Primordial black hole formation from primordial perturbations

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Gravitational Wave Probes beyond Standard Model @Osaka Metropolitan University, 6-9 Nov 2023

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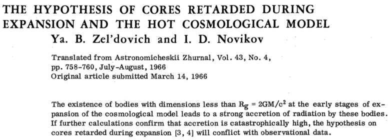
Introduction

- Primordial black holes (PBHs) are black holes formed in the early Universe (Zeldovich & Novikov (1967), Hawking (1971)).
    - Fossils of the early Universe
    - Dark matter candidate
    - Hawking evaporation
    - High-energy physics
    - GW sources

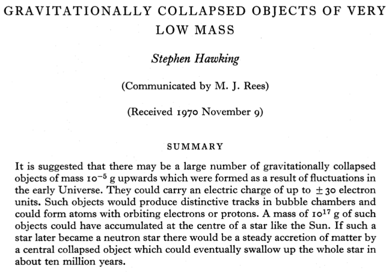


THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel'dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4,
pp. 758–760, July–August, 1966
Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.



GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

(Communicated by M. J. Rees)

(Received 1970 November 9)

SUMMARY

It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ±30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

Introduction

- Observational constraint on the abundance of PBHs
  - Dark matter mass windows: $\sim 10^{16} - 10^{23}$ g and $\sim 1 - 10^{3} M_{\odot}$, while severely constrained in other mass ranges.

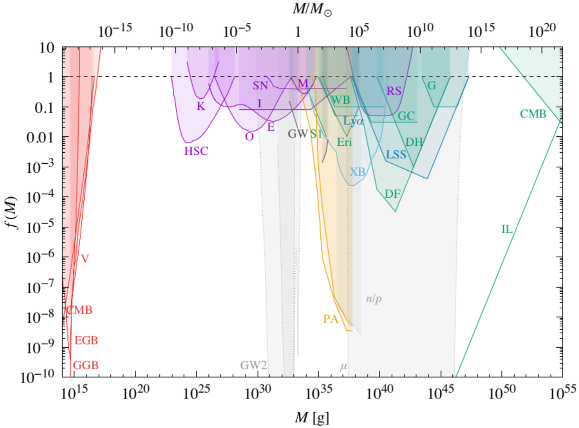


Figure: $f(M) = \Omega_{PBH}/\Omega_{CDM}$ (Carr et al. (2021))

GW Observations

- Many BBHs of $\sim 30 M_\odot$ discovered by GW observation
  - Those BHs may be of cosmological origin. (Sasaki et al. (2016), Bird et al. (2016), Clesse & Garcia-Bellido (2017)).
  - Constraint on the spin parameter $\chi_{\rm eff}$ of LIGO BBHs (Abbott et al. (2017)), Search for PBH population in LIGO-Virgo BBHs (Franciolini et al. (2022))
- Evidence or detection of nHz GWs by NANOGrav (Agazie et al. (2023), ...) and other PTAs.
  - Maybe consistent with the secondary GWs of scalar perturbations that may have produced PBHs of solar mass or subsolar masses (Kohri & Terada (2021), Inomata et al. (2023), ...).

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Mass of PBHs

- PBH mass is approximately equal to the mass enclosed within the cosmological horizon at the formation.

$$M \simeq M_H(t_f) \simeq \frac{c^3}{G} t_f \simeq 1 M_\odot \left(\frac{t_f}{10^{-5}\ \mathrm{s}}\right), \quad R_g \simeq 1\ \mathrm{km} \left(\frac{M}{M_\odot}\right).$$

- The mass accretion does not significantly affect the initial mass (Carr & Hawking (1974) ...).
- Hawking evaporation (Hawking (1974))

$$\begin{aligned} T_H &= \frac{\hbar c^3}{8\pi G M k} \simeq 100\ \mathrm{MeV} \left(\frac{M}{10^{15}\mathrm{g}}\right)^{-1}, \\ \frac{dM}{dt} &= -\frac{g_{\mathrm{eff}} \hbar c^4}{15360 \pi G^2 M^2}, \quad t_{\mathrm{ev}} \simeq \frac{G^2 M^3}{g_{\mathrm{eff}} \hbar c^4} \simeq 10\ \mathrm{Gyr} \left(\frac{M}{10^{15}\mathrm{g}}\right)^3. \end{aligned}$$

Thus, they have dried up until now for $M \lesssim 10^{15}$ g.

Abundance of PBHs

- $\beta(M)$: The fraction of the Universe which goes into PBHs
- $f(M)$: The fraction of PBHs to all of the CDM at the present time

$$f(M) = \left.\frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{CDM}}}\right|_{t=t_0}.$$

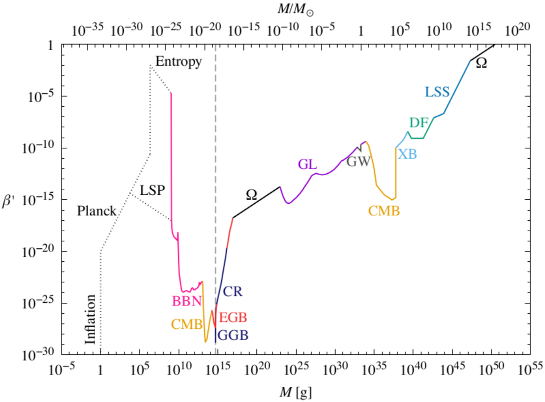
- PBHs are condensed during the radiation-dominated (RD) era because they behave as nonrelativistic particles, so

$$\beta(M) \simeq 2 \times 10^{-18} \left(\frac{M}{10^{15}\ \mathrm{g}}\right)^{1/2} f(M)$$

for $M > 10^{15}$ g if they are formed in the RD era.

Constraint on $\beta(M)$

- Constraint on $\beta(M)$ (Carr (1975), Carr et al. (2021))



Taken from Carr et al. (2021). The dotted lines below 10^9 g involve less secure assumptions.

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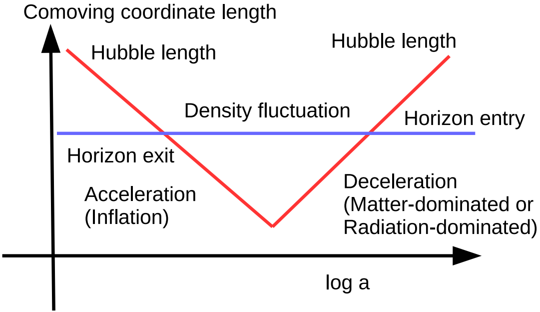
Formation studies of PBHs

- Central question: Can we predict $\beta(M)$ and other properties of PBHs for a given cosmological scenario?
- Possible mechanism
  - Conventional: growth of primordial fluctuations generated by inflation
  - New physics: collapse of domain walls, bubble nucleation, collision of bubbles, phase transitions, ...
- We here focus on the conventional scenario. Key ideas are
  - Primordial fluctuation generated by inflation
  - Cosmological long-wavelength solutions
  - Formation threshold
  - Critical behaviour
  - Abundance estimate

Fluctuation generated in inflation

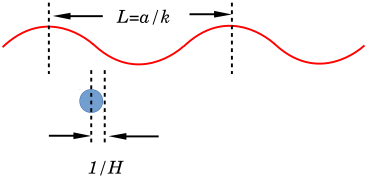
- The scales of perturbations of super-horizon scale generated in inflation enter the horizon in the decelerated expansion.



- Inflation gives the power spectrum $P_\zeta(k)$ and the statistics of curvature perturbations ζ and thereby the standard deviation $\sigma(k)$ and the statistics of density perturbation δ. See Sasaki et al. (2018).

Cosmological longwave-length solutions

- Initial data: long-wavelength solutions obtained by the gradient expansion in powers of $\epsilon = k/(aH) \ll 1$ (Shibata & Sasaki (1999), Polnarev & Musco (2007), Harada, Yoo, Nakama & Koga (2015))



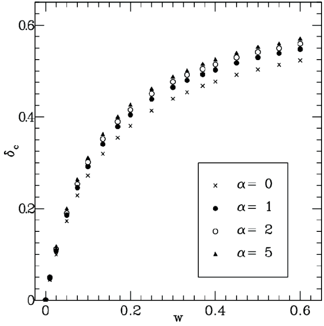
- $3+1$ decomposition of spacetime

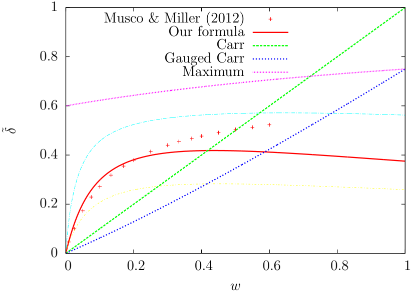
$$ds^2 = -\alpha^2 dt^2 + e^{2\zeta} a^2(t) \tilde{\gamma}_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

  where $\tilde{\gamma}_{ij}$ is chosen so that $\det(\tilde{\gamma}_{ij}) = \det(\eta_{ij})$ and ζ in the uniform-density slice is called curvature perturbation.
- The long-wavelength solutions have $\zeta = O(1)$ but $\delta = O(\epsilon^2)$.
- Only a nonliearly large amplitude of perturbation can form a PBH.

Formation threshold

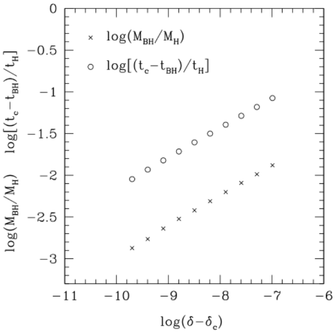
- PBH formation needs numerical relativity even in spherical symmetry.
- Threshold $\delta_{\rm th} \simeq \mathbf{0.45}$ for the RD $p = \rho/3$ for the averaged density perturbation δ_H. Alternatively, $C \simeq \mathbf{0.4}$ in terms of the compaction function C. (Carr (1975), Shibata & Sasaki (1999), ...)
- EOS dependence ($p = w\rho$), implying enhancement for a soft EOS. The Jeans criterion works. (Musco & Miller (2013), Harada, Yoo & Kohri (2013))





Critical behaviour

- The perturbation collapse to a BH if $\delta_H > \delta_{\rm th}$, while it doesn't if $\delta_H < \delta_{\rm th}$. In such a situation, there appears critical behaviour with universality, self-similarity and power-law scaling laws. (Choprtuik 1993, ...)
- PBH critical behaviour (Niemeyer & Jedamzik (1999), Musco & Miller (2013)) There appears mass scaling law. $M_{BH} \approx kM_H(\delta - \delta_{\rm th})^\gamma, \quad \gamma \simeq 0.36$

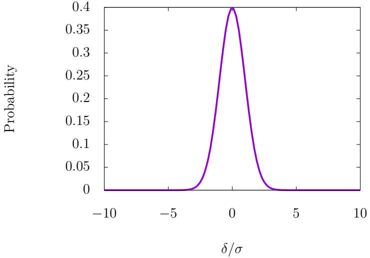


Abundance estimate

- Assuming δ_H obeys a Gaussian distibution, Carr (1975) obtained

$$\beta \simeq 2\frac{1}{\sqrt{2\pi}\sigma}\int_{\delta_{\rm th}}^{\delta_{\rm max}} d\delta e^{-\frac{\delta^2}{2\sigma^2}} \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma}{\delta_{\rm th}}e^{-\delta_{\rm th}^2/(2\sigma^2)}.$$

  This is also called Press-Schechter. So, $\sigma \gtrsim 0.05$ or $P_\zeta \gtrsim 0.01$ is needed to have cosmologically interesting amount of abundance.
- This means typically $\delta_{\rm th} \sim 8\sigma$. Only the tail of the distribution is responsible. The non-Gaussianity is crucial.



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Revisiting the compaction function

- PBH formation needs numerical relativity even in spherical symmetry. Shibata & Sasaki (1999) introduced the compaction function to intend

$$C_{\rm SS} := \frac{1}{R}\int_V d\Sigma\delta\rho\bigg|_{\rm CMC} \overset{?}{=} \frac{\delta M}{R}\bigg|_{\rm CMC}$$

  in place of δ_H. The threshold $\simeq 0.4$ is empirically very robust. (Musco (2018), Escriva et al. (2020), Escriva et al. (2021)).

- However, it turns out the second equality is not correct but instead

$$C_{\rm SS} \approx \frac{1}{2}[1-(1+r\zeta')^2] \approx \frac{3w+5}{3(1+w)}\frac{\delta M}{R}\bigg|_{\rm com} \neq \frac{\delta M}{R}\bigg|_{\rm CMC},$$

  with the curvature perturbation $\zeta(r)$ for $p = w\rho$. (Harada, Yoo, Nakama & Koga (2015), Harada, Yoo & Koga (2023))

Spins of PBHs formed in RD

- The spins of PBHs formed in RD era are estimated to $a_* = O(10^{-3})$ but can be larger for $M \ll M_H$. (De Luca et al. 2020, Harada, Yoo, Kohri, Koga & Monobe (2021)).
- The distribution of the effective spin parameter $\chi_{\rm eff}$ of BBHs (Koga, Harada, Tada, Yokoyama & Yoo (2022)): This can be compared with the data of GW observation.

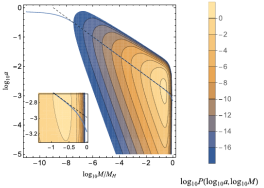


Figure 1. A contour plot of $\log_{10}P(\log_{10}a, \log_{10}M)$ for $\nu_{\rm th} = 10$ and $\sigma^0 = 0.192$ (see discussions in Section 2.3). The solid line shows the expected value $\langle a \rangle$ for each M, while the dashed line is its power-law fitting $\propto (M/M_{\rm H})^{-1/3}$

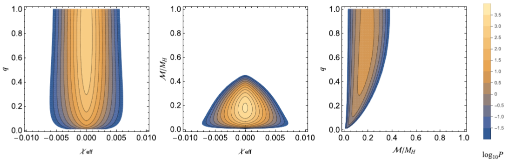
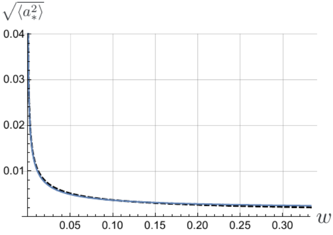
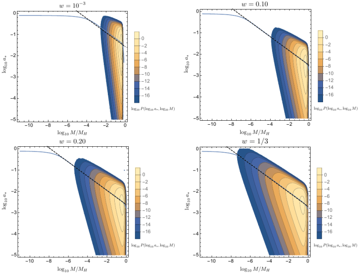


Figure 2. Contour plots of $\log_{10}P(\chi_{\rm eff}, q)$ (left), $\log_{10}P(\chi_{\rm eff}, \mathcal{M})$ (middle), and $\log_{10}P(\mathcal{M}, q)$ (right) for $\nu_{\rm th} = 10$ and $\sigma^0 = 0.192$ which correspond to $f_{\rm PBH} \sim 0.1\%$ for $M_{\rm H} \sim M_{\odot}$.

Spins of PBHs formed with a soft EOS

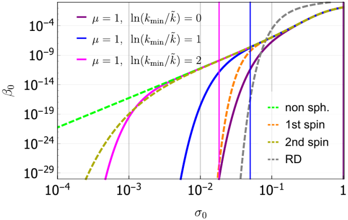
- The spins of PBHs formed in MD era ($w = 0$) can be very large (Harada, Yoo, Kohri & Nakao (2017)).
- δ_{th} drops for a soft EOS. The PBH production is significantly enhanced in the QCD crossover (Musco et al. (2023)).
- So, what happens to spins of PBHs formed with a soft EOS? The rms of the spin is a decreasing function of w. The spin is only modestly enhanced for the QCD crossover. (Saito, Harada, Koga & Yoo (2023))

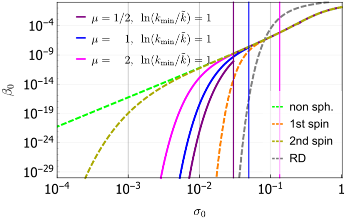




Velocity dispersion in (early) MD era

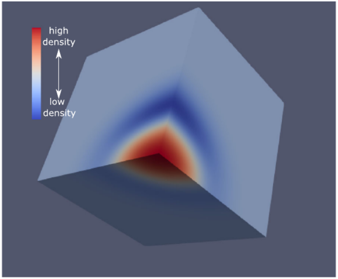
- In the MD era, the threshold is very small ($\delta_{\rm th} \simeq 0$) for spherical collapse. In this case, nonspherical effects play important roles such as anisotropies, spins and inhomogeneities (Khlopov & Polnarev (1980), Harada, Yoo, Kohri, Nakao & Jhingan (2016), Harada, Yoo, Kohri, Nakao (2017), Kokubu, Kohri & Harada (2018)).
- Dark matter is not dust ($p = 0$) but with small velocity dispersion, which effectively acts as pressure and can impede PBH formation (Harada, Kohri, Sasaki, Terada & Yoo (2023)).

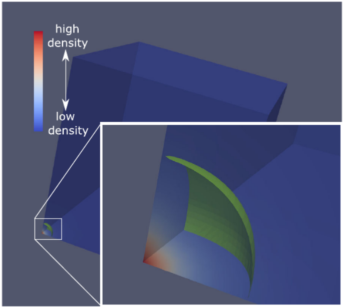




Nonspherical PBH formation

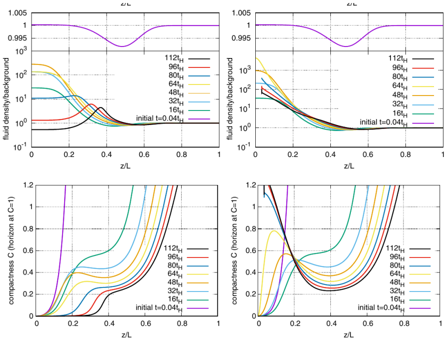
- Nonsphericity in the initial condition should be typically very small from the prediction of peak theory for PBHs formed in RD era.
- 3D numerical relativity simulations show that nonsphericity affects the threshold very little (Yoo, Harada & Okawa (2020)).





PBH formation from isocurvature perturbation

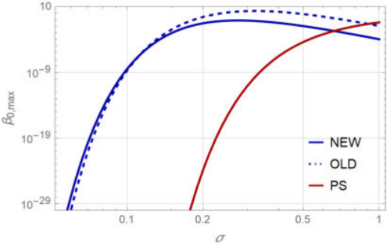
- Isocurvature perturbations may be generated in some cosmological scenarios. We assume a radiation fluid + a massless scalar field ϕ and construct isocurvature long-wavelength solutions with large $\delta\phi$ but with $\delta\rho_{\mathrm{rad}} + \delta\rho_{\phi} \approx 0$. Numerical relativity simulations show that PBHs really form (Yoo, Harada, Hirano, Okawa & Sasaki 2023).



- Massless scalar field + massive scalar field (De Jong et al. (2021, 2023))

Abundance of PBHs in peak theory

- Carr's formula assumes that δ_H takes Gaussian distribution but that is questionable for nonlinearly large amplitude.
- The peak theory combined with the PBH threshold would give a better estimate of β if we take the Laplacian of ζ as a random field. The new estimate is much larger than that by Carr's (labelled PS) by several orders of magnitude (Yoo, Harada, Garriga & Kohri (2018), Yoo, Harada, Yoo, Harada, Hirano & Kohri (2021)).



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Summary

- PBHs are fossils of the early Universe
    - LVK's BBHs might be of cosmological origin.
    - A window for PBH dark matter is now wide open.
    - PTA nHz GWs give a new perspective.
- Basics of PBHs
    - PBH mass ~ Mass contained within the horizon at the formation
    - PBHs have dried up due to Hawking evaporation for $M \lesssim 10^{15}$ g.
    - The abundance can be transformed to the probability of PBH formation.
- Formation and abundance studies of PBHs
    - Key words: long-wavelength solutions, numerical relativity, threshold, critical behaviour, statistics, etc.
    - Many open problems mainly due to difficulties in nonlinearity
    - Perspective on PBH studies with new physics