Hellings-Downs curve deformed by ultra-light vector dark matter

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@Gravitational Wave Probes of Physics Beyond Standard Model

Introduction

- NANOGrav found evidence of the stochastic gravitational wave background!

The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

NANOGrav Collaboration • Gabriella Agazie (Marquette U.) Show All(114)
Jun 28, 2023

24 pages
Published in: *Astrophys.J.Lett.* 951 (2023) 1, L8
Published: Jun 29, 2023
e-Print: 2306.16213 [astro-ph.HE]
DOI: 10.3847/2041-8213/acdac6
Experiments: NANOGRAV
View in: ADS Abstract Service

pdf cite claim reference search 284 citations

- Many talks about PTAs in this workshop.

Introduction

Stochastic Gravitational Wave Background (SGWB)

- Superposition of many unresolvable gravitational wave.
- Important observational target of current observation
- Origin: Binary Black hole, Inflation, Phase transition, Cosmic String,……

Pulsar Timing Array can also probe ultra-light dark matter. (Porayko+, 2014)

- What if both gravitational wave background and ultra-light dark matter exist at the same time?

Pulsar Timing Array & GW

- Pulsar emits a pulse periodically.
- When a pulse travels in gravitational waves, its period gets redshift.

$$z_a(t) \equiv \frac{\nu_0 - \nu_{\mathrm{obs}}(t)}{\nu_0} = \frac{1}{2} u_a^i u_a^j \int_0^t dt' \partial_{t'} h_{ij}(t', x)\,|_{x=x(t')}$$

$\boldsymbol{u}_a$:unit vector pointing from the pulsar to Earth

$$\Delta T_{\mathrm{GW},a}(t) = \int_0^t dt' z_a(t') \ ,$$ Timing residual

Correlation Analysis

SGWB seen by a single pulsar is “noise”

→ Taking the correlation between pulsars can distinguish SGWB and noise

Total timing residual: $\Delta T_{\mathrm{tot},a} = \underline{\Delta T_{\mathrm{GW},a}} + \underline{N_a}$

SGWB　　Noise

Cross-Correlation:

$$< \Delta T_{\mathrm{tot},a} \Delta T_{\mathrm{tot},b} > = < \Delta T_{\mathrm{GW},a} \Delta T_{\mathrm{GW},b} > + < N_a N_b >$$

$$= < \Delta T_{\mathrm{GW},a} \Delta T_{\mathrm{GW},b} >$$ (Noise are independent)

Only SGWB remain
(relatively amplified)

Hellings-Downs curve

(Hellings&Downs, 1983)

One can analytically calculate the angular correlation between the pulsar.

$$< \Delta T_{\mathrm{GW},a} \Delta T_{\mathrm{GW},b} > = \int df \;\; \Gamma_{\mathrm{HD}}(\xi) \Phi_{\mathrm{GW}}(f) \cos 2\pi f t$$

ξ:angle between pulsar, Φ_{GW}:amplitude of GW

$$\Gamma_{\mathrm{HD}}(\xi) = \frac{1}{2} + \frac{3}{2}\left(\frac{1-\cos\xi}{2}\right)\log\left(\frac{1-\cos\xi}{2}\right) - \frac{1}{4}\left(\frac{1-\cos\xi}{2}\right) \;,$$

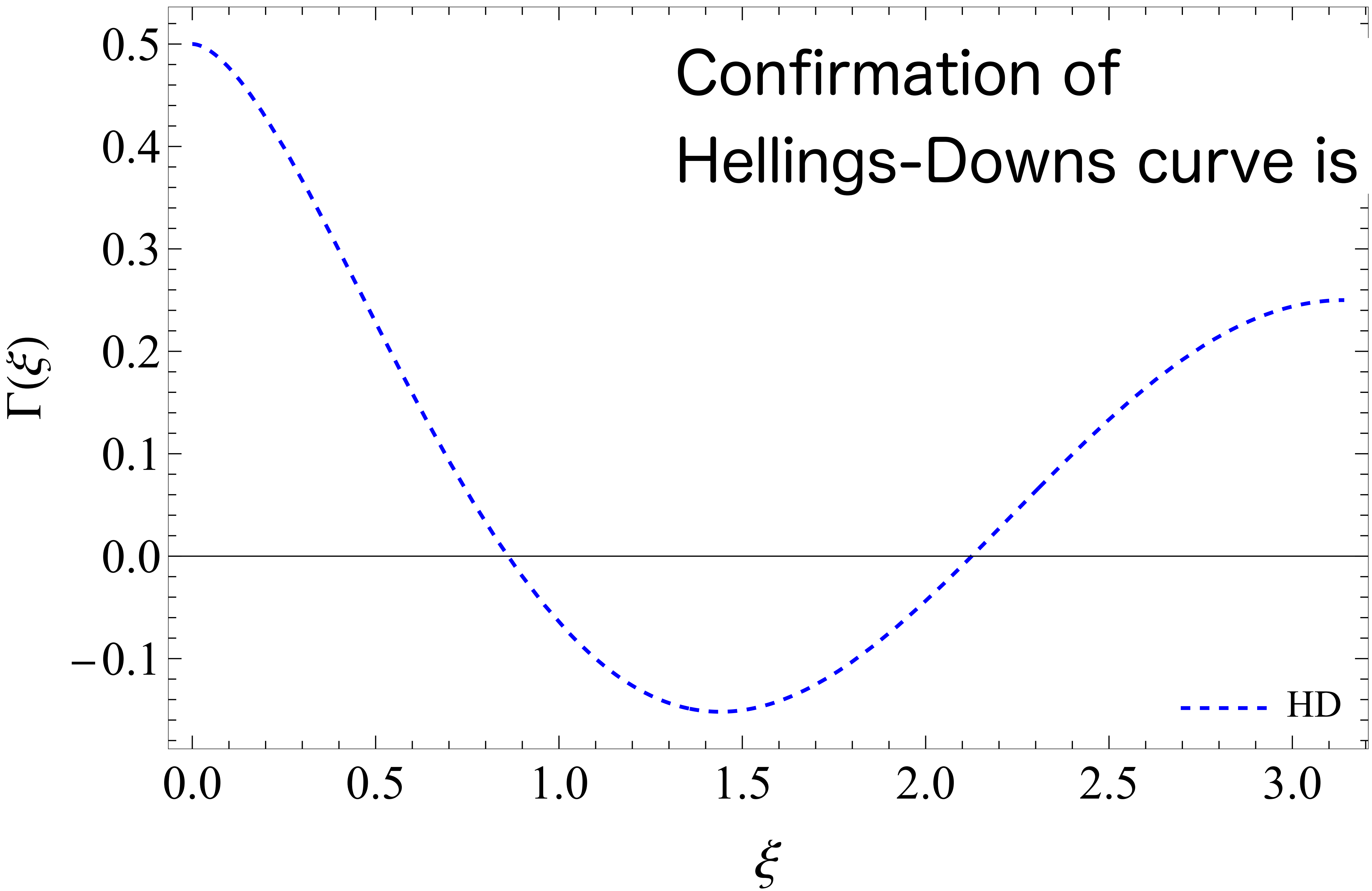
$$\Phi_{\mathrm{GW}}(f) = \frac{A_{\mathrm{GW}}^2}{12\pi^2}\frac{1}{T_{\mathrm{obs}}}\left(\frac{f_i}{f_{\mathrm{ref}}}\right)^{-\gamma} f_{\mathrm{ref}}^{-3} \;.$$

NANOGrav:$A_{\mathrm{GW}} \sim 2.4 \times 10^{-15}$ with $f_{\mathrm{ref}} = 1\mathrm{yr}^{-1}, \gamma = 13/3$

Hellings-Downs curve

(Hellings&Downs, 1983)

Confirmation of
Hellings-Downs curve is key



Ultra-Light Dark Matter

Not only gravitational waves, but ultra-light dark matter can produce timing residual. (Porayko+, 2014, Nomura+, 2019)

Ultra-light dark matter

- Bosonic particle with mass ~10^{-24} - 1 eV
- Behaves as cold dark matter on a large scale
- Wave-like properties appear on a small scale
- Spin-0 (or axion-like particle) is considered in many literature but the higher spin field is also possible

In this talk, we focus on vector-type dark matter.

Configuration of ULVDM

(Nomura+, 2019)

Coherent Length

$$k^{-1} = \frac{2\pi}{\mu v_{\rm DM}} \sim 0.4\,{\rm kpc}\left(\frac{10^{-22}{\rm eV}}{\mu}\right)\left(\frac{10^{-3}}{v_{\rm DM}}\right) ,$$

Equation of Motion

$$\nabla_\mu F^{\mu\nu} - \mu^2 A^\nu = 0 \quad , \quad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

$$\longrightarrow \quad A_0 \sim 0 \ , \quad \boldsymbol{A}(t, \boldsymbol{x}) \sim \boldsymbol{\Omega}_A(\boldsymbol{x}) A \cos\left(\mu t + \boldsymbol{k}\cdot\boldsymbol{x}\right) \ .$$

$\boldsymbol{\Omega}_A$:Direction of the vector field

A:Amplitude of the vector field

・Coherent length is much longer than the Compton wavelength. Thus, the spatial derivative is much smaller than the time derivative.

Configuration of ULVDM

(Nomura+, 2019)

Stress-Energy tensor

$$T_{00} \sim \frac{1}{2}\mu^2 A^2 \ ,$$

$$T_{ij} \sim -\frac{1}{2}\mu^2 A^2 \left(\delta_{ij} - 2\Omega_{A,i}\Omega_{A,j}\right) \underline{\cos(2\mu t + 2\boldsymbol{k}\cdot\boldsymbol{x})} \ ,$$

Time-dependent

・Large amplitude for small mass (energy density is fixed).

・Time-dependent pressure exists.

→ Time-dependent metric perturbation is produced!

Metric perturbation by ULVDM

(Nomura+, 2019)

Solving linearized Einstein equation……

$$
\begin{aligned}
ds^2 = &-(1-2\Psi(t,\boldsymbol{x}))dt^2 \\
&+ \left[(1+2\Psi(t,\boldsymbol{x}))\delta_{ij} + \gamma_{ij}(t,\boldsymbol{x})\right] dx^i dx^j \ ,
\end{aligned}
$$

$$
\Psi(t,\boldsymbol{x}) = \Psi_{\rm osc}(\boldsymbol{x}) \cos\left(2\mu t + 2\boldsymbol{k}\cdot\boldsymbol{x}\right) \ ,
$$

$$
\gamma_{ij}(t,\boldsymbol{x}) = h_{\rm osc}(\boldsymbol{x}) \left(\delta_{ij} - 3\Omega_{A,i}(\boldsymbol{x})\Omega_{A,j}(\boldsymbol{x})\right) \cos(2\mu t + 2\boldsymbol{k}\cdot\boldsymbol{x}) \ ,
$$

Similar situation as gravitational waves.

Timing Residual by ULVDM

(Nomura+, 2019)

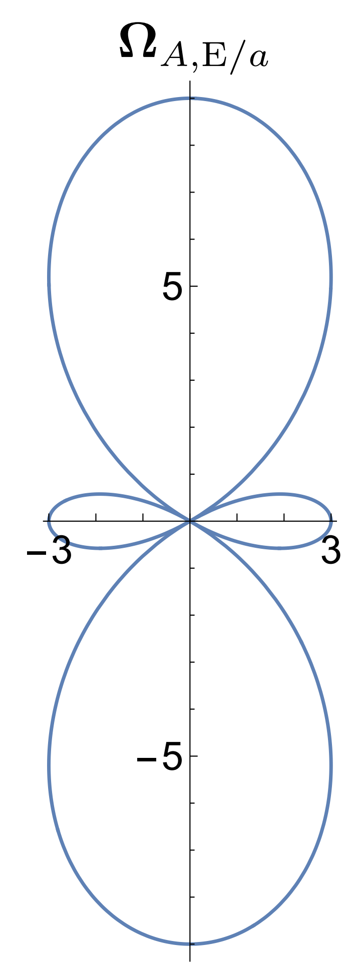
$$\Delta T_{\mathrm{DM},a} = \frac{1}{2\mu}\left(F^{\mathrm{DM}}_{a,\mathrm{E}}\Psi_{\mathrm{osc}}(\boldsymbol{x}_\mathrm{E})\sin(2\mu t + 2\boldsymbol{k}\cdot\boldsymbol{x}_\mathrm{E})\right.$$

Contribution from perturbation@Earth

$$\left.-F^{\mathrm{DM}}_{a,\mathrm{P}}\Psi_{\mathrm{osc}}(\boldsymbol{x}_a)\sin(2\mu t - 2\mu L_a + 2\boldsymbol{k}\cdot\boldsymbol{x}_a)\right)$$

Contribution from perturbation@Pulsar

$$F^{\mathrm{DM}}_{a,\mathrm{E}} = -3\left(1 - 4(\boldsymbol{u}_a\cdot\boldsymbol{\Omega}_{A,\mathrm{E}})^2\right) \ ,$$
$$F^{\mathrm{DM}}_{a,\mathrm{P}} = -3\left(1 - 4(\boldsymbol{u}_a\cdot\boldsymbol{\Omega}_{A,a})^2\right) \ .$$



Response of the PTA to the ULVDM

Response is Monopole+Quadrupole

Cross correlation

(HO+, 2023)

$$< \Delta T_{\mathrm{DM},a} \Delta T_{\mathrm{DM},b} >$$

$$\begin{aligned}
&= \frac{1}{2(2\mu)^2} \Big[\langle \Psi_{\mathrm{osc}}(\boldsymbol{x}_a) \Psi_{\mathrm{osc}}(\boldsymbol{x}_b) \rangle \langle F^{\mathrm{DM}}_{a,\mathrm{P}} F^{\mathrm{DM}}_{b,\mathrm{P}} \rangle \\
&\qquad \times \cos\left(2\mu\tau + 2\mu(L_a - L_b) - 2\boldsymbol{k} \cdot (\boldsymbol{x}_a - \boldsymbol{x}_b)\right) \\
&\quad - \langle \Psi_{\mathrm{osc}}(\boldsymbol{x}_\mathrm{E}) \Psi_{\mathrm{osc}}(\boldsymbol{x}_b) \rangle \langle F^{\mathrm{DM}}_{a,\mathrm{E}} F^{\mathrm{DM}}_{b,\mathrm{P}} \rangle \\
&\qquad \times \cos(2\mu\tau - 2\mu L_b - 2\boldsymbol{k} \cdot (\boldsymbol{x}_\mathrm{E} - \boldsymbol{x}_b)) \\
&\quad - \langle \Psi_{\mathrm{osc}}(\boldsymbol{x}_\mathrm{E}) \Psi_{\mathrm{osc}}(\boldsymbol{x}_a) \rangle \langle F^{\mathrm{DM}}_{a,\mathrm{P}} F^{\mathrm{DM}}_{b,\mathrm{E}} \rangle \\
&\qquad \times \cos(2\mu\tau + 2\mu L_a - 2\boldsymbol{k} \cdot (\boldsymbol{x}_a - \boldsymbol{x}_\mathrm{E})) \\
&\quad + \langle \Psi^2_{\mathrm{osc}}(\boldsymbol{x}_\mathrm{E}) \rangle \langle F^{\mathrm{DM}}_{a,\mathrm{E}} F^{\mathrm{DM}}_{b,\mathrm{E}} \rangle \cos 2\mu\tau \Big]
\end{aligned}$$

"Pulsar term"

・We take an average over observed pulsars.

→ "Pulsar term" is relatively suppressed by $\cos \mu L$ or $\sin \mu L$ if we observe a large number of pulsars.

※$10^3 \lesssim \mu L \lesssim 10^5$ for $\mu \sim 10^{-22}$eV.

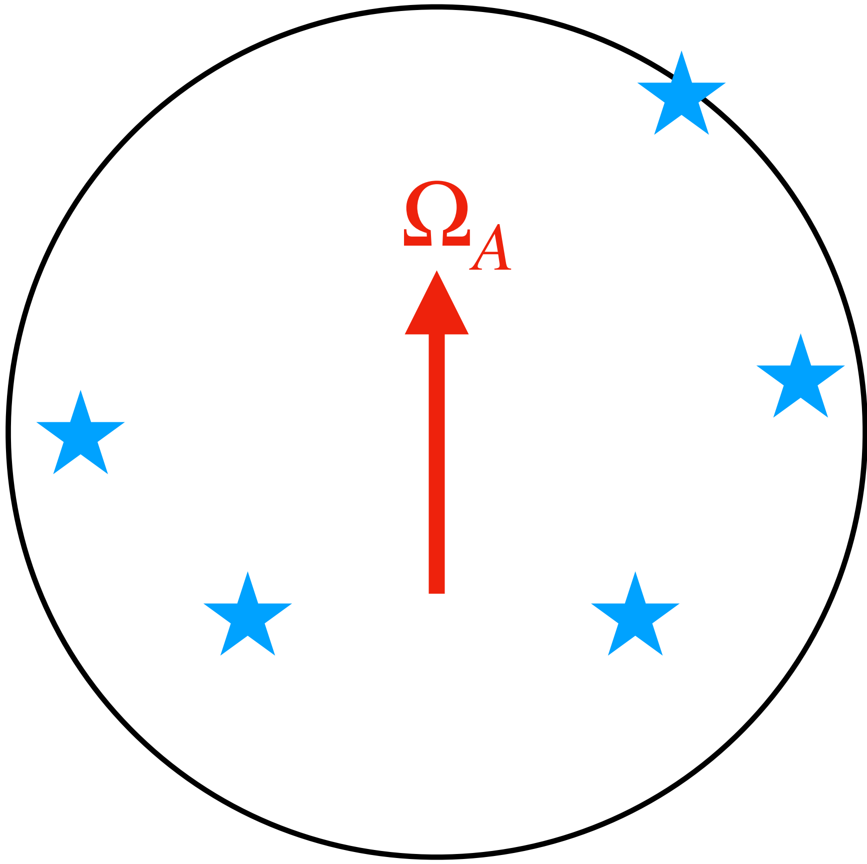
Angular correlation

Dropping pulsar terms

$$\langle \Delta T_a(t) \Delta T_b(t+\tau) \rangle$$
$$\approx \frac{1}{2(2\mu)^2} \langle \Psi^2_{\rm osc}(\boldsymbol{x}_{\rm E}) \rangle \langle F^{\rm DM}_{a,{\rm E}} F^{\rm DM}_{b,{\rm E}} \rangle \cos 2\mu\tau \ .$$

Averaging over pulsars all over the sky

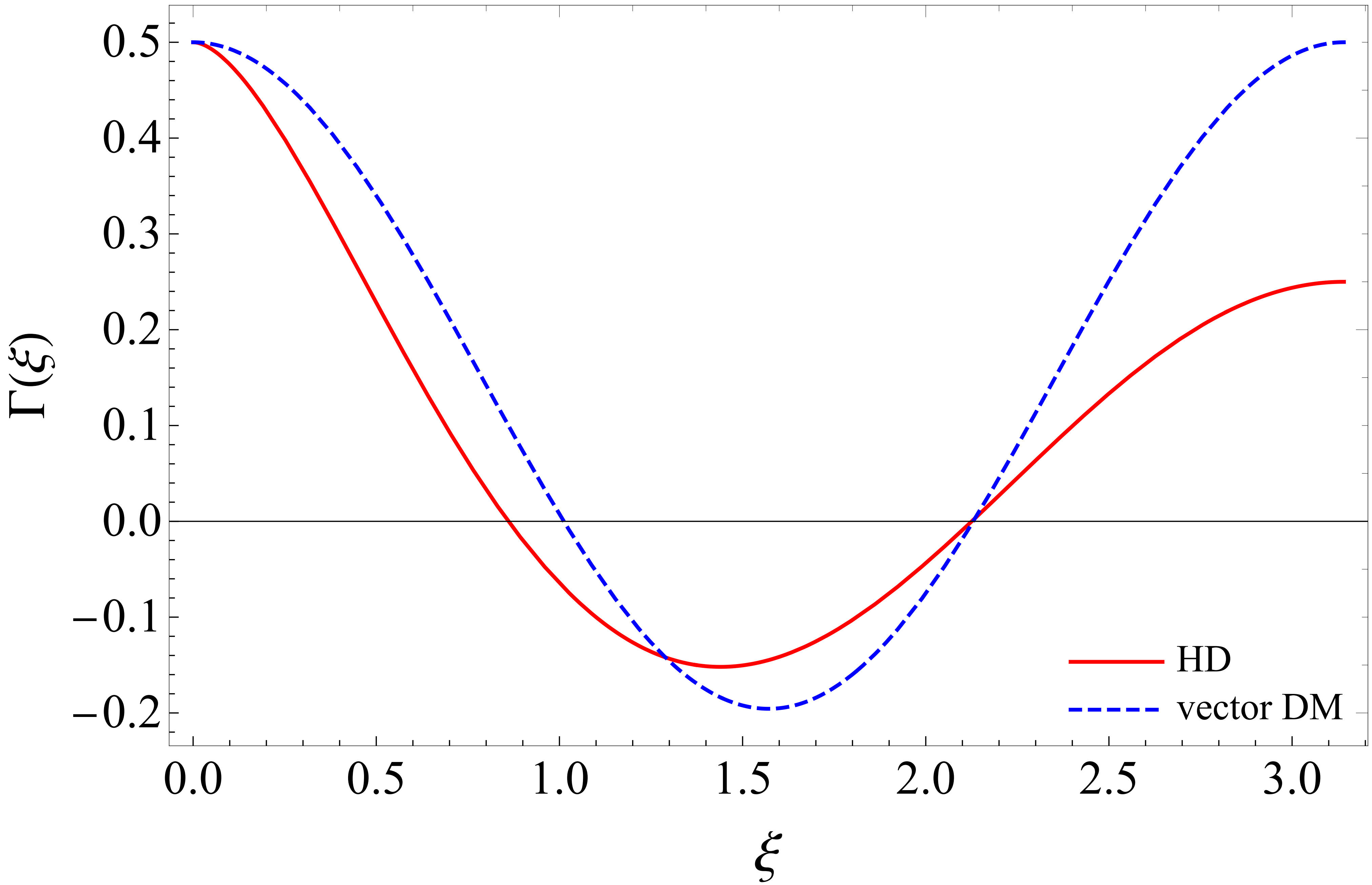
⟷ Average over direction of the vector field.



$$\langle F^{\rm DM}_{a,{\rm E}} F^{\rm DM}_{b,{\rm E}} \rangle = 9 \int \frac{d\Omega_A}{4\pi} \left(1 - 4(\boldsymbol{u}_a \cdot \boldsymbol{\Omega}_{A,{\rm E}})^2\right) \times \left(1 - 4(\boldsymbol{u}_b \cdot \boldsymbol{\Omega}_{A,{\rm E}})^2\right)$$
$$= \frac{3}{5}(7 + 16\cos 2\xi) \equiv \frac{138}{5}\Gamma_{\rm DM}(\xi) \ .$$

$$\Gamma_{\rm DM}(\xi) = \frac{5}{138} P_0(\cos\xi) + \frac{64}{138} P_2(\cos\xi)$$

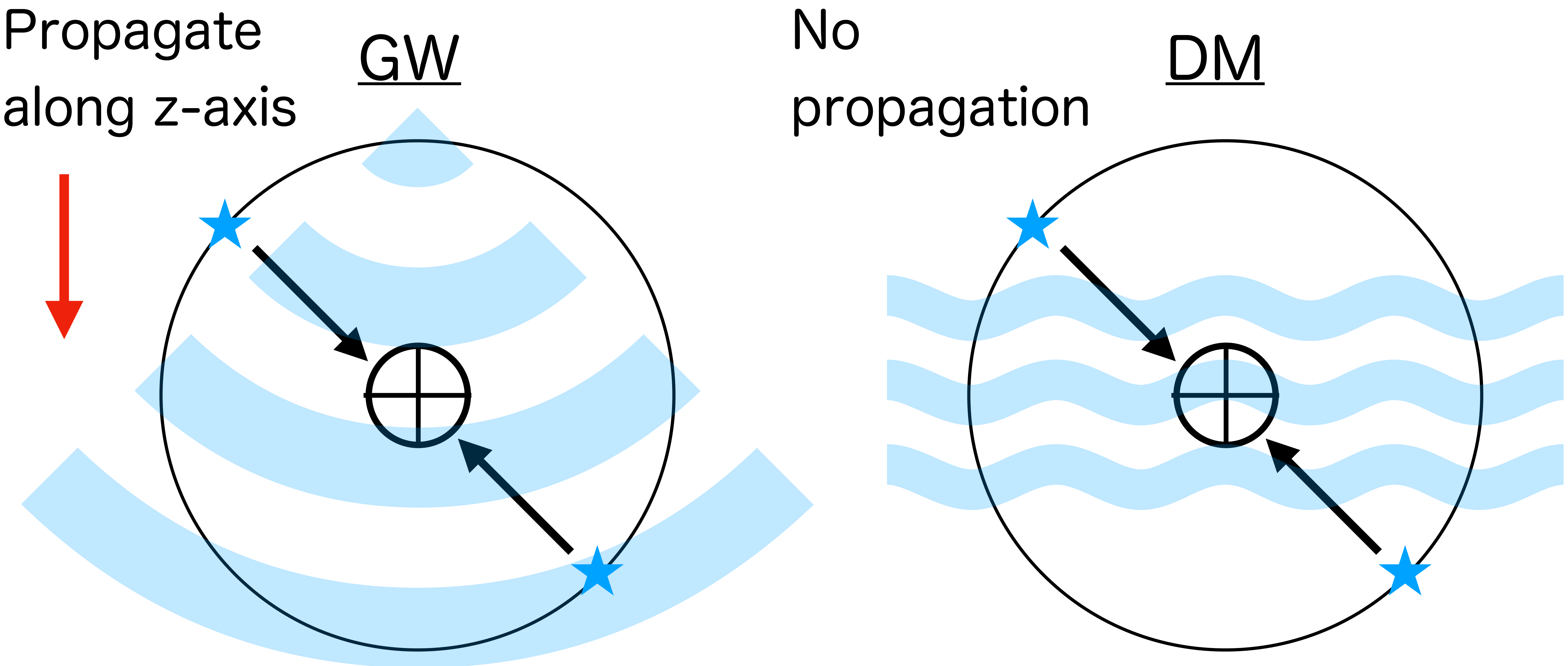
Angular correlation



・Quadrupole component dominantes for both case.

・Higher multipoles exist in SGWB signal.

Why Different?



Since ULDM is just a standing wave, pulses from opposite direction feels the same perturbation.

Total Angular correlation

Total timing residual is sum of all components

$$\Delta T_{\mathrm{tot},a} = \Delta T_{\mathrm{GW},a} + \Delta T_{\mathrm{DM},a} + N_a$$

GW signal and DM signal are independent

$$C_{ab}(\tau) = < \Delta T_{\mathrm{tot},a}(t+\tau)\Delta T_{\mathrm{tot},b}(t) >$$

$$\sim < \Delta T_{\mathrm{GW},a}\Delta T_{\mathrm{GW},b} > + < \Delta T_{\mathrm{DM},a}\Delta T_{\mathrm{DM},b} >$$

$$= \sum_i \Gamma_{\mathrm{HD}}(\xi)\Phi_{\mathrm{GW}}(f_i)\cos 2\pi f_i \tau$$

$$+ \Gamma_{\mathrm{DM}}(\xi)\Phi_{\mathrm{DM}}\cos 2\mu\tau \ .$$

When the frequency of dark matter is contained in the frequency bin, angular correlation is modified!

Total Angular correlation

Define effective angular correlation as

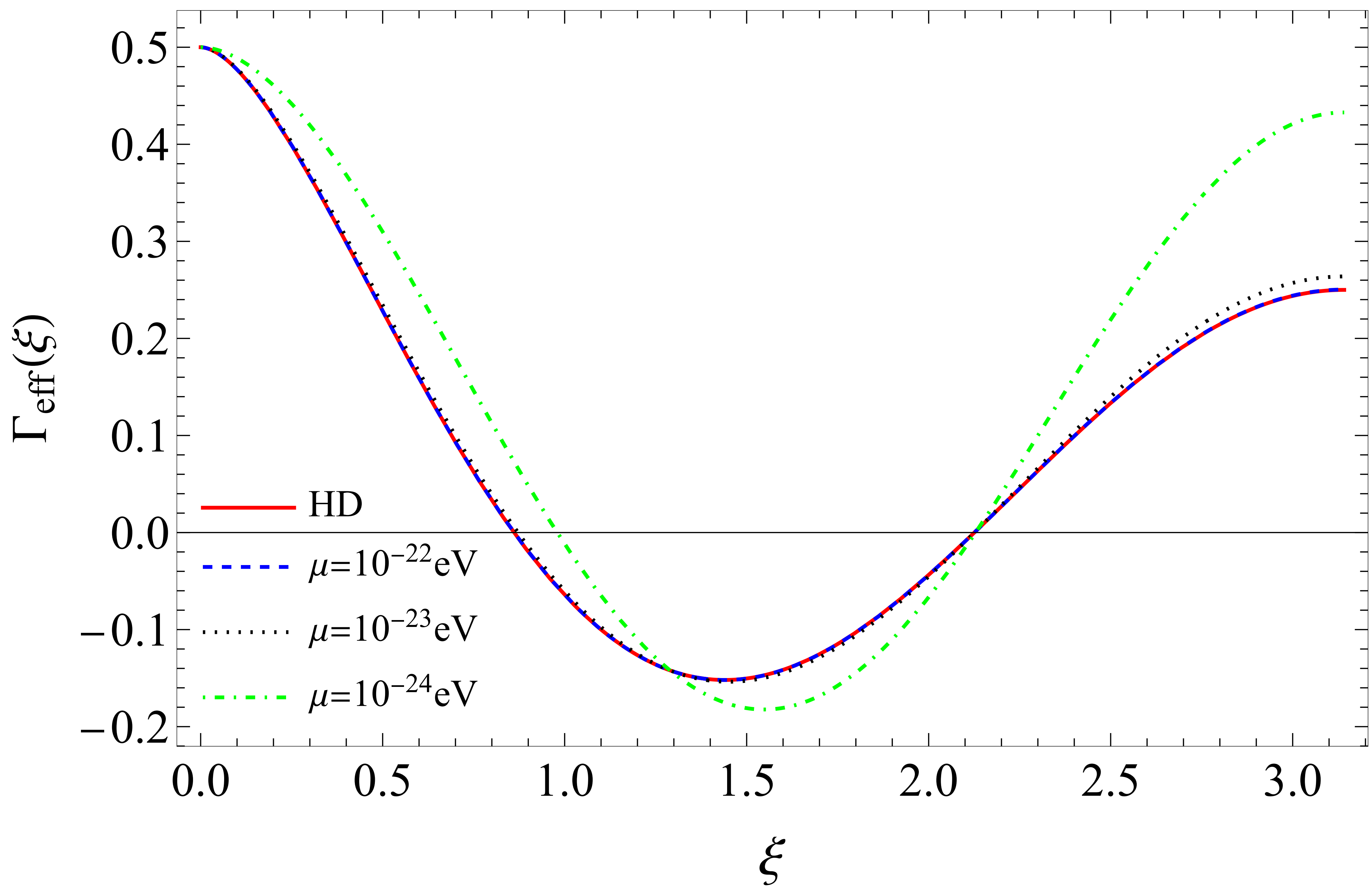
$$\Gamma_{\rm eff}(\xi) = \frac{\Phi_{\rm GW}(\mu/\pi)}{\Phi_{\rm GW}(\mu/\pi) + \Phi_{\rm DM}} \times \left(\Gamma_{\rm HD}(\xi) + \frac{\Phi_{\rm DM}}{\Phi_{\rm GW}(\mu/\pi)}\Gamma_{\rm DM}(\xi)\right) .$$

Amplitude of GW and DM is given by

$$\Phi_{\rm GW}(\mu/\pi) \sim 5 \times 10^{-34} {\rm yr}^2 \left(\frac{\mu}{10^{-22}{\rm eV}}\right)^{-13/3} \left(\frac{15{\rm yr}}{T_{\rm obs}}\right) .$$

$$\Phi_{\rm DM} \sim 7 \times 10^{-37} {\rm yr}^2 \left(\frac{\rho_{\rm DM}}{0.4{\rm GeV}\cdot{\rm cm}^{-3}}\right)^2 \left(\frac{10^{-22}{\rm eV}}{\mu}\right)^6 .$$

Total Angular correlation



Hellings-Downs curve is deformed due to the ULVDM

Summary

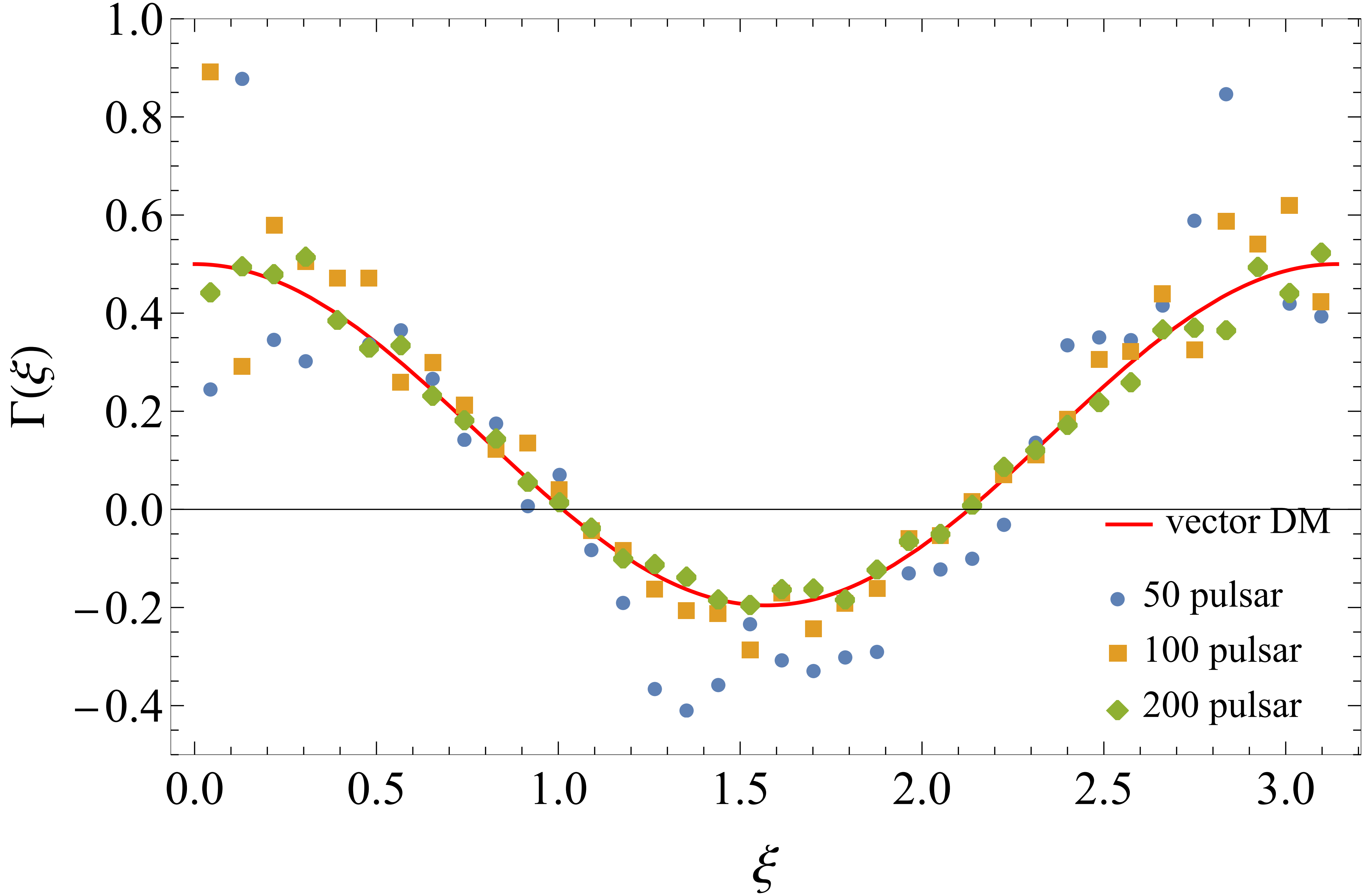
- We considered angular correlation of PTA, produced by the ultra-light vector dark matter
- The angular correlation produced by ULVDM is different from that of GWs.
  - The difference comes from the standing wave nature of ULDM.
  - For dark matter mass $\mu \lesssim 10^{-23}\mathrm{eV}$, the deformation of the Hellings-Downs curve is obvious by eye.
  - For mass $\mu \gtrsim 10^{-22}\mathrm{eV}$, the amplitude of the ultra-light dark matter is too small to deform the Hellings-Downs curve.

Back up

Average over pulsars

Generate pulsars randomly with uniform distribution.

Average timing residual in each bin (5°).



Average over pulsars

With pulsar term.