Search for cosmological phase transitions through their gravitational wave signals

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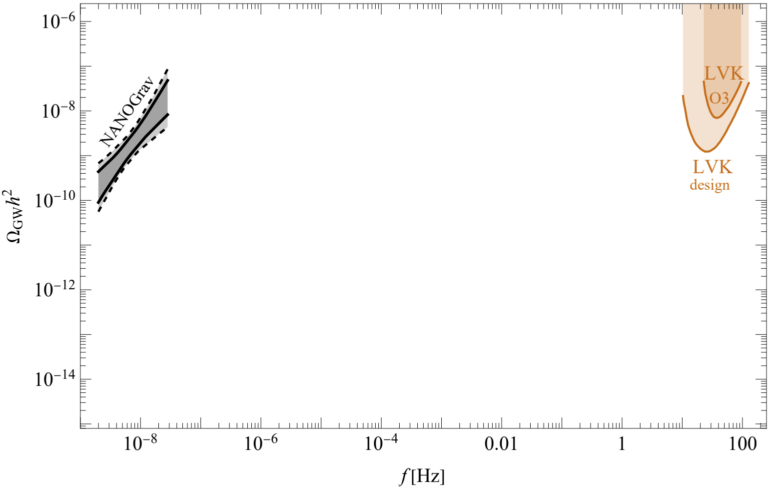

10^{-6}
10^{-8}
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$\Omega_{GW}h^2$
NANOGrav
LVK
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LVK
design
10^{-8}
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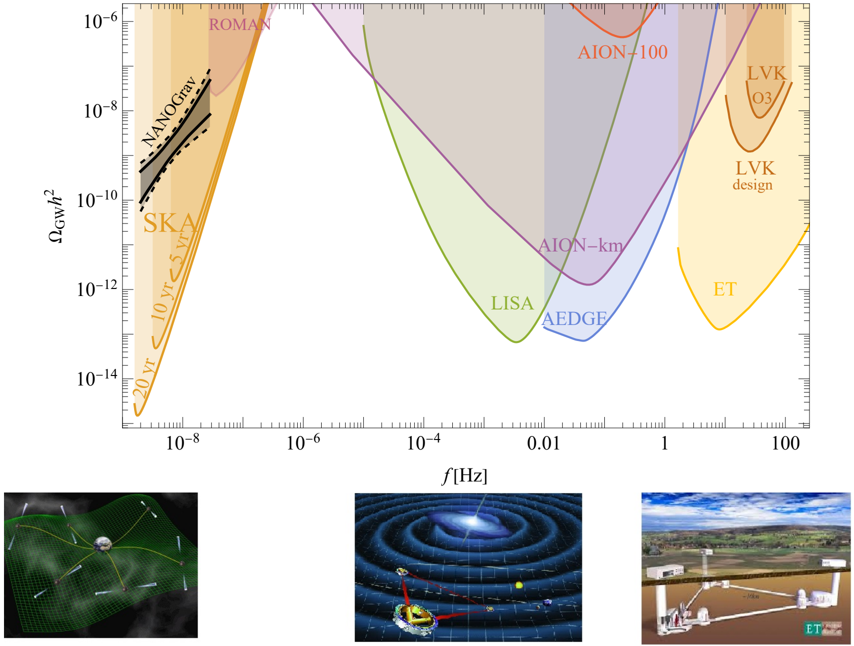
f [Hz]



ROMAN
NANOGrav
SKA
5 yr
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20 yr
AION-100
AION-km
LISA
AEDGE
ET
LVK
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0.01
1
100
f[Hz]
ET

First Order Phase Transition: bubble nucleation

- Temperature corrections to the potential

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4$$

- EOM → bubble profile

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial\phi} = 0,$$

$$\phi(r \to \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

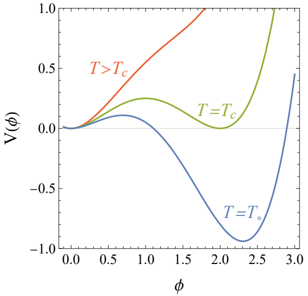
- $\mathcal{O}(3)$ symmetric action

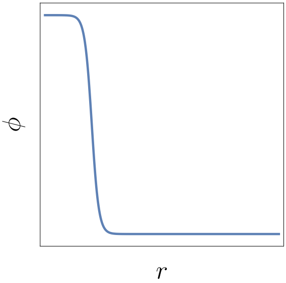
$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- nucleation temperature

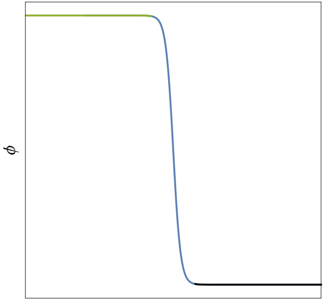
$$\frac{\Gamma}{H^4} \approx \left(\frac{T}{H}\right)^4 \exp\left(-\frac{S_3(T)}{T}\right) \approx 1$$

Linde ’81 ’83

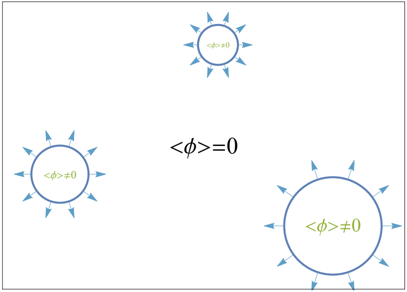




First Order Phase Transition



$\Longrightarrow$



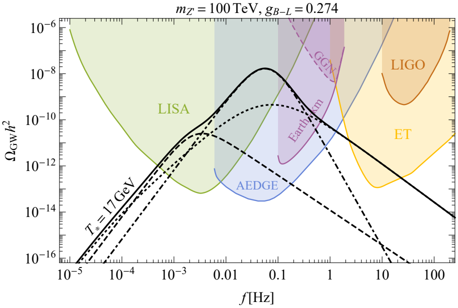
- Strength of the transition

$$\alpha \approx \left. \frac{\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}}{\rho_R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Average size of bubbles upon collision (Characteristic scale)

$$H R_* = (8\pi)^{\frac{1}{3}} \left(\frac{\beta}{H} \right)^{-1}$$

Gravitational waves from a PT



- Gravitational wave signals are produced by three main mechanisms:
  - collisions of bubble walls $\Omega_{\rm col} \propto \left(\kappa_{\rm col}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)^2$

    Kamionkowski '93, Konstandin '08 '17, Hindmarsh '18 '20, Lewicki '19 '20 '22,
  - sound waves $\Omega_{\rm sw} \propto \left(\kappa_{\rm sw}\frac{\alpha}{\alpha+1}\right)^2 (HR_*)(H\tau_{sw})$

    Hindmarsh '13 '15 '17 '19 '21 '22, Ellis '18 '19 '20, Jinno '20 '22 Lewicki '22
  - turbulence $\Omega_{\rm turb} \propto\ ?$

    Caprini '06 '09 '20, Brandenburg '10 '12 '17, Roper-Pol '17 '19 '21, Ellis '19 '20

Energy Budget

- Energy of the bubble

$$\mathcal{E} = 4\pi R^2 \sigma \gamma - \frac{4\pi}{3} R^3 p \,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

$$p_0 = \Delta V$$

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- Leading order plasma contribution
Bodeker '09 Caprini '09

$$p_1 = \Delta V - \Delta P_{\rm LO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} \,,$$

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- Next-To-Leading order plasma contribution
  Bodeker '17 Gouttenoire '21

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma\, g^2 \Delta m_V T^3\,.$$

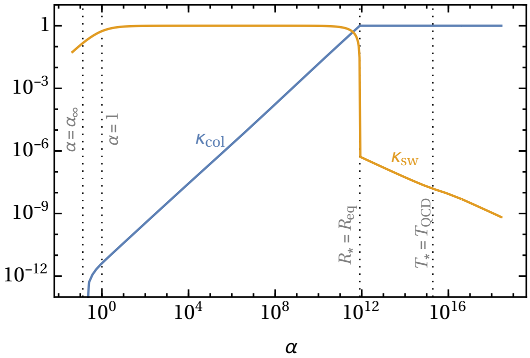
- Next-To-Leading order plasma contribution with resummation
  Hoche '20

$$P = \Delta V - P_{1\to 1} - \gamma^2 P_{1\to N} \approx \Delta V - 0.04 \Delta m^2 T^2 - 0.005 g^2 \gamma^2 T^4\,.$$

- Terminal velocity corresponds to $\gamma_{\rm eq}$
- Without friction we would find γ_*

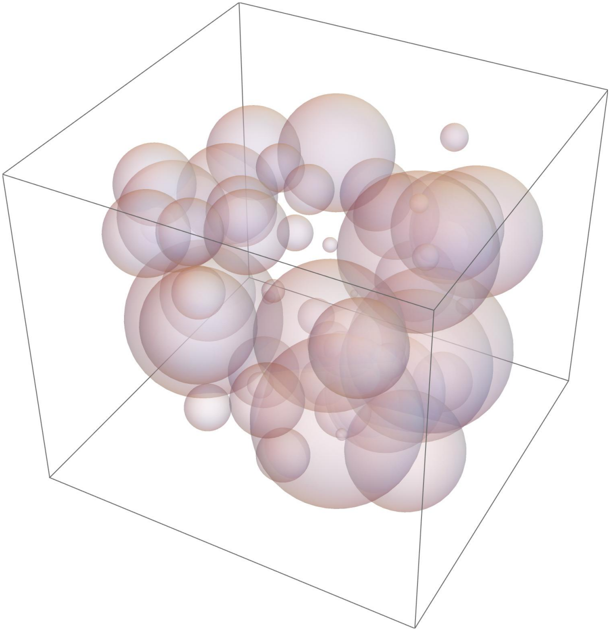
$$\kappa_{\rm col} = \frac{E_{\rm wall}}{E_V} = \begin{cases} \left[1 - \frac{1}{3}\left(\frac{\gamma_*}{\gamma_{\rm eq}}\right)^2\right]\left[1 - \frac{P_{1\to 1}}{\Delta V}\right], & \gamma_* < \gamma_{\rm eq}\,, \\ \frac{2}{3}\frac{\gamma_{\rm eq}}{\gamma_*}\left[1 - \frac{P_{1\to 1}}{\Delta V}\right], & \gamma_* > \gamma_{\rm eq}\,, \end{cases}$$

$$\kappa_{\rm sw} = \frac{\alpha_{\rm eff}}{\alpha}\frac{\alpha_{\rm eff}}{0.73 + 0.083\sqrt{\alpha_{\rm eff}} + \alpha_{\rm eff}}\,, \quad \text{with} \quad \alpha_{\rm eff} = \alpha(1 - \kappa_{\rm col})\,.$$

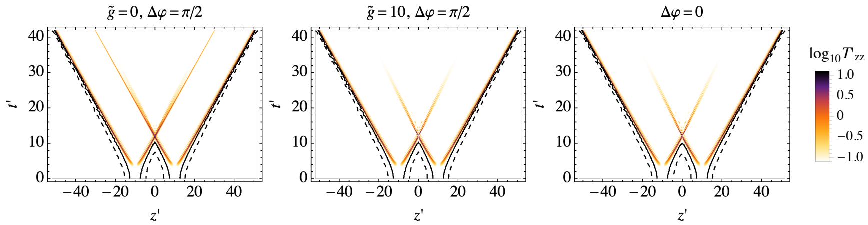


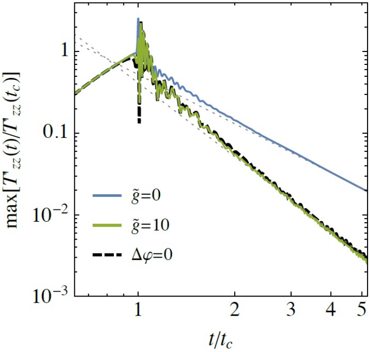
John Ellis, ML, José Miguel No, Ville Vaskonen, JCAP **06** (2019), 024, arXiv:1903.09642

Strong transitions: computation of the GW spectrum



Abelian Higgs Model: Energy Scaling





- scaled gauge coupling:

$$\tilde{g} = \frac{g v^2}{\sqrt{\Delta V}}$$

- Global Symmetry breaking:

$$T_{zz} \propto R^{-2}$$

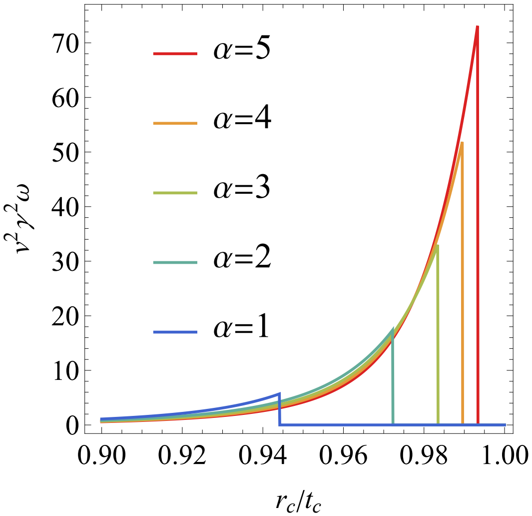
- Gauge Symmetry breaking:

$$T_{zz} \propto R^{-3}$$

ML, Ville Vaskonen arXiv: 2007.04967

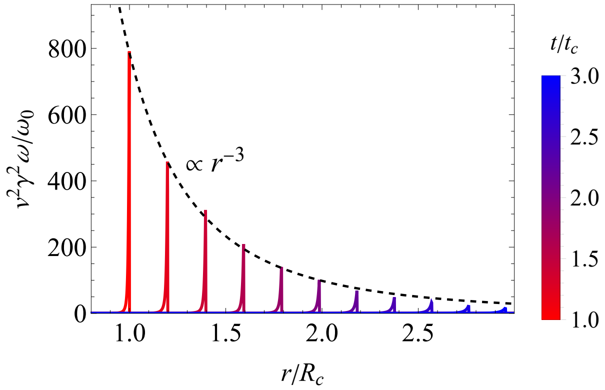
Fluid Shells

- Plasma profiles for $v_w \gtrsim v_J$



Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 20$ and $\gamma_w = 50$



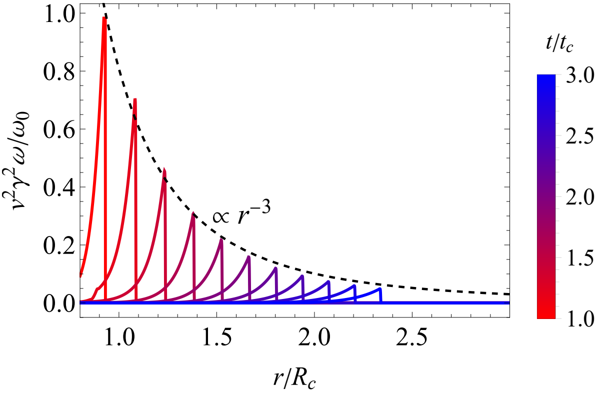
- Fluid shells with $\alpha \gg 1$:

$$T_{zz} \propto R^{-3}$$

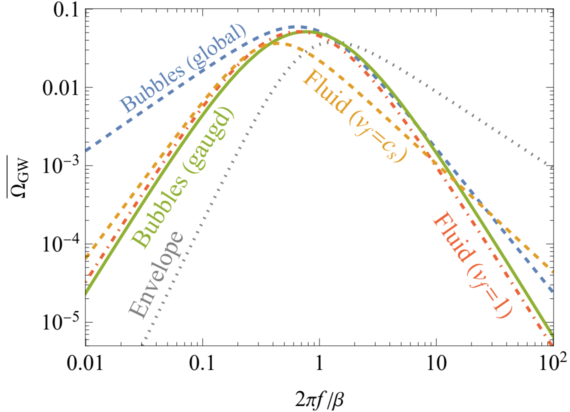
ML, Ville Vaskonen arXiv:2208.11697

Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 0.5$ and $\gamma_w = 3$



ML, Ville Vaskonen arXiv:2208.11697



- Resulting spectrum:

$$\overline{\Omega_{GW}} = \frac{A\,(a+b)^c}{\left[b\left(\frac{f}{f_p}\right)^{-\frac{a}{c}} + a\left(\frac{f}{f_p}\right)^{\frac{b}{c}}\right]^c}$$

	Bubbles		Fluid	
	Global($T \propto R^{-2}$)	Gauged ($T \propto R^{-3}$)	$v_{\text{fluid}} = 1$	$v_{\text{fluid}} = c_s$
$100\,A$	5.93 ± 0.05	5.13 ± 0.05	5.14 ± 0.04	3.64 ± 0.02
a	1.03 ± 0.04	2.41 ± 0.10	2.36 ± 0.09	2.02 ± 0.08
b	1.84 ± 0.17	2.42 ± 0.11	2.36 ± 0.09	1.38 ± 0.06
c	1.91 ± 0.29	1.45 ± 0.34	3.69 ± 0.48	1.48 ± 0.32
$2\pi f_p/\beta$	1.33 ± 0.19	0.64 ± 0.09	0.66 ± 0.04	0.44 ± 0.04

ML, Ville Vaskonen arXiv: 2208.11697
ML, Ville Vaskonen, arXiv: 2007.04967

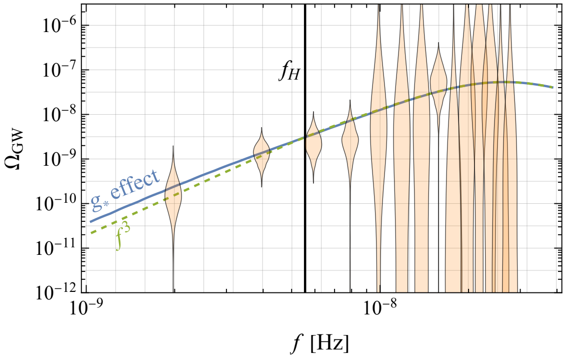
- Spectrum today

$$\Omega_{\rm GW}(f,T_*)h^2 \approx 1.6\times 10^{-5} S_H(f,f_H(T_*)) \left[\frac{\beta}{H}\right]^2 \frac{A(a+b)^c}{\left(b\left[\frac{f}{f_p}\right]^{-\frac{a}{c}} + a\left[\frac{f}{f_p}\right]^{\frac{b}{c}}\right)^c}$$

- Superhorizon modes

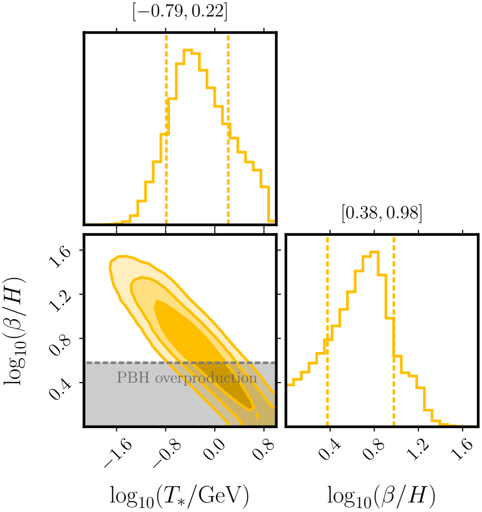
Gabriele Franciolini, Davide Racco, Fabrizio Rompineve arXiv: 2306.17136

$$f_H(T) = \frac{a(T)}{a_0}\frac{H(T)}{2\pi}\,, \quad S_H(f,f_H) = \left(1+\left[\frac{\Omega_{\rm CT}(f)}{\Omega_{\rm CT}(f_H)}\right]^{-\frac{1}{\delta}}\left[\frac{f}{f_H}\right]^{\frac{a}{\delta}}\right)^{-\delta}$$



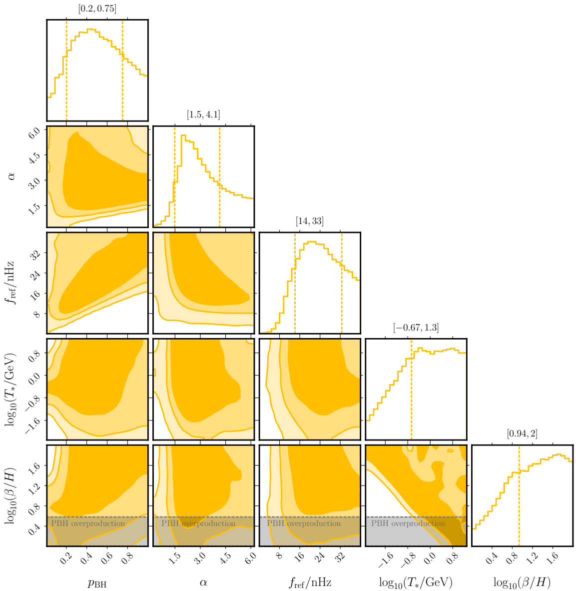
Fit to NANOGrav

- Constraint from PBH overproduction $\beta/H \gtrsim 3.9$
  ML, Piotr Toczek, Ville Vaskonen arXiv: 2305.04924



John Ellis et al. arXiv: 2308.08546

Fit to NANOGrav including SMBH

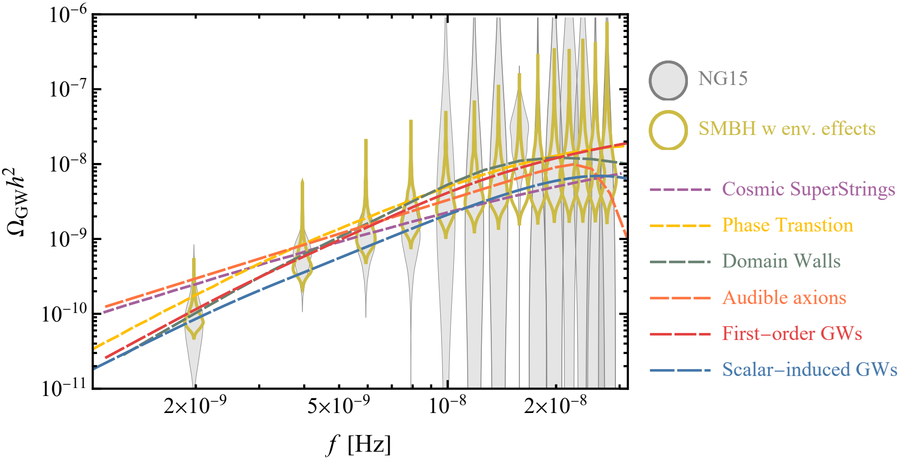


John Ellis et al. arXiv: 2308.08546

Scenario	Best-fit parameters	ΔBIC
GW-driven SMBH binaries	$p_{\rm BH} = 0.25$	6.0
GW + environment-driven SMBH binaries	$p_{\rm BH} = 1$ $\alpha = 3.8$ $f_{\rm ref} = 12$ nHz	(BIC = 53.9)
Cosmic (super)strings (CS)	$G\mu = 2 \times 10^{-12}$ $p = 6.3 \times 10^{-3}$	-1.2 (4.6)
Phase transition (PT)	$T_* = 0.24$ GeV $\beta/H = 6.0$	-4.9 (2.9)
Domain walls (DWs)	$T_{\rm ann} = 0.79$ GeV $\alpha_* = 0.026$	-5.7 (2.2)
Scalar-induced GWs (SIGWs)	$k_* = 10^{7.6}/{\rm Mpc}$ $A = 0.08$ $\Delta = 0.28$	-2.1 (5.8)
First-order GWs (FOGWs)	$\log_{10} r = -16$, $n_{\rm t} = 2.9$ $T_{\rm rh} = 0.35\,{\rm GeV}$	-2.0 (6.0)
"Audible" axions	$m_a = 3.1 \times 10^{-11}$ eV $f_a = 0.87\,M_{\rm P}$	-4.2 (3.7)

For each model, we tabulate their best-fit values, and the Bayesian information criterion $BIC \equiv -2\Delta\ell + k \ln 14$ relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the ΔBIC for the best-fit combined SMBH+cosmological scenario.

Best Fits to NANOGrav including SMBH



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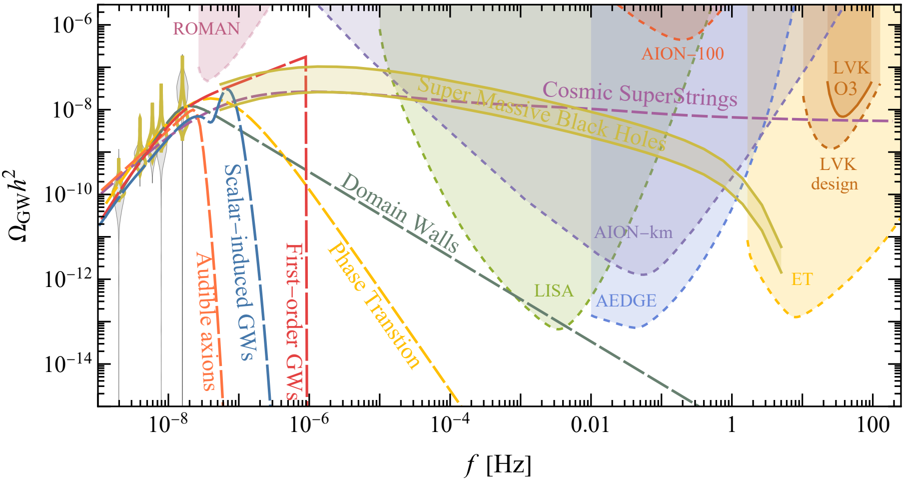

John Ellis et al. arXiv: 2308.08546

Conclusions

- Observable bubble collision signal is produced in extremely strong transitions $\alpha > 10^{10}$, however, also fluid shells in a strong transition $\alpha \gg 1$ would produce the same spectrum.
- A very strong phase transition is one of the best explanations for the current PTA data.

Thank you for your attention!