Gravitational waves from Gauss-Bonnet-corrected single-field inflation

[based on 2308.13272 and 2108.01340 (with Shinsuke Kawai)]

Jinsu Kim
Tongji University, Shanghai, China

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Inflation with Gauss-Bonnet coupling

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{P}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) - \frac{1}{16}\xi(\varphi)R_{\mathrm{GB}}^2\right]$$

$$R_{\mathrm{GB}}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- Higher-curvature correction [Weinberg (2008)]
- 1-loop correction to some superstring models [Antoniadis, Rizos, and Tamvakis (1994)], [Rizos and Tamvakis (1994)]
- Widely studied in the context of cosmology, including slow-roll inflation [Kawai, Sakagami, and Soda (1998)], [Kawai, Sakagami, and Soda (1999)], [Kawai and Soda (1999)], [Satoh and Soda (2008)], [Satoh (2010)], [Kawai and JK (2019)], ...

Inflation with Gauss-Bonnet coupling

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$

- Potential term $\gg$ GB term:
  - usual slow-roll inflation with GB coupling as a small correction
- Potential term $\approx$ GB term:
  - Balance; cancellation when $V_{,\varphi}\xi_{,\varphi} < 0$
  - first part of the talk
- Potential term $\ll$ GB term:
  - GB dominance
  - second part of the talk

Balance regime

Potential term $\approx$ GB term with $V_{,\varphi}\xi_{,\varphi} < 0$

$$
\begin{aligned}
0 &= \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi} \\
&\implies \ddot{\varphi} + 3H\dot{\varphi} \approx 0\,.
\end{aligned}
$$

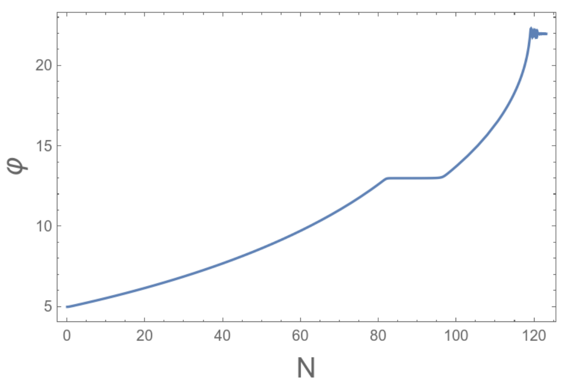
- Ultra-slow-roll inflation

[Inoue and Yokoyama (2002)], [Kinney (2005)],... see also [Kawaguchi and Tsujikawa (2022)]

Ultra-slow-roll with Gauss-Bonnet coupling

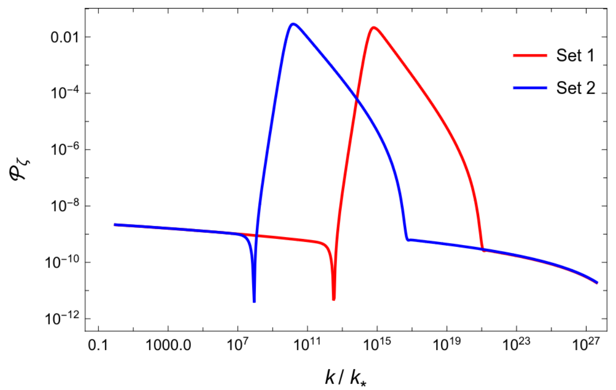
For concreteness, let us consider

$$V = \Lambda^4 \left(1 + \cos\frac{\varphi}{f}\right) , \quad \xi = \xi_0 \tanh\left[\xi_1(\varphi - \varphi_c)\right] .$$



$$\{f[M_{\rm P}], \varphi_c[M_{\rm P}], \xi_0, \xi_1[M_{\rm P}^{-1}]\}$$
$$= \{7, 13.0, 6.044 \times 10^7, 15.0\}$$

Curvature power spectrum



Curvature perturbation enhanced

- $A_s \sim 1/\epsilon_1$ or $\sim 1/\dot{\varphi}$
- production of primordial black holes
- scalar-induced gravitational waves

3 free parameters for a given f:

- φ_c : peak position
- ξ_1 : width of the peak
- ξ_0 : magnitude of the peak

Primordial black holes

When very large density fluctuations re-enter the horizon, primordial black holes may form due to the gravitational collapse. [Zel'dovich and Novikov (1967)], [Hawking (1971)], [Carr and Hawking (1974)], [Polnarev and Khlopov (1985)], ...

Abundance of the PBHs:

$$f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH,0}}{\Omega_{\rm DM,0}} \approx \left(\frac{\beta(M)}{3.27\times 10^{-8}}\right)\left(\frac{0.2}{\gamma}\right)^{3/2}\left(\frac{106.75}{g_{*,f}}\right)^{1/4}\left(\frac{0.12}{\Omega_{\rm DM,0}h^2}\right)\left(\frac{M}{M_\odot}\right)^{-1/2}$$

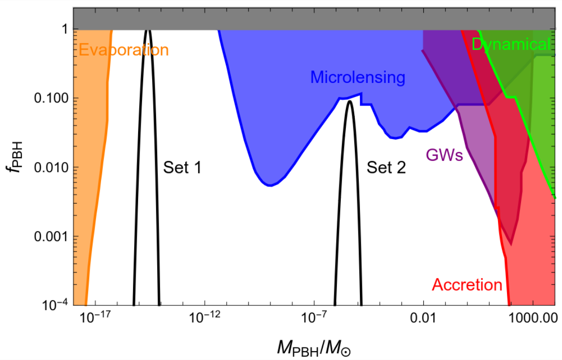
$$\beta = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\delta^2}{2\sigma^2}}, \quad \sigma^2 = \frac{16}{81}\int_0^\infty \frac{dq}{q}\left(\frac{q}{k}\right)^4 W^2\left(\frac{q}{k}\right)\mathcal{P}_\zeta(q)$$

Assumptions:

- Density fluctuation follows a Gaussian distribution.
- PBH formation occurs during radiation-dominated era.

Primordial black holes

constraints data : [Green and Kavanagh (2021)]



- Set 1 : $f_{\rm PBH}^{\rm tot} \approx 1$
- Set 2 : $f_{\rm PBH}^{\rm tot} \approx 0.087$

Scalar-induced gravitational waves

[Matarrese, Mollerach, and Bruni (1998)], [Mollerach, Harari, and Matarrese (2004)], [Ananda, Clarkson, and Wands (2007)], [Baumann, Steinhardt, Takahashi, and Ichiki (2007)], [Kohri and Terada (2018)], [Domènech (2020)]

Enhanced curvature perturbation : a source, $S_{\mathbf{k}}$, for the tensor perturbation

$$h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = S_{\mathbf{k}}$$

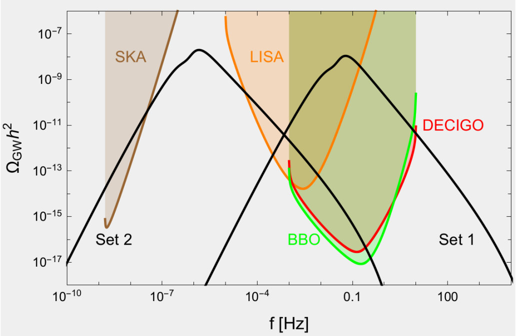
Scalar-induced second-order gravitational waves

Scalar-induced gravitational waves

[Kohri and Terada (2018)]

$$
\begin{aligned}
\Omega_{\mathrm{GW,f}}(k) = &\frac{1}{12}\int_0^\infty dv \int_{|1-v|}^{1+v} du \\
&\times \left(\frac{4v^2 - (1+v^2-u^2)^2}{4uv}\right)^2 \\
&\times \mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)\left(\frac{3(u^2+v^2-3)}{4u^3v^3}\right)^2 \\
&\times \left[\left(-4uv + (u^2+v^2-3)\log\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 \right. \\
&\left. + \pi^2(u^2+v^2-3)^2\theta(v+u-\sqrt{3})\right].
\end{aligned}
$$

sensitivity curves : [Schmitz (2021)]



Inflation with Gauss-Bonnet coupling

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$

- Potential term $\approx$ GB term:
  - Balance; cancellation when $V_{,\varphi}\xi_{,\varphi} < 0$
  - SR $\rightarrow$ USR $\rightarrow$ SR
  - primordial black hole formation
  - scalar-induced gravitational waves
- Potential term $\ll$ GB term:
  - GB dominance
  - SR $\rightarrow$ GB domination $\rightarrow$ SR
  - with $V_{,\varphi}\xi_{,\varphi} > 0$

Gauss-Bonnet domination regime

$$0 = \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} + \frac{3}{2}H^2\left(\dot{H} + H^2\right)\xi_{,\varphi}$$

We again consider

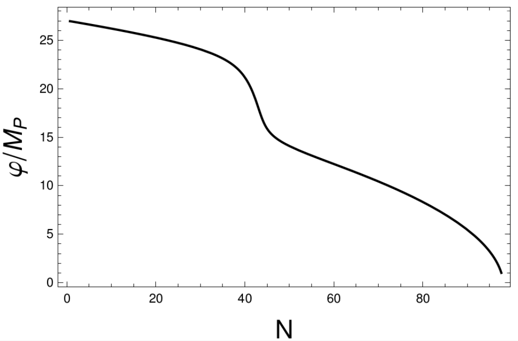
$$\xi = \xi_0 \tanh\left[\xi_1(\varphi - \varphi_c)\right] \, .$$

As $\xi_{,\varphi} \sim \mathrm{sech}^2[\xi_1(\varphi - \varphi_c)]$, the GB term may become dominant.

φ away from φ_c	φ close to φ_c	φ away from φ_c
SR	GB domination	SR

Gauss-Bonnet domination regime

For completeness, let us consider $V = m^2\varphi^2/2$.



$$\{\xi_0, \xi_1, \varphi_c\} = \{0.196 M_{\rm P}^2/m^2, 0.5/M_{\rm P}, 18.5 M_{\rm P}\}$$

- sudden acceleration → deceleration
- opposite to the USR case
- surge of gravitational wave

Tensor perturbation during the Gauss-Bonnet domination regime

$$v''_{\mathbf{k}} + \left(k^2 C_t^2 - \frac{A_t''}{A_t} \right) v_{\mathbf{k}} = 0$$

$$A_t^2 \equiv a^2 \left(1 - \frac{\sigma_1}{2} \right) , \quad C_t^2 \equiv 1 + \frac{a^2 \sigma_1}{2 A_t^2} (1 - \sigma_2 - \epsilon_1) , \quad \sigma_1 \equiv \frac{H \dot{\xi}}{M_{\mathrm{P}}^2} , \quad \sigma_2 \equiv \frac{\dot{\sigma}_1}{H \sigma_1}$$

- Away from φ_c :

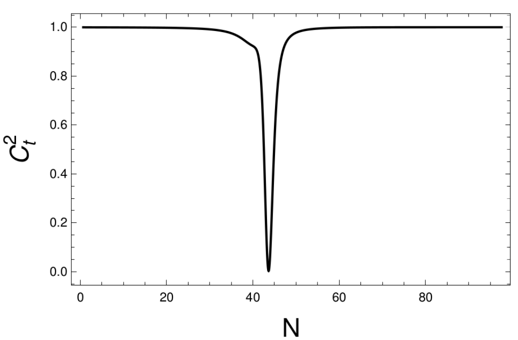
$$A_t \approx a , \quad C_t^2 \approx 1 \quad \Longrightarrow \quad \text{standard tensor perturbation}$$

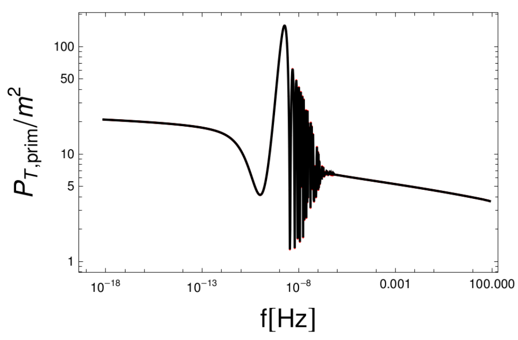
- Near φ_c :

$$C_t^2 \ll 1$$

Primordial tensor power spectrum

$$\{\xi_0, \xi_1, \varphi_c\} = \{0.196 M_\mathrm{P}^2/m^2, 0.5/M_\mathrm{P}, 18.5 M_\mathrm{P}\}$$



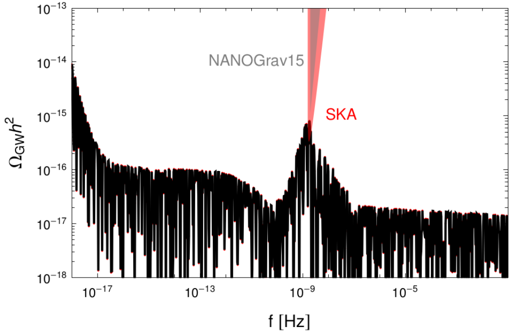


see also [Satoh, Kanno, and Soda (2008)], [Guo and Schwarz (2009)], [Cai, Wang, and Piao (2015, 2016)], [Cai and Piao (2022)]

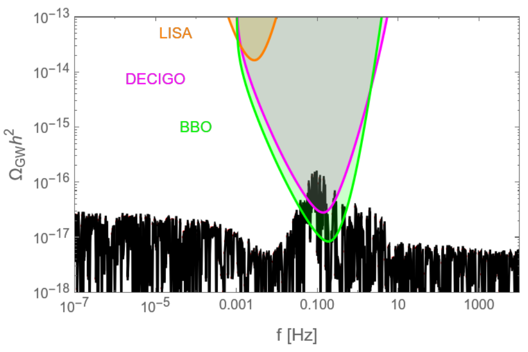
Gravitational wave spectrum

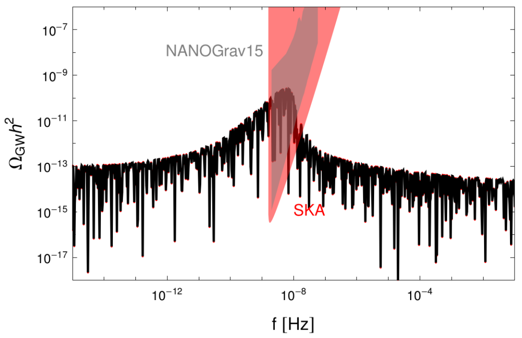
[Guzzetti, Bartolo, Liguori, and Matarrese (2016)], [Kuroyanagi, Takahashi, and Yokoyama (2015, 2021)], [Boyle and Steinhardt (2008)], ...

$$\Omega_{\mathrm{GW}}(k) = \frac{1}{12}\left(\frac{k}{a_0 H_0}\right)^2 T^2(k)\mathcal{P}_{\mathrm{T,prim}}$$



Gravitational wave spectrum





Summary

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- Balance; $V_{,\varphi}\xi_{,\varphi} < 0$
- SR $\to$ USR $\to$ SR
- PBH formation
- induced gravitational waves

Potential term $\ll$ GB term

- GB dominance; $V_{,\varphi}\xi_{,\varphi} > 0$
- SR $\to$ GB domination $\to$ SR
- primordial inflationary gravitational waves enhanced

Thank you