# *Gravitational waves from an axion cloud around a rotating black hole*

Hidetoshi Omiya (Kobe University)

Takuya Takahashi (Kyoto University)

Takahiro Tanaka (Kyoto University)

Hirotaka Yoshino (Osaka Metropolitan University)

JCAP06 (2023) 016 and ongoing work

also old works with Hideo Kodama (YITP)

"Gravitational wave probes of physics beyond standard model"
@ Osaka Metropolitan University (November 9, 2023)

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- Adiabatic evolution of axion cloud
- Brief discussion on gravitational waves
- Summary

# Introduction

# Axion field (Sine-Gordon field)

- QCD axion
- String axion

$$\nabla^2 \varphi - \mu^2 \sin \varphi = 0$$

- Strong CP problem in QCD

$$\mathcal{L}_{\mathrm{QCD}} = \bar{Q}_i(i\gamma^\mu D_\mu - m_{ij})Q_j - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \frac{g^2\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

CP-violating term

from experiment $|\theta| \lesssim 10^{-9}$

Peccei-Quinn theory

# Axion field (Sine-Gordon field)

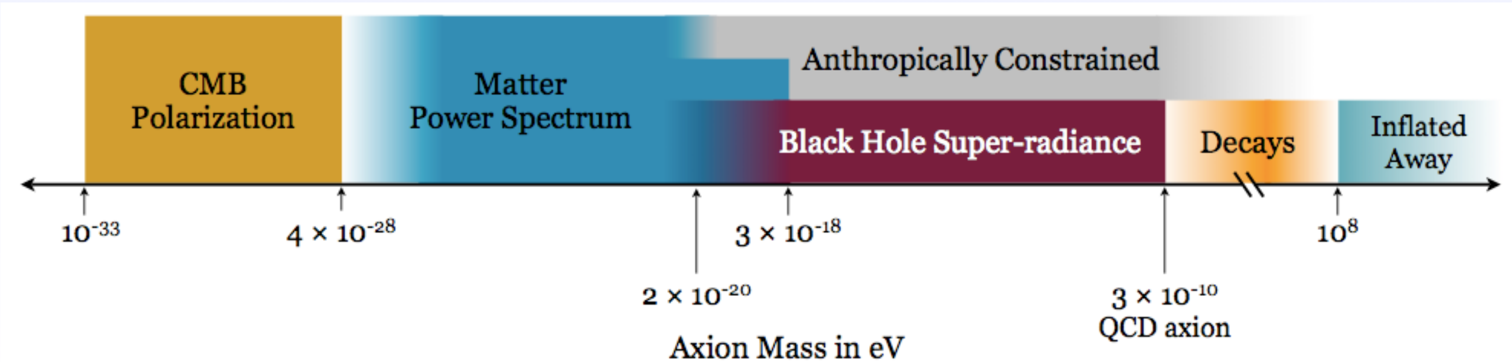
- QCD axion
- String axion

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0$$

Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

In string theory, many moduli appear when the extra dimensions get compactified.

Some of them (10-100) are expected to behave like scalar fields with very tiny mass, which are called string axions.



# Axion field (Sine-Gordon field)

$$\mathcal{L} = -\frac{1}{2}\left(\nabla_a \Phi \nabla^a \Phi + V(\Phi)\right) - \frac{1}{4} g_{a\gamma\gamma} \Phi F_{ab}{}^{*}F^{ab} + \cdots$$

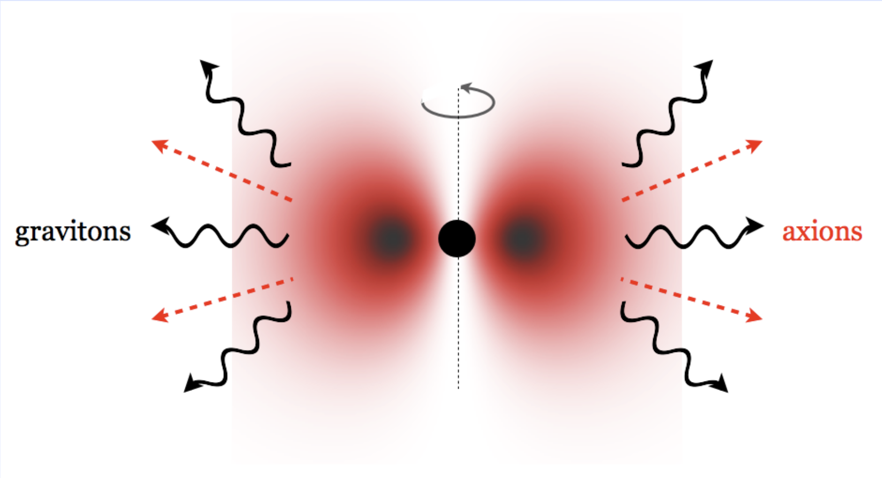
$$V = f_a^2 \mu^2 [1 - \cos(\Phi / f_a)]$$

$$\boxed{\nabla^2 \varphi - \mu^2 \sin\varphi = 0}$$

$$\varphi \equiv \frac{\Phi}{f_a}$$

# Issues to be explored

- String axion field forms an axion cloud around a rotating astrophysical BH by extracting BH's rotation energy.




- Superradiant instability
- Nonlinear self-interaction

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0 \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- GW emission
- Long-term evolution of BH parameters

# Kerr BH

- Metric

$$ds^2 = -\left(\frac{\Delta - a^2 \sin^2\theta}{\Sigma}\right) dt^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma} dt d\phi$$
$$+ \left[\frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right] \sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 + a^2 - 2Mr.$$

$$J = Ma$$

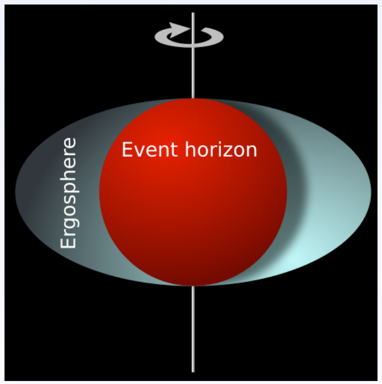
- Ergo region

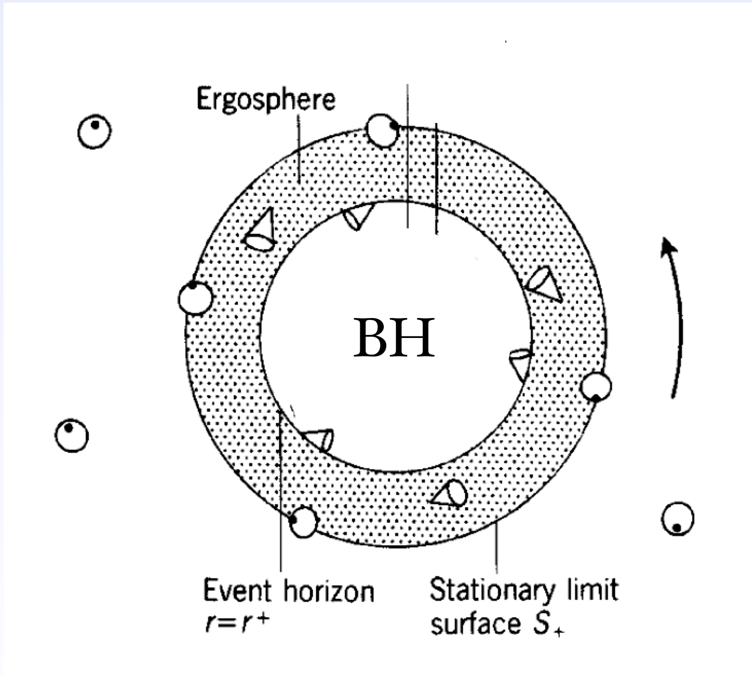
$\xi = \partial_t$ becomes spacelike:

$$\xi_a \xi^a = g_{tt} > 0$$

$$E = -p_a \xi^a$$

can be negative



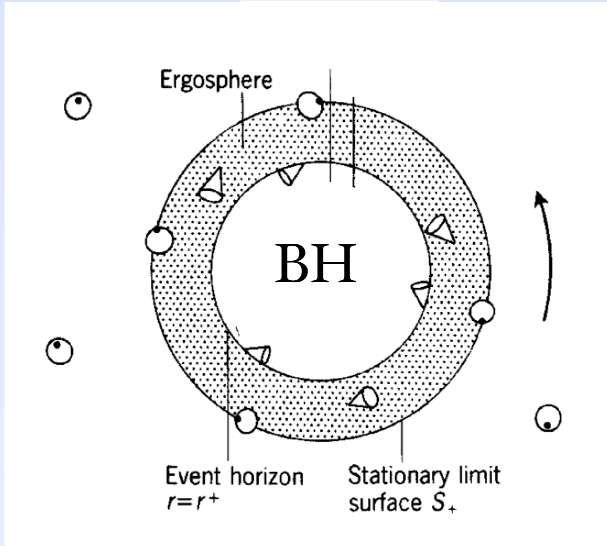



# Energy extraction

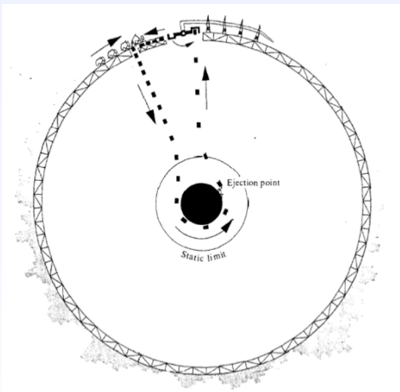
- BH's rotational energy

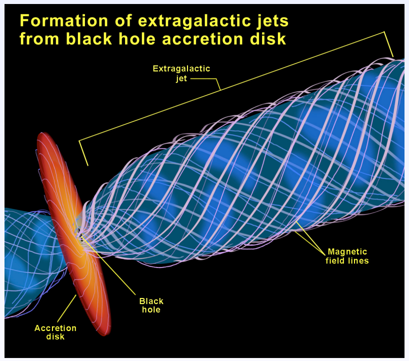
$$M_{\mathrm{rot}} = M - M_{\mathrm{irr}}$$

$$M_{\mathrm{irr}} = \sqrt{\frac{A_H}{16\pi}}$$




- Various methods of energy extraction
  - Penrose process
  - Blandford-Znajek process
  - Superradiance (Next slide)



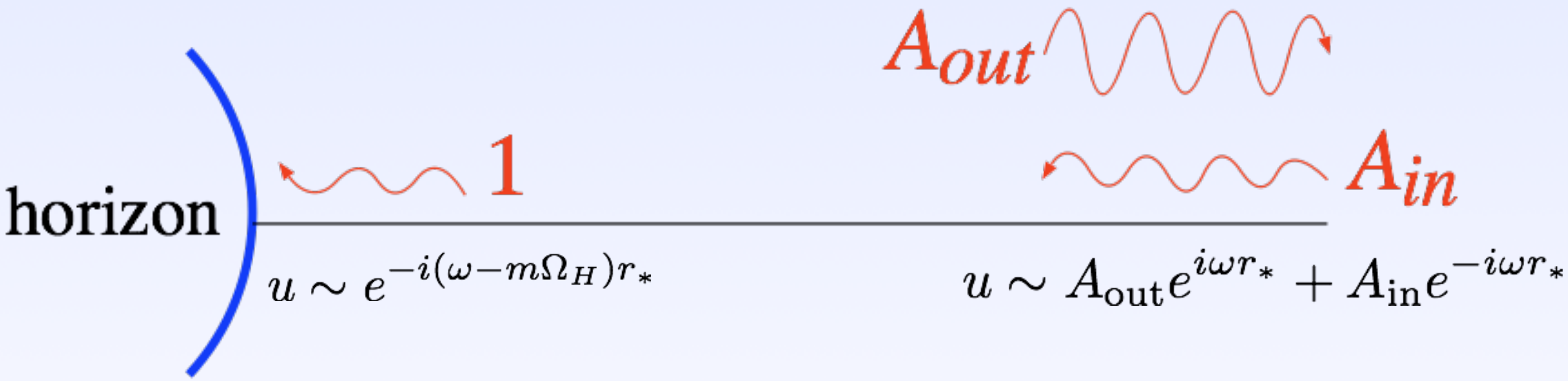



# Superradiance

- Massless Klein-Gordon field $\nabla^2\Phi = 0$ Zel'dovich (1971)

$$\Phi = \mathrm{Re}[e^{-i\omega t}R(r)S(\theta)e^{im\phi}]$$

$$R = \frac{u}{\sqrt{r^2+a^2}} \quad \Rightarrow \quad \frac{d^2u}{dr_*^2} + \left[\omega^2 - V(\omega)\right]u = 0$$




$$\left(1 - \frac{m\Omega_H}{\omega}\right)|T|^2 = 1 - |R|^2$$

Superradiant condition:

$$\omega < \Omega_H m$$

# Gravitational Atom

- Massive Klein-Gordon field

$$\nabla^2\Phi - \mu^2\Phi = 0$$

$$\Phi = \mathrm{Re}[e^{-i\omega t}R(r)S(\theta)e^{im\phi}]$$

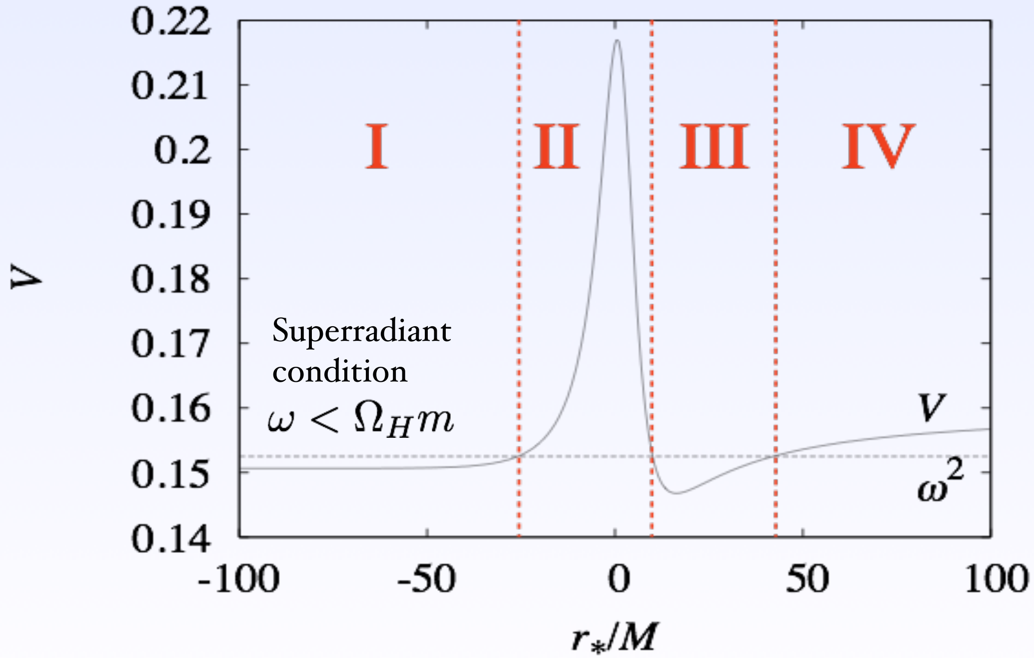
$$R = \frac{u}{\sqrt{r^2+a^2}}$$

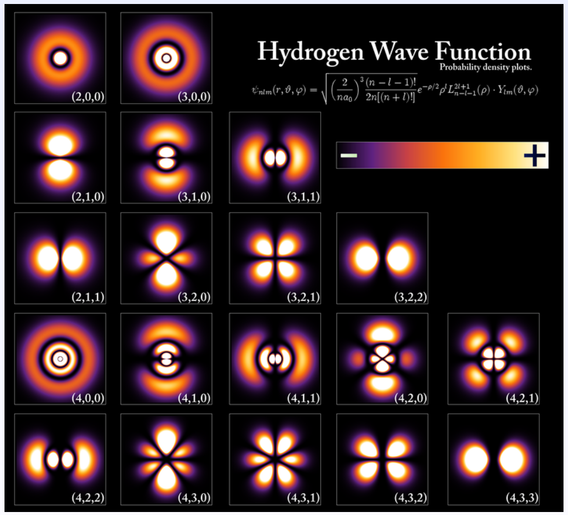
$$\frac{d^2u}{dr_*^2} + \left[\omega^2 - V(\omega)\right]u = 0$$

$$\omega = \omega_R + i\,\omega_I$$ ← Unstable if positive

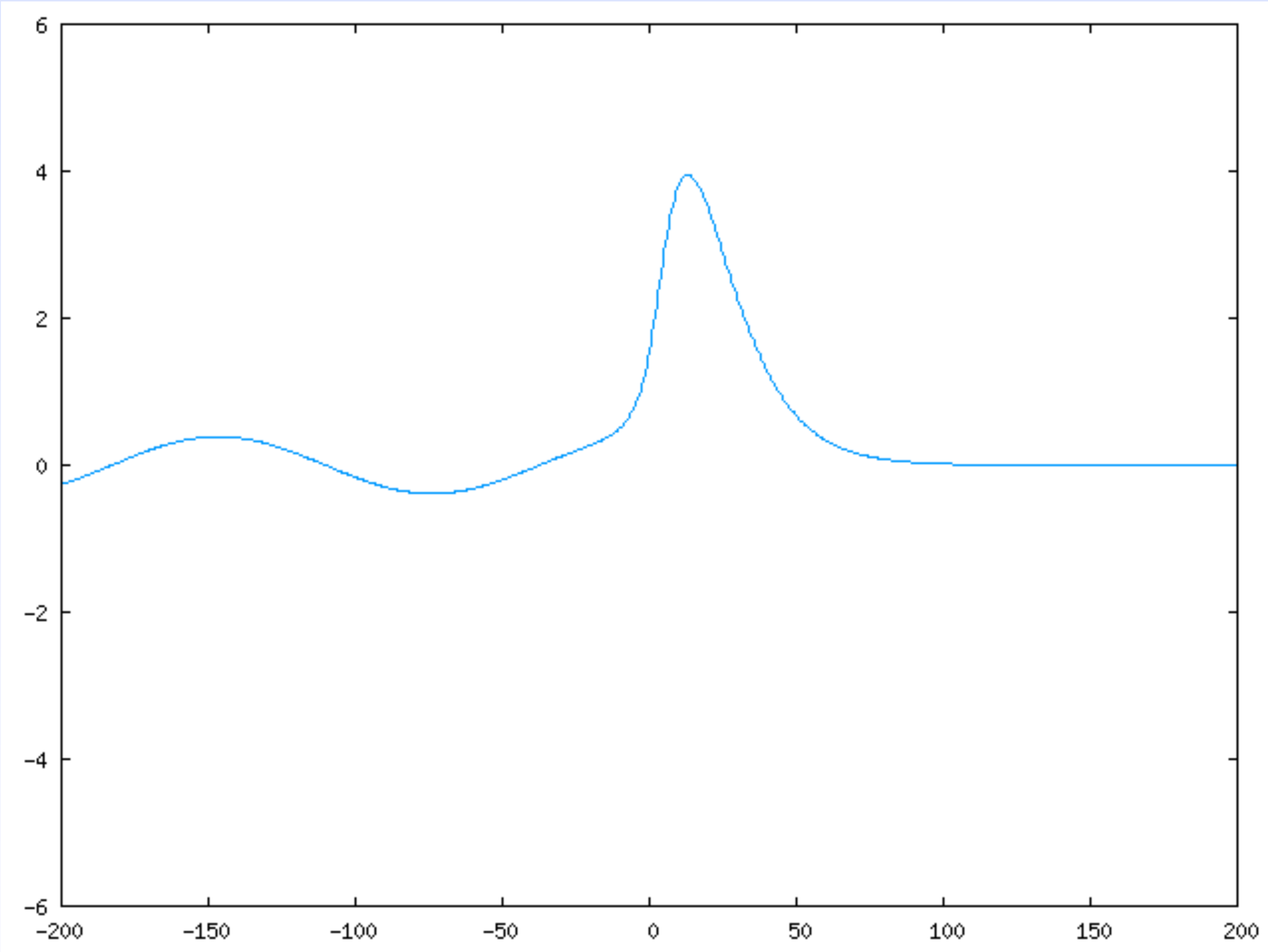
Quantum numbers:

$$n,\ \ell,\ m$$



# Bound State

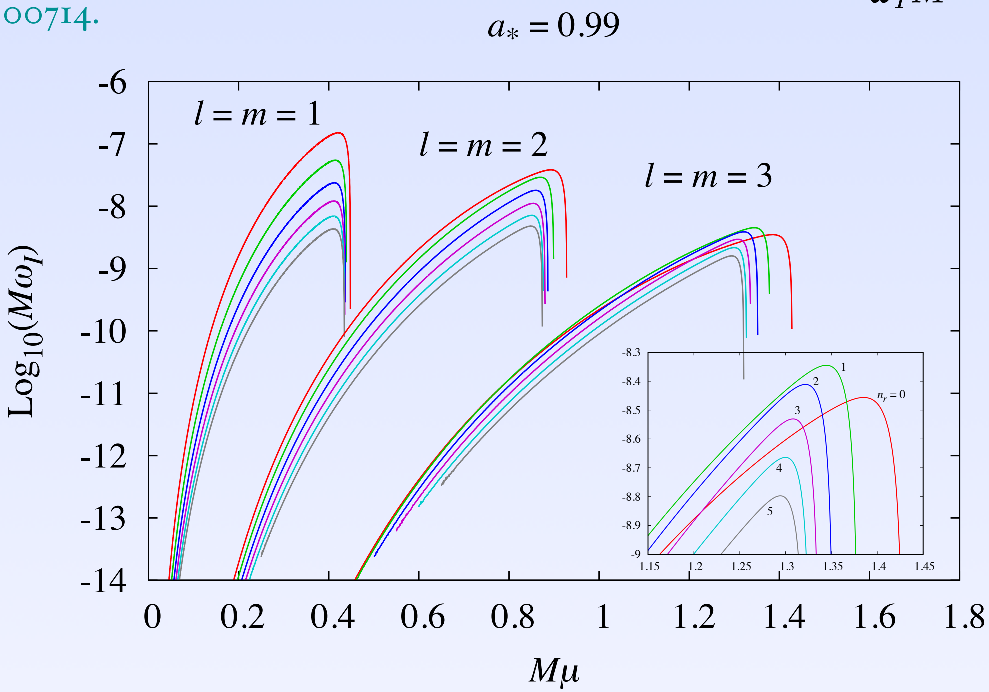


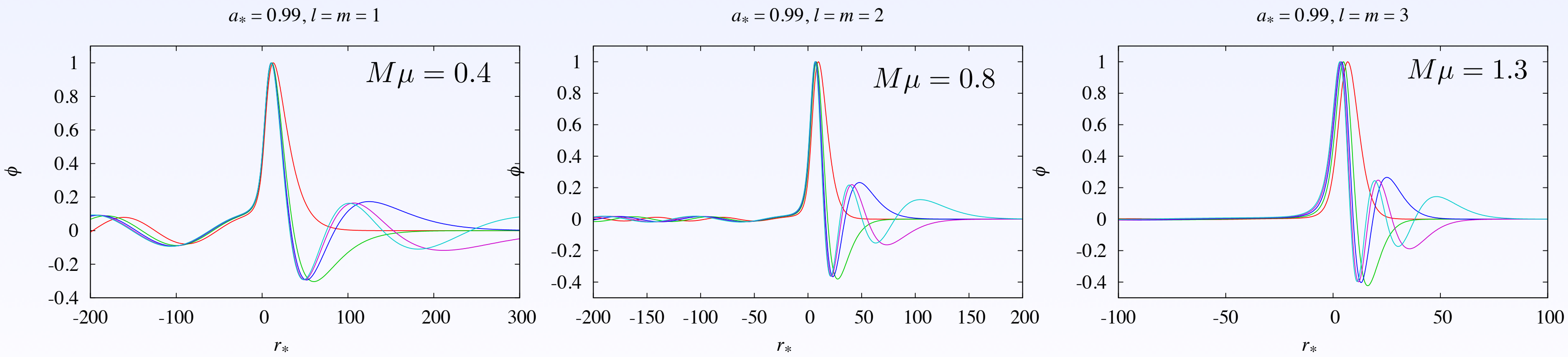
# Wave functions and growth rate

Time scale: $M = M_{\odot}$

$\omega_I M \sim 10^{-7} \quad \Rightarrow \quad \sim 1$ min.

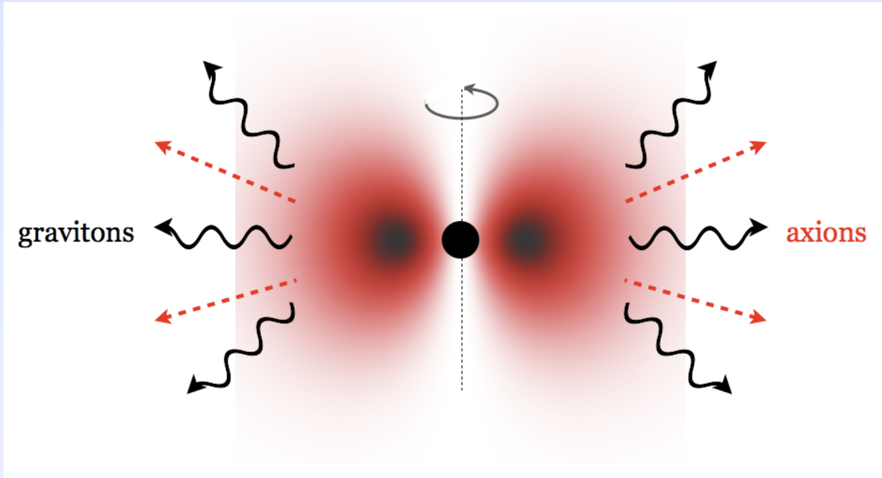
HY and Kodama, arXiv:1505.00714.



# Issues to be explored

- String axion field forms an axion cloud around a rotating astrophysical BH by extracting BH's rotation energy.



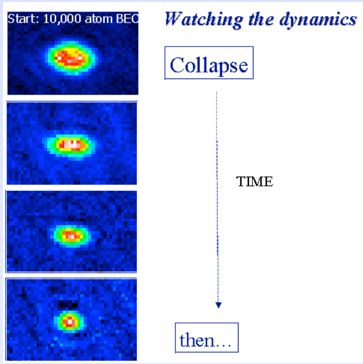

- Superradiant instability
- Nonlinear self-interaction

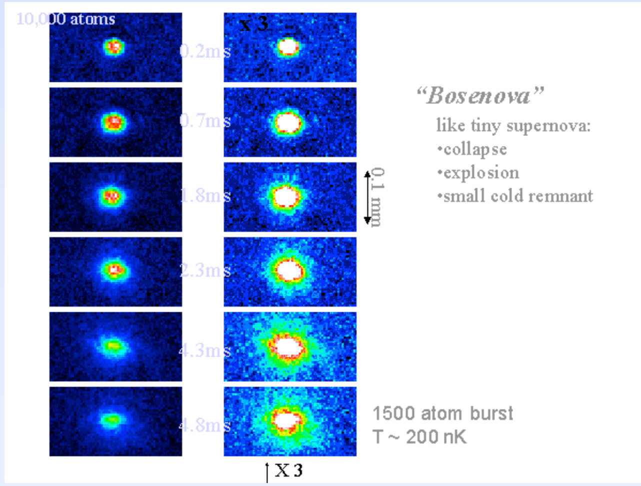
$$\nabla^2\varphi - \mu^2 \sin\varphi = 0 \qquad \varphi \equiv \frac{\Phi}{f_a}$$

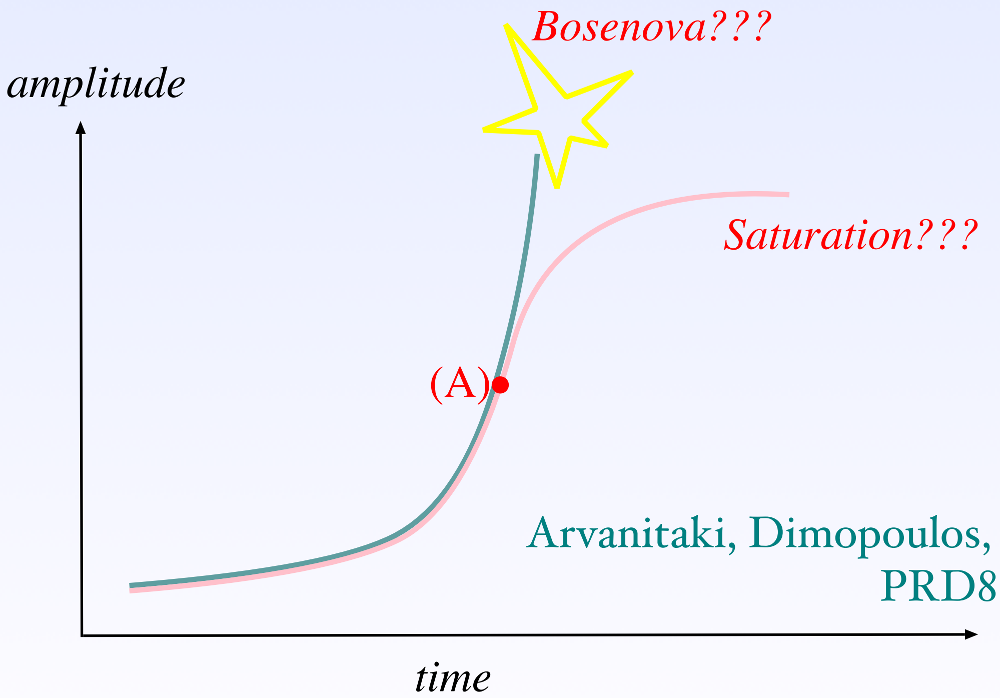
- GW emission
- Long-term evolution of BH parameters

# Final state??

Wieman et al., Nature 412 (2001), 295



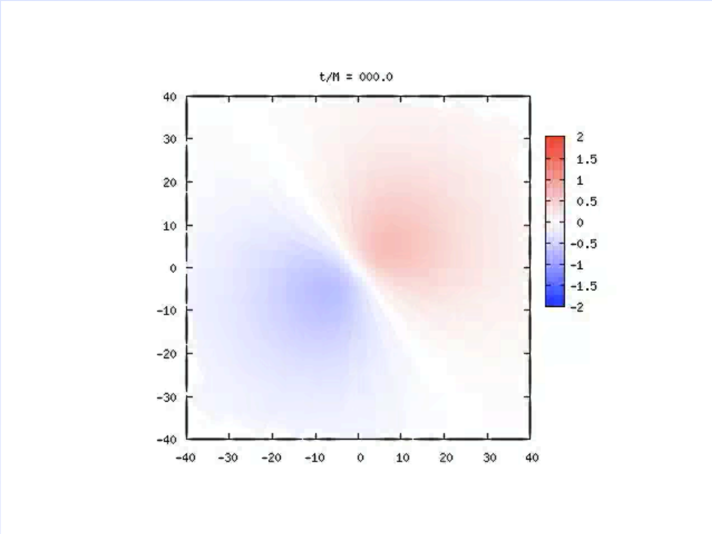

Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

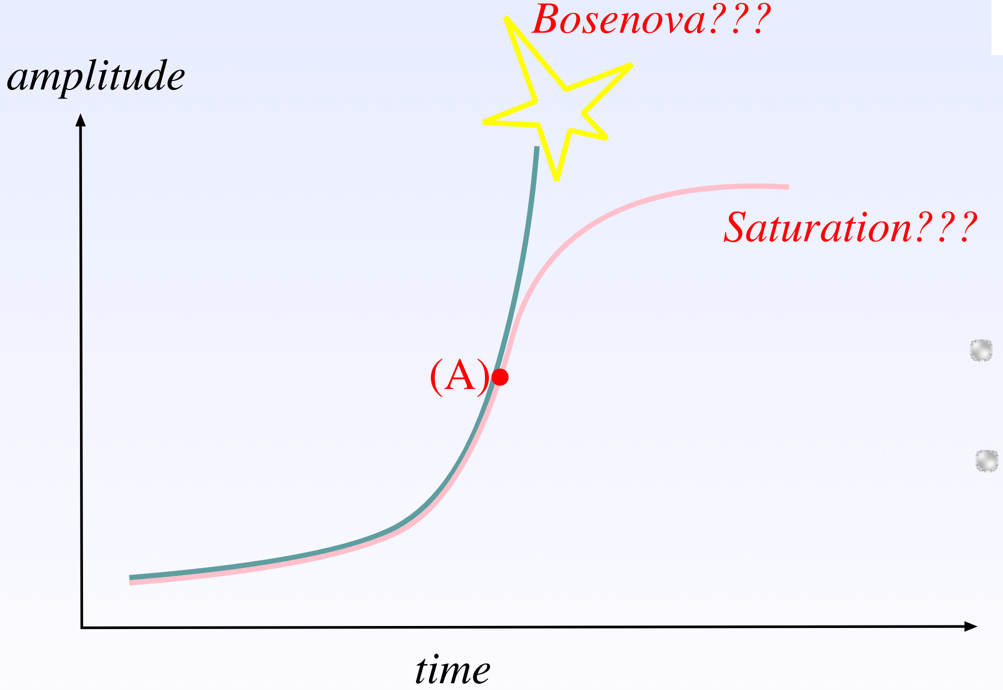
# Final state??

3D simulation

HY and Kodama, CQG32, 214001 (2015)

HY and Kodama, PTP128, 153 (2012)



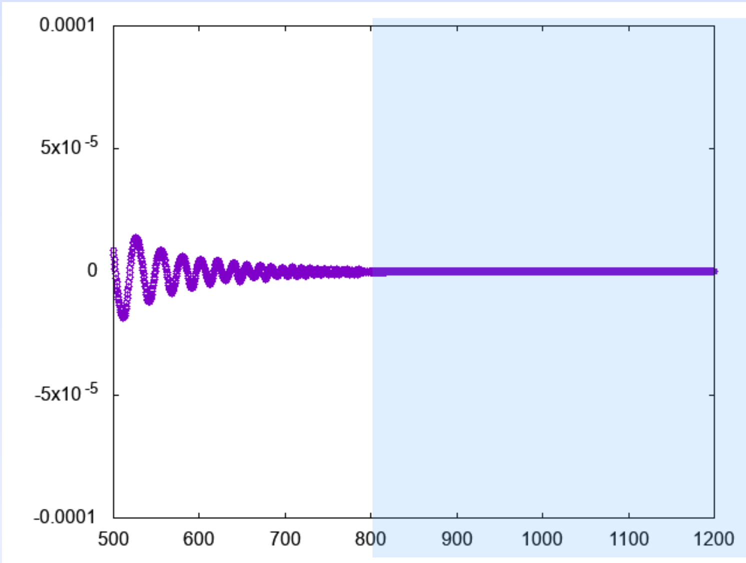



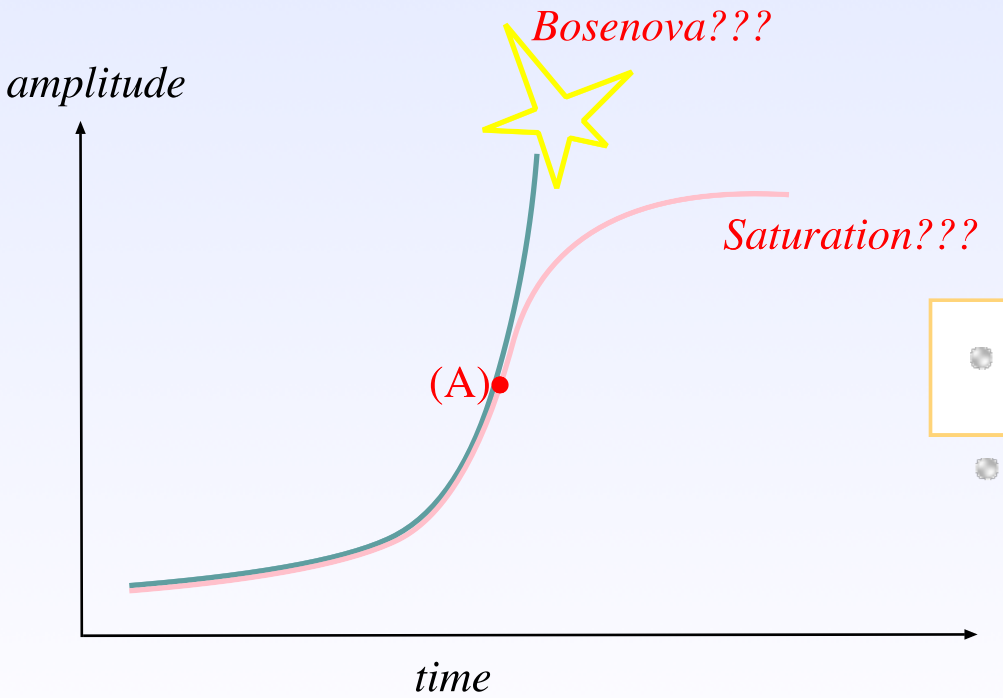

- The outer boundary condition
- Mode interactions

# Final state??

Previously, I imposed the fixed boundary condition at the outer boundary.

A few years ago, I have improved the outer boundary condition



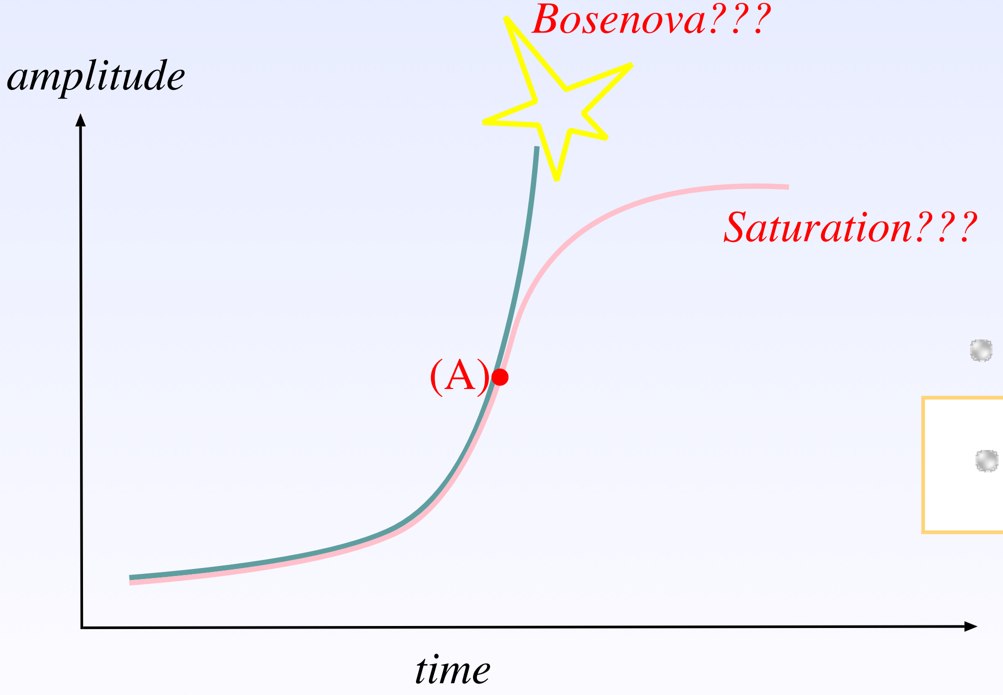




- The outer boundary condition
- Mode interactions

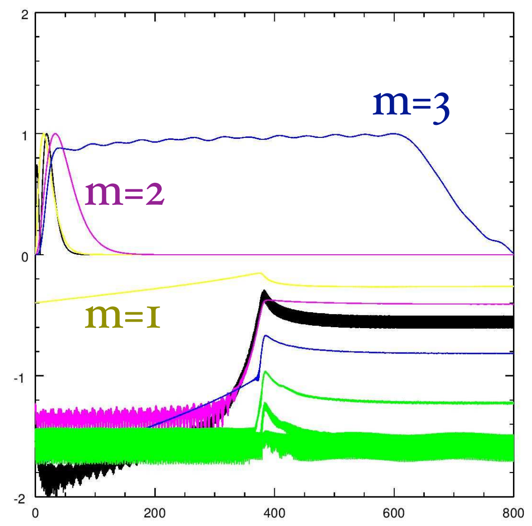
# Final state??

Previously, I killed l=m=2, 4, 6,... modes, in order to save the computation time.

However, it was pointed out that interaction between l=m=1 and 2 modes are very important




Gruzinov, arXiv:1604.06422




- The outer boundary condition
- Mode interactions

# Final state??

Baryakhtar *et al.*, PRD103 (2021) 095019

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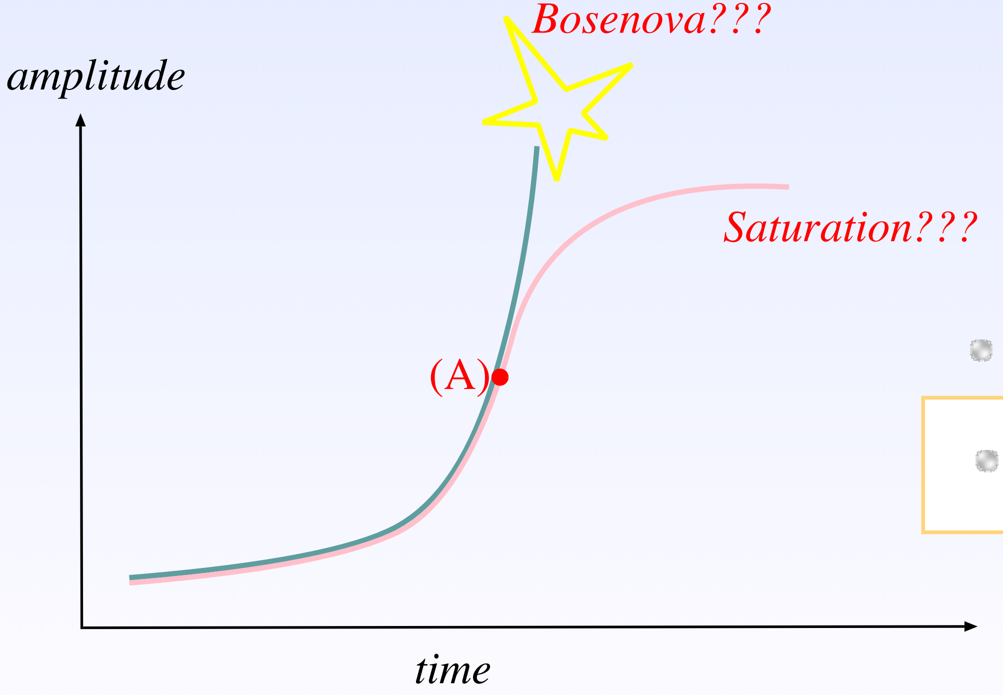
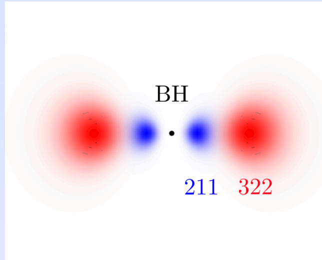
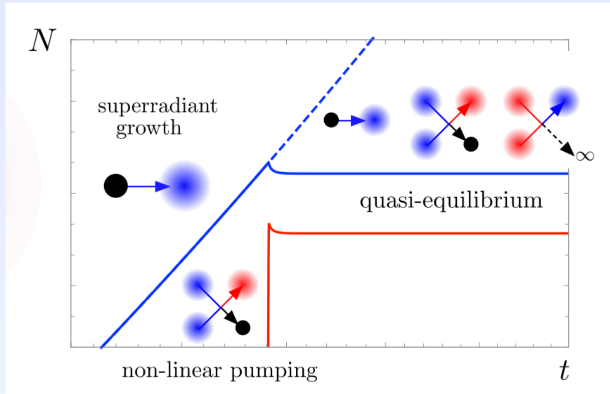


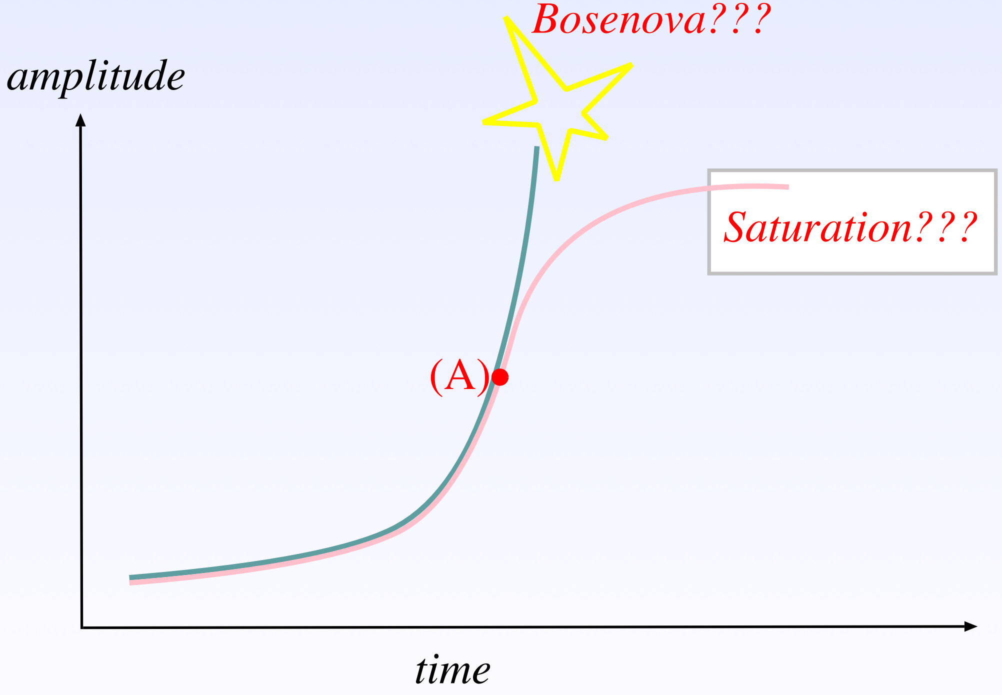

- The outer boundary condition
- Mode interactions

# Final state??

- Determining the final state by numerical simulation is very difficult because the time scale is very long.

⇒ Development of the new effective method is necessary.

The method of the adiabatic approximation     JCAP06 (2023) 016.



# Adiabatic evolution of axion cloud

# Adiabatic evolution

- Perturbative method
- Nonlinear method

$$\nabla^2\varphi - \mu^2\varphi = -\frac{\mu^2}{3!}\varphi^3$$

$$\varphi = \varphi^{(0)} + \varphi^{(1)} + \cdots$$

$$\nabla^2\varphi^{(0)} - \mu^2\varphi^{(0)} = 0$$

$$\nabla^2\varphi^{(1)} - \mu^2\varphi^{(1)} = -\frac{\mu^2}{3!}\left(\phi^{(0)}\right)^3$$

$$\varphi^{(0)} = \sqrt{E_1}\, e^{-i\omega_1 t}\, e^{i\phi}\, \Phi_{211}(r,\theta) + \sqrt{E_2}\, e^{-i\omega_2 t}\, e^{2i\phi}\, \Phi_{322}(r,\theta) + \text{c.c.}$$

$$3E_1\sqrt{E_2}\,\Phi_{211}^2\Phi_{322}^* e^{-i(2\omega_1-\omega_2)t} + \text{c.c.}$$

falls into the BH

$$+3E_2\sqrt{E_1}\,\Phi_{211}^*\Phi_{322}^2 e^{-i(2\omega_2-\omega_1)t}\, e^{3i\phi} + \text{c.c.} + \cdots$$

escapes to infinity

→ Energy flux and angular momentum flux to infinity and to the horizon can be calculated

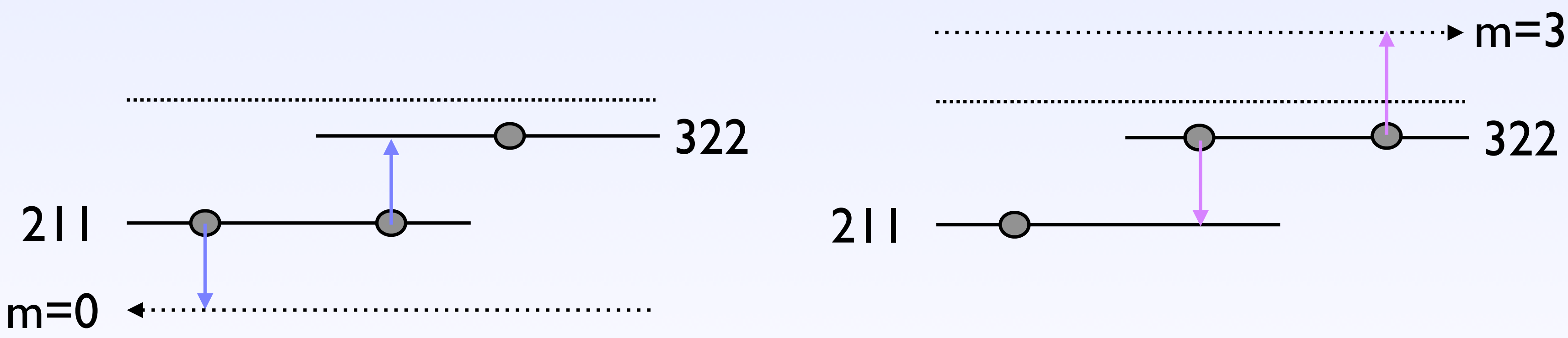
→ Evolution of $E_1(t)$ and $E_2(t)$ can be calculated assuming the conservation of energy and angular momentum

# Adiabative evolution

- Perturbative method
- Nonlinear method

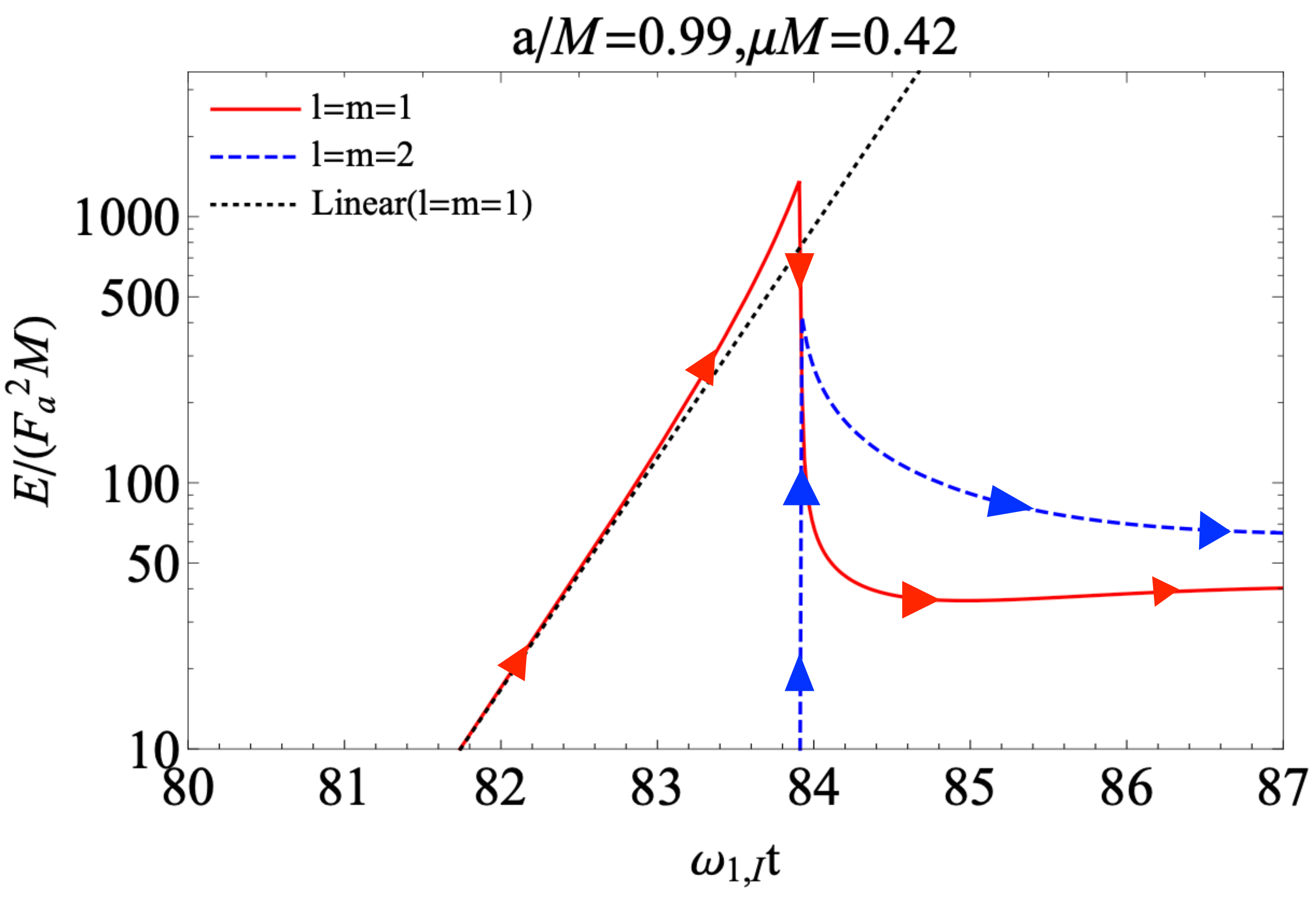
$$\frac{dE_1}{dt} + \frac{dE_2}{dt} = -F^E_{tot}$$

$$\frac{dJ_1}{dt} + \frac{dJ_2}{dt} = -F^J_{tot}$$



# Adiabative evolution

- Perturbative method
- Nonlinear method



# Adiabative evolution

- Perturbative method
- Nonlinear method

$$\varphi = \varphi_1(A_1) + A_2\varphi_2(A_1) + \varphi_r(A_1, A_2)$$

- $\nabla^2\varphi_1 - \mu^2 \sin\varphi_1 = 0$ $\qquad \varphi_1 = e^{-i\omega_1 t}e^{i\phi}\Phi_1(r,\theta)$

  ⇨ Eigenvalue problem for $\omega_1(A_1)$

- $\nabla^2\varphi_2 - \mu^2(\cos\varphi_1)\varphi_2 = 0$ $\qquad \varphi_2 = e^{-i\omega_2 t}e^{2i\phi}\Phi_2(r,\theta)$

  ⇨ Eigenvalue problem for $\omega_2(A_1)$

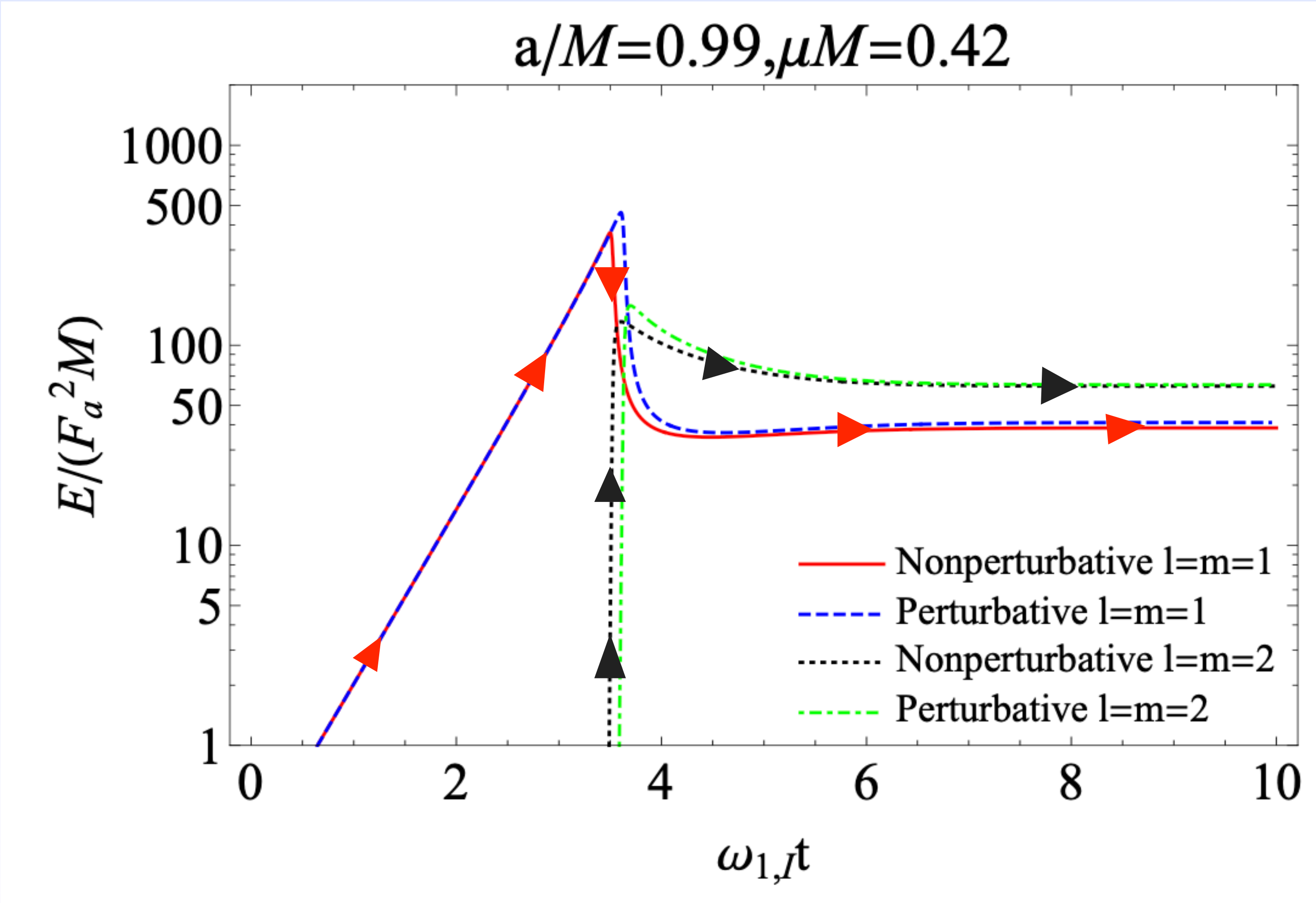
- $\nabla^2\varphi_r - \mu^2(\cos\varphi_1)\varphi_r = \mu^2(\cos\varphi_1)\tilde{\varphi}_2 - \frac{\mu^2}{2}(\sin\varphi_1)\tilde{\varphi}_2^2 + \cdots$

⇨ Calculate energy and angular momentum flux to infinity and horizon

⇨ Determine $A_1(t)$ and $A_2(t)$ so that conservation of energy and angular momentum is satisfied

# Adiabative evolution

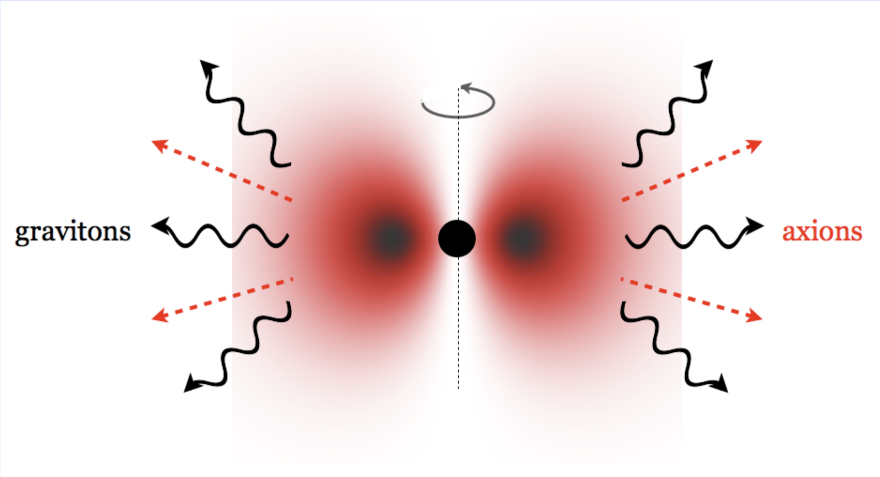
- Perturbative method
- Nonlinear method



# Brief discussion on gravitational waves

# Issues to be explored

- String axion field forms an axion cloud around a rotating astrophysical BH by extracting BH's rotation energy.




- Superradiant instability
- Nonlinear self-interaction

$$\nabla^2 \varphi - \mu^2 \sin \varphi = 0 \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- GW emission
- Long-term evolution of BH parameters

# Teukolsky equation

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\phi^2}$$

$$-\Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{d\psi}{dr}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\phi}$$

$$-2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T$$

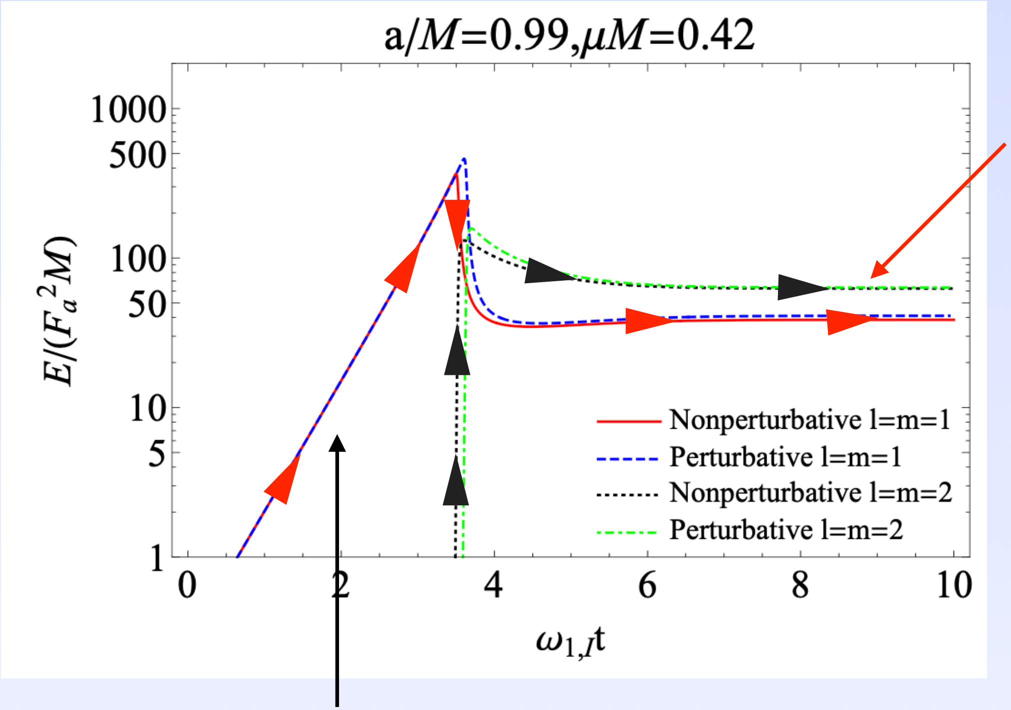
source term

$$T_{ab} = \nabla_a\Phi\nabla_b\Phi - \frac{1}{2}g_{ab}(\nabla_c\Phi\nabla^c\Phi + 2U(\Phi))$$

2 axion annihilation　squared term of m=1 mode

Level transition　cross term of m=1 and 2 modes

Omiya-kun, ongoing

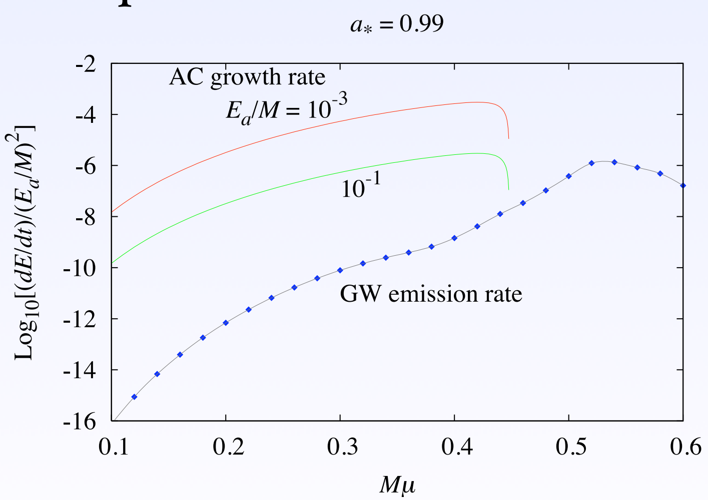



2 axion annihilation $\omega_{\mathrm{GW}} \fallingdotseq 2\mu$

+ level transition

$$\omega_{\mathrm{GW}} \fallingdotseq \omega_2 - \omega_1$$

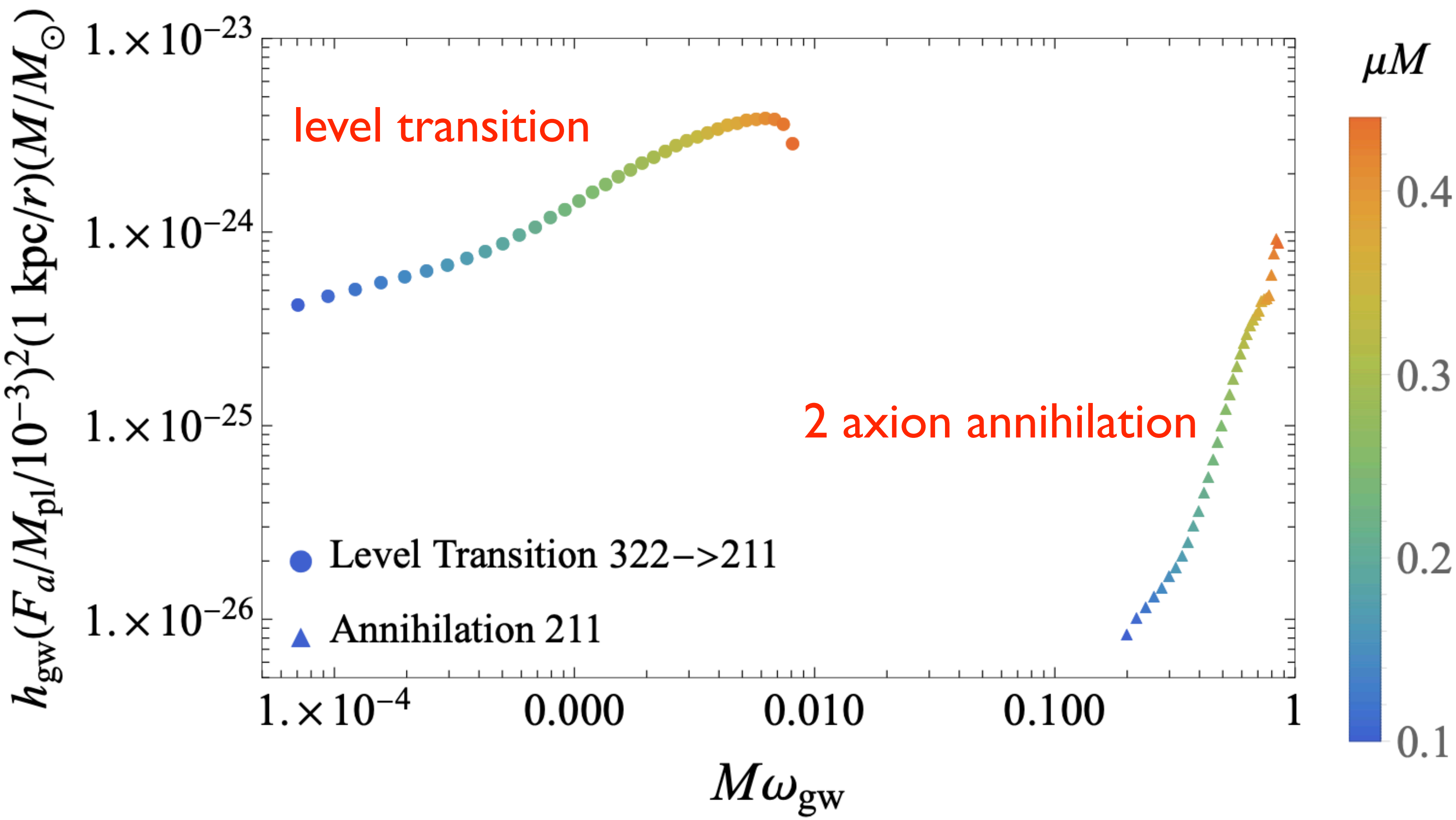
Superradiant phase




HY and Kodama, PTEP (2014) 043E02

2 axion annihilation

$$\omega_{\mathrm{GW}} \fallingdotseq 2\mu$$

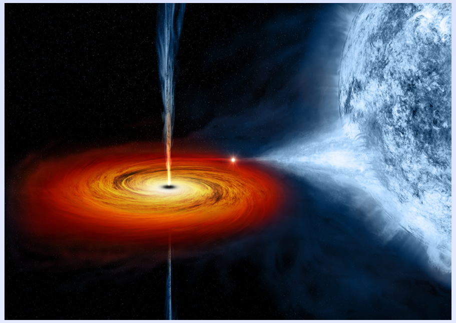



Preliminary

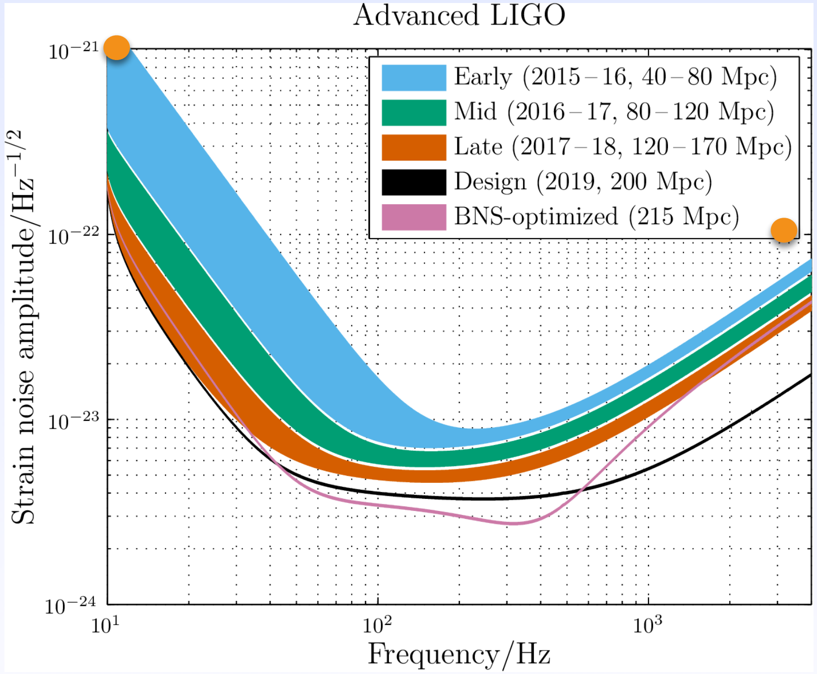
# Possible constraints from Cygnus X-1

- $M \approx 15 M_{\odot}$
- $a_* \gtrsim 0.983$
- $d \approx 1.86$ kpc

McClintock, et al., arXiv:1106.3688-3690{astro-ph}



- In the case of $\mu = 2.4 \times 10^{-12}$eV $(M\mu = 0.3)$




- Constraint from GW observation

$\Rightarrow$ $f_a \lesssim 10^{15}$ GeV

- Constraint from BH parameter evolution

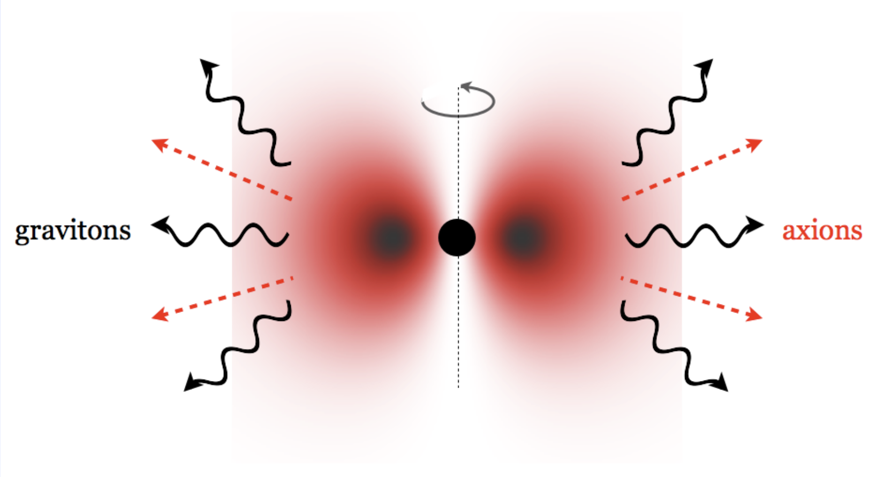
$\Rightarrow$ $\Delta a_* \ll 1$ $\Rightarrow$ $f_a \lesssim 10^{11}$ GeV

Preliminary

# Summary

# The most difficult part has been solved by Omiya-kun

- String axion field forms an axion cloud around a rotating astrophysical BH by extracting BH's rotation energy.

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- GW emission
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*Thank you!*