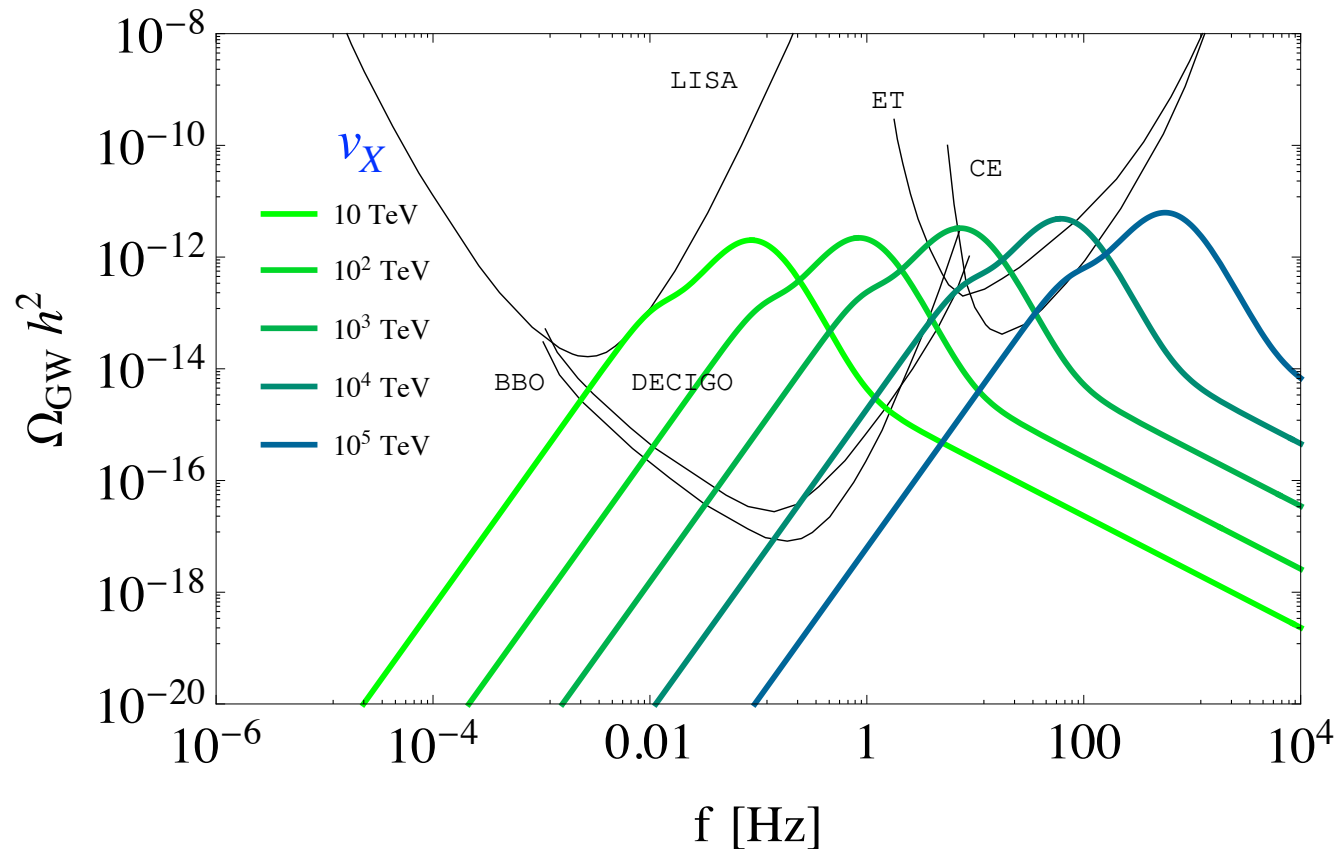
from 3 main sources: **bubble collisions (coll)**, **sound waves (sw)**
after bubble collisions, and **turbulence (turn)**

Fitting formulas for the spectrum are obtained by simulations

Huber et al., 0806.1828; Hindmarsh et al., 1504.03291; Caprini et al., 0909.0622, ..

Minimal U(1)_X Model ($x_H = -4/5$)

NO, Seto & Uchida (2021)



- Probing the seesaw scale with GWs from 1st order PT!
even if $\nu_X \gg$ LHC energy scale
- $\nu_X \lesssim 10^5$ TeV for detection

5. Summary

- The classically conformal gauged U(1) B-L ($U(1)_X$) extended SM can solve several problems of the SM:
 - ☑ What drives **Electroweak Symmetry Breaking**?
 - ☑ Why are **Neutrino Masses** non-zero and so tiny?
 - ☑ What is the nature of **Dark Matter**?
 - ☑ What drives **Cosmic Inflation** before Big Bang?
 - ☑ What is the origin of **Matter-Antimatter asymmetry** in the Universe?

- In the model, physics is controlled by only 2 (3) free parameters: g_{BL} & v_{BL} (g_{BL} & v_{BL} & x_H)
- Inflaton with $0.3 \lesssim m_\phi [\text{GeV}] \lesssim 3$ can be searched by FASER. This search is complementary with CMB measurements.
- In the U(1) Higgs inflation scenario, the reheating temperature is not a free parameter, and thus the inflationary predictions (n_s, r) are determined by (g_{BL}, v_{BL}) .
- More precise measurements of (n_s, r) can exclude the model or pin down (g_{BL}, v_{BL}) values.
- Gravitational wave prob of the classically conformal extended SM even if U(1) symmetry breaking scale exceeds the LHC energy

*Thank you
for your attention!*