



Relation between higher-dimensional gauge theories and gravitational waves from first-order phase transitions

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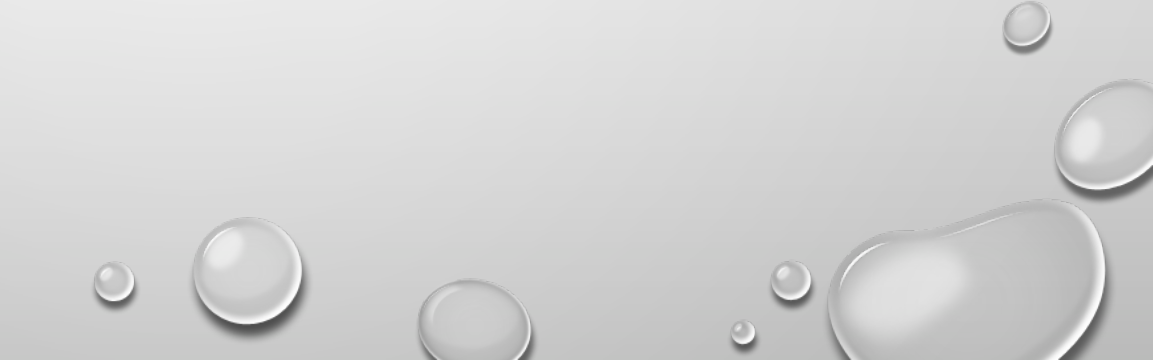
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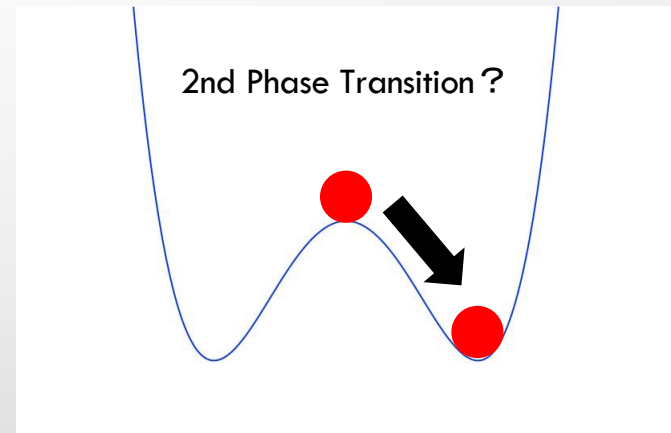
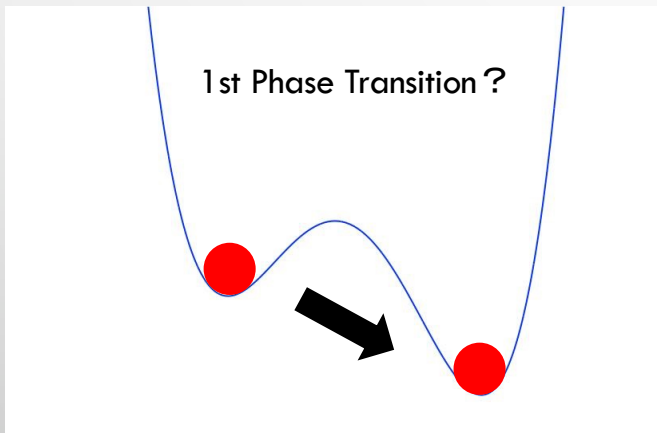
1. Introduction

Electroweak Phase Transition (EWPT) occurs
in the universe at EW scale ($\mathcal{O}(10^2)$ GeV)

However



We do not know the type of PT



The types are determined by scalar potential and thermal effect

1. Introduction

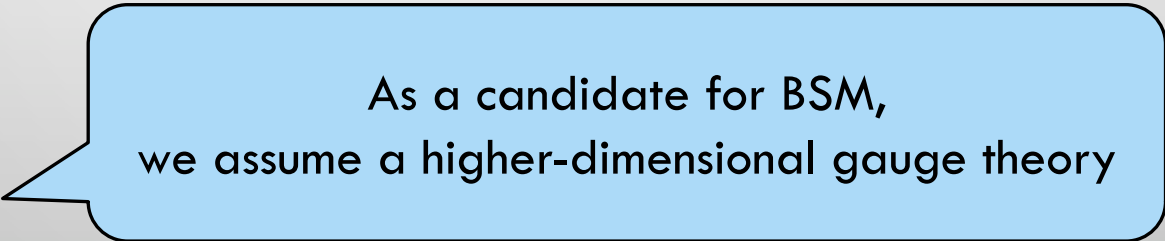
If first-order phase transition occurs,

Gravitational Waves are generated !!

[Kosowsky, Turner, Watkins (1992)]

If 1st order PT occurs in Beyond the Standard Model (BSM),

**Parameters in BSM and the spectrum of GW
must be related !!**



As a candidate for BSM,
we assume a higher-dimensional gauge theory



2. 5D SU(3) gauge theory

5D SU(3) gauge theory [Haba, Hosotani, Kawamura, Yamashita (2004)]

- 4D Minkowski spacetime $\times S^1/Z_2$

- Periodic b.c.: $A_M(x^\mu, y + 2\pi R) = U A_M(x^\mu, y) U^\dagger$

- Z_2 parity : $A_\mu(x^\mu, -y) = P_0 A_\mu(x^\mu, y) P_0^{-1}$

$$A_\mu(x^\mu, \pi R - y) = P_1 A_\mu(x^\mu, \pi R + y) P_1^\dagger$$

$$A_y(x^\mu, -y) = -P_0 A_y(x^\mu, y) P_0^{-1}$$

$$A_y(x^\mu, \pi R - y) = -P_1 A_y(x^\mu, \pi R + y) P_1^\dagger$$

4D gauge fields

5th gauge fields

- gauge group: **SU(3)**

- Extra components of gauge field = Scalar field in 4D

$$A_y \Rightarrow \phi$$

2. 5D SU(3) gauge theory

Effective potential

$$\langle A_y \rangle = \frac{a}{g_4 R} \frac{\lambda^6}{2}$$

$$V_{\text{eff}}(a, T) = V_{\text{eff}}^{T=0}(a) + V_{\text{eff}}^{T \neq 0}(a, T)$$

[Maru, Takenaga (2005)]

- Example of the potential

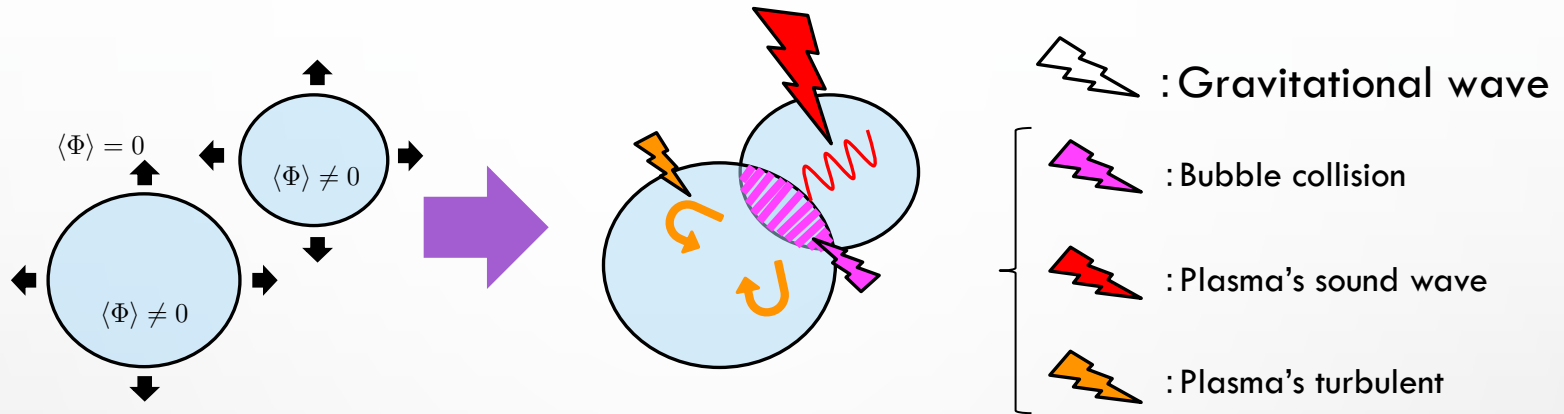
$$V_{\text{eff}}^{T=0}(a) = -3C \sum_{n=1}^{\infty} \frac{1}{n^5} \left(\cos(2\pi na) + 2 \cos(\pi na) \right) + \dots$$

$$V_{\text{eff}}^{T \neq 0}(a, T) = 2C \sum_{l=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{[n^2 + l^2 / (2\pi R T)^2]^{5/2}} (-3) \left(\cos(2\pi na) + 2 \cos(\pi na) \right) + \dots$$

$C = 3 / (64\pi^6 R^4)$

- R : Radius of compact space S^1
- Tree-level potential vanishes because of gauge symmetry

3. GW from First-Order Phase Transition



- Nucleation temperature T_{nuc}

$$\left. \frac{\Gamma}{H^4} \right|_{T=T_{nuc}} \simeq 1 \quad \longrightarrow \quad \left. \frac{S_3(T)}{T} \right|_{T=T_{nuc}} \simeq 140$$

- Normalized latent heat

$$\alpha = \frac{\epsilon(T_{nuc})}{\rho_{rad}(T_{nuc})}$$

- Duration of PT

$$\tilde{\beta} = \frac{\beta}{H(T_{nuc})} = T \frac{d}{dT} \left(\frac{S_3}{T} \right) \Big|_{T=T_{nuc}}$$

GW spectrums depend on these parameters

4. Results

- We investigate two cases

Case 1 : $N_f^{(+)} = 3$, (otherwise) = 0

Case 2 : $N_{ad}^{(+)} = 2$, $N_f^{(-)} = 8$, $N_f^{s(+)} = 4$, $N_f^{s(-)} = 2$

[Haba, Hosotani, Kawamura, Yamashita (2004)]

[Maru, Takenaga (2005)]

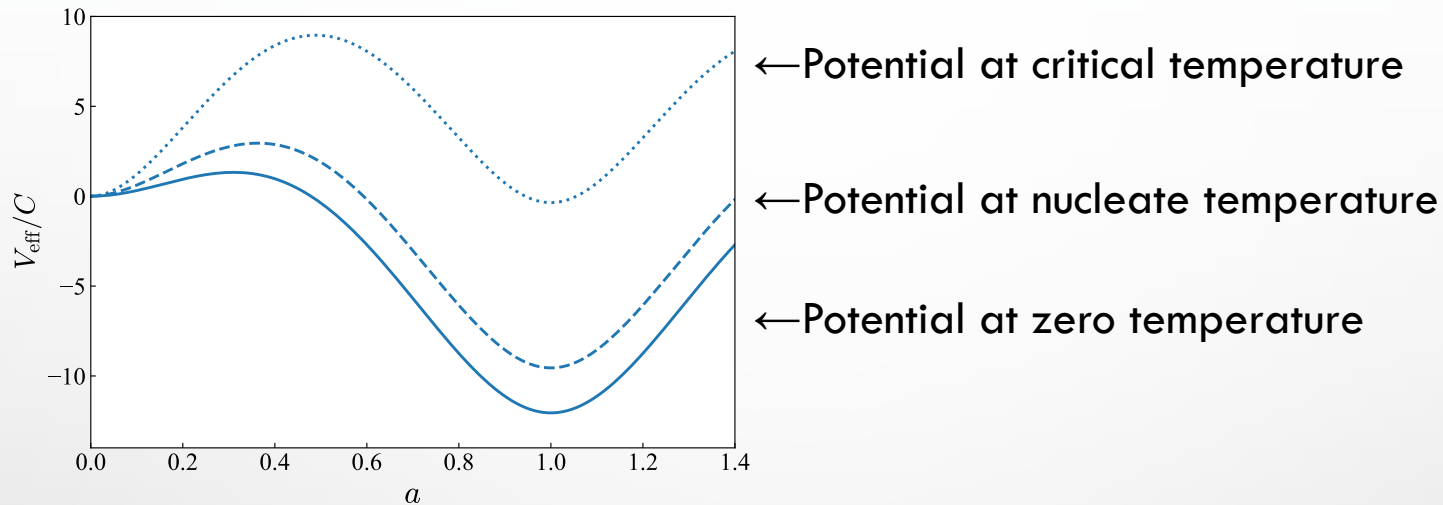
- Use the numerical results of the spectrum of GW

[Caprini et al. (2015)]

- Parameters are compact scale and coupling in 4D

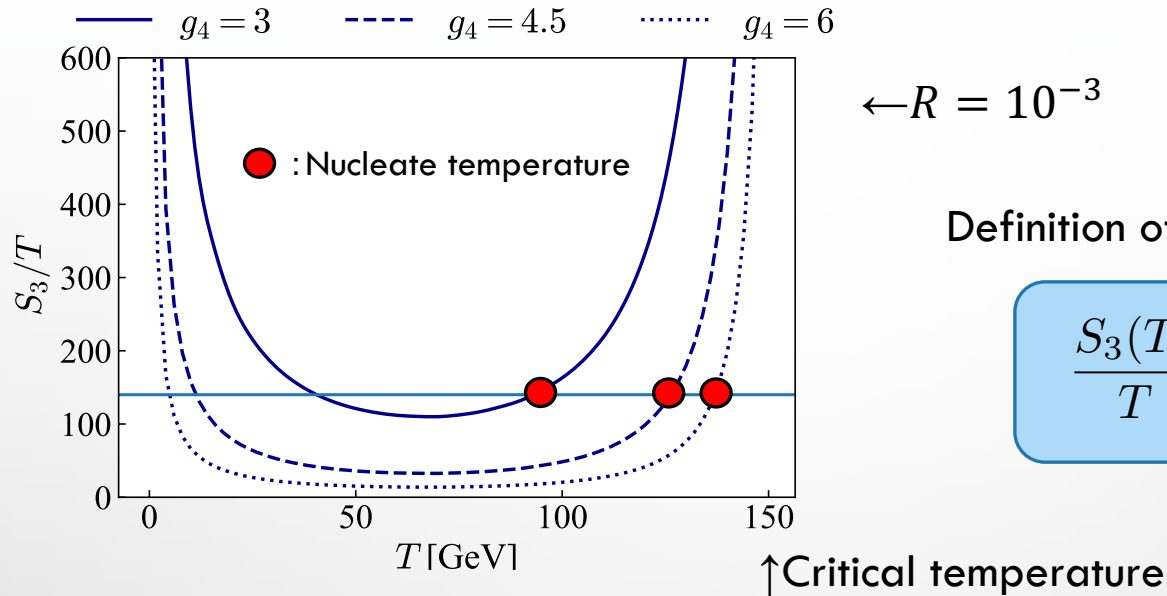
$$\langle A_y \rangle = \frac{a \lambda^6}{g_4 R 2}$$

4. Results



- Introduce only fermions in fundamental representation
- **A potential barrier** even at zero temperature → 1st PT
- critical temperature: $T_c = 150 \text{ GeV} @ R = 10^{-3}$

4. Results

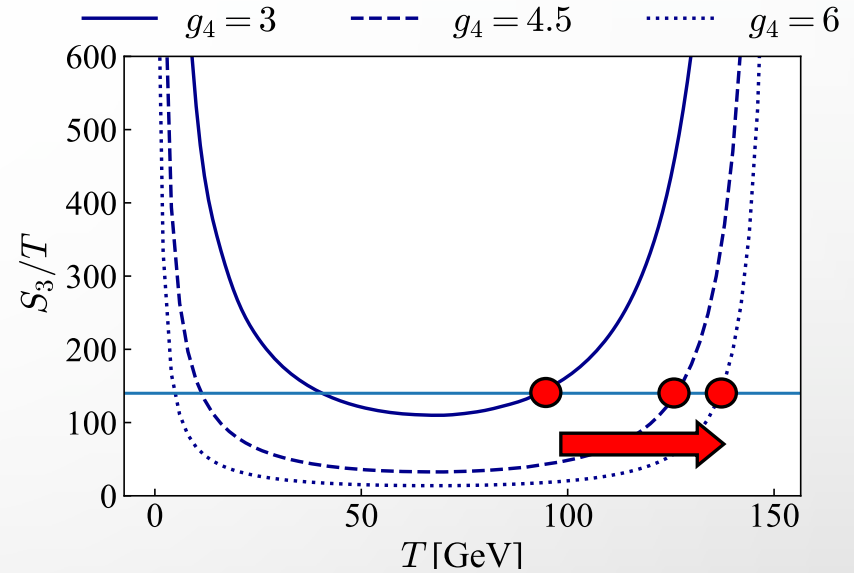
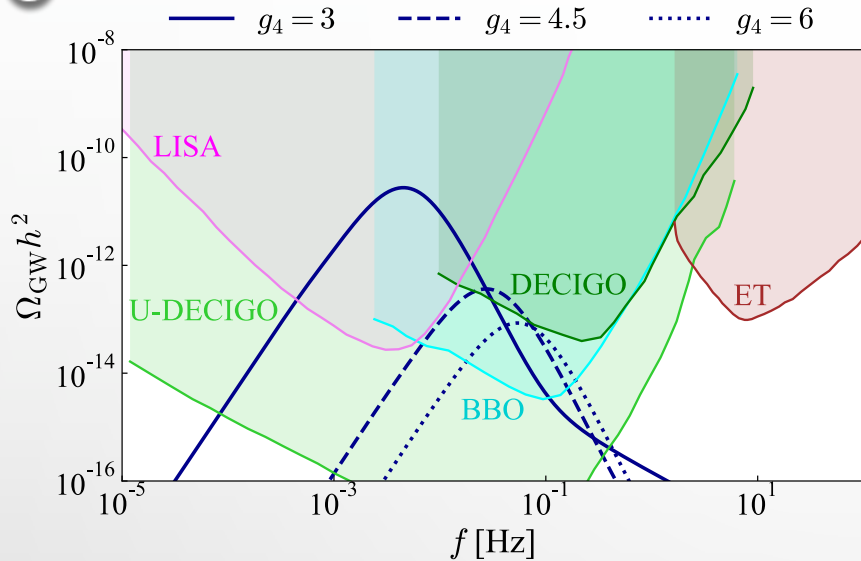


Definition of nucleate temperature

$$\left. \frac{S_3(T)}{T} \right|_{T=T_{nuc}} \simeq 140$$

- Whether 1st PT occurs depends on the coupling
- $g_4 \leq 3 \rightarrow$ 1st PT **does not occur**
- Increase $g_4 \rightarrow T_{nuc}$ approaches T_c

4. Results

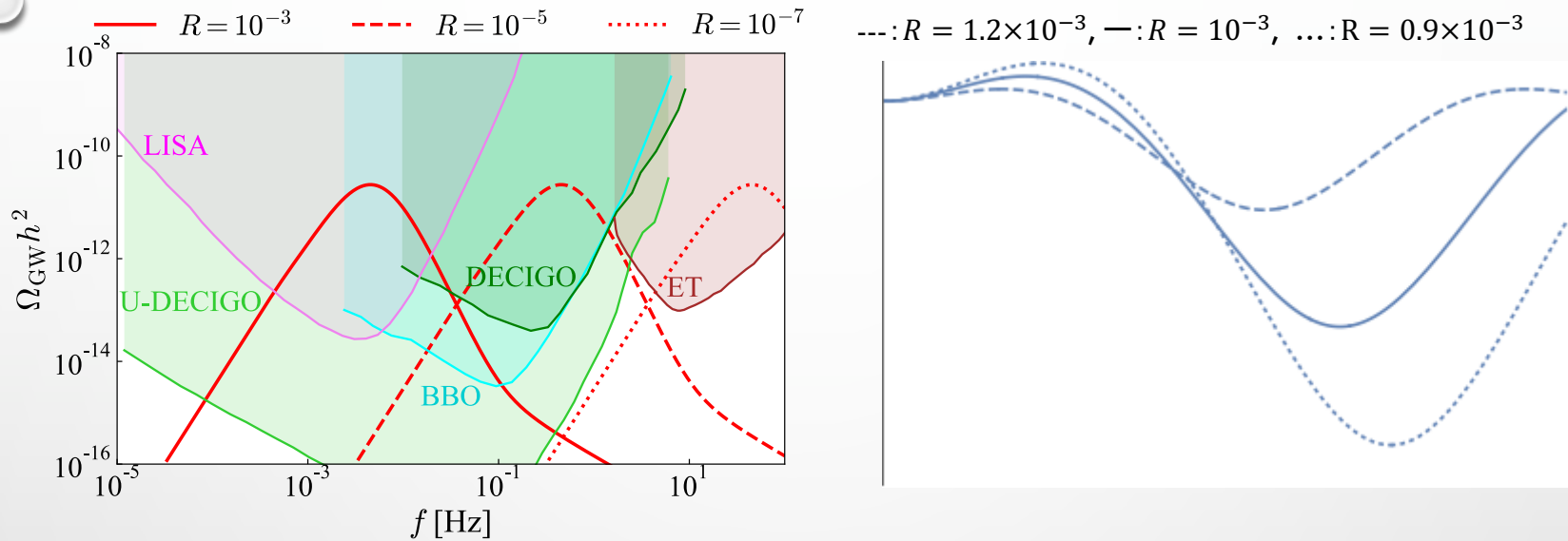


Increase $g_4 \rightarrow$ Spectrums move to lower right (left panel)

- Increase frequency \rightarrow Increase T_{nuc} , the tilt of S_3/T (right panel)
- Decrease energy density \rightarrow decrease latent heat α

$$\alpha = \frac{\epsilon(T_{nuc})}{\rho_{rad}(T_{nuc})}$$

4. Results



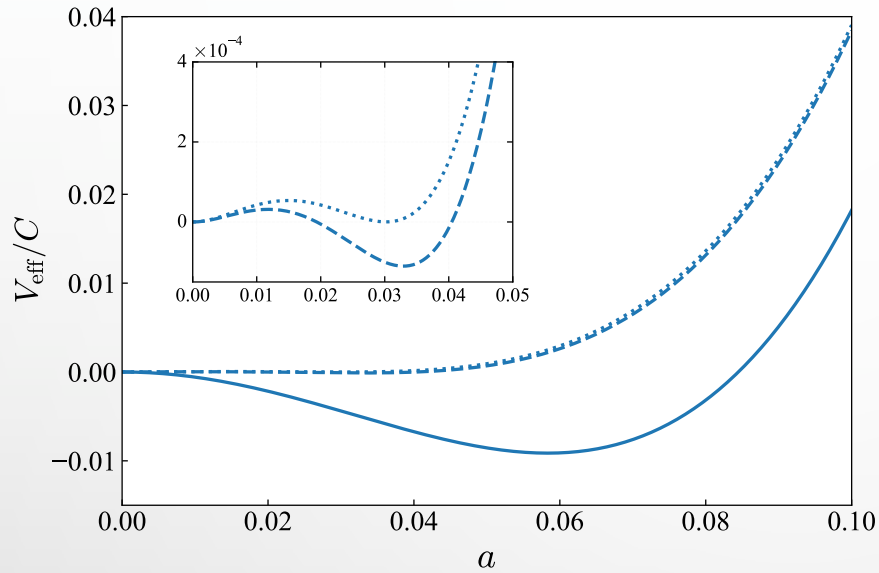
Increase $R^{-1} \rightarrow$ Spectrums shifts sideways (left panel)

- Energy density \rightarrow Potential changes with similarity (right panel)

$\rightarrow \alpha, \tilde{\beta}$ do not depend on R

- Increase frequency \rightarrow Because $T_{\text{nuc}} \sim R^{-1}$ increase

4. Results



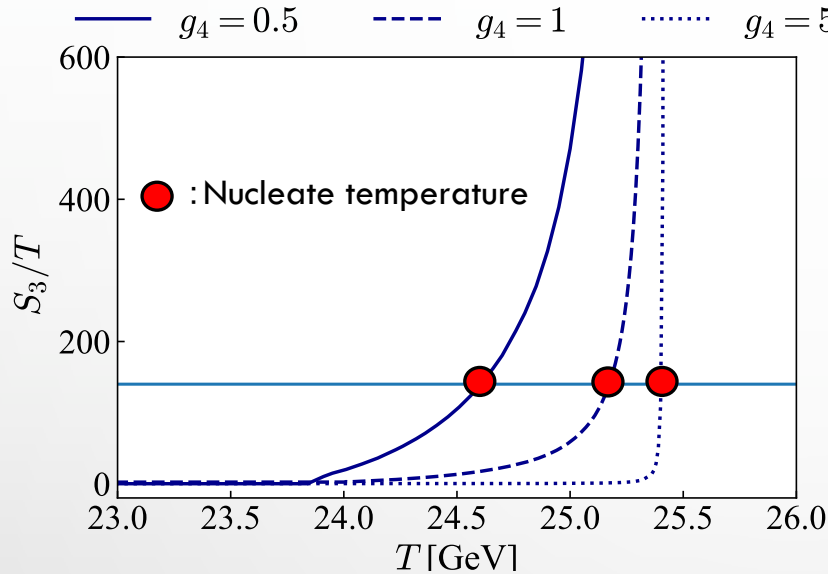
... : potential at critical temperature

--- : potential at nucleate temperature

— : potential at zero temperature

- To realize Higgs mass, introduce several fields
- **No potential barrier** at zero temperature
- critical temperature : $T_c = 25 \text{ GeV} @ R = 10^{-3}$

4. Results



$$\leftarrow R = 10^{-3}$$

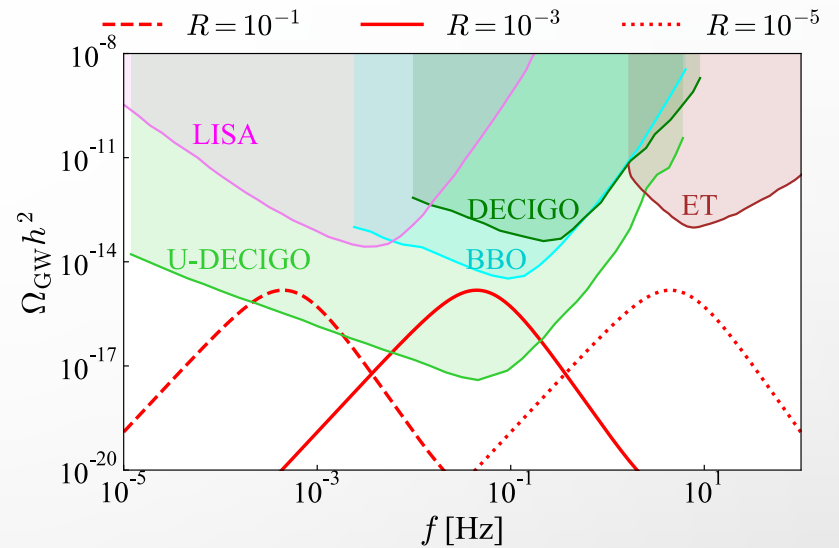
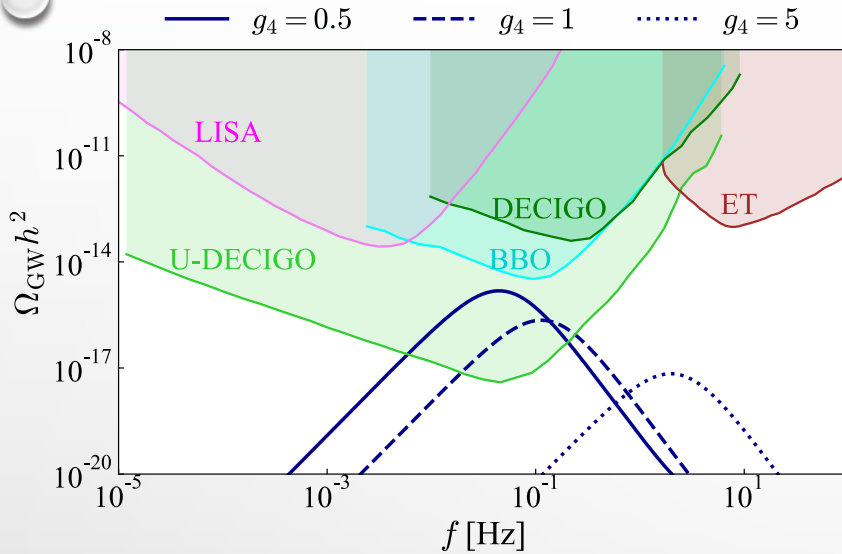
Definition of nucleate temperature

$$\left. \frac{S_3(T)}{T} \right|_{T=T_{nuc}} \simeq 140$$

- Increase $g_4 \rightarrow T_{nuc}$ approaches T_c
- T_{nuc} slightly change \rightarrow Because of small potential barrier
- Decrease temperature \rightarrow approach to potential at zero temperature

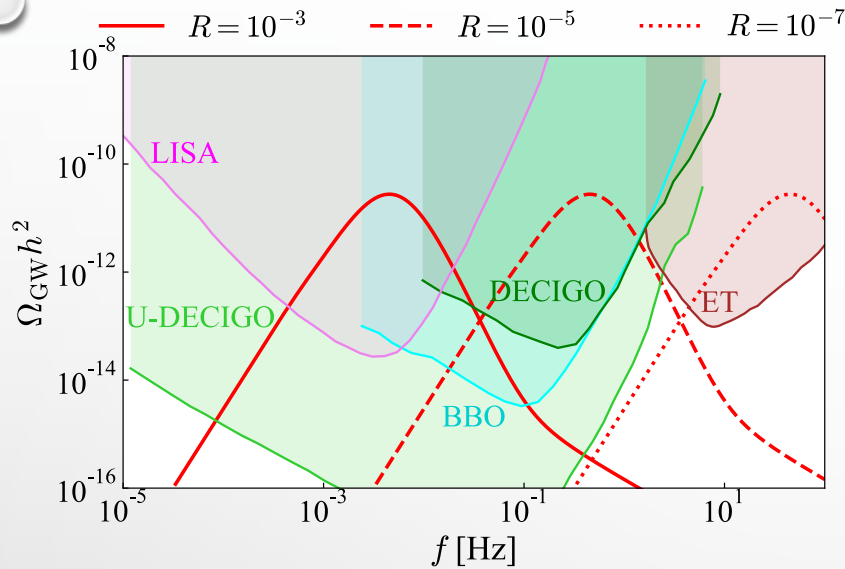
$\rightarrow S_3/T$ is always less than 140

4. Results



- Energy density is low because potential barrier (latent heat) is small
- In $g_4 \leq 3$, GW is detected by U-DECIGO ?
 → At $g_4 = 0.5$, $10^{-4} \text{ GeV} \leq R \leq 10^{-1} \text{ GeV}$

4. Results



	R [GeV ⁻¹]	v [GeV]
LISA	$10^{-4} - 10^{-1}$	$10^0 - 10^3$
DECIGO	$10^{-6} - 10^{-3}$	$10^2 - 10^5$
ET	$10^{-8} - 10^{-6}$	$10^5 - 10^7$

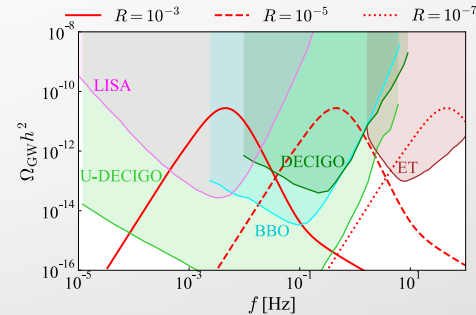
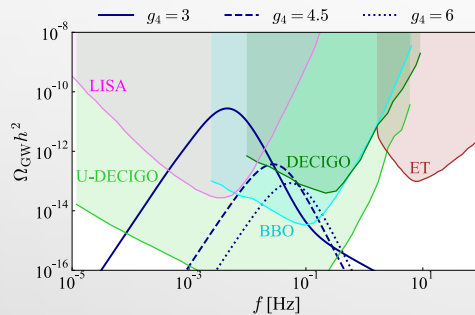
Possibility of verification LISA, DECIGO, Einstein telescope (ET)

Future Work: Extension to electroweak 1st PT



5. Summary

- We focus on a 5D SU(3) gauge theory and its 1st phase transition
- Relation between higher-dimensional gauge theory and 1st PT mainly depends on compact scale R and coupling in 4D g_4
- Possibility of verification in future experiments (LISA, DECIGO, ET...)



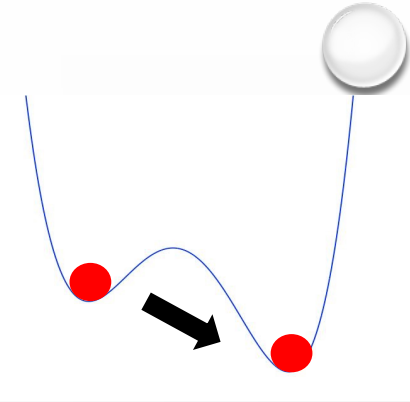
◆ Extension to electroweak 1st PT or another physics

- ✓ Application to gauge-Higgs unification
- ✓ Application to grand unified theory

The image features a light gray gradient background with several realistic water droplets of various sizes scattered in the corners. The droplets have highlights and shadows, giving them a three-dimensional appearance. The text 'Back up' is centered in the middle of the page.

Back up

Appendix

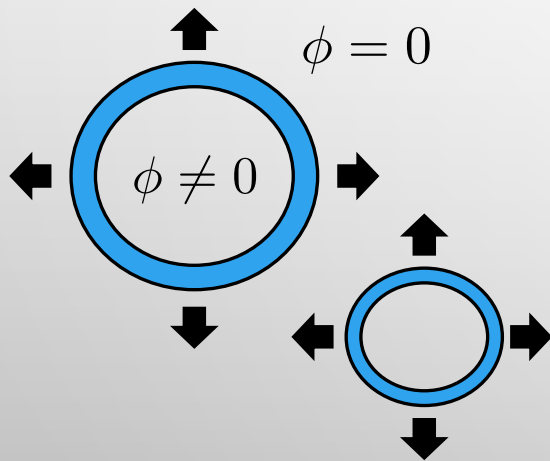


First-order Phase Transition

- The potential barrier exists between two minima
- The value of potential becomes discontinuous (the tunneling effect)



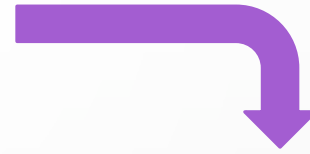
in the universe



- Bubbles ($\Phi \neq 0$) appear
- The bubbles expand and collide with other bubbles

Appendix

orbifold
 S^1/Z_2



Periodic boundary condition

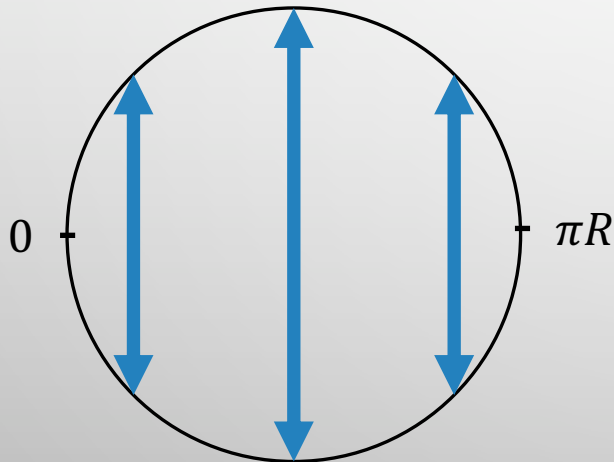
$$A_M(x^\mu, y + 2\pi R) = U A_M(x^\mu, y) U^\dagger$$

Z_2 Parity

$$A_\mu(x^\mu, -y) = P_0 A_\mu(x^\mu, y) P_0^{-1}$$

$$A_\mu(x^\mu, \pi R - y) = P_1 A_\mu(x^\mu, \pi R + y) P_1^\dagger$$

U, P_0, P_1 : Unitary matrices



Identification



Appendix

Symmetry Breaking Pattern

1. Explicitly breaking by P_0, P_1

$$P_0 = P_1 = \text{diag}(1, 1, -1)$$

$$SU(3) \rightarrow SU(2) \times U(1)$$

2. Spontaneously breaking by the VEV of 5th gauge field

[Hosotani (1983, 1989)]

$$SU(2) \times U(1) \rightarrow U'(1) \times U(1) \text{ or } U(1)$$

$$A_\mu = \left(\begin{array}{cc|c} \text{SU}(2) & & \\ \hline (+, +) & (+, +) & (-, -) \\ (+, +) & (+, +) & (-, -) \\ \hline (-, -) & (-, -) & \text{U}(1) \\ & & (+, +) \end{array} \right)$$

$$A_y = \left(\begin{array}{cc|c} \text{Scalar doublet} & & \\ \hline (-, -) & (-, -) & (+, +) \\ (-, -) & (-, -) & (+, +) \\ \hline (+, +) & (+, +) & (-, -) \end{array} \right)$$

VEV

$(P_0, P_1) = (+, +)$: zero modes of massless fields

Appendix

$$W = P \exp \left(ig \oint_{S^1} dy A_y \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i \sin(\pi a) \\ 0 & i \sin(\pi a) & \cos(\pi a) \end{pmatrix}$$

$SU(3)$ ^{orbifold} \rightarrow $SU(2) \times U(1)$ ^{spontaneous} \rightarrow change by a

- $a = 0$: $SU(2) \times U(1)$
- $a = 1$: $SU(2) \times U(1) \rightarrow U'(1) \times U(1)$
- $a \neq 0, 1$: $SU(2) \times U(1) \rightarrow U(1)$

Appendix


Periodic boundary condition & Z_2 parity

➔ Kaluza-Klein (KK) expansion

ex.)

$$A_\mu(x^\mu, y)_{(+,+)} = \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)}(x^\mu)_{(+,+)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\mu)_{(+,+)} \cos\left(\frac{ny}{R}\right)$$

We find KK mass from E.O.M

$$(\square + \partial_y \partial_y) A_\mu = 0$$

$$m_{KK}^2 = \frac{n^2}{R^2}, \frac{(n \pm a)^2}{R^2}, \dots$$

n : KK mode (integer)

Appendix

Effective potential (zero temperature)

$$V_{\text{eff}}^{T=0}(a) \equiv \underbrace{V_{\text{eff}}^g(a)}_{\text{gauge fields}} + \underbrace{V_{\text{eff}}^m(a)}_{\text{matter fields}}$$

$$V_{\text{eff}}^g(a) = -3C \sum_{n=1}^{\infty} \frac{1}{n^5} \left(\cos(2\pi na) + 2 \cos(\pi na) \right)$$

$$\begin{aligned} V_{\text{eff}}^m(a) &= (4N_f^{(+)} - 2N_f^{s(+)})C \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(\pi na) + (4N_f^{(-)} - 2N_f^{s(-)})C \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(\pi n(a-1)) \\ &\quad + 4N_{\text{ad}}^{(+)} C \sum_{n=1}^{\infty} \frac{1}{n^5} \left(\cos(2\pi na) + 2 \cos(\pi na) \right) \\ &\quad + 4N_{\text{ad}}^{(-)} C \sum_{n=1}^{\infty} \frac{1}{n^5} \left(\cos(2\pi n(a-1/2)) + 2 \cos(\pi n(a-1)) \right) \end{aligned}$$

$N_f^{(+)}$ など : degree of freedom

$$C = 3/(64\pi^6 R^4)$$

Appendix

Effective potential (finite temperature) [Maru, Takenaga (2005)]

Compactify S^1 in the direction of time

→ Introduce T as its radius.

$$\begin{aligned} V_{\text{eff}}^{T \neq 0}(a, T) = & 2C \sum_{l=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{[n^2 + l^2 / (2\pi RT)^2]^{5/2}} \left[\left(-3 + 4(-1)^l N_{\text{ad}}^{(+)} \right) \left(\cos(2\pi na) + 2 \cos(\pi na) \right) \right. \\ & + 4(-1)^l N_{\text{ad}}^{(-)} \left(\cos(2\pi n(a - 1/2)) + 2 \cos(\pi n(a - 1)) \right) \\ & + \left(4(-1)^l N_f^{(+)} - 2N_f^{s(+)} \right) \cos(\pi na) \\ & \left. + \left(4(-1)^l N_f^{(-)} - 2N_f^{s(-)} \right) \cos(\pi n(a - 1)) \right] \end{aligned}$$

Appendix

Introduce matter fields [Haba, Hosotani, Kawamura, Yamashita (2004)]

$$\begin{aligned}\phi(x, -y) &= \eta P_0 \phi(x, y) \quad , \quad \phi(x, \pi R - y) = \eta' P_1 \phi(x, \pi R + y) \quad , \\ \psi(x, -y) &= \eta P_0 \gamma^5 \psi(x, y) \quad , \quad \psi(x, \pi R - y) = \eta' P_1 \gamma^5 \psi(x, \pi R + y) \quad , \\ \psi^a(x, -y) &= \eta P_0 \gamma^5 \psi^a(x, y) P_0^\dagger \quad , \quad \psi^a(x, \pi R - y) = \eta' P_1 \gamma^5 \psi^a(x, \pi R + y) P_1^\dagger\end{aligned}$$

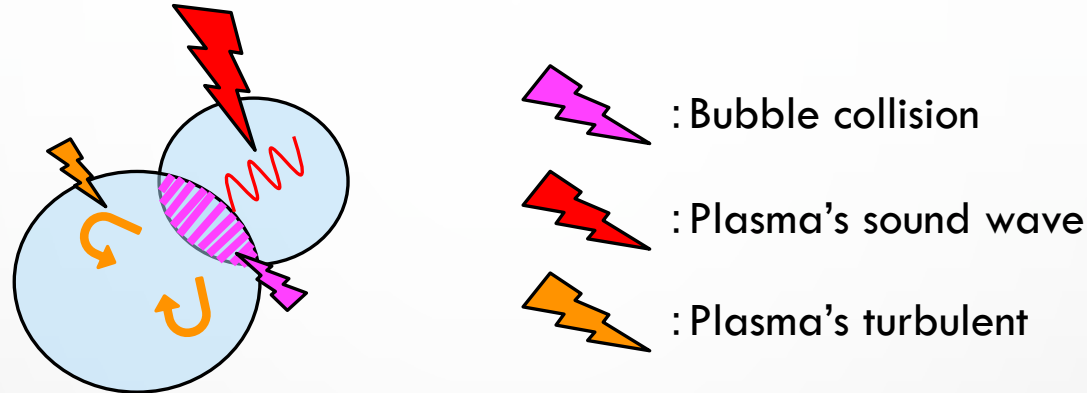
$(\eta, \eta' = \pm)$

ϕ : Scalar in fundamental rep.

ψ : Fermion in fundamental rep.

ψ^a : Fermion in adjoint rep.

3. GW from First Phase Transition



- Contributions to GW is the sum of above three sources
- To computing the spectrum of GW, need numerical calculation
- Approximate calculation has known [Caprini et al. (2015)]

✧ Plasma's sound wave (main contribution)

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left(\frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{\frac{7}{2}}$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \text{Hz} v_w^{-1} \tilde{\beta} \left(\frac{T_{\text{nuc}}}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}}$$

Appendix

Detail for approximate calculation [Caprini et al. (2015)]

- Peak of energy density

$$\Omega_{\text{env}} h^2 = 1.67 \times 10^{-5} \tilde{\beta}^{-2} \left(\frac{\kappa_{\varphi} \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) \frac{3.8 (f/f_{\varphi})^{2.8}}{1 + 2.8 (f/f_{\varphi})^{3.8}},$$

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left(\frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{\frac{7}{2}},$$

$$\Omega_{\text{turb}} h^2 = 3.35 \times 10^{-4} \tilde{\beta}^{-1} \left(\frac{\kappa_{\text{turb}} \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left(\frac{100}{g_*} \right)^{\frac{1}{3}} v_w \frac{(f/f_{\text{turb}})^3}{[1 + (f/f_{\text{turb}})]^{\frac{11}{3}} (1 + 8\pi f/h_n)}$$

- Peak of gravitational frequency

$$f_{\text{env}} = 1.65 \times 10^{-5} \text{Hz} \tilde{\beta} \left(\frac{0.62}{1.8 - 0.1 v_w + v_w^2} \right) \left(\frac{T_{\text{nuc}}}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}},$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \text{Hz} v_w^{-1} \tilde{\beta} \left(\frac{T_{\text{nuc}}}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}},$$

$$f_{\text{turb}} = 2.7 \times 10^{-5} \text{Hz} v_w^{-1} \tilde{\beta} \left(\frac{T_{\text{nuc}}}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}}$$