Generalization of Z string and its application

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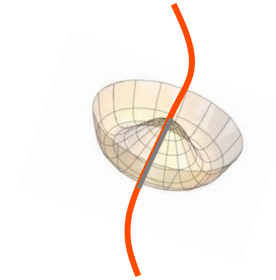
Based on
YK, N. Maekawa, PRD 107 (2023) 9, 096007 [arXiv: 2303.09517]

Gravitational Wave Probes of Physics Beyond Standard Model
2023/11/9, Osaka Metropolitan University



Introduction

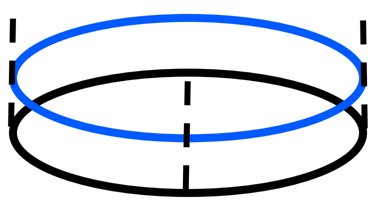
Cosmic strings are

- linear defects produced during some symmetry breakings
- probed by gravitational wave observations
- cannot be produced in the SM

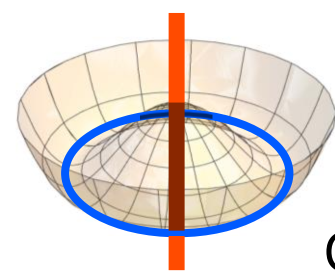
Find cosmic string through GWs ⇒ **Physics BSM is detected**

Cosmic strings are produced when the vacuum manifold of symmetry breaking has a non-contractible loop. [Kibble (1976)]

e.g. $U(1) \to \times$



Vacuum manifold $\mathcal{V} \simeq S^1$

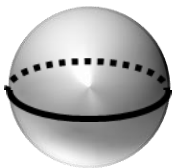
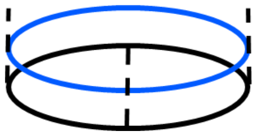
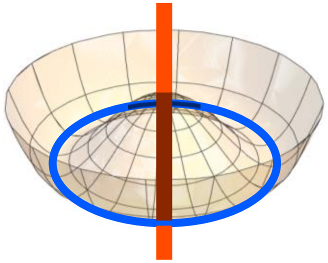


Classical solution having a linear excited region

Embedded string

Can cosmic strings not be produced if there are no non-contractible loop on the vacuum manifold? ⇒ NO!

When $G \to H$,

$\mathcal{V} \simeq G/H \simeq$  $\supset$  ⇒  String like classical solution

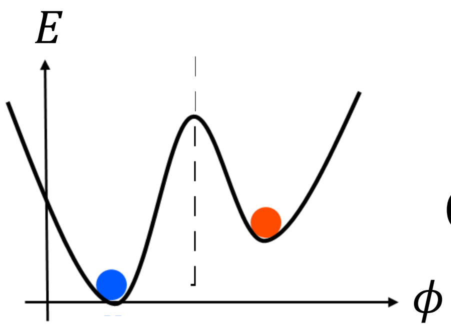
There are no non-contractible loop

$\mathcal{V}_{sub} \simeq S^1$

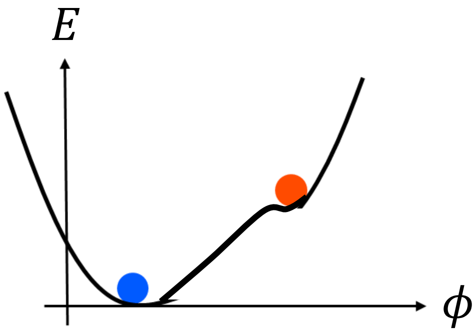
"Embedded string"

[Vachaspati, Barriola (1992)]
[Vachaspati, Barriola, Bucher (1994)]

Embedded string's stability is not topologically guaranteed



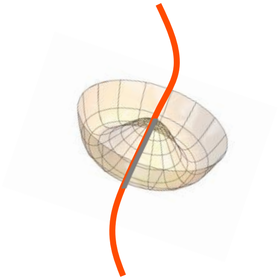
or



● : constant vacuum solution
● : Embedded string solution

⇐ Determined by the parameters

Motivation



Embedded string is classically stable.

⇒ it is produced during a symmetry breaking.

However, it has been studied only for one embedded string (Z string).

[James, Perivolaropoulos, Vachaspati (1993)]

To probe physics BSM through GW from cosmic strings,
it is important to study the stability of other embedded strings.

Today's talk

- We have studied generalization of the Z string, and found that its stability is determined only by the mass ratios of the Higgs and the gauge bosons.
- We have applied this result to cases of gauge group unification.

YK, N. Maekawa (2023)

- We comments on the relevance to the recent result of pulsar timing array collaborations. Our ongoing work

Z string

$SU(2) \times U(1) \xrightarrow{H:\,(\mathbf{2}, 1/2)} U(1)$

$\cup \qquad\qquad\qquad\qquad \cup$

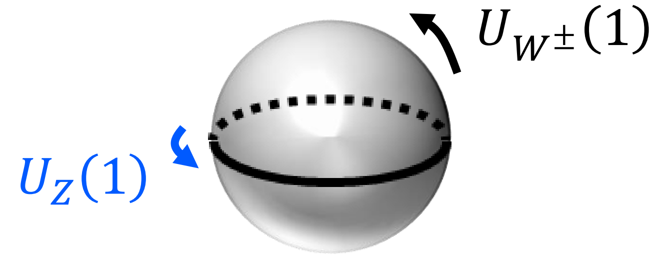
$U_Z(1) \longrightarrow \times$

related to
the Z boson

Consider
embedded string

$\mathcal{V} \simeq SU(2) \times U(1)/U(1) \simeq S^3$

No non-contractible loops on S^3



Z-string [Vachaspati (1992)]

$$H = \begin{pmatrix} 0 \\ f(r) v e^{i\theta} \end{pmatrix}, \qquad \vec{Z} = -\frac{2z(r)}{\alpha r}\,\vec{e}_\theta, \qquad (\text{others}) = 0$$

in cylindrical coordinate (r, θ, z), $\alpha^2 = g_1^2 + g_2^2$

- $f(r) v e^{i\theta}, -\frac{2z(r)}{\alpha r}$ are string solution of $U(1)$ breaking

$$(f(0) = z(0) = 0, f(\infty) = z(\infty) = 1\,)$$

Stability of the Z string

[James, Perivolaropoulos, Vachaspati (1993)]

Consider all the perturbations from the Z-string

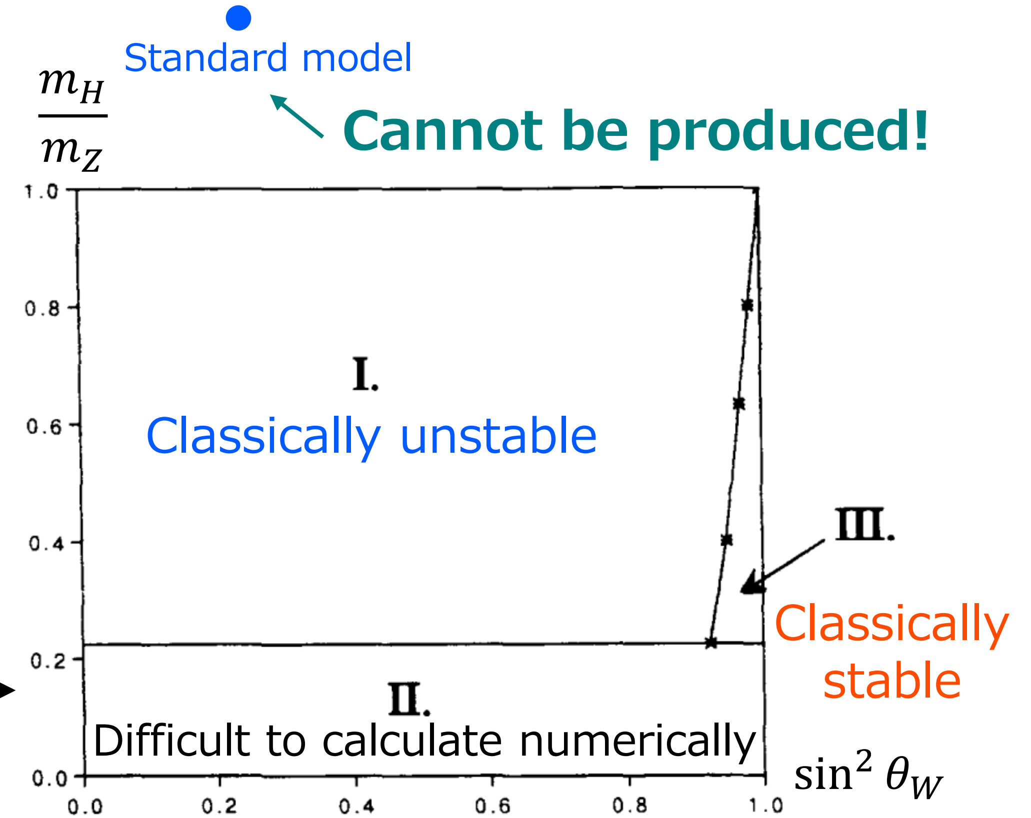
$$H = \begin{pmatrix} \phi(x) \\ f(r)ve^{i\theta} + \delta h(x) \end{pmatrix}, \vec{Z} = -\frac{z(r)}{\alpha r}\vec{e}_\theta + \delta\vec{Z}(x), \vec{W}^{\pm}(x), \vec{A}(x)$$

Calculate the variations of the energy $\delta\mu$ and find modes decreasing it

Only one mode can destabilize the Z string

$$\delta\mu = \int RdR\ \zeta(R)\mathcal{O}\left(R; \frac{m_H}{m_Z}, \theta_W\right)\zeta(R)$$

$R \equiv \frac{\alpha v}{2} r$, $\tan\theta_W \equiv g_1/g_2$
m_H, m_Z: mass of scalar and Z boson
ζ: perturbation mode



Overview of our work

- The stability of the Z string was studied in 1990s.
- The stability of other embedded strings were not studied.
  (∵ motivated by the electroweak breaking)

⇩ But now,

- Models beyond the standard model have various symmetry breakings.
- If cosmic strings are produced during a symmetry breaking, they can be probed by GW observation.

Our work

We consider the generalization of the Z string for $SU(N) \times U(1)_X \to SU(N-1) \times U(1)_Q$ and study the stability of them.

Generalization of Z string

We consider $SU(N) \times U(1)_X \xrightarrow{\phi: \left(\boldsymbol{N}, \frac{1}{2}\right)} SU(N-1) \times U(1)_Q$

Scalar potential: $V(\phi) = \lambda(|\phi|^2 - v^2)^2 \longleftarrow \mathcal{V} \simeq S^{2N-1}$ No non-contractible loop

➡ There is a neutral massive gauge boson $\tilde{Z}_\mu$

$$\tilde{Z}_\mu \equiv \sqrt{\frac{2(N-1)}{N}} \frac{g_N}{\alpha_N} G_\mu^{N^2-1} - \frac{g_1}{\alpha_N} B_\mu$$

$G_\mu^a, B_\mu : SU(N), U(1)$ gauge bosons

$T^{N^2-1} = \frac{1}{\sqrt{2N(N-1)}} \mathrm{diag}(1, \ldots, 1, 1-N)$

$\alpha_N^2 \equiv \frac{2(N-1)}{N} g_N^2 + g_1^2$

⬇ Make an embedded string

Generalized Z-string

$f(0) = z(0) = 0, f(\infty) = z(\infty) = 1$

$$\phi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f(r) v e^{i\theta} \end{pmatrix}, \quad \vec{\tilde{Z}} = -\frac{2z(r)}{\alpha_N r} \vec{e}_\theta, \quad \text{(others)=0}$$

Note that it is the Z-string when $N = 2$

Calculation of the stability

Scalar: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_{N-1}(x) \\ f(r)e^{in\theta} + \delta\phi(x) \end{pmatrix}$, Gauge boson: $\begin{pmatrix} \vec{G}^a(x) & \vec{G}^+(x) \\ \vec{G}^-(x) & \end{pmatrix}$,

$\vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\vec{e}_\theta + \delta\vec{\tilde{Z}}(x)$,

$\vec{\tilde{A}}(x) \equiv \frac{g_1}{\alpha_N}\vec{G}^{N^2-1} + \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}\vec{B}$

Diagonal part

- (light blue): $SU(N-1)$ adjoint
- (orange): $SU(N-1)$ fundamental
- (blue): $SU(N-1)$ singlet

⇩ Calculate the variations of the energy $\delta\mu$

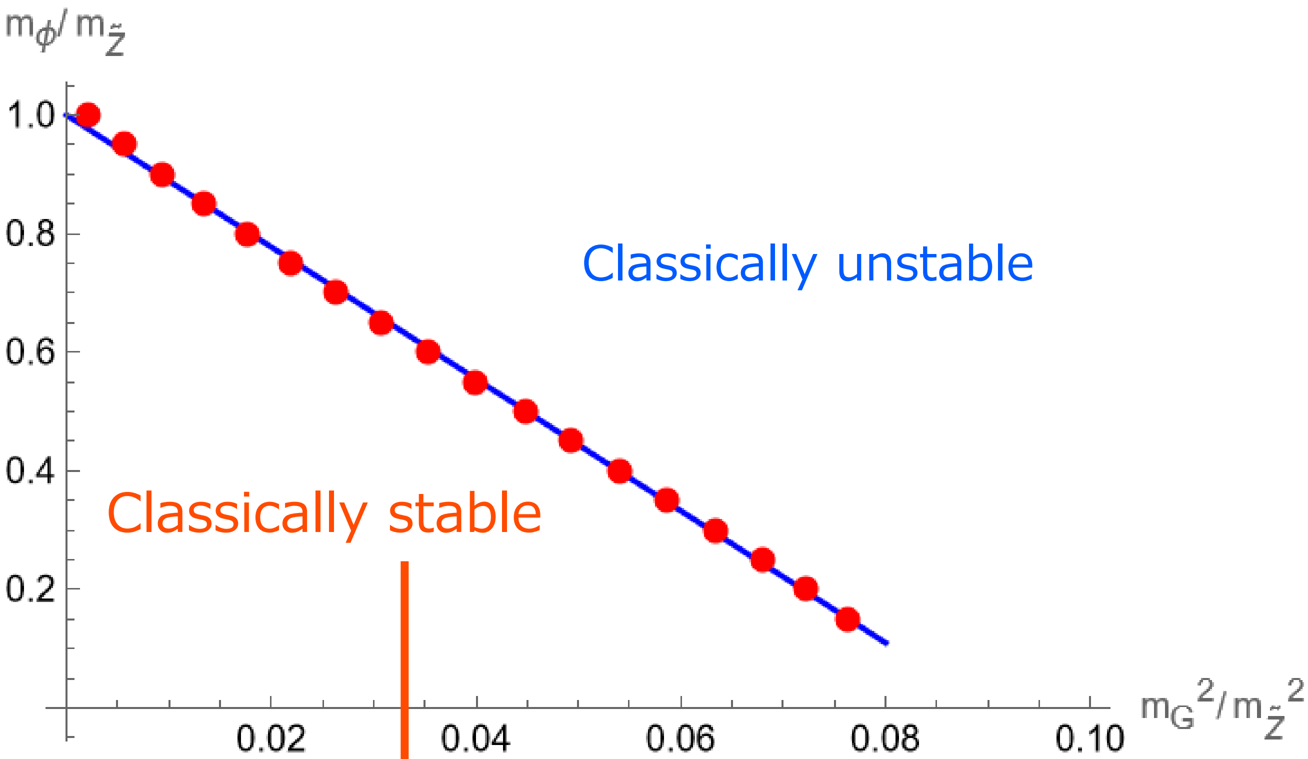
Only fundamental modes can destabilize the generalized Z string.
$\delta\mu$ is divided into $N-1$ parts which are similar to $\delta\mu$ of the Z string.

$$\delta\mu = \sum_{k=1}^{N-1} \delta\mu_k(r; m_\phi/m_{\tilde{Z}}, m_G/m_{\tilde{Z}})$$

Consistent with the Z string (cf. $\frac{m_G}{m_{\tilde{Z}}} = \sqrt{\frac{N}{2(N-1)}}\cos\theta_G$)

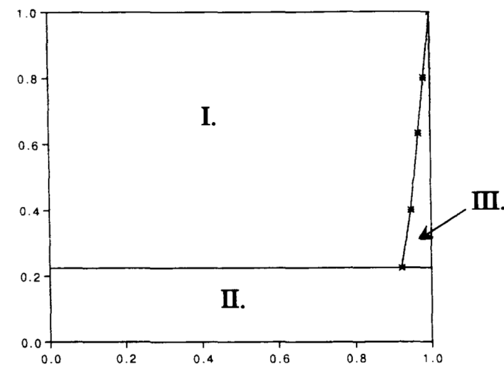
$m_\phi, m_{\tilde{Z}}, m_G$: the mass of scalar, neutral gauge boson, charged gauge boson
θ_G: mixing angle

Result

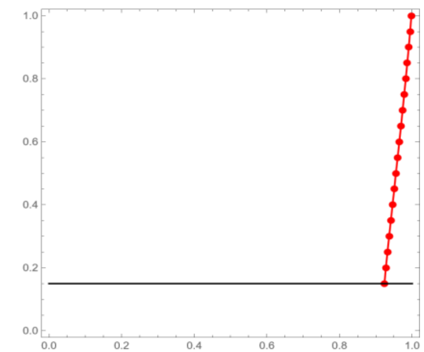


<u>$N = 2$</u>

[James, Perivolaropoulos, Vachaspati (1993)]



Ours result



approximately evaluation

$$\frac{m_\phi}{m_{\tilde{Z}}} \leq 1 - 11\frac{m_G^2}{m_{\tilde{Z}}^2} \iff g_1 \geq \sqrt{\frac{11}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N$$

Application for unification

We consider the case that $SU(N)$ and $U(1)$ have the same origin

$$\phi = (\boldsymbol{N}, q, \boldsymbol{1})\Big|_{g_1'} = (\boldsymbol{N}, 1/2, \boldsymbol{1})\Big|_{g_1}$$

$$G \to \cdots \to SU(N) \times U(1) \times H \to SU(N-1) \times U(1) \times H$$

$$g_U = g_N = g_1' \xrightarrow{\text{RG running}} g_N = \alpha_{RG}\, g_1' = \frac{\alpha_{RG}}{2q} g_1$$

The generalized Z-strings are formed when g_N and g_1 satisfy

$$g_1 \geq \sqrt{\frac{11}{1-m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N \quad \Rightarrow \quad q^2 \geq \alpha_{RG}^2 \left[\frac{2.75}{1-m_\phi/m_{\tilde{Z}}} - \frac{N-1}{2N}\right]$$

Condition for the rep. of ϕ in G

We apply it for

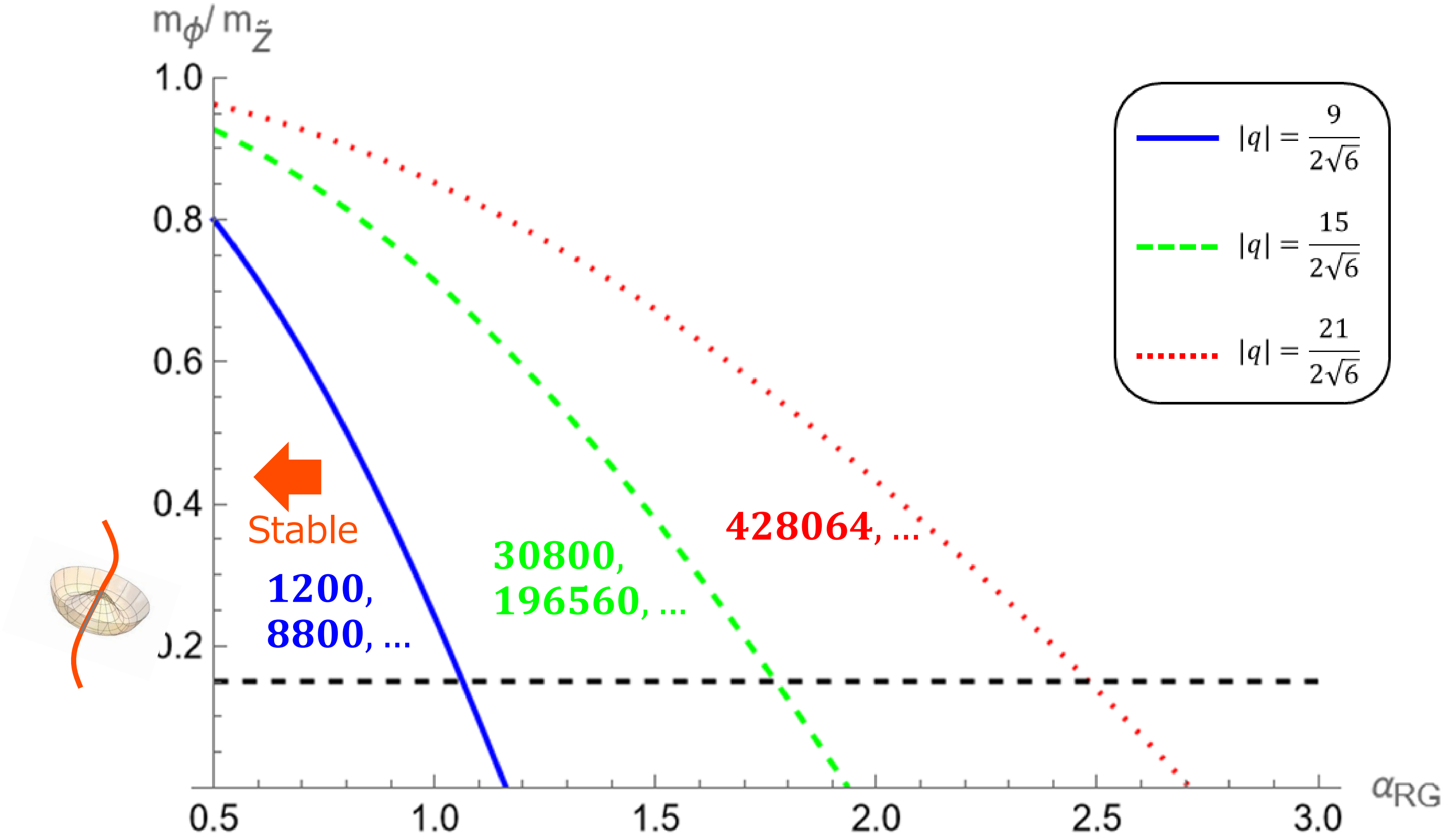
$$\boldsymbol{SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \to SU(3)_C \times SU(2)_L \times U(1)_Y}$$

$$\phi = (\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{2}, q)$$

$SO(10) \to SU(3)_C \times SU(2)_L \times \underline{SU(2)_R \times U(1)_X} \to SU(3)_C \times SU(2)_L \times \underline{U(1)_Y}$

($\alpha_{RG} = g_{2R}/g_{1X}$ at the breaking scale)

$\uparrow$ $\phi = (\mathbf{1}, \mathbf{1}, \mathbf{2}, q)$



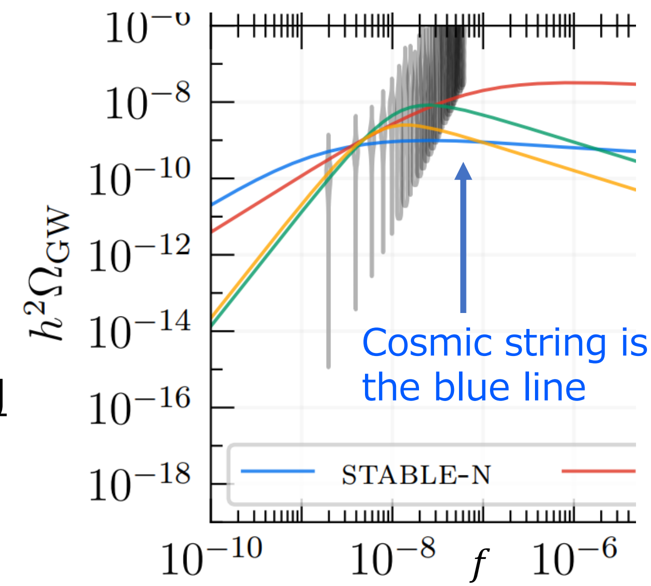
Very big representation Higgs is needed!

⇒ If generalized Z string is found, it can be a constraint for GUT.

PTA results and metastability

In June 2023, PTA collaborations reported a detection of stochastic gravitational waves.

NANOGrav [2306.16219], EPTA [2306.16214], PPTA [2306.16215], CPTA [2306.16216]



<u>PTA result</u>		<u>stable local string</u>
Blue tilted	← tension →	**Flat spectrum**

Metastable strings can solve this tension.

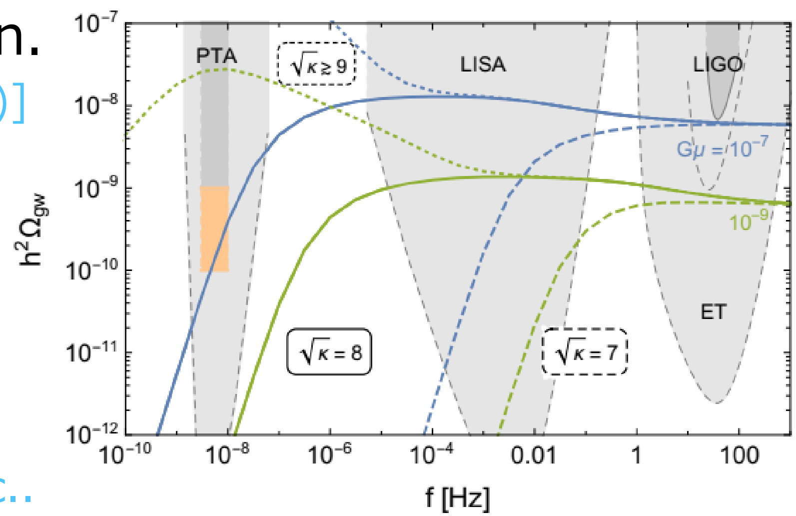
[Buchmuller, Domcke, Schmitz (2020)]

Decay probability $P \sim \frac{\mu}{2\pi} e^{-\pi\kappa}$

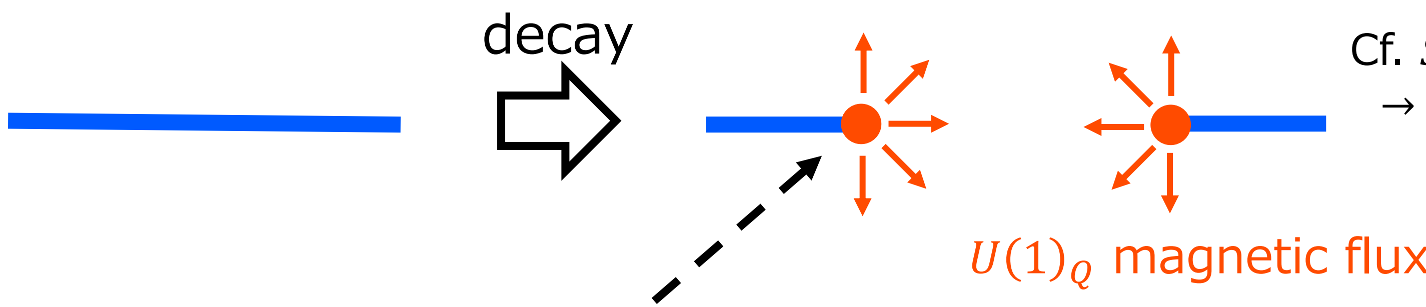
(μ: string tension)

$\sqrt{\kappa} \sim 8$ is preferred to solve the tension.

[Buchmuller, Domcke, Schmitz (2023)], etc..



Estimation of decay probability



Cf. $SU(N) \times U(1)_X$
$\rightarrow SU(N-1) \times U(1)_Q$

For $N = 2$, "Nambu monopole" [Nambu (1977)]

Decay probability can be estimated as $P \sim \frac{\mu}{2\pi} e^{-\pi m^2/\mu}$ [Preskill, Vilenkin (1993)]

where Nambu-monopole mass $m \sim \frac{4\sqrt{2}\pi}{3\alpha_N} \tan^{\frac{3}{2}} \theta_G \sqrt{\frac{m_\phi}{m_{\tilde{Z}}}}\, v \times \mathcal{O}(1)$

String tension $\mu \sim 2\pi \frac{m_\phi}{m_{\tilde{Z}}} v^2 \times \mathcal{O}(1)$

$$\sqrt{\kappa} \sim \frac{4\sqrt{\pi}}{3\alpha_N} \tan^{\frac{3}{2}} \theta_G \sim \frac{4\sqrt{\pi}}{3\alpha_N} \left(\frac{N}{2(N-1)} \frac{m_{\tilde{Z}}^2}{m_G^2} - 1 \right)^{\frac{3}{4}} \xrightarrow[]{e.g.\ \alpha_N = 1.3,\ N = 7,\ m_G^2/m_{\tilde{Z}}^2 = 0.07} 8.1 \ldots$$

Large α_N, N and $m_G/m_{\tilde{Z}}$ are preferred to explain the PTA results.

Summary

- Embedded strings are the classical solutions having 1-dimensional excited region (= cosmic string), but their stability is not guaranteed by a topological feature of the vacuum manifold.

- We have generalized the Z string to $SU(N) \times U(1)_X \to SU(N-1) \times U(1)_Q$, and studied its stability. We have found that it is needed that the $U(1)$ gauge coupling is larger than $SU(N)$ to the formation.

- We have applied the formation condition to the case that $SU(N)$ and $U(1)$ have the same origin, and found that a very large representation of scalar is needed for the formation.

- To see the relevance to the recent results of PTA collaborations, we focus on the metastability of the generalized Z string.
The decay probability may be good for explaining the PTA result for large gauge couplings, mixing angle and N, but more precise calculations are needed. (Ongoing work)

Back up