

Primordial Black Holes

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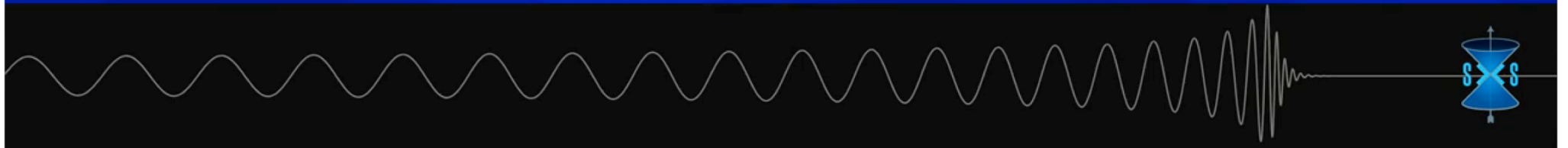
Why PBHs?

- We can probe high-energy physics, the early Universe, and gravity with **PBHs** through recent and future gravitational wave observations

LIGO and Virgo have detected gravitational wave signals from Binary Black Holes

<https://www.youtube.com/watch?v=1agm33iEAuo>

-0.76s

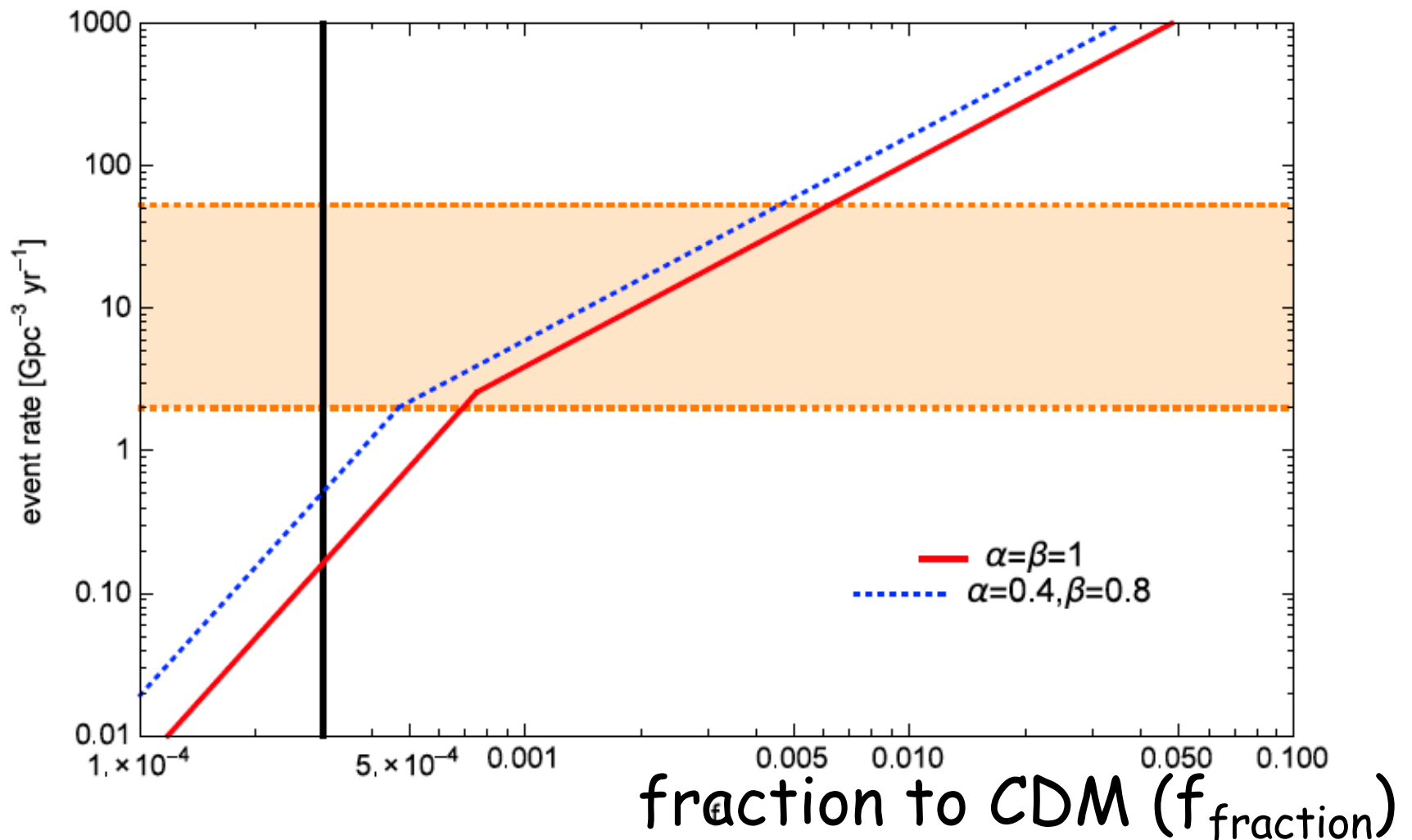


GW150914 and its merger rates for 30 M_{solar} masses BBH

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

A 3-body effect is important for the BBH formations

Rate of GW150914



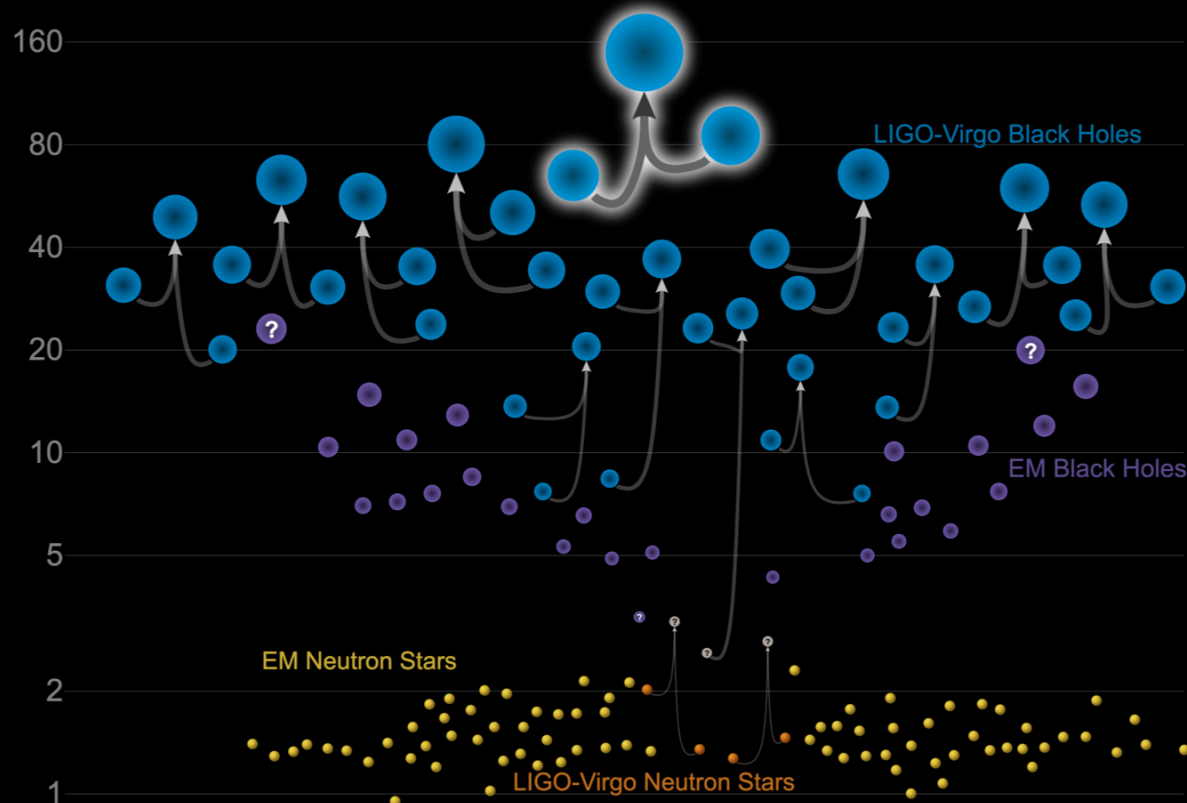
How to produce the binary black holes (BBH) with masses of $O(10) M_{\odot}$?

- **Astrophysics:** Large uncertainties on **gravitational frictions** through common envelope phases, mechanisms of supernovae (SNe) and appropriate **kick velocities** after SNe for **Pop III/Pop II** stars [astrophysically-model dependent]
- **Cosmology:** large uncertainties on numbers of PBHs, which depend on **inflation models** [cosmologically-model dependent]

Mass Gap

$M = 2 - 5 M_{\odot}$ or $M \gg O(10) M_{\odot}$ would be no longer neutron stars or astrophysical BHs

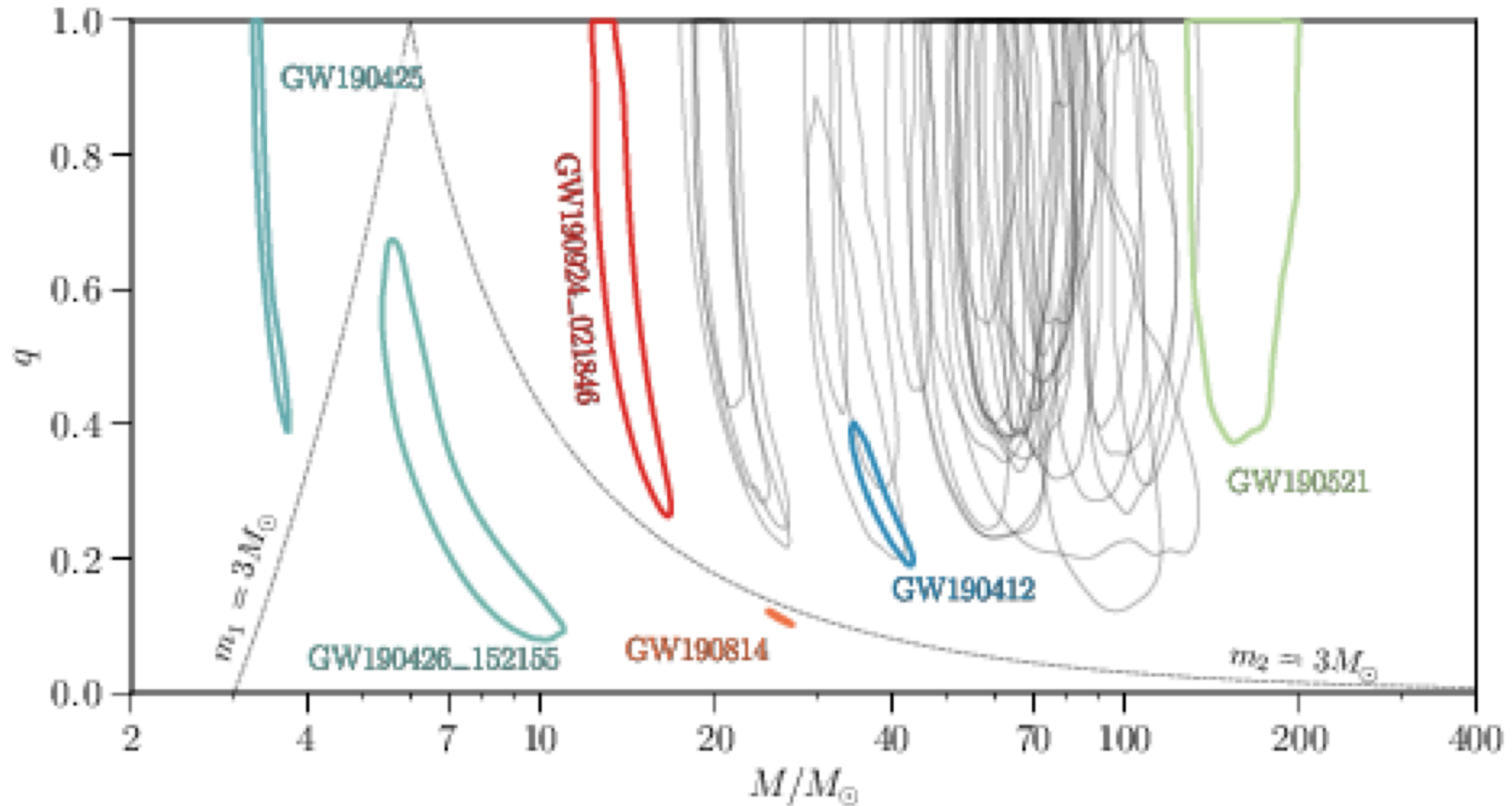
Masses in the Stellar Graveyard
in Solar Masses



Updated 2020-09-02

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Mass ratio q v.s. total mass

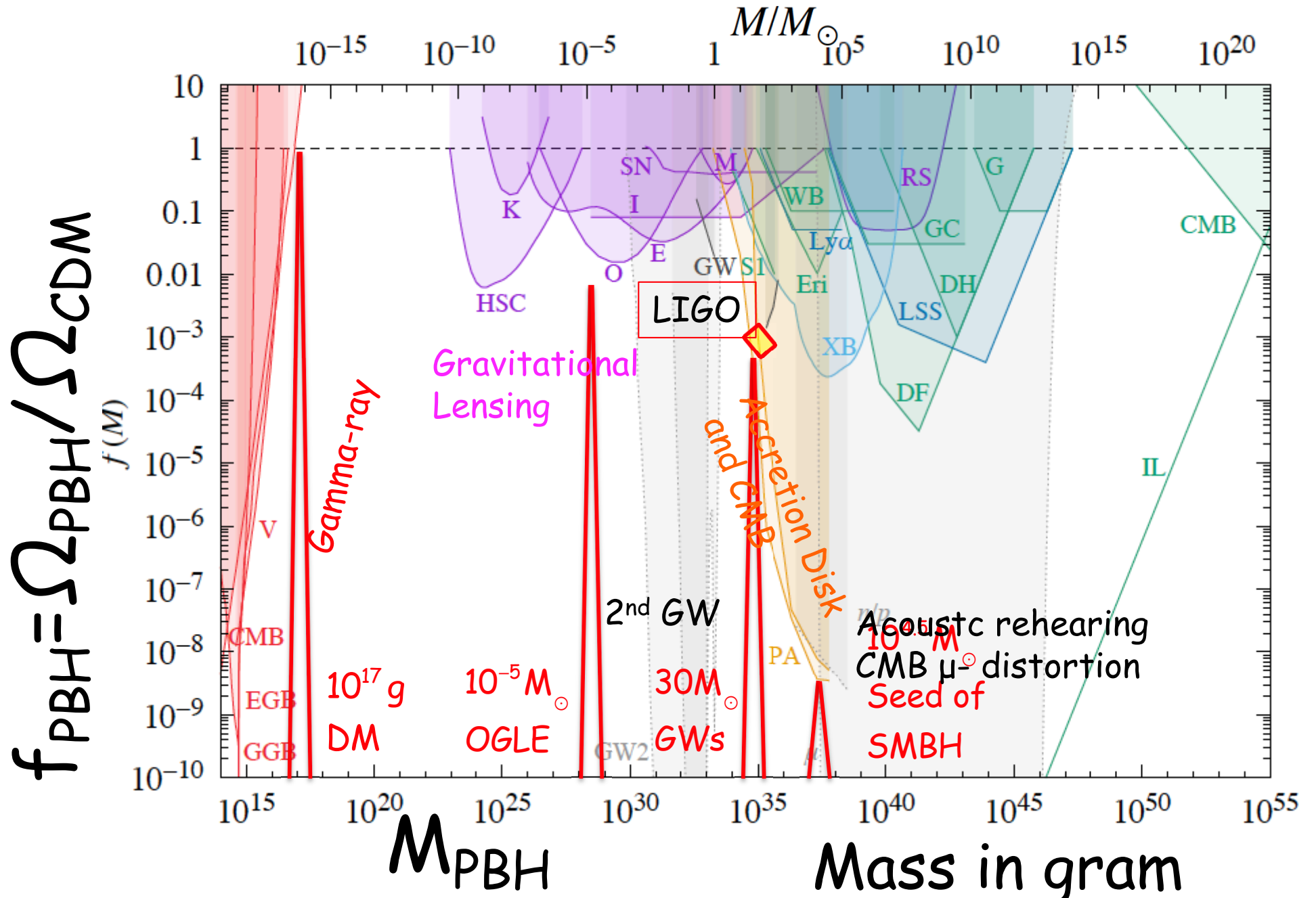


Credible region contours for all candidate events in the plane of total mass M and mass ratio q . Each contour represents the 90% credible region for a different event. We highlight the previously published notable candidate events: `\protect\NAME{GW190412A}`, `\protect\NAME{GW190425A}`, `\protect\NAME{GW190521A}` and `\protect\NAME{GW190814A}`, the potential NSBH `\protect\NAME{GW190426A}`, and finally `\protect\NAME{GW190924A}`, which is most probably the least massive system with both masses $>3 \backslash, \backslash M_{\text{sun}}$. The dashed lines delineate regions where the primary/secondary can have a mass below $3 \backslash, \backslash M_{\text{sun}}$. For the region above the $m_2=3 \backslash, \backslash M_{\text{sun}}$ line, both objects in the binary have masses above $3 \backslash, \backslash M_{\text{sun}}$.

R. Abbott, et al, LSC P&P Committee, arXiv:2010.14527 [gr-qc]

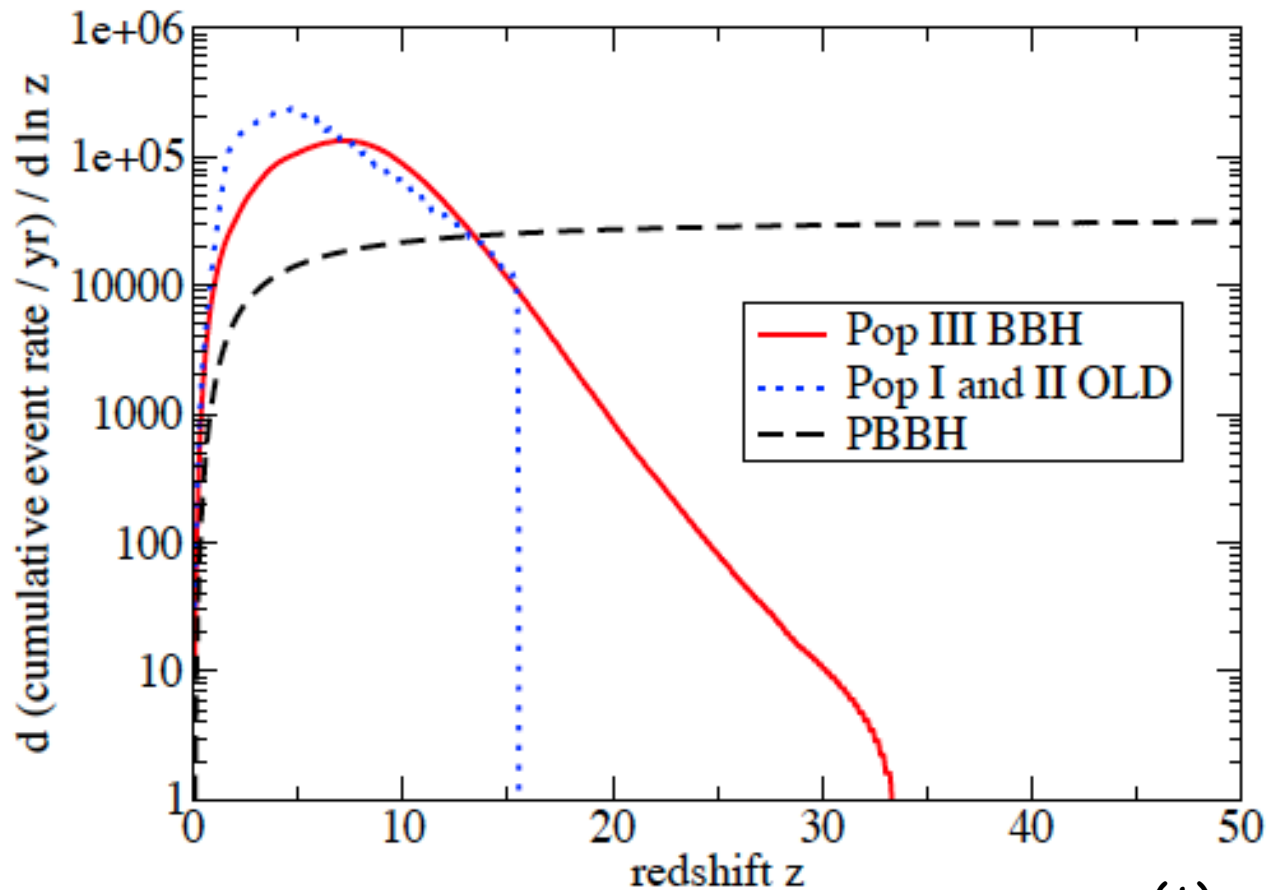
Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



DECIGO discriminates BPBHs from the normal BBHs

[Takashi Nakamura et al, arXiv:1607.00897 \[astro-ph.HE\]](#)

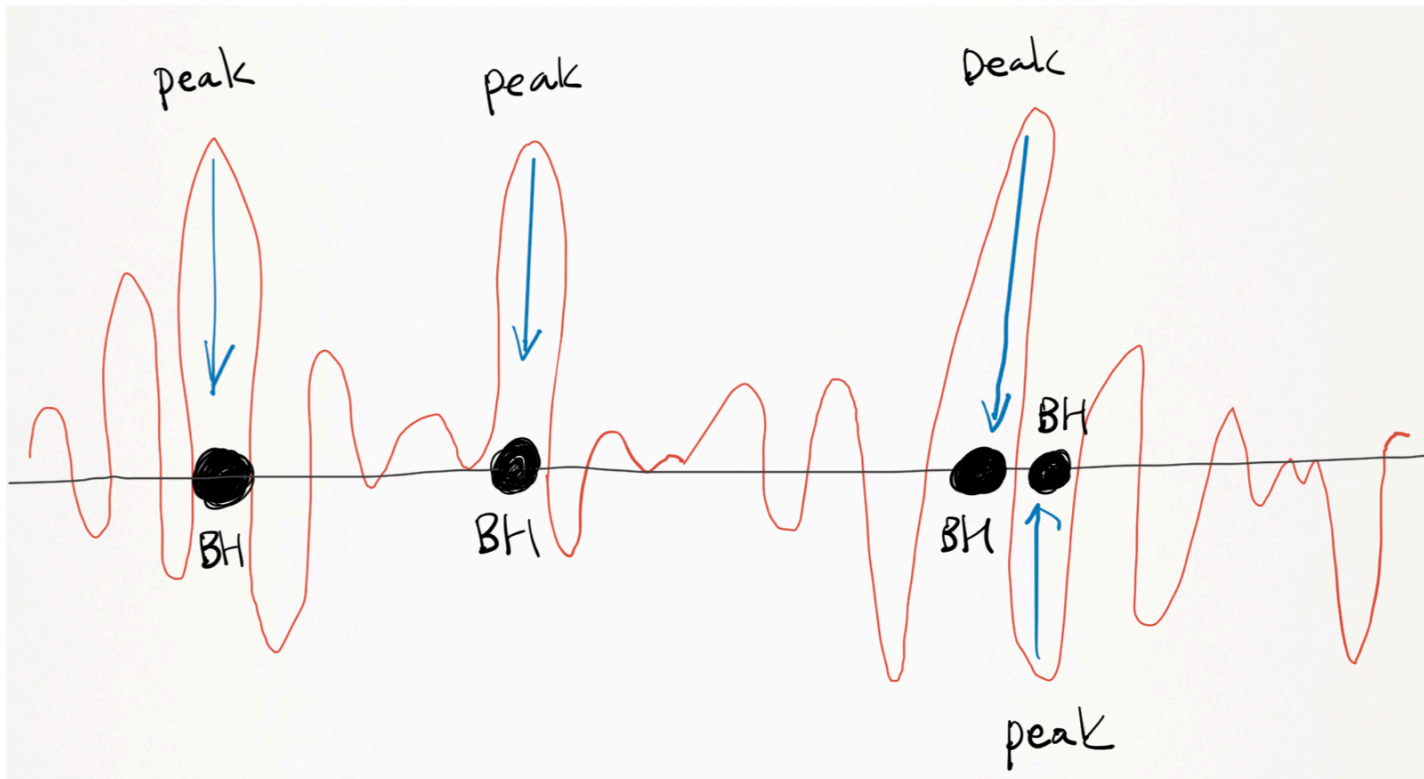


$$1/z \sim \frac{a(t)}{a(t_0)} \sim \left(t / 10\text{Gyr}\right)^{2/3}$$

Formation

Primordial Black Hole (PBH)

- Large perturbation at small scales was produced by Inflation at around $> 10^{-38}$ second



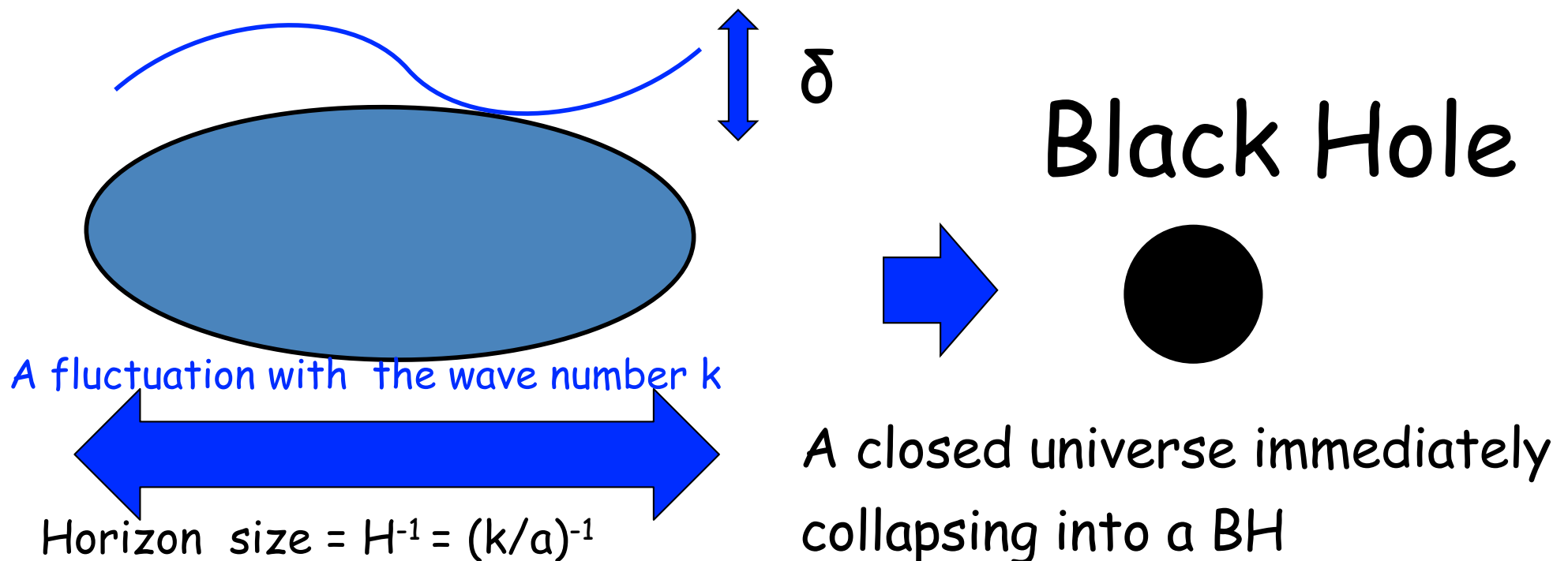
Conditions for a PBH formation in Radiation dominated (RD) Universe

Zel'dovich and Novikov (1967), Hawking (1971), Carr (1975)

Harada, Yoo and KK (2013)

- Gravity could be stronger than pressure

$$\delta > \delta_c \sim p / \rho \sim c_s^2 = w = 1/3$$



$P_\zeta(k)$ and PBH abundance $\beta(M)$

- Fraction of PBH to the total with Press Schechter formalism

For Peak Statistics,
e.g., see Yoo, Harada, KK et al (2018)(2020)

$$\beta(M) \equiv \frac{\rho_{\text{PBH}}(M)}{\rho_{\text{tot}}} = 2 \int_{\delta_{\text{th}}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \text{erfc}\left(\frac{\delta_{\text{th}}}{\sqrt{2}\sigma}\right)$$

$\delta_{\text{th}} \sim 1/3 - 0.5$

$\sigma \sim \overline{\delta\rho/\rho}$

- Relation between β and fluctuation σ (or β and Ω)

$$\beta(M) \sim \text{erfc}\left(\frac{\delta_{\text{th}}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}} \frac{\sigma}{\delta_{\text{th}}} \exp\left(-\frac{\delta_{\text{th}}^2}{2\sigma^2}\right)$$

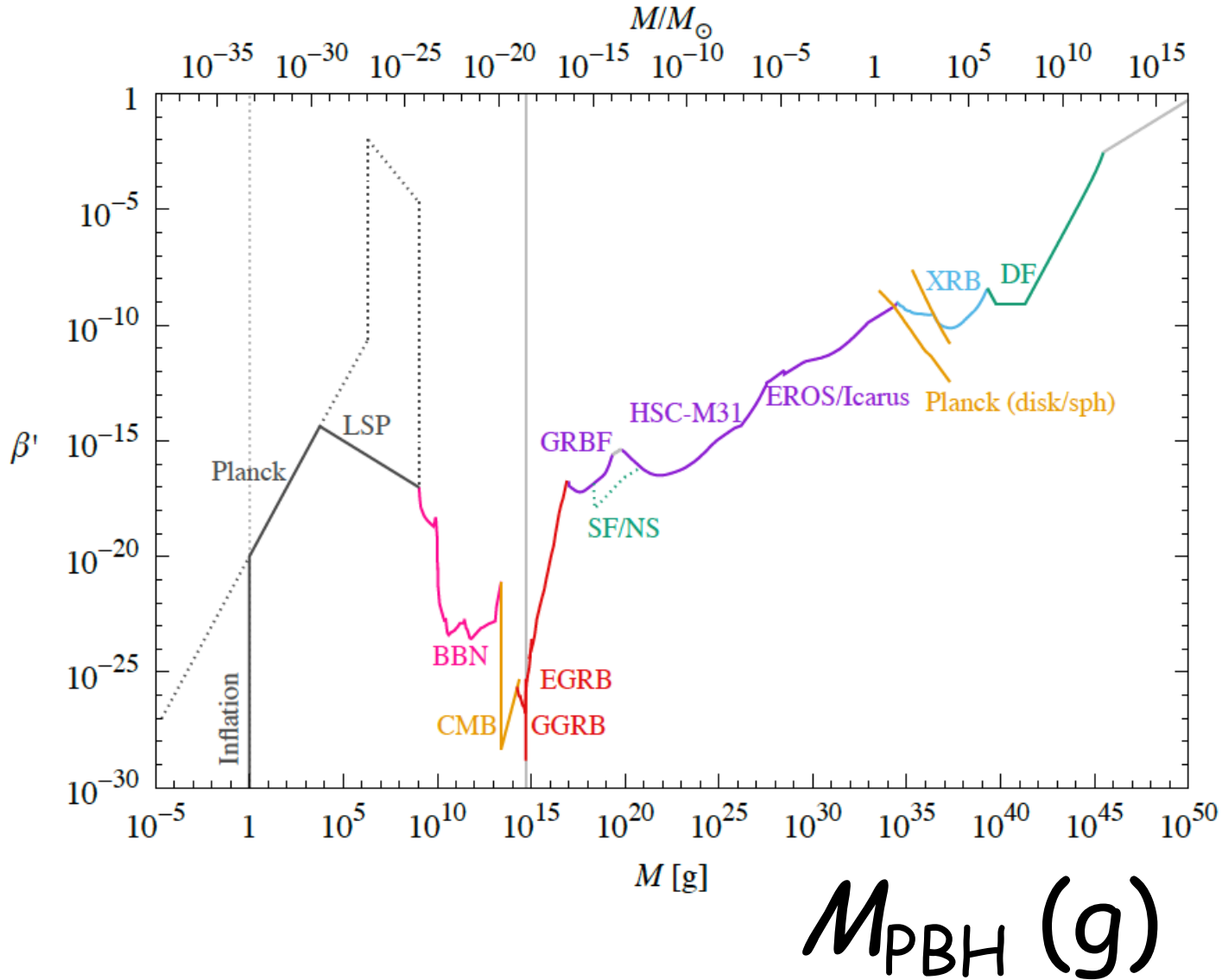
$$= 1.5 \times 10^{-18} \left(\frac{m_{\text{PBH}}}{10^{15} \text{ g}}\right)^{1/2} \left(\frac{\Omega_{\text{PBH}} h^2}{0.1}\right)$$

$\sim P_\zeta$

$$\beta = \rho_{\text{PBH}} / \rho_{\text{tot}} \text{ vs } M_{\text{PBH}}$$

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)

$$\beta = \rho_{\text{PBH}} / \rho_{\text{tot}}$$



Typical quantities of PBHs in RD

- Mass (horizon mass = $\rho(t_{\text{form}}) H(t_{\text{form}})^{-3}$)

$$M_{\text{PBH}} \sim \rho(H_{\text{form}}^{-1})^3 \sim M_{\text{pl}}^2 t_{\text{form}} \sim \frac{M_{\text{pl}}^3}{T_{\text{form}}^2} \sim 10^{15} \text{ g} \left(\frac{T_{\text{form}}}{3 \times 10^8 \text{ GeV}} \right)^{-2} \sim 30 M_{\odot} \left(\frac{T_{\text{form}}}{40 \text{ MeV}} \right)^{-2}$$

- Lifetime

$$\tau_{\text{PBH}} \sim \frac{M_{\text{PBH}}^3}{M_{\text{pl}}^4} \sim 4 \times 10^{17} \text{ sec} \left(\frac{M_{\text{PBH}}}{10^{15} \text{ g}} \right)^3 \sim 3 \times 10^{68} \text{ yrs} \left(\frac{M_{\text{PBH}}}{30 M_{\odot}} \right)^3$$

- Hawking Temperature

$$T_{\text{PBH}} \sim \frac{M_{\text{pl}}^2}{M_{\text{PBH}}} \sim 0.1 \text{ MeV} \left(\frac{M_{\text{PBH}}}{10^{15} \text{ g}} \right)^{-1} \sim 3 \times 10^{-11} \text{ K} \left(\frac{M_{\text{PBH}}}{30 M_{\odot}} \right)^{-1}$$

- Wave number of horizon length

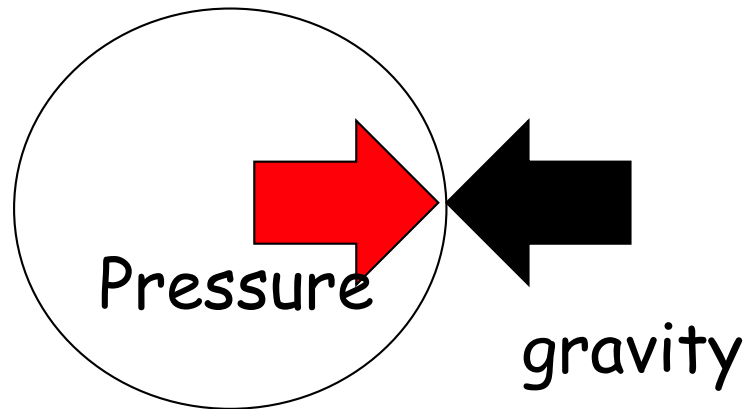
$$k = aH \sim 10^5 \text{ Mpc}^{-1} \left(\frac{M_{\text{PBH}}}{10^4 M_{\odot}} \right)^{-1/2} \sim 10^5 \text{ Mpc}^{-1} \left(\frac{T_{\text{form}}}{\text{MeV}} \right)^{+1}$$

- Fraction to CDM

$$f_{\text{fraction}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} \sim \left(\frac{\beta}{10^{-18}} \right) \left(\frac{M_{\text{PBH}}}{10^{15} \text{ g}} \right)^{-1/2} \sim \left(\frac{\beta}{10^{-8}} \right) \left(\frac{M_{\text{PBH}}}{30 M_{\odot}} \right)^{-1/2} \sim 10^8 \left(\frac{M_{\text{PBH}}}{30 M_{\odot}} \right)^{-1/2} \sqrt{P_{\delta}} \exp \left[-\frac{1}{18 P_{\delta}} \right]$$

Features of PBH formations in RD

- Spherical due to radiation pressure



$$(w \equiv p / \rho \sim 1/3)$$

- Negligible evolutions of density perturbations
- Quite a small angular momentum

See, T.Chiba and S.Yokoyama, 2017

De Luca et al, 2019

Minxi He and Suyama, 2019

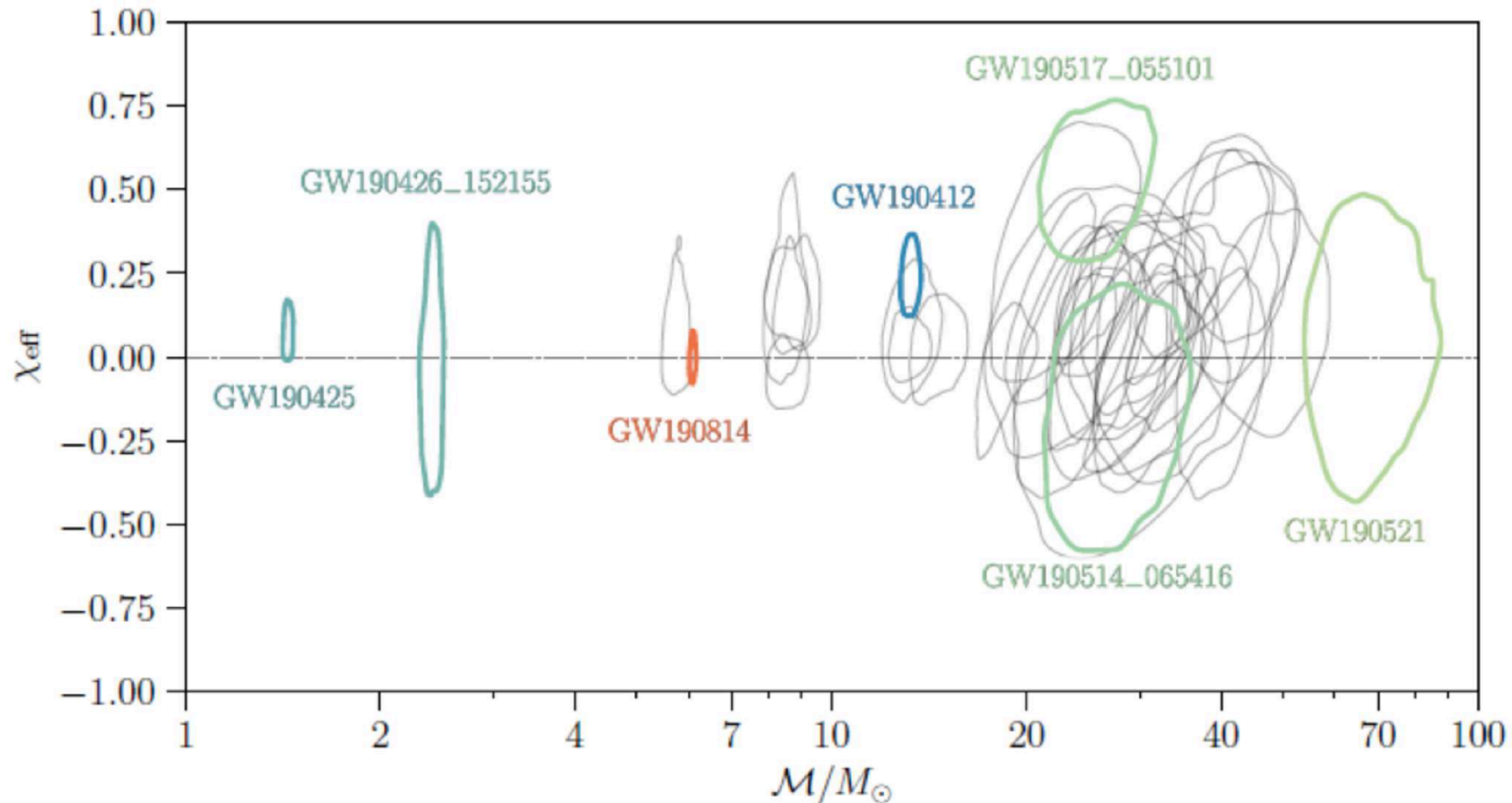
Harada, Yoo, Kohri, Koga and Monobe, 2020

(dimensionless Kerr parameter)

$$\sqrt{\langle a_*^2 \rangle} \simeq 6.5 \times 10^{-4} \left(\frac{M}{M_H} \right)^{-1/3}$$

Effective inspiral spin parameter of the observed BHs

$$\chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2}$$

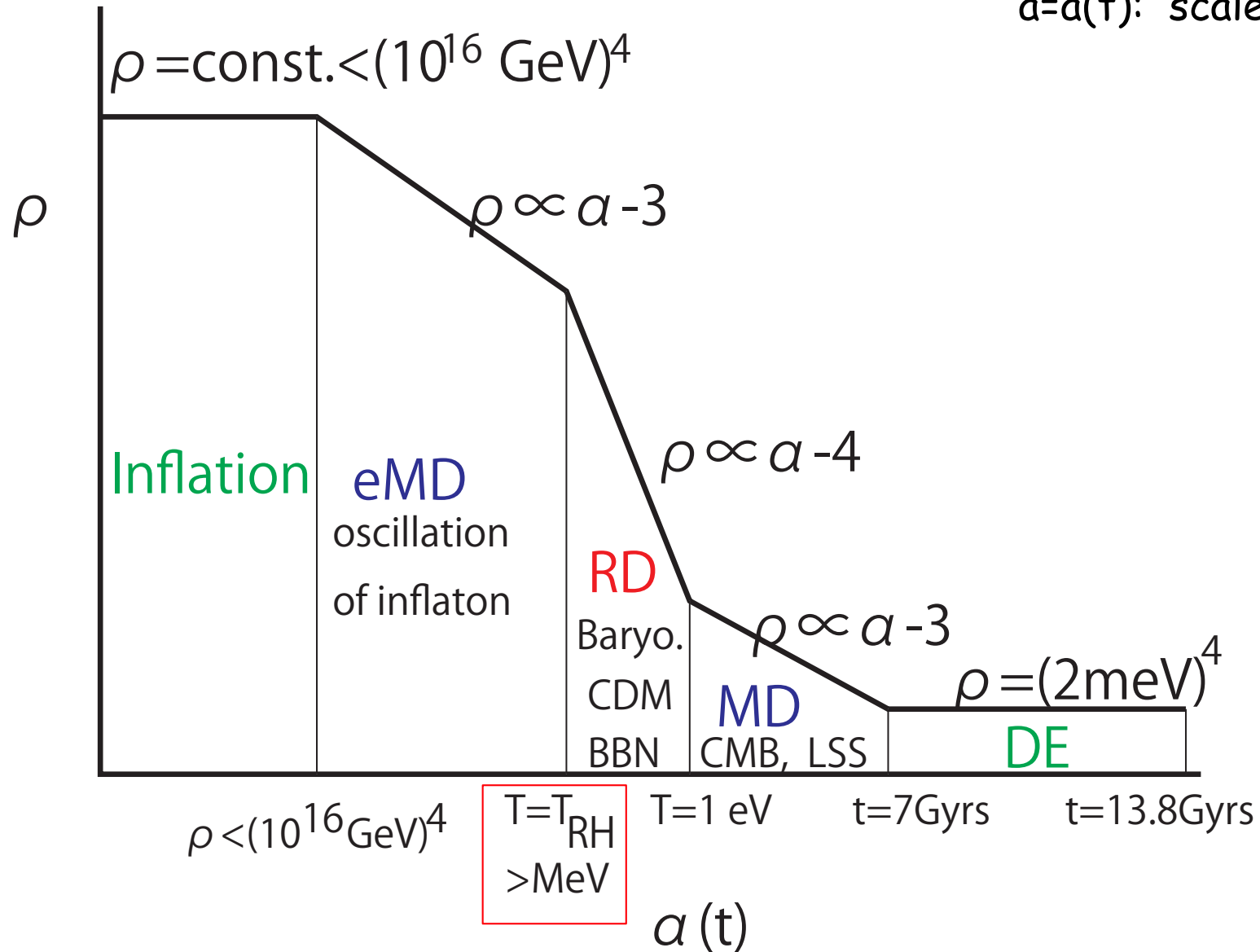


Credible region contours for all candidate events in the plane of chirp mass \mathcal{M} and effective inspiral spin χ_{eff} . Each contour represents the 90% credible region for a different event. We highlighted the previously published candidate events (cf. Fig.~\ref{fig:mtotqpost}), as well as `\protect\NAME{GW190517A}` and `\protect\NAME{GW190514A}`, which have the highest probabilities of having the largest and smallest χ_{eff} respectively.

R. Abbott, et al, LSC P&P Committee, arXiv:2010.14527 [gr-qc]

Cosmic history of energy density

$a=a(t)$: scale factor



PBH formation at the (early) matter dominated (MD) Universe

Polnarev and Khlopov (1982)

Harada, Yoo, KK, Nakao, Jhingan (2016)

1. **Pressure is negligible**, which could induce an immediate collapse and producing more PBHs?
2. **Density perturbations can evolve**, which produces non-spherical objects and cannot be enclosed by the Horizon. That means less PBHs can be produced?

Matter Domination

- Three radius in Lagrangian coordinate q_i

$$r_1 = (a - \alpha b)q_1$$

Zel'dovich Approximation

$$r_2 = (a - \beta b)q_2$$

$$r_3 = (a - \gamma b)q_3$$

- Eccentricity $e^2 = 1 - \left(\frac{r_2(t_c)}{r_3(t_c)}\right)^2 = 1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2$

- Hoop with 2nd Elliptic function $E(x)$

$$\mathcal{C} = 16 \left(1 - \frac{\gamma}{\alpha}\right) E \left(\sqrt{1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2} \right) r_f,$$

- Hoop conjecture for PBH production

$$\mathcal{C} \lesssim 2\pi r_g.$$

Abundance of PBHs formed in MD

- Probability distribution by peak statistics (BBKS)

Doroshkevich (1970)

$$\begin{aligned}
 & w(\alpha, \beta, \gamma) d\alpha d\beta d\gamma \\
 &= -\frac{27}{8\sqrt{5}\pi\sigma_3^6} \exp \left[-\frac{1}{10\sigma_3^2} (\alpha + \beta + \gamma)^2 - \frac{1}{4\sigma_3^2} \{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2\} \right] \\
 & \cdot (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha) d\alpha d\beta d\gamma.
 \end{aligned}$$

$\sigma_H = \sqrt{5}\sigma_3$

- Probability

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^\alpha d\beta \int_{-\infty}^\beta d\gamma \theta(1 - h(\alpha, \beta, \gamma)) w(\alpha, \beta, \gamma)$$

$$h(\alpha, \beta, \gamma) = \frac{2}{\pi} \frac{\alpha - \gamma}{\alpha^2} E \left(\sqrt{1 - \left(\frac{\alpha - \beta}{\alpha - \gamma} \right)^2} \right)$$

$h(\alpha, \beta, \gamma) := C / (2\pi r_g)$

Angular momentum produced by perturbations

Harada, Yoo, KK, and Nakao (2017)

- Angular momentum

$$\mathbf{L}_c = \int_{a^3V} \rho \mathbf{r} \times \mathbf{v} d^3\mathbf{r} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x} \delta \times \mathbf{u} d^3\mathbf{x} \right)$$

- Density perturbation δ 1st order effects 2nd order effects
- (Peculiar) Velocity perturbation $\mathbf{u} := aD\mathbf{x}/Dt$
 $\mathbf{u}_1 = -\frac{t}{a} \nabla \psi_1$
- Potential perturbation $\psi := \Psi - \Psi_0$

Effects by finite angular momentum

Harada, Yoo, KK, Nakao (2017)

- Probability distribution

$$a_* := L/(GM^2/c)$$
$$f_{\text{BH}(2)}(a_*) da_* \propto \frac{1}{a_*^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}} \left(\frac{2}{5}\mathcal{I}\right)^{4/3} \frac{1}{a_*^{4/3}}\right) da_*$$

- Probability

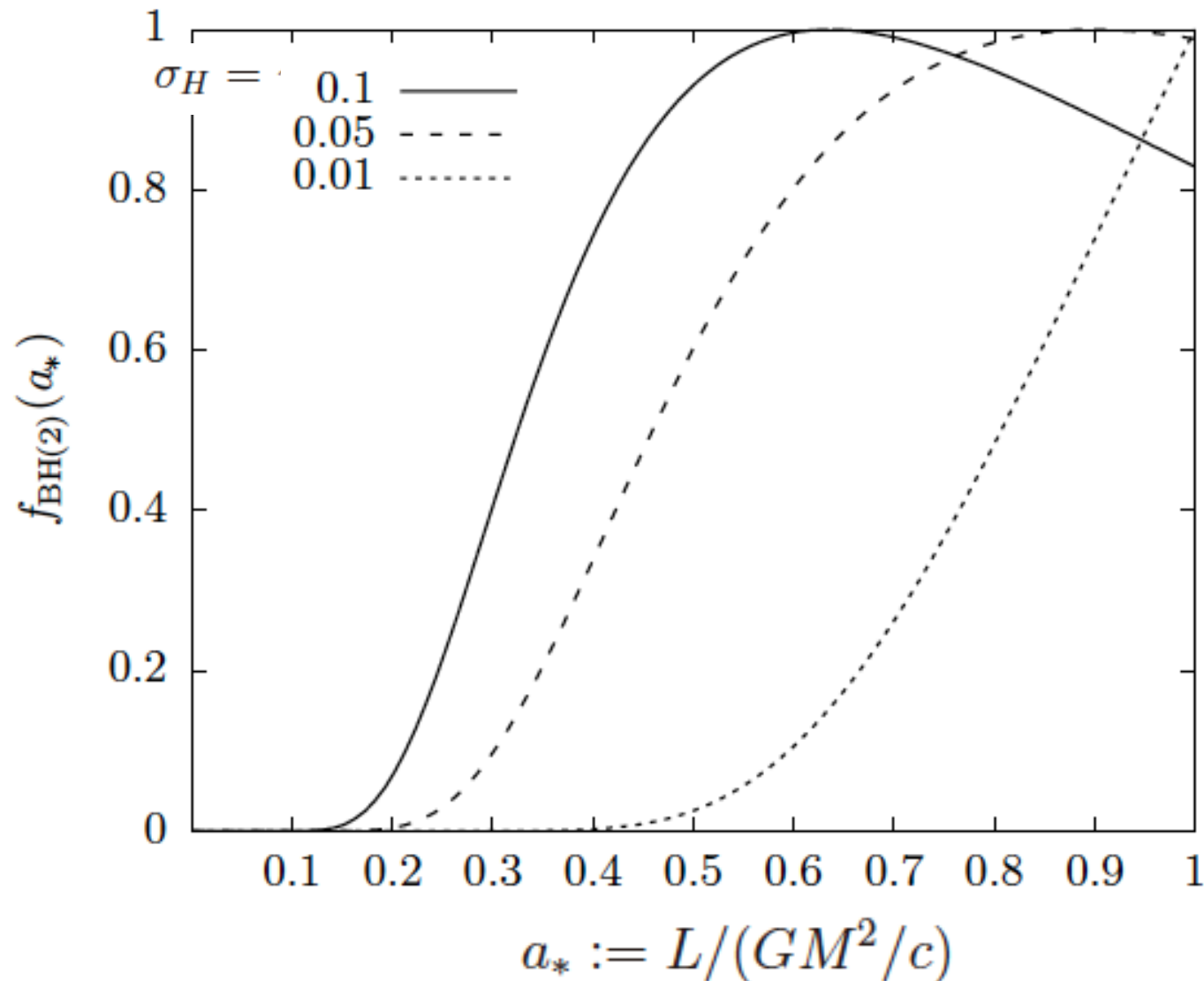
$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^\alpha d\beta \int_{-\infty}^\beta d\gamma \theta[\delta_H(\alpha, \beta, \gamma) - \delta_{\text{th}}] \theta[1 - h(\alpha, \beta, \gamma)] w(\alpha, \beta, \gamma)$$

$$\delta_{,H}(\alpha, \beta, \gamma) = \alpha + \beta + \gamma \quad \delta_{\text{th}} := \left(\frac{2}{5}\mathcal{I}\sigma_H\right)^{2/3}$$

Spin distribution

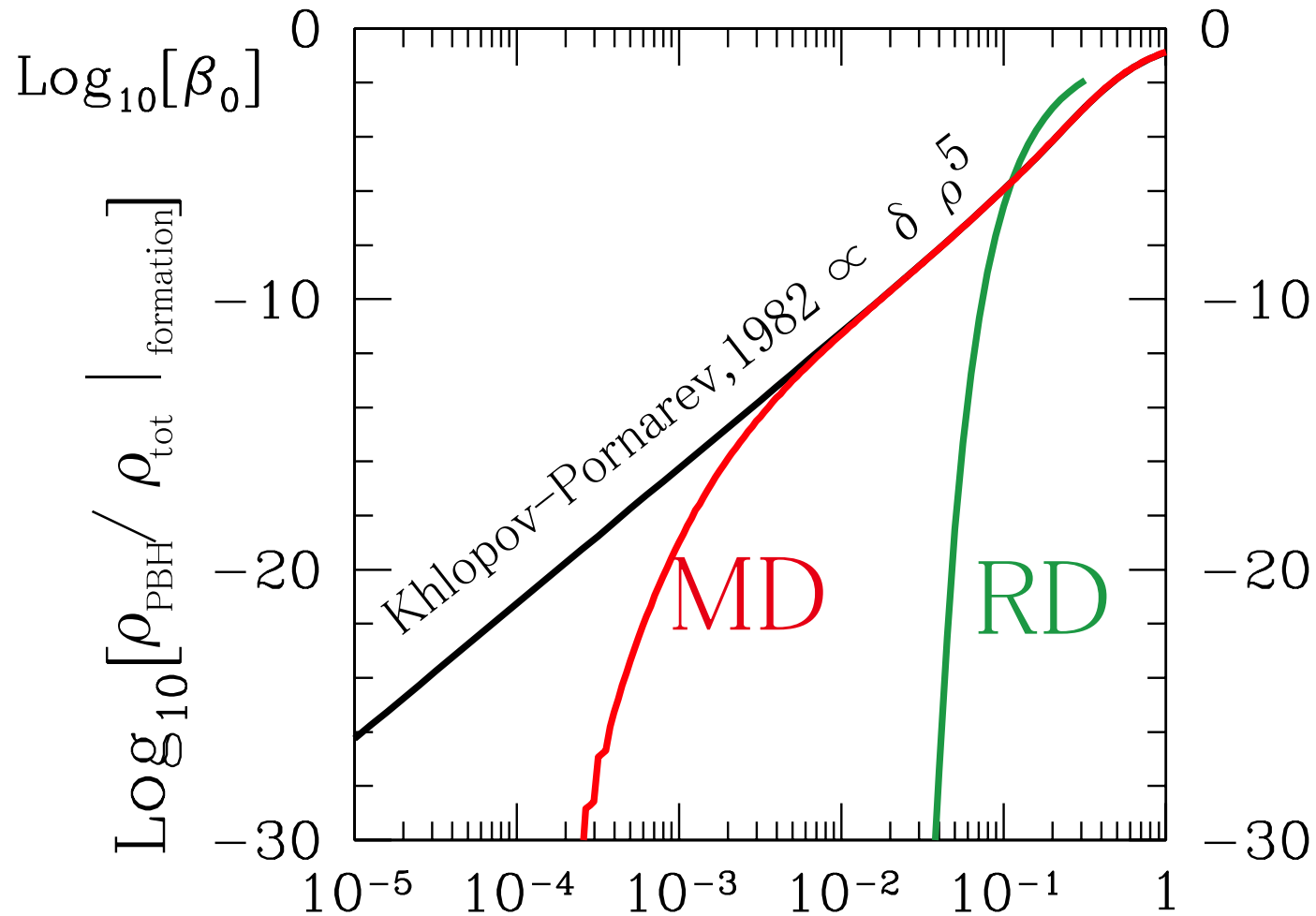
More highly-spinning halos cannot collapse into PBHs, which means that the PBHs produced tend to have high spins in MD

Harada, Yoo, KK, Nakao (2017)



Beta in matter-domination

Harada, Yoo, KK, Nakao (2017)

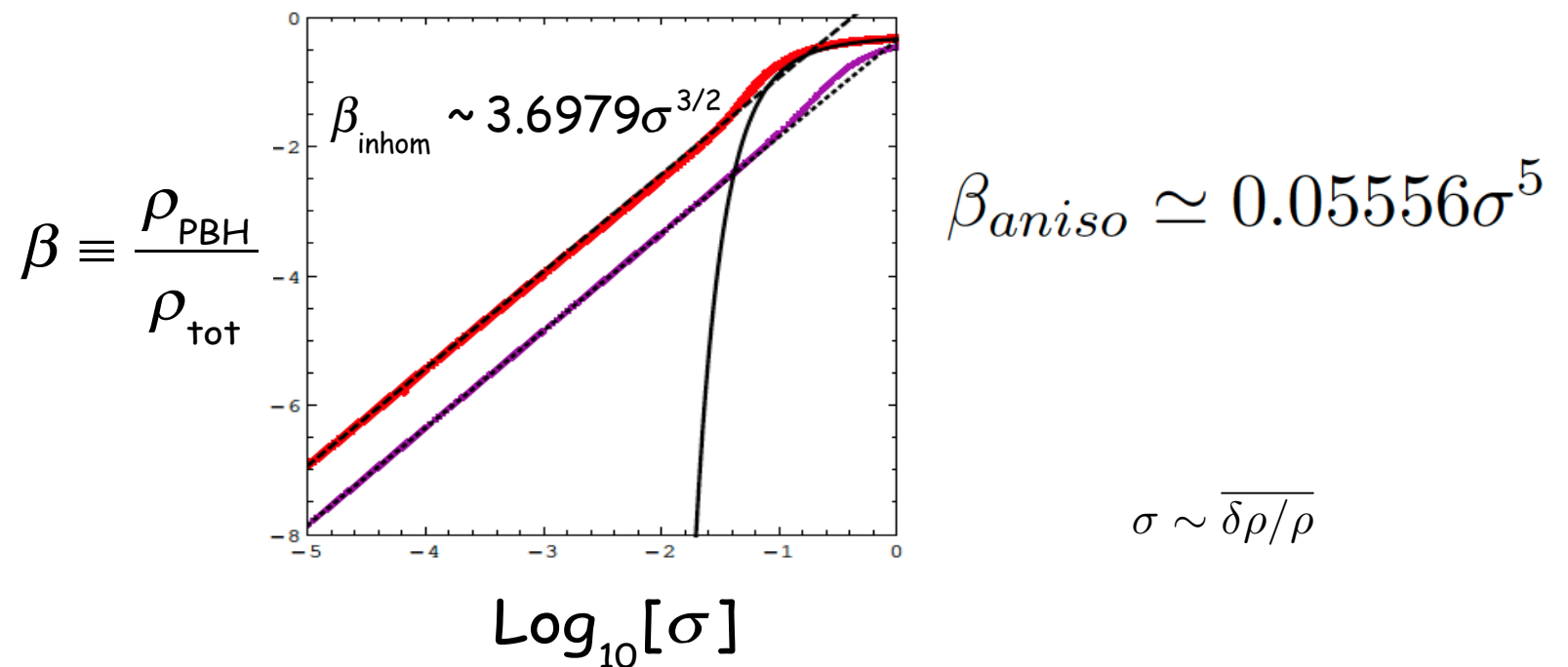


$$\sigma_H = \sqrt{5}\sigma_3 = \delta \rho / \rho$$

Effects of Inhomogeneity on PBH formations in Matter Domination

T.Kokubu, K.Kyutoku, K.Kohri, T.Harada, arXiv:1810.03490

Singularity should be enclosed by (apparent) horizon



$$\beta_{\text{inhom+aniso}} \simeq \beta_{\text{inhom}} \times \beta_{\text{aniso}} = 0.2055\sigma^{13/2}$$

Inflation models

PBH formation and Inflation models

- **Multi-field Inflation** *Kawasaki, Sugiyama, Yanagida (1998)*
- **At the end of inflation** *Lyth, Malik, Sasaki, Zabarra (2006)*
- **Preheating** *Green and Malik (1999)*
Taruya (1998)
- **Blue-tilted spectral** *Kohri, Lyth and Melchiorri (2007)*
- **Curvaton** *Kawasaki, Kitajima, Yanagida (2012)*
Kohri, Lin, Matsuda (2012)
- ...

Simple parameterization of running of spectral indexes of curvature perturbation

- Curvature perturbation

$$P_\zeta(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + \frac{\alpha_s}{2} \ln\left(\frac{k}{k_*}\right) + \frac{\beta_s}{6} \left(\ln\left(\frac{k}{k_*}\right) \right)^2}$$

$$A_s \equiv P_\zeta|_* \sim \frac{V}{m_{\text{pl}}^4 \epsilon} \Big|_* \sim (\delta T/T)^2$$

$$\epsilon \equiv \frac{1}{2} \left(m_{\text{pl}} \frac{V'}{V} \right)^2$$

- spectral index

$$n_s - 1 = dP_\zeta/d \ln k = 2\eta - 6\epsilon$$

$$\eta \equiv m_{\text{pl}}^2 \frac{V''}{V}$$

- running of n_s

$$\alpha_s = dn_s/d \ln k = -24\epsilon^2 + 16\epsilon\eta - \xi^{(2)}$$

$$\xi^{(2)} \equiv m_{\text{pl}}^4 \frac{V'V'''}{V^2}$$

- running of running of n_s

$$\sigma^{(3)} \equiv m_{\text{pl}}^6 \frac{(V')^2 V'''}{V^3}$$

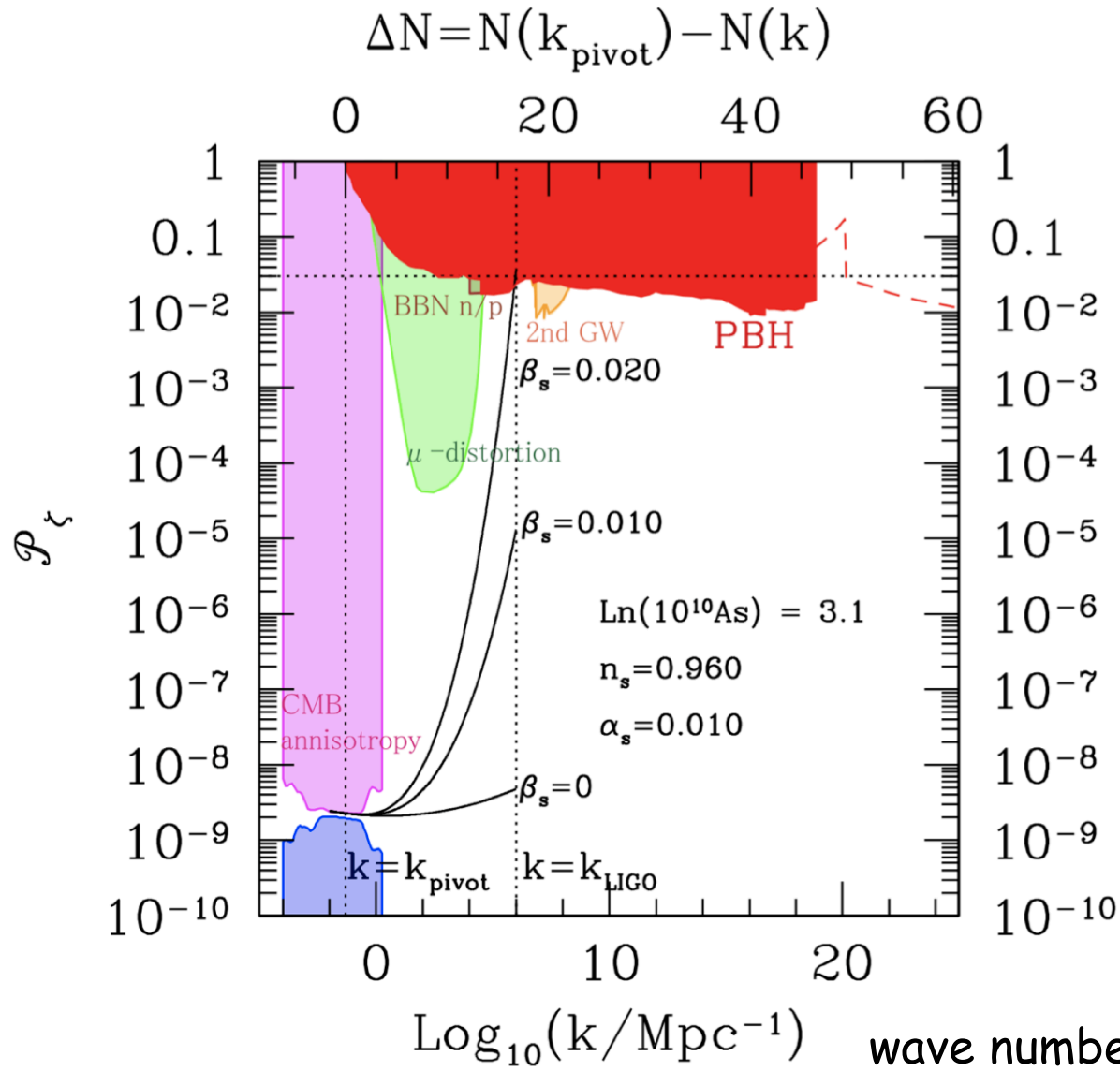
$$\beta_s = d\alpha_s/d \ln k = 192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(2)}$$

Curvature perturbation $P_\zeta(k)$

KK and T.Terada, 2018

Alabidi, Kohri, Sendouda, Sasaki, 2013

Amplitude of curvature perturbation



Planck (2015)

$$n_s = 0.9586 \pm 0.0056,$$

$$\alpha_s = 0.009 \pm 0.010,$$

$$\beta_s = 0.025 \pm 0.013.$$

at 68% C.L.

For inflation models with a big running, see Kohri, Lin Lyth (2008)

Type-III Hilltop inflation models

German, Ross, Sarkar (01)

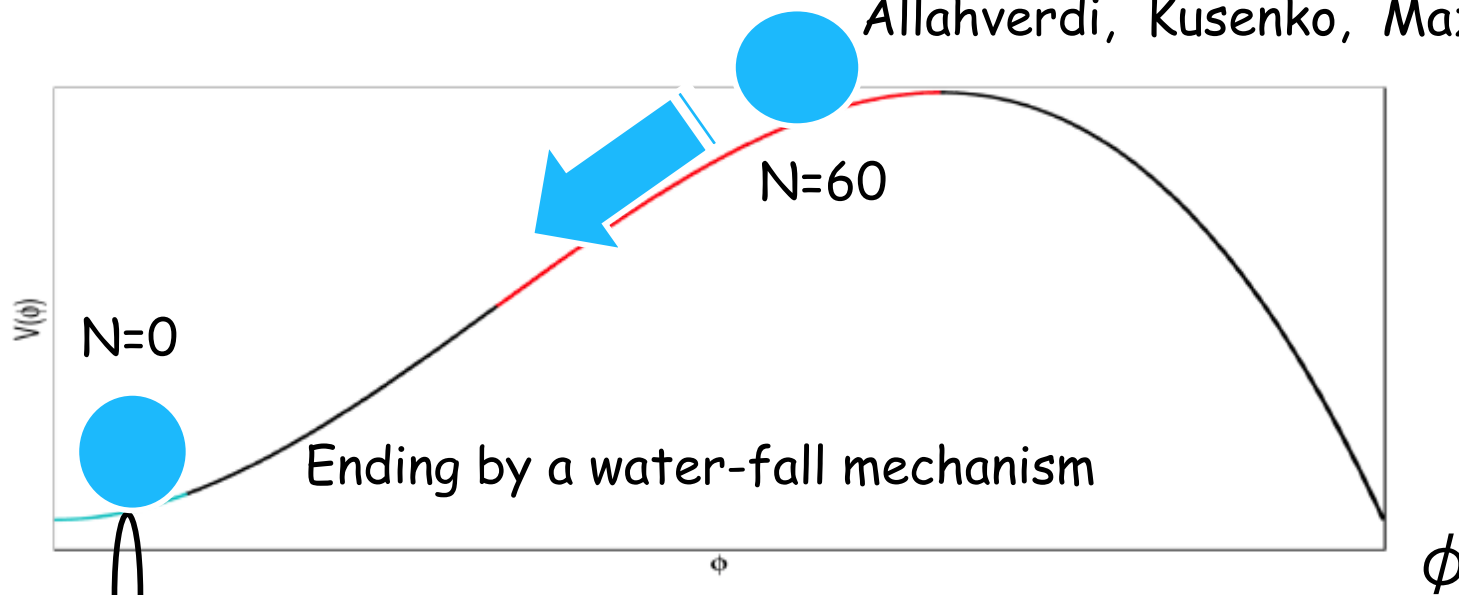
KK, Lin and Lyth (07)

- Potential in supergravity, e.g.,

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 - \lambda\frac{\phi^p}{M_{\text{P}}^{p-4}} + \dots$$

$$W = C\frac{\phi^p}{M_{\text{pl}}^{p-3}}, \quad \lambda \sim Cm_{3/2}/M_{\text{pl}} \quad \text{in SUGRA}$$

Allahverdi, Kusenko, Mazumdar (06)



Large running spectral index

KK, Lin and Lyth (07)

- **Spectrum**

$$P_\zeta \sim \frac{V}{m_{\text{pl}}^4 \epsilon}$$

- **Enhanced curvature perturbation at small scales due to a large running of running**

$$\epsilon \equiv \frac{1}{2} \left(m_{\text{pl}} \frac{V'}{V} \right)^2 \rightarrow 0 \text{ for } \phi \downarrow$$

$$\beta_s = \frac{d^3 P_\zeta}{d(\ln k)^3} = 192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(3)}$$

Could be large!

Observational constraints on PBHs

High-energy particles from evaporating PBHs : $M < 10^{-16} M_{\odot}$

To be dark matter : $10^{-16} < M < 10^{-10} M_{\odot}$

Gravitational lensing : $10^{-10} < M < 10^0 M_{\odot}$

GWs from merging PBHs : $10^{-4} < M < 10^1 M_{\odot}$

Emission from accretion disk around a PBH : $10^0 < M < 10^4 M_{\odot}$

CMB distortions by dissipating density perturbation : $10^4 < M < 10^{11} M_{\odot}$

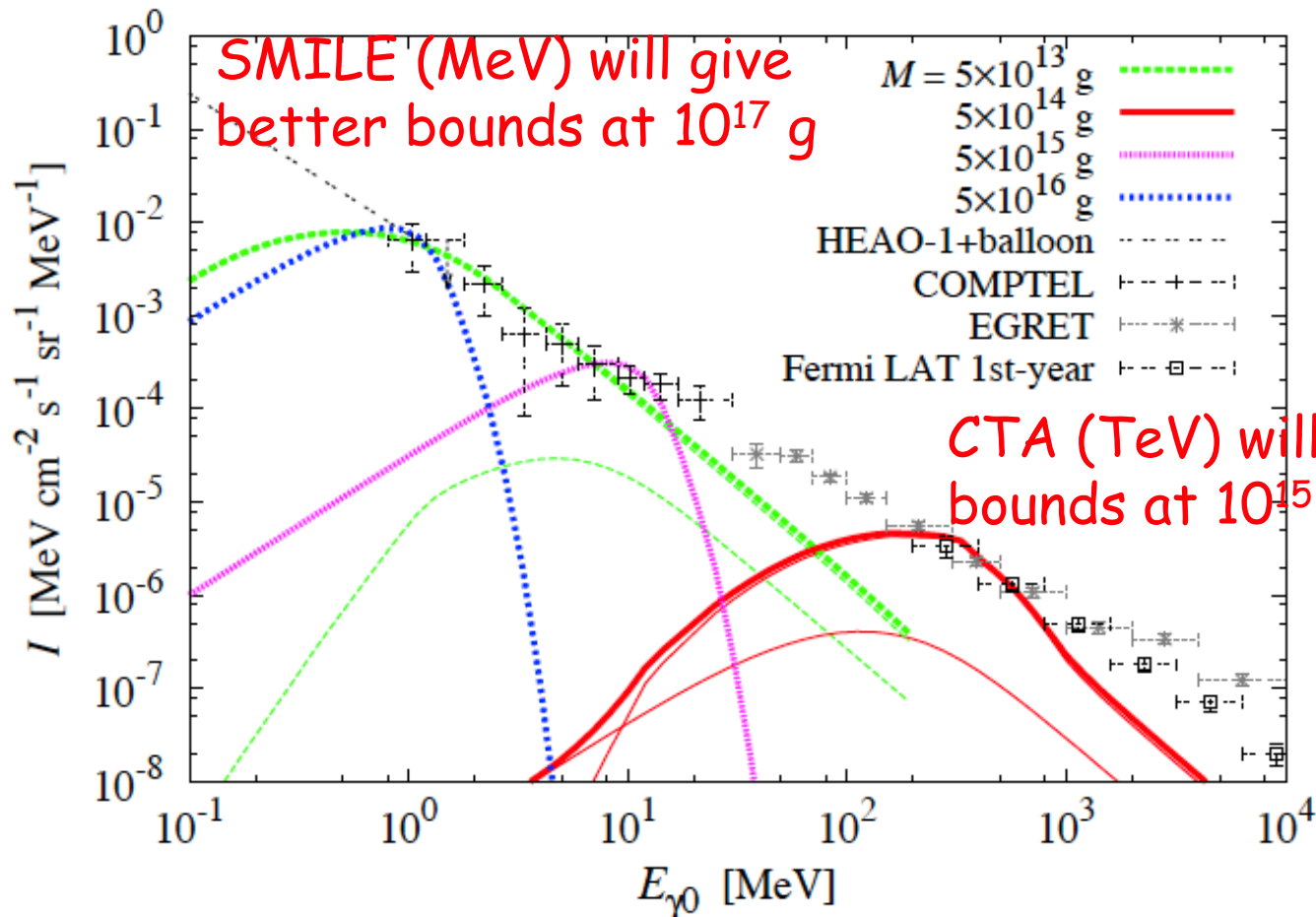
Secondary GWs produced from large perturbation : $10^{-8} < M < 10^0 M_{\odot}$

...

Evaporating PBHs through Hawking Process

Carr, Kohri, Sendouda and Yokoyama (2010)

$$d\dot{N}_s = \frac{dE}{2\pi} \frac{\Gamma_s}{e^{E/T_{\text{BH}}} - (-1)^{2s}}$$



M31 lensing on PBHs modified by size-distribution and finite-size effects on bright star sources

Nolan Smyth, Stefano Profumo, Samuel English, Tesla Jeltema, Kevin McKinnon, Puragra Guhathakurta, arXiv:1910.01285 [astro-ph.CO]

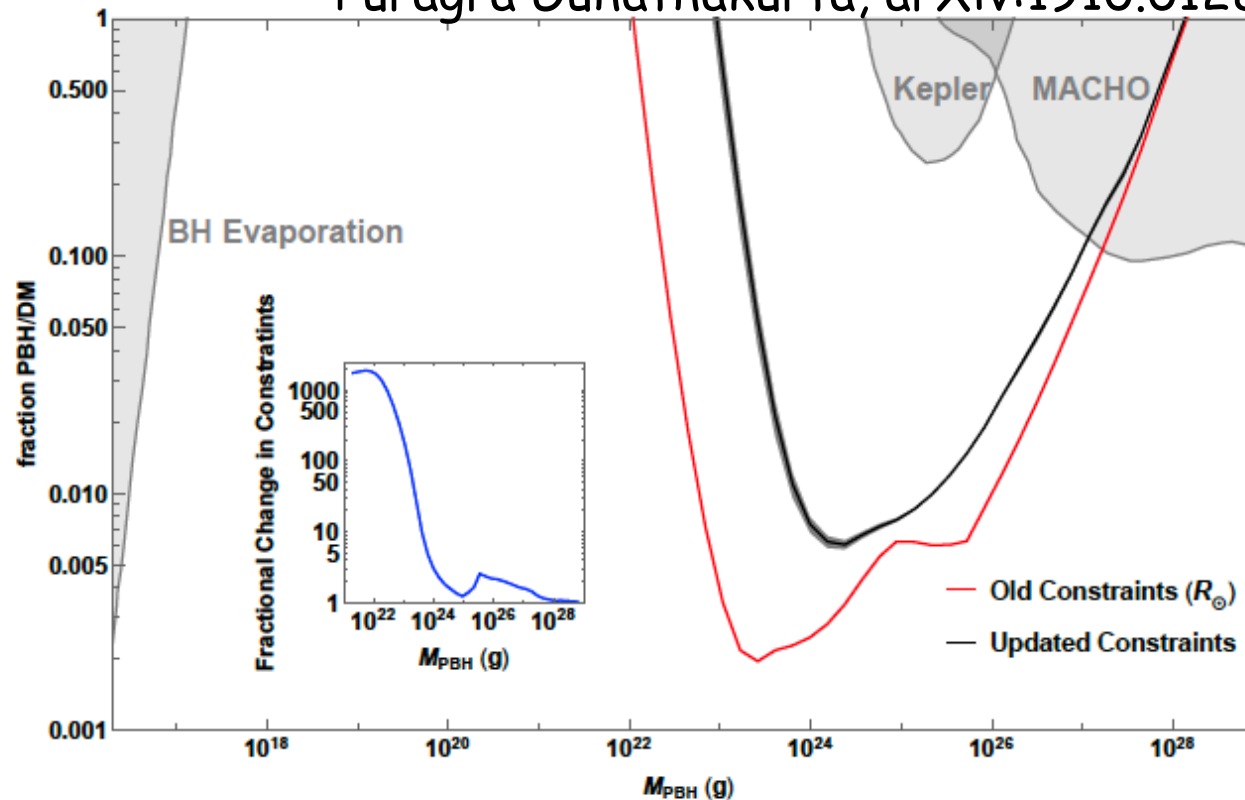
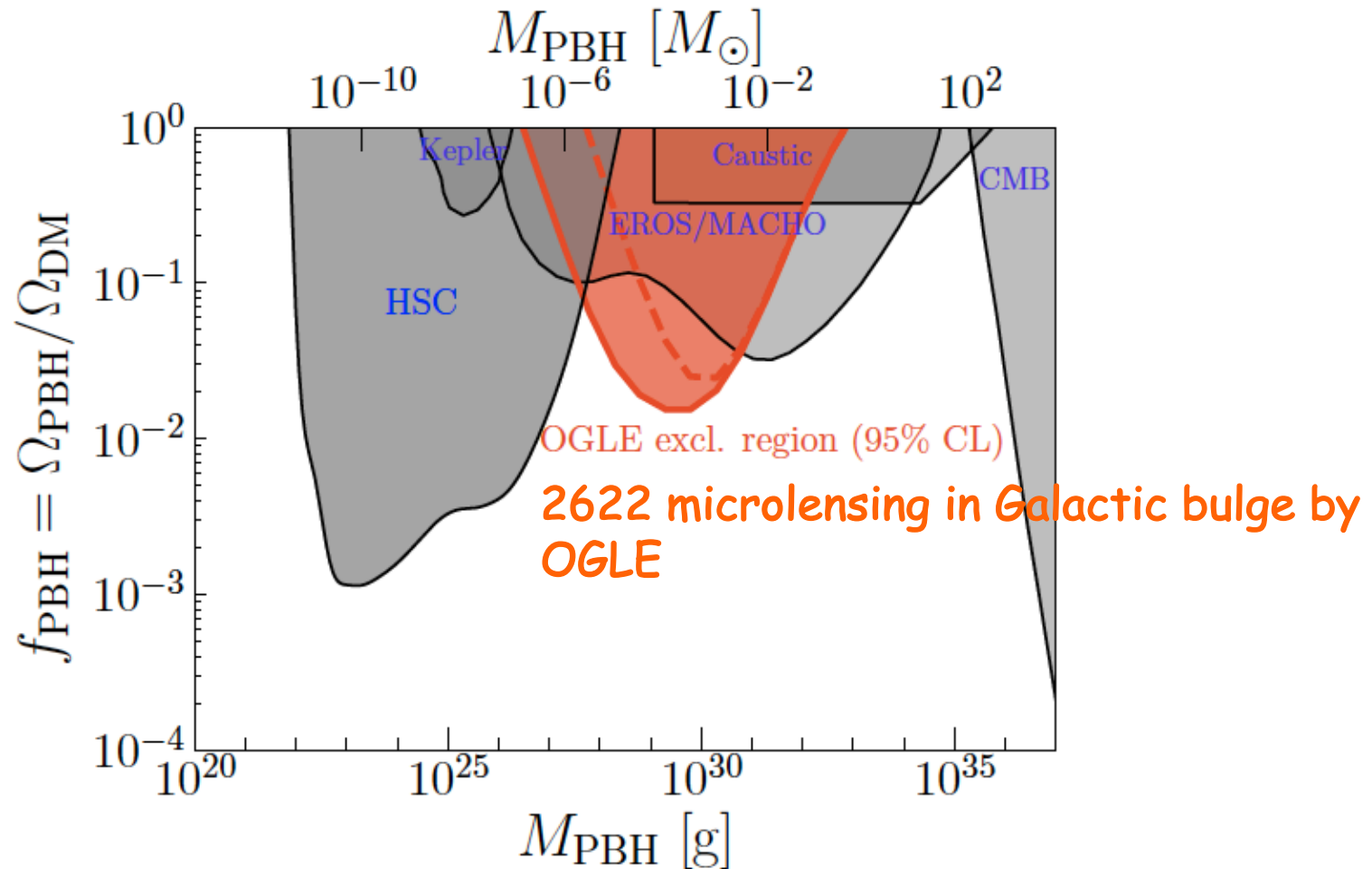


Figure 2. The constraints on primordial black holes as dark matter. The black line is the benchmark constraint and the primary result of this paper. The gray shading comes from the uncertainty in determining the stellar size distribution. The red line is the previous constraint which includes finite size effects but assumes that all stars in M31 have a radius of R_{\odot} .

Gravitational lensing constrains on PBHs

Hiroko Niikura, Masahiro Takada, Shuichiro Yokoyama, Takahiro Sumi, Shogo Masaki,
arXiv:1901.07120 [astro-ph.CO]



CMB bound on PBHs by disk-accretion in the late MD epoch

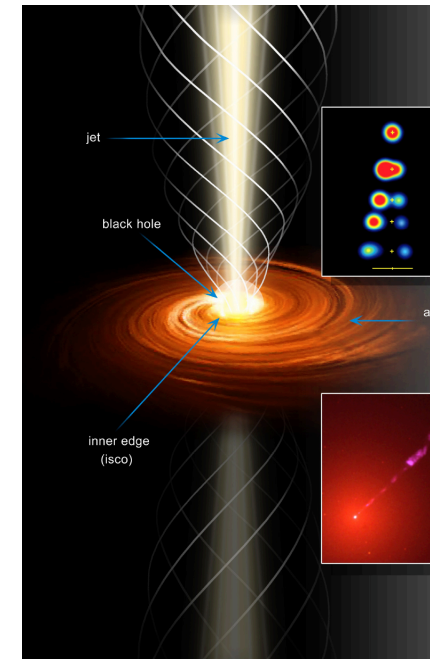
Poulin, Serpico, Calore, Clesse, KK (2017)

- A non-spherical accretion disk (ADAF(slim) + Standard disk) around a PBH caused by an angular momentum emits radiation

$$\dot{M}_{\text{HB}} \equiv 4\pi\lambda\rho_{\infty}v_{\text{eff}}r_{\text{HB}}^2 \equiv 4\pi\lambda\rho_{\infty}\frac{(GM)^2}{v_{\text{eff}}^3}$$
$$l \simeq \omega r_{\text{HB}}^2 \simeq \left(\frac{\delta\rho}{\rho} + \frac{\delta v}{v_{\text{eff}}}\right)v_{\text{eff}}r_{\text{HB}}$$

- CMB anisotropies are affected

- From observations, we can constrain the number density of PBHs



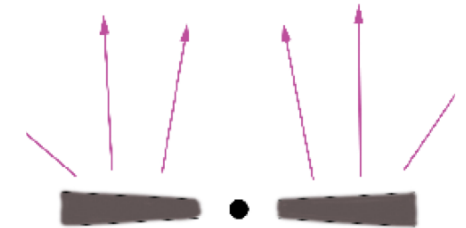
An accretion disk around a black hole

Kohri, Mineshige, 2002
Kohri, Narayan, Piran, 2005

Viscous heating process \leftrightarrow Various cooling processes

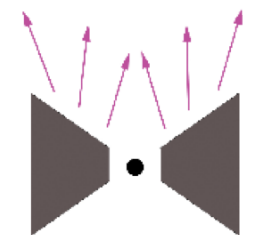
i. Standard Accretion Disk (Standard Disk)

- Radiative Cooling



ii. Advection Dominated Accretion Flow ($AD_{\text{H}\Gamma}$)

- Advective cooling (entropy going into BH) gives RIAF (optically thin) or Slim Disk (optically-thick)

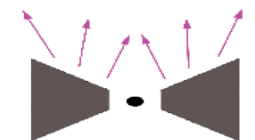


iii. Convection Dominated Accretion Flow (CDAF)

- Convective cooling

iv. Neutrino-Dominated Accretion Disk (NDAF)

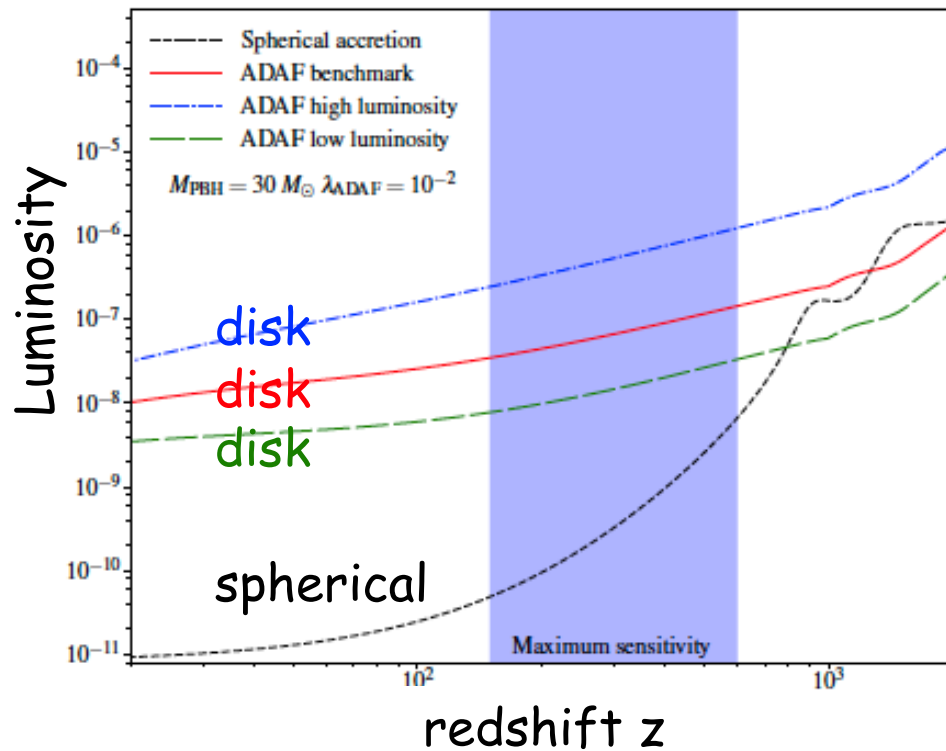
- Neutrino Cooling



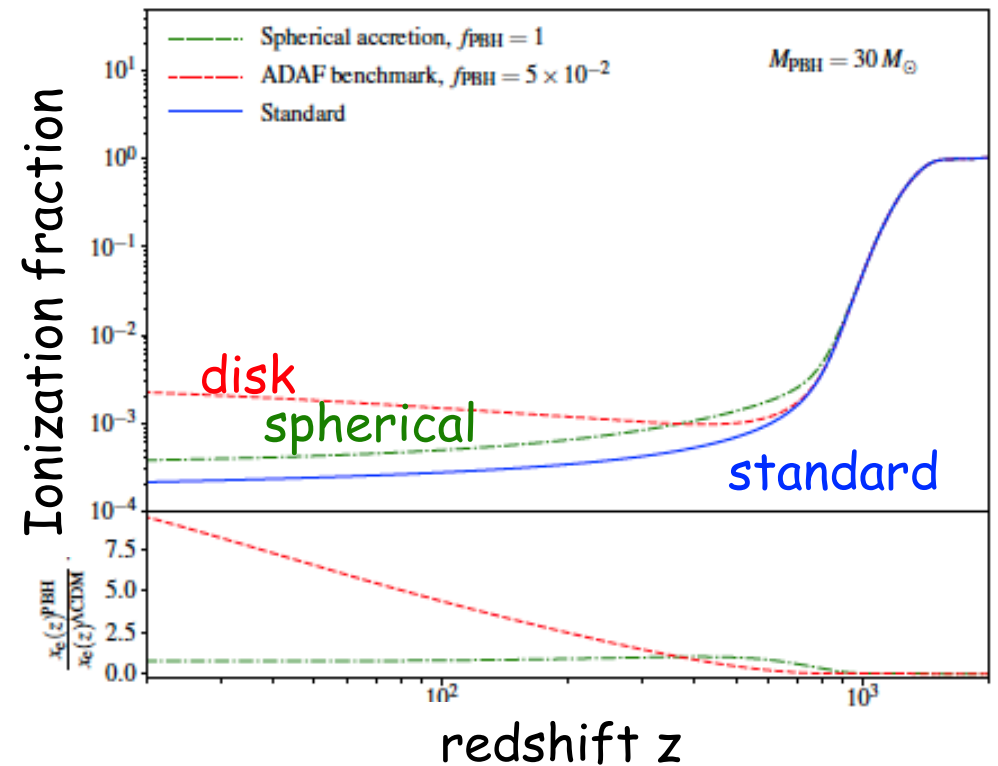
v. ...

Modified CMB anisotropy

Poulin, Serpico, Calore, Clesse, Kohri (2017)



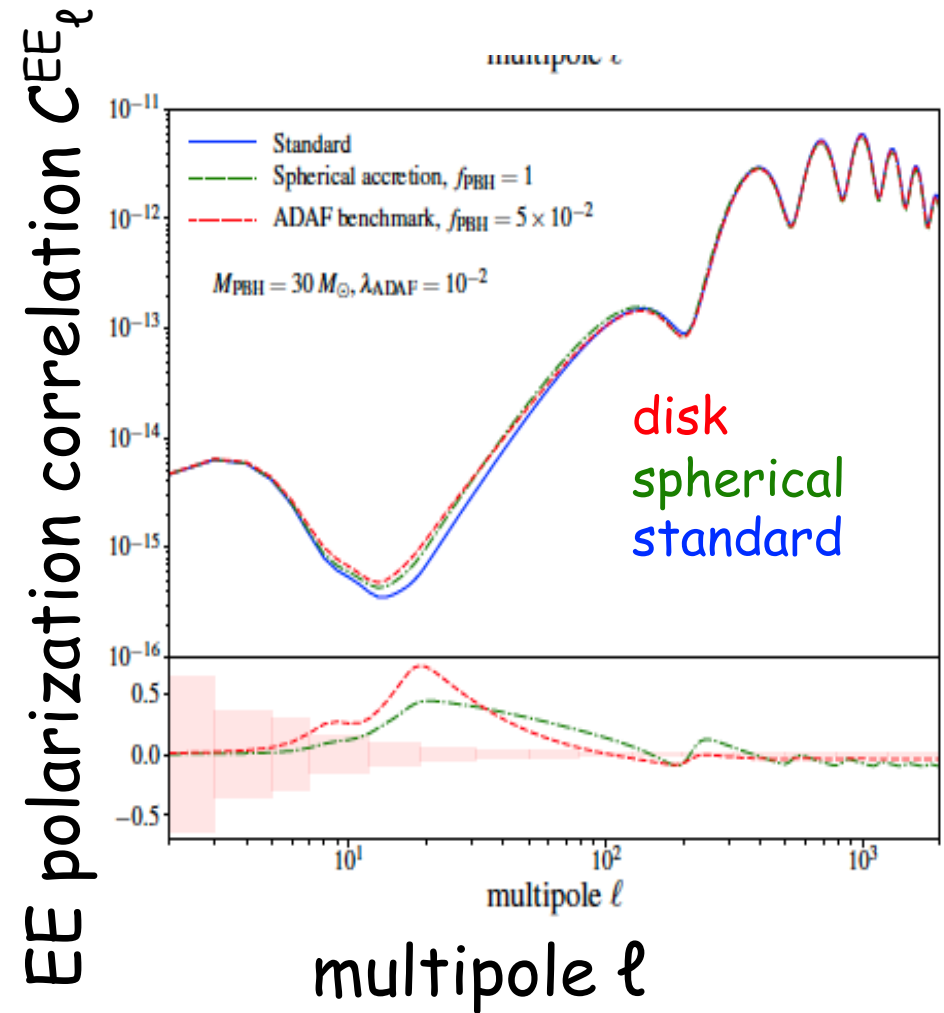
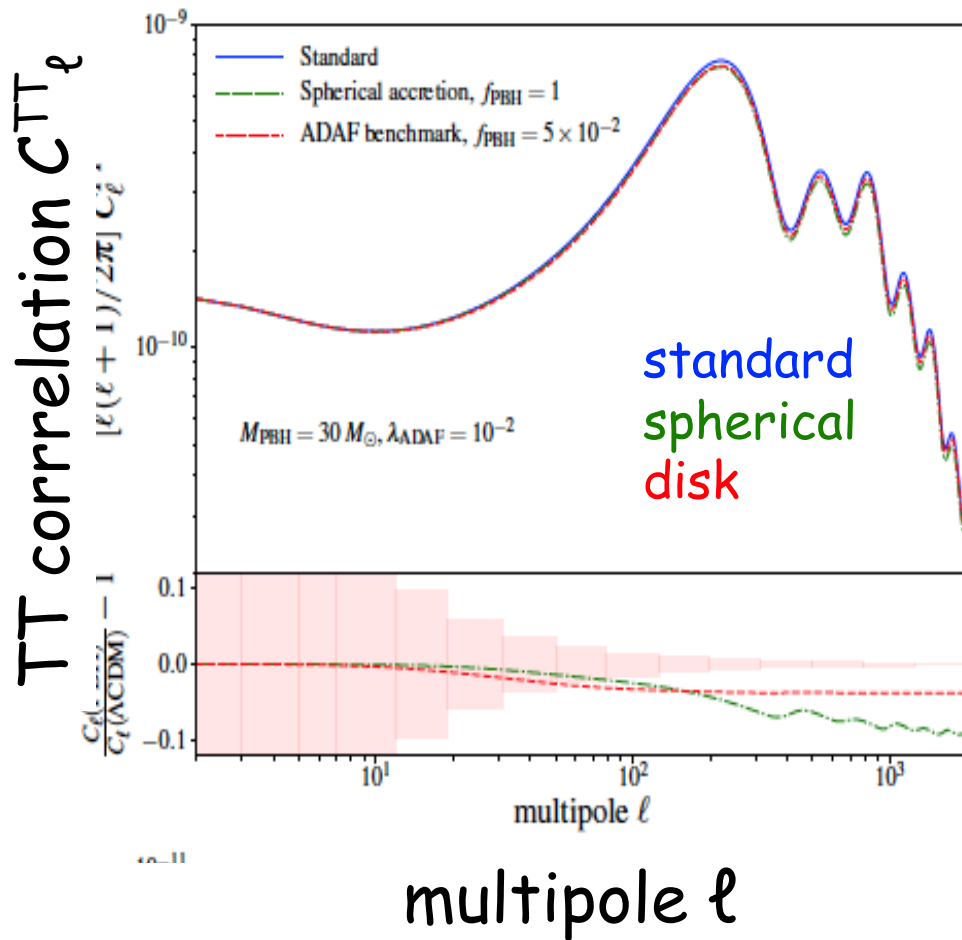
Luminosity



Ionization fraction

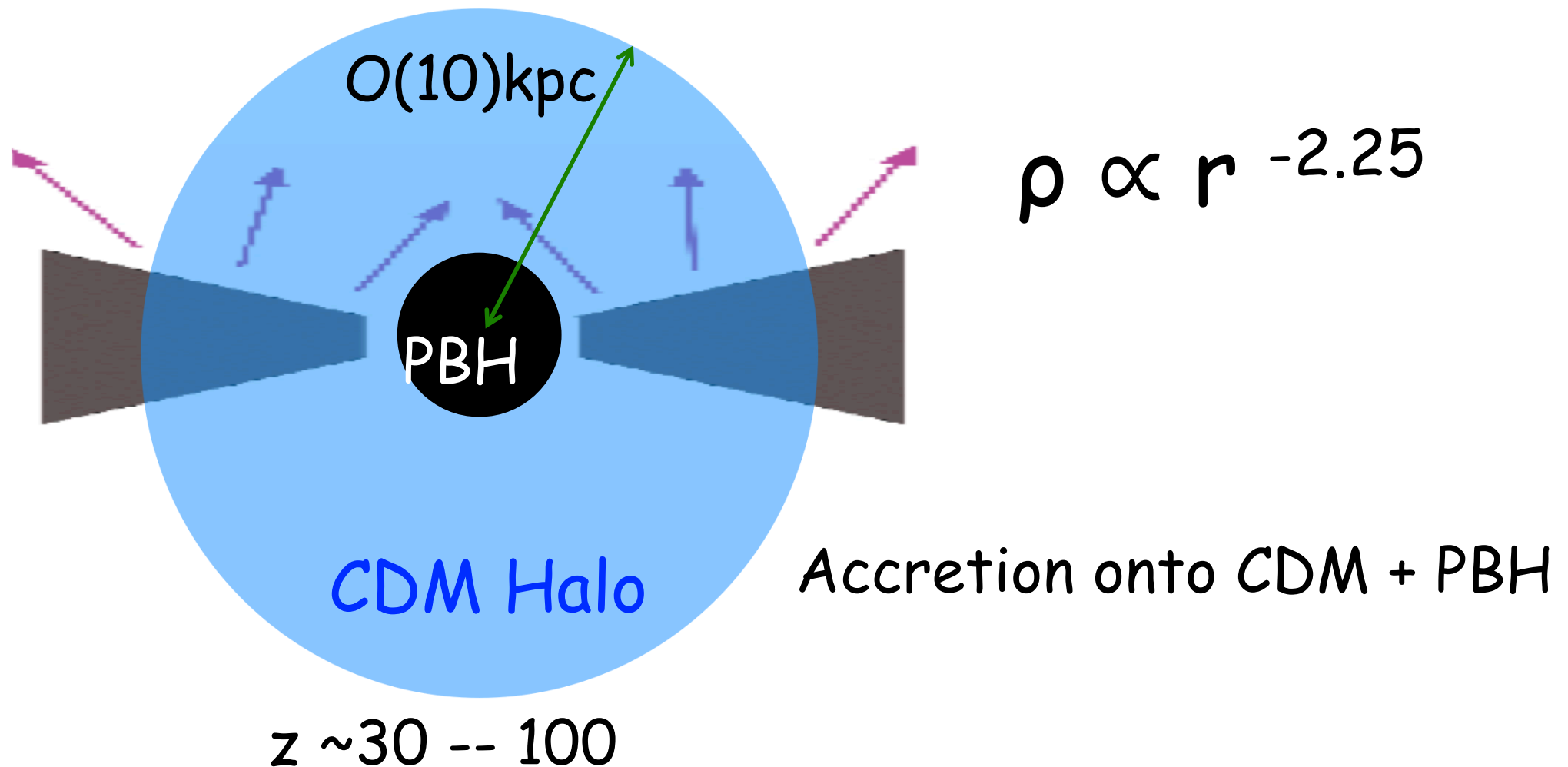
Modified CMB anisotropy

Poulin, Serpico, Calore, Clesse, Kohri (2017)



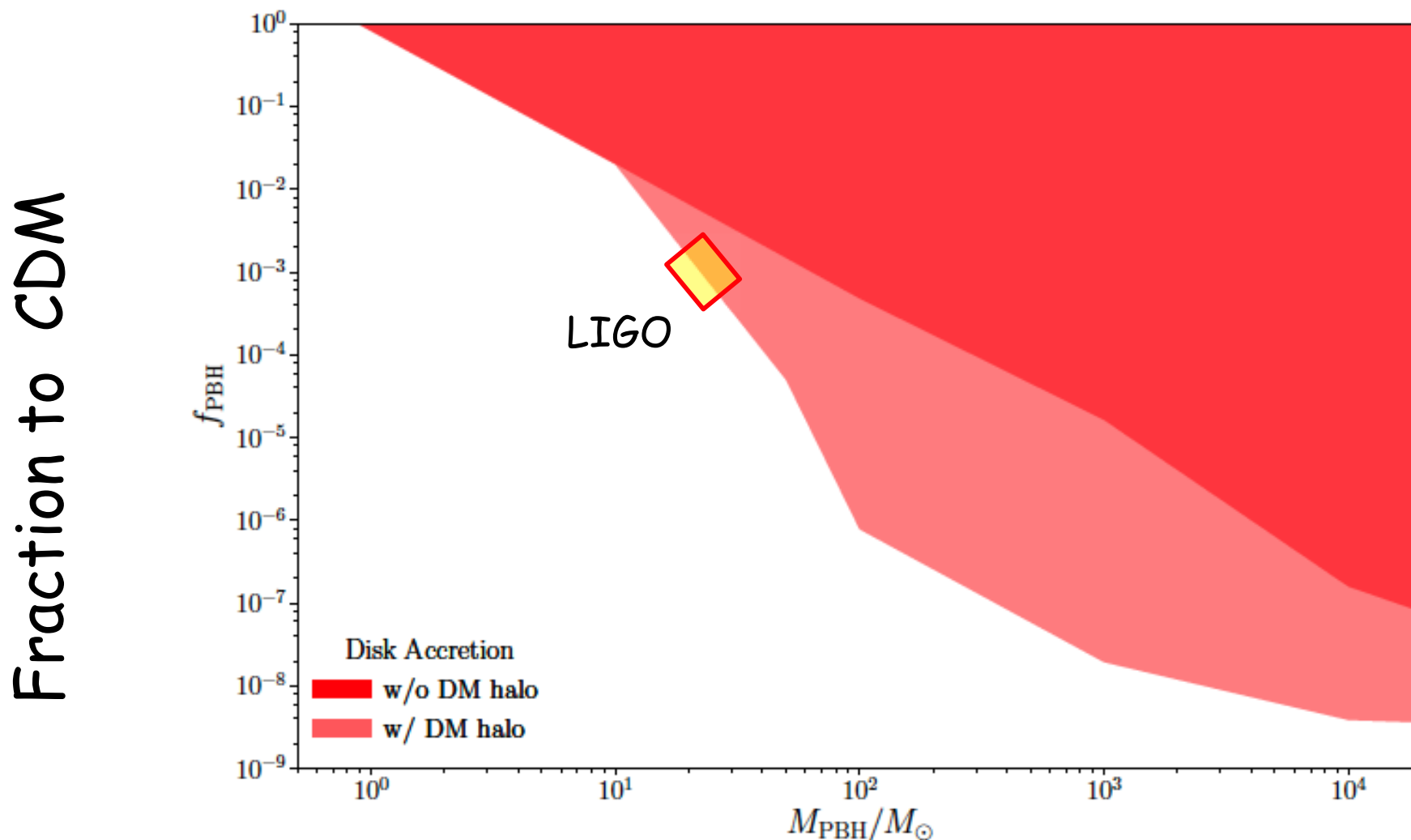
Cosmological baryon accretion onto the PBH + CDM halo system

Poulin, Serpico, Inman, Kohri (2020)



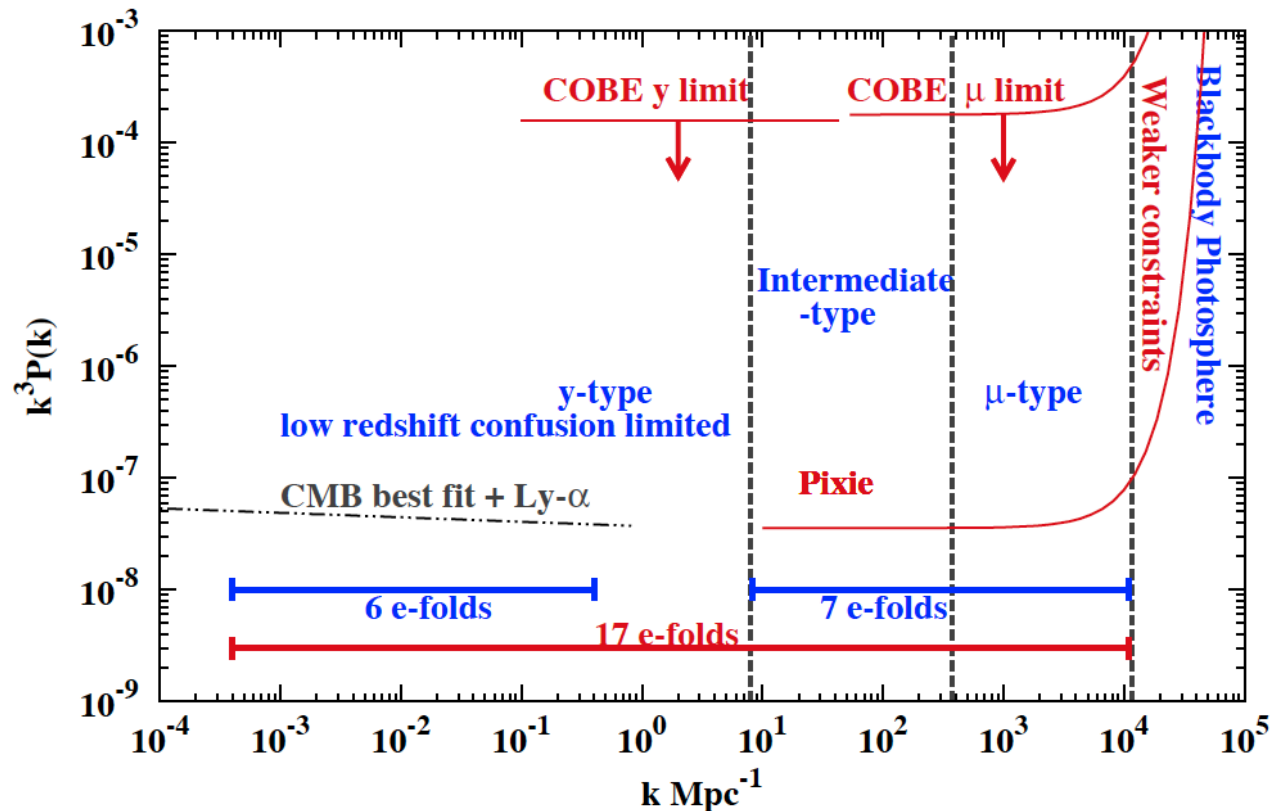
CMB bound by disk-accretion in the latest MD epoch

Poulin, Serpico, Inman, Kohri (2020)



CMB μ - and y - distortion by Large P_z

- Large amplitude of fluctuation will be dissipated and **produce μ - and y - distortion** of CMB after decoupling of double-Compton scatterings



Khatri (2013)

Acoustic reheating

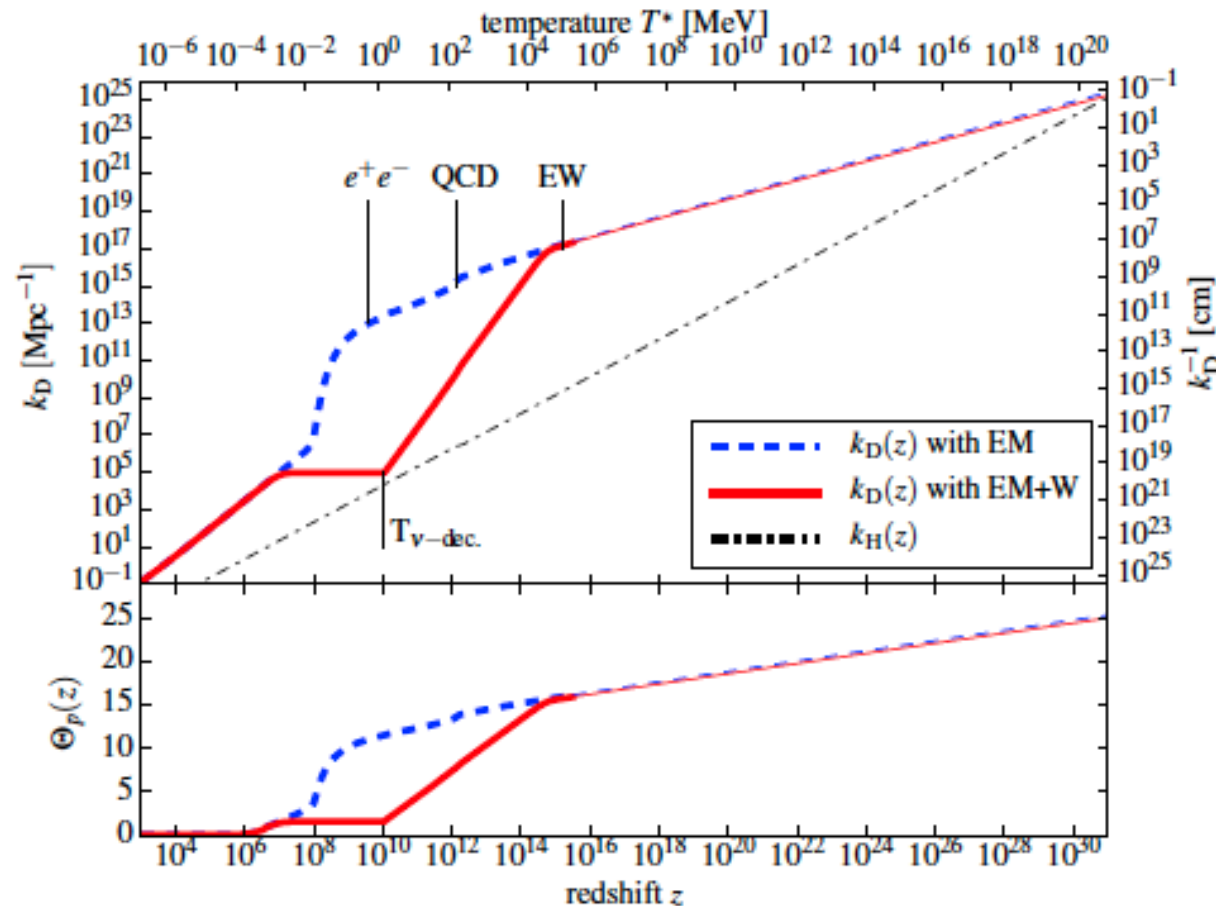
- Nonthermal heating by Silk dumping which can be constrained by BBN and CMB

Jeong, Kamionkowski, Chluba and Pradler (2014)

Nakama, Suyama, Yokoyama (2014)

Inomata, Tada, Kawasaki (2016)

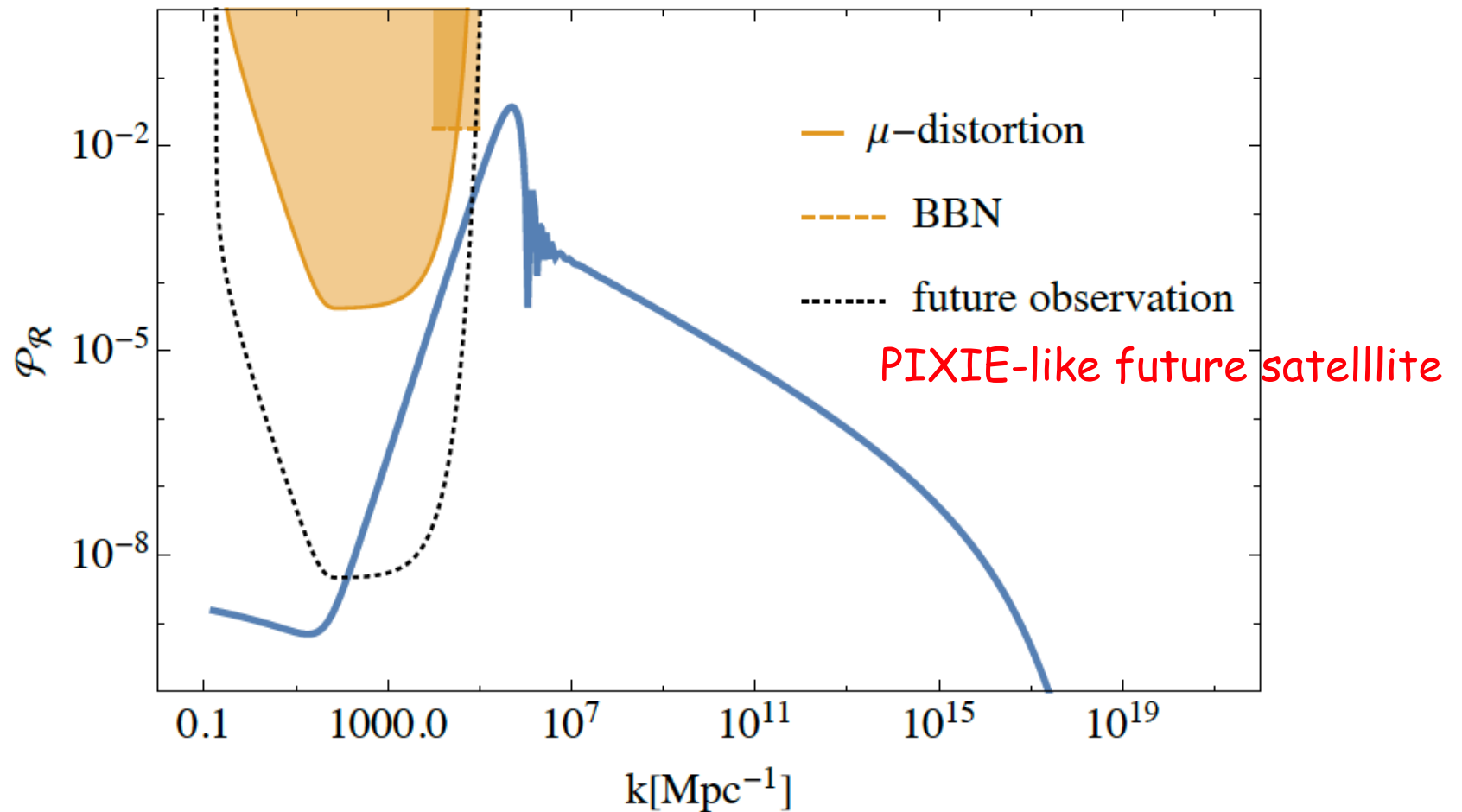
Diffusion scale



μ -distortion and acoustic reheating

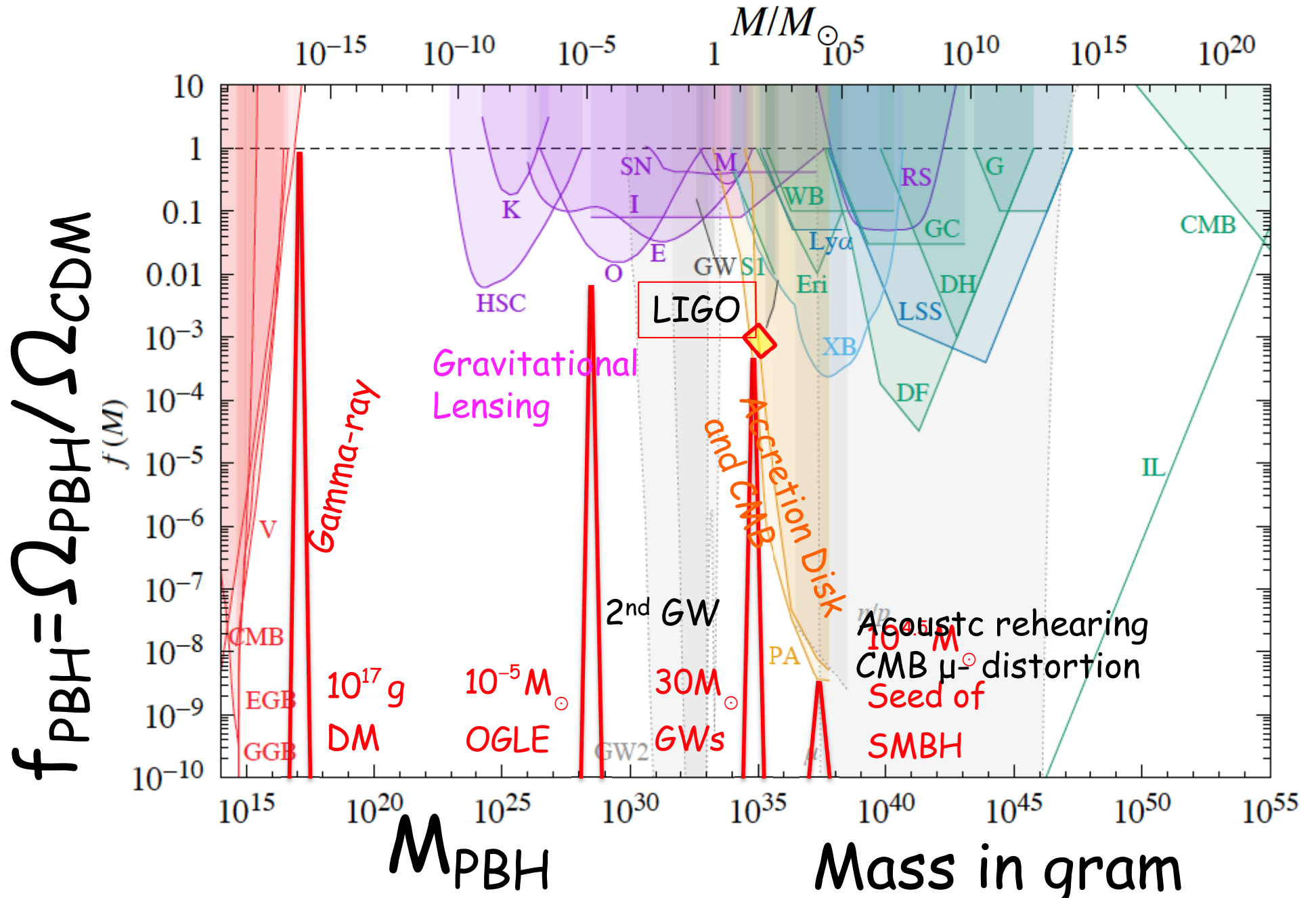
Kohri, Nakama, Suyama (2014)

Inomata, Kawasaki, Mukaida, Tada, Yanagida (2017)



Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)

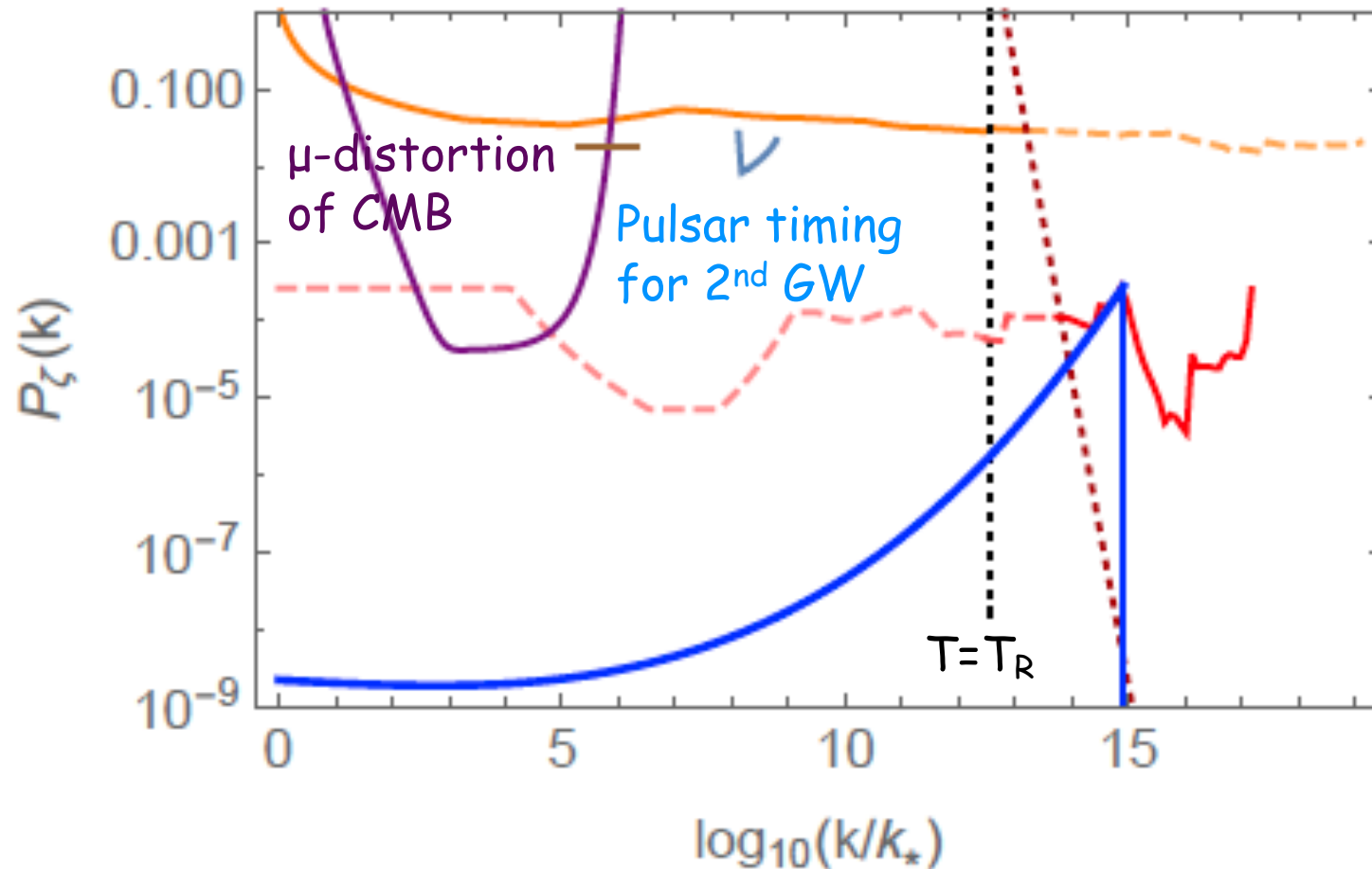


100 % Dark Matter by PBHs with 10^{17} g masses

KK and T.Terada, 2018

$$T_R = 10^4 \text{ GeV},$$

$$n_s = 0.96, \alpha_s = 0, \beta_s = 0.0019485.$$

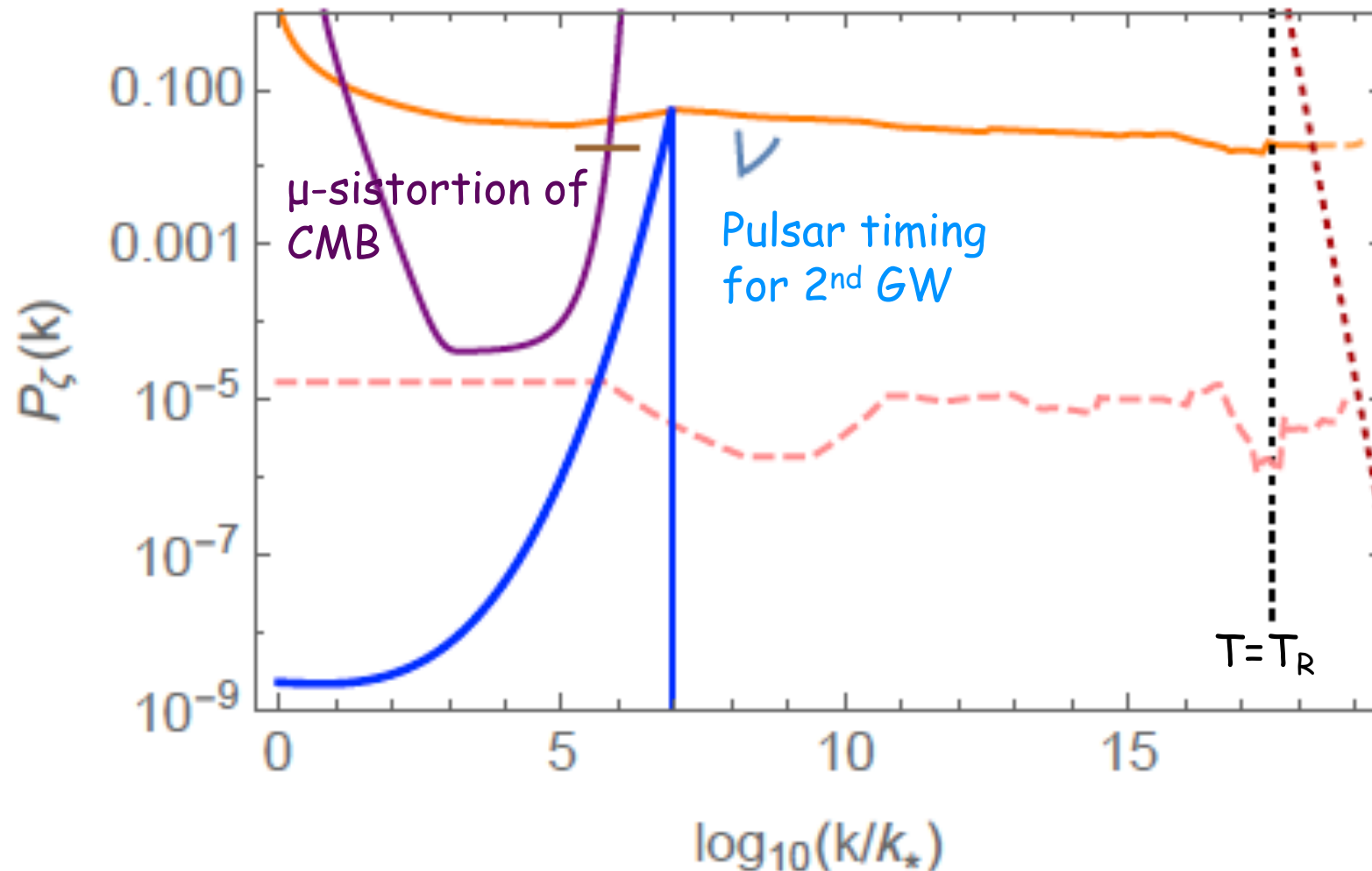


LIGO/VIRGO event with 30 Msolar

KK and T.Terada, 2018

$$T_R = 10^9 \text{ GeV}$$

$$n_s = 0.96, \alpha_s = 0, \beta_s = 0.026.$$



2nd order GWs enhanced at a sudden transition from MD to RD

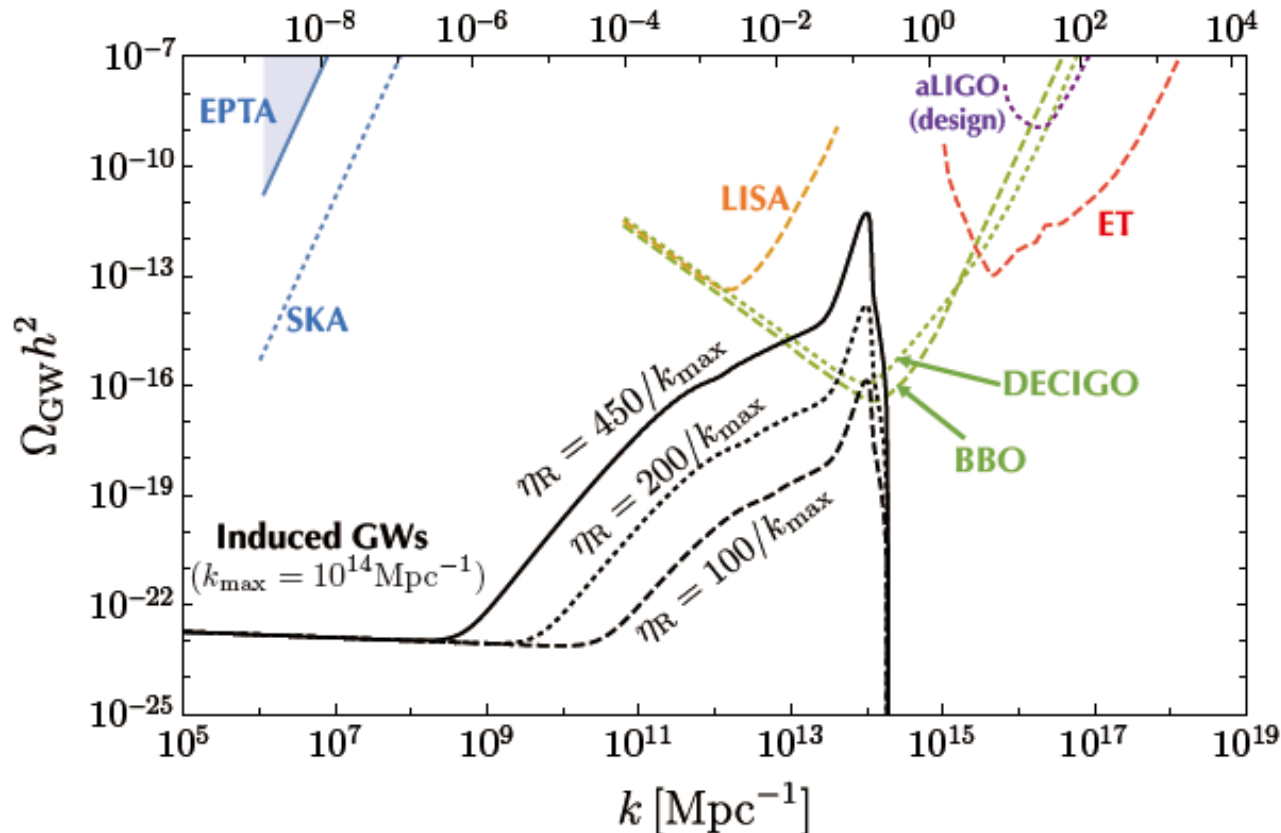
Inomata, Kohri, Nakama, Terada, 2019

See also, S. Kuroyanagi's talk in 2015

$$\overline{\mathcal{P}_h(\eta, k)} \sim \int \int f^2(u, v, \bar{x}, x_R)$$

$$f(u, v, \bar{x}, x_R) = \frac{3(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x}))}{25(1+w)}$$

This is big!



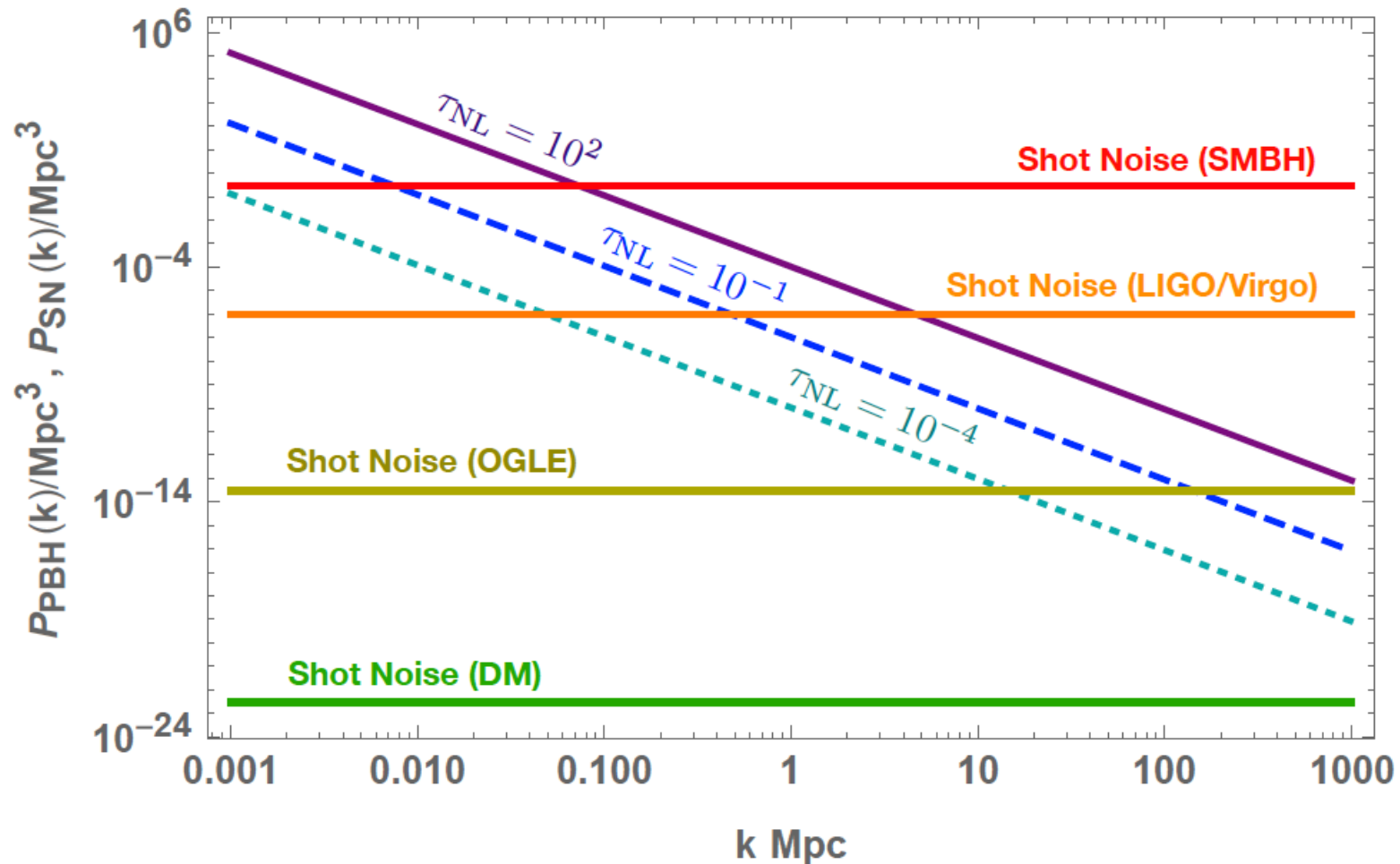
PBHs are clustering?

Matsubara, Terada, Kohri, S. Yokoyama, 1909.06048

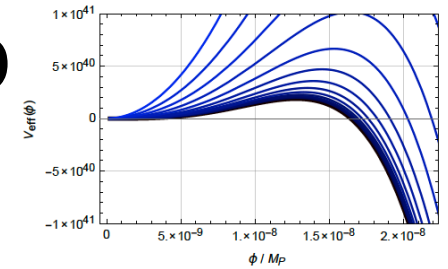
See also, Suyama and S. Yokoyama (2019)

See also **Suyama's talk**

Tada and S. Yokoyama (2015)



Higgs stabilization due to evaporating PBHs?



Kohri and Matsui (2017)

- Potential with finite-temperature corrections

$$V_{\text{eff}}(\phi) \simeq \frac{1}{2} (\lambda_{\text{eff}} T_{\text{H}}^2 + \kappa^2 T_{\text{H}}^2) \phi^2 + \frac{\lambda_{\text{eff}}}{4} \phi^4$$

$$\phi_{\text{max}}^2 / T_{\text{H}}^2 \approx \mathcal{O}(10)$$

- Probability to get over the potential

$$P(\phi > \phi_{\text{max}}) \simeq \frac{\sqrt{2 \langle \delta \phi^2 \rangle_{\text{ren}}}}{\pi \phi_{\text{max}}} \exp\left(-\frac{\phi_{\text{max}}^2}{2 \langle \delta \phi^2 \rangle_{\text{ren}}}\right) \quad \langle \delta \phi^2 \rangle_{\text{ren}} / T_{\text{H}}^2 \simeq \mathcal{O}(0.1)$$

- This gives,

$$\phi_{\text{max}}^2 / \langle \delta \phi^2 \rangle_{\text{ren}} \sim 10^2$$

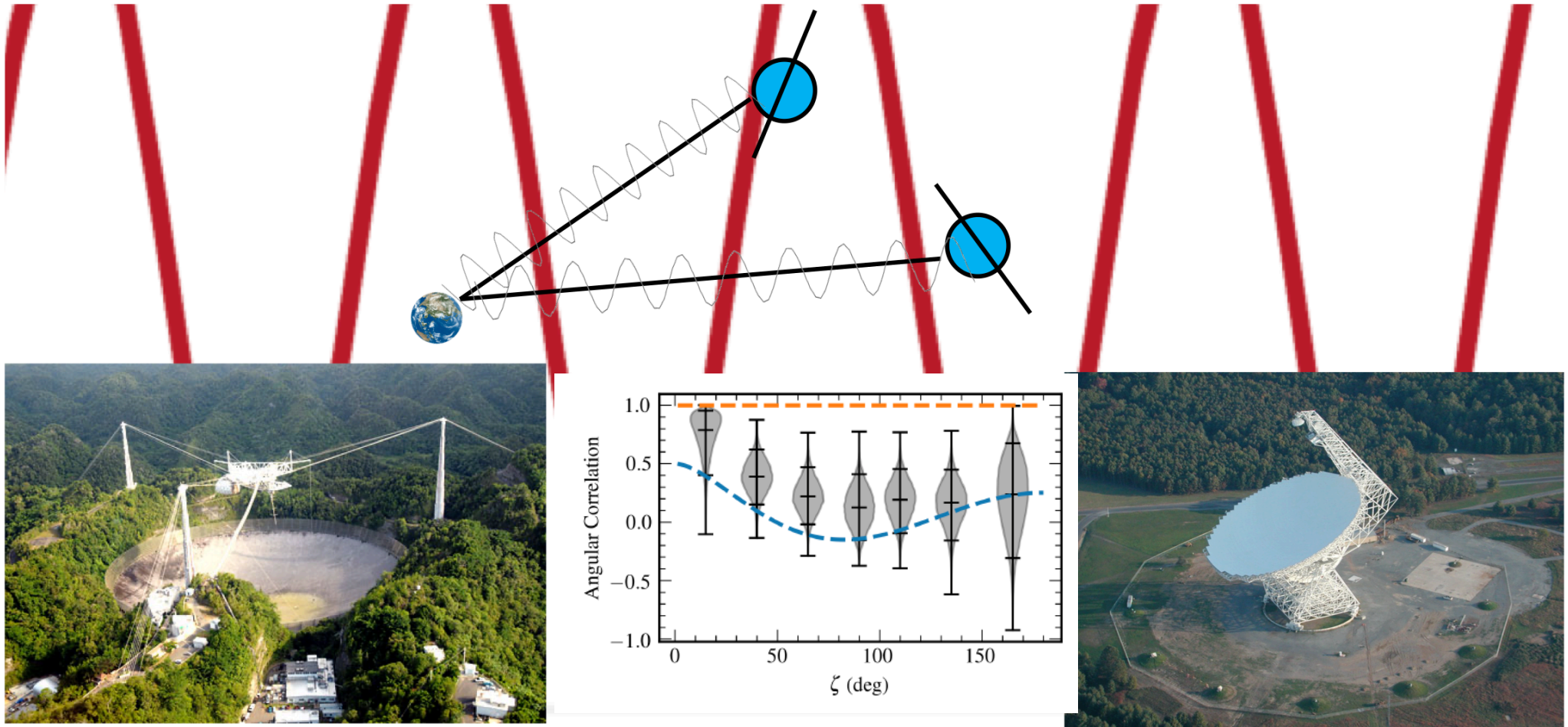
$$\mathcal{N}_{\text{PBH}} \cdot P(\phi > \phi_{\text{max}}) \lesssim 1$$

or

$$\beta \lesssim \mathcal{O}(10^{-21}) \left(\frac{m_{\text{PBH}}}{10^9 \text{g}}\right)^{3/2}$$

NANOGrav 12.5 yr

(North American Nanohertz Observatory for Gravitational Waves)
finds stochastic GWs through pulsar timing ?

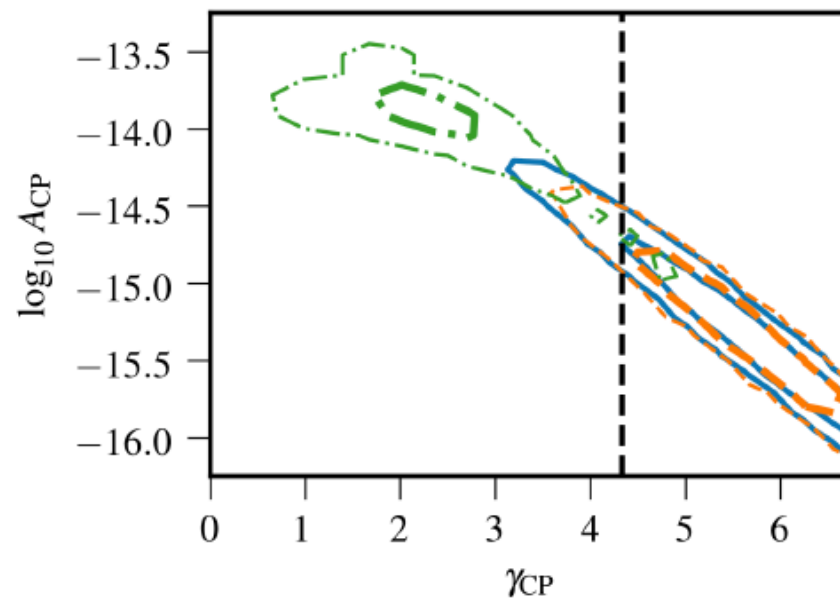
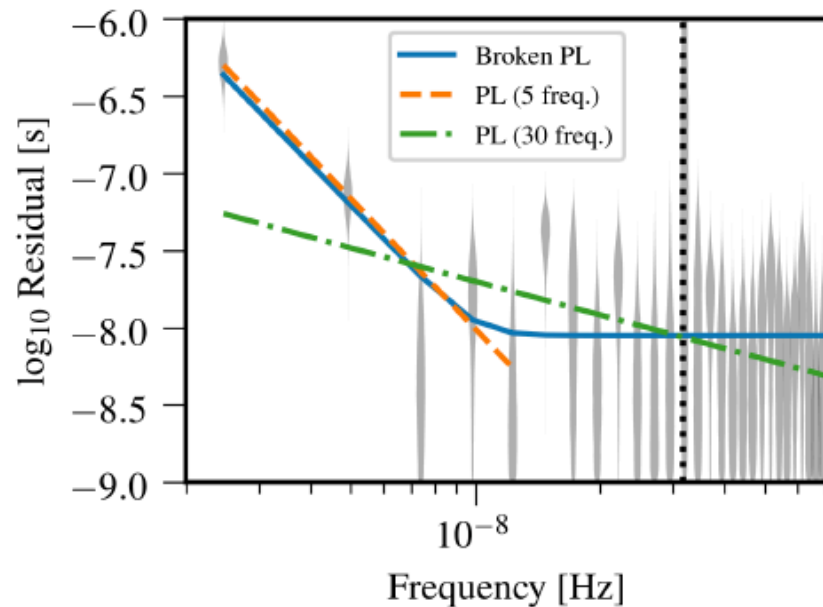


The 305-meter dish of the William E. Gordon Telescope, The Arecibo Obs.

The 100-meter Green Bank Telescope

The NANOGrav 12.5-year Pulsar-timing Data for An Isotropic Stochastic Gravitational-Wave Background

Zaven Arzoumanian, et al, The NANOGrav Collaboration, arXiv:2009.04496



$$h_c(f) = A_{GWB} \left(\frac{f}{f_{yr}} \right)^\alpha$$

$$\Omega(f) = \frac{2\pi}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr} \left(\frac{f}{f_{yr}} \right)^\beta$$

$$\gamma = 3 - 2\alpha = 5 - \beta$$

$$\beta = 5 - \gamma$$

Models to fit NANOGrav 12.5 yr

1. Astrophysics

- Binary SMBHs

2. Early Universe

- 2ndary GWs from large ζ at small scales
- Cosmic string produced by GUT phase transition
- 1st order phase transition of unknown scalar
- ...

Secondary GWs from large curvature perturbation ζ at small scales

K. Kohri and T. Terada, arXiv:arXiv:2009.11853

Secondary gravitational wave induced from large curvature perturbation ($P_\zeta \gg r$) at small scales

K. N. Ananda, C. Clarkson, and D. Wands, 2006

D. Baumann, P. J. Steinhardt, K. Takahashi and K. Ichiki, 2007

R. Saito and J. Yokoyama, 2008

KK and T. Terada, 2018

R.-G. Cai, S. Pi, and M. Sasaki, 2019

- Power spectrum of the tensor mode

$$\langle h_{\mathbf{k}}^r(\eta) h_{\mathbf{k}'}^s(\eta) \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_h(k, \eta) \delta(\mathbf{k} + \mathbf{k}') \delta^{rs}, \quad h_{ij}(\mathbf{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} [h_{\mathbf{k}}^+(\eta) e_{ij}^+(\mathbf{k}) + h_{\mathbf{k}}^\times(\eta) e_{ij}^\times(\mathbf{k})]$$

- Omega parameter well inside the horizon

$$\Omega_{\text{GW}}(k, \eta) = \frac{1}{3} \left(\frac{k}{\mathcal{H}} \right)^2 \mathcal{P}_h(k, \eta).$$

- Substituting the solution into this

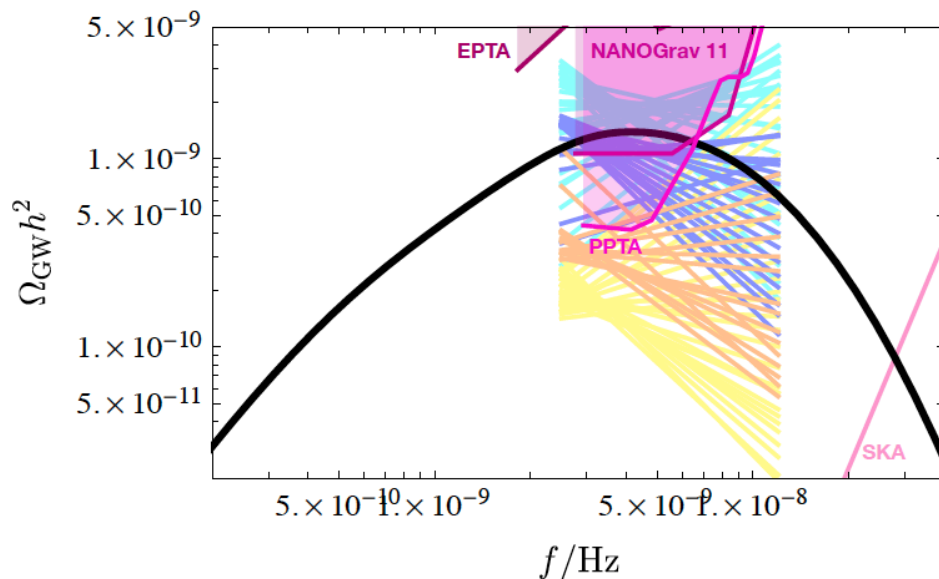
$$\Omega_{\text{GW},c}(f) = \frac{1}{12} \left(\frac{f}{2\pi aH} \right)^2 \int_0^\infty dt \int_{-1}^1 ds \left[\frac{t(t+2)(s^2-1)}{(t+s+1)(t-s+1)} \right]^2 \times \overline{I^2(t, s, k\eta_c)} \mathcal{P}_\zeta \left(\frac{(t+s+1)f}{4\pi} \right) \mathcal{P}_\zeta \left(\frac{(t-s+1)f}{4\pi} \right)$$

Large P_ζ at $k \sim 10^7 \text{Mpc}^{-1}$ produces both GWs at nHz and $O(1) M_\odot$ PBHs

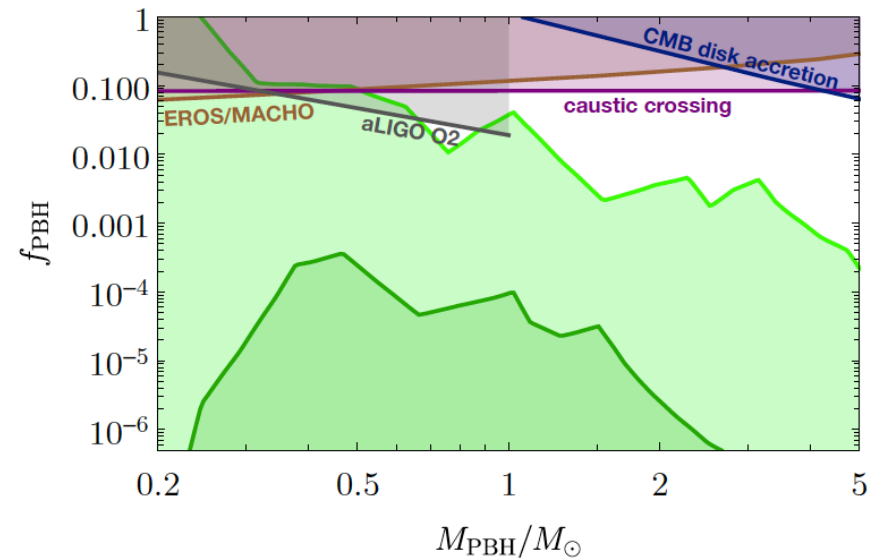
R.Saito and J.Yokoyama, Phys. Rev. Lett. 102 (2009) 161101

K. Kohri and T. Terada, arXiv:arXiv:2009.11853

$$M_{\text{PBH}} \sim O(1)M_\odot \left(\frac{k}{10^7 \text{Mpc}^{-1}} \right)^{-2} \sim O(1)M_\odot \left(\frac{5 \times 10^{-9} \text{Hz}}{f} \right)^2$$



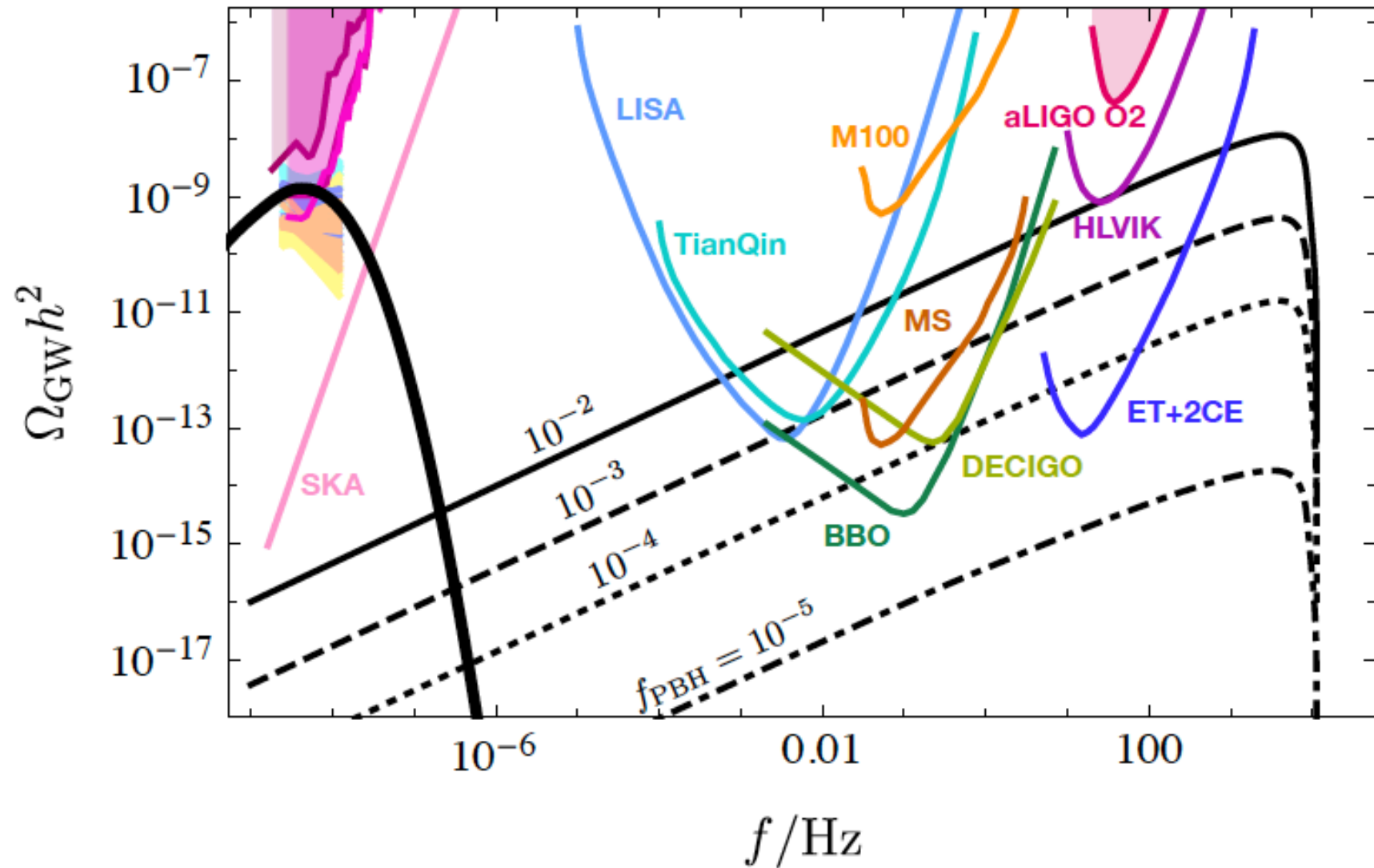
GWs at nHz



PBHs of $O(1) M_\odot$

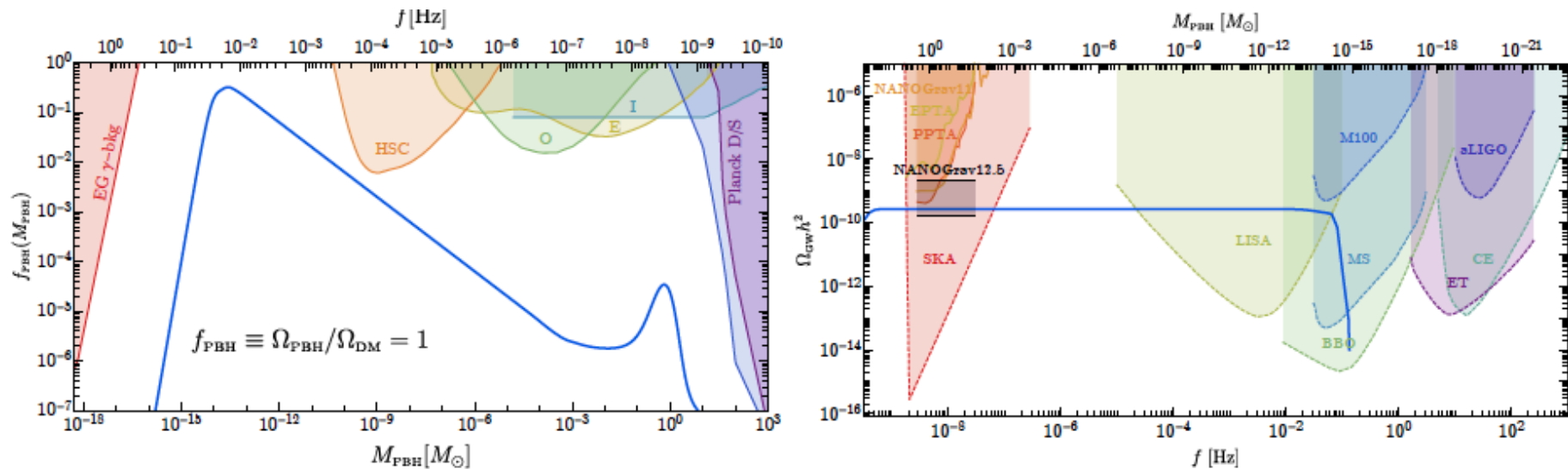
NANOGrav12.5yr and solar mass PBHs

K. Kohri and T. Terada, arXiv:arXiv:2009.11853



NANOGrav Hints to Primordial Black Holes as Dark Matter

V. De Luca, G. Franciolini, A. Riotto, arXiv:2009.08268 [astro-ph.CO]



$$\mathcal{P}_{\zeta}(k) \approx A_{\zeta} \Theta(k_s - k) \Theta(k - k_l), \quad k_s \gg k_l \quad (12)$$

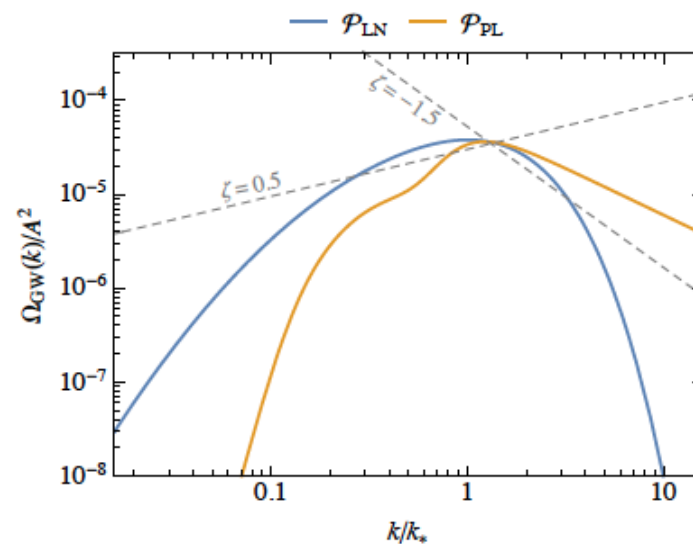
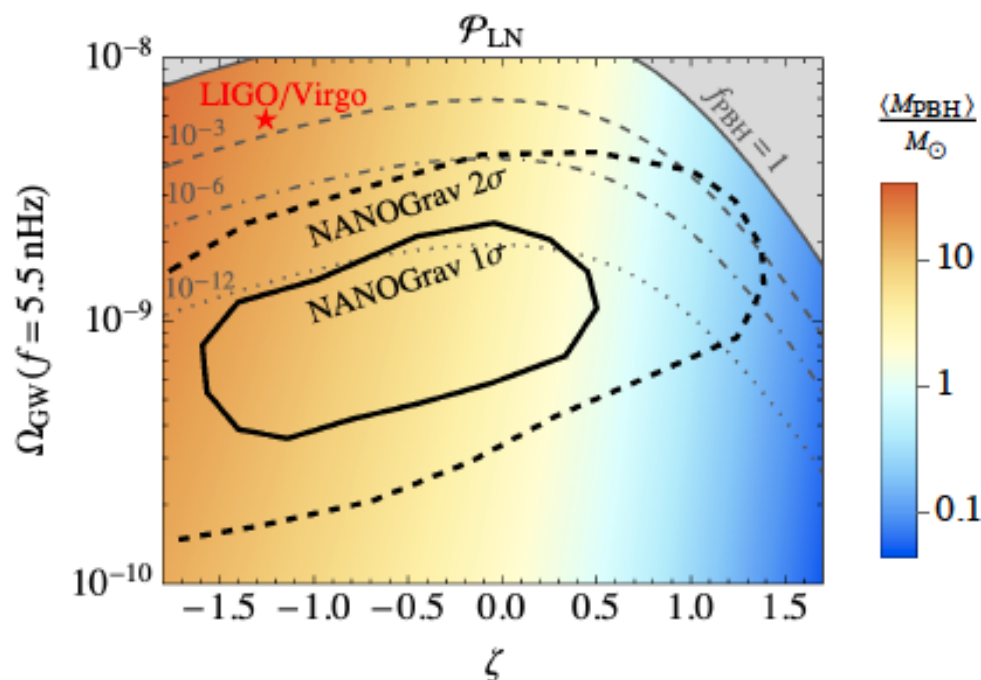
$$M_H \simeq 33 \left(\frac{10^{-9} \text{ Hz}}{f} \right)^2 M_{\odot}.$$

$$M_{\text{PBH}} = \kappa M_H (\delta - \delta_c)^{\gamma_c}$$

PBHs seed for SMBHs and NANOGrav12.5yr

Ville Vaskonen and Hardi Veerae, arXiv:2009.07832v2

The potential GW signal can be fitted by a power-law $\Omega_{\text{GW}} \propto f^\zeta$ with an amplitude $\Omega_{\text{GW}}(f = 5.5 \text{ nHz}) \in (3 \times 10^{-10}, 2 \times 10^{-9})$ and exponent $\zeta \in (-1.5, 0.5)$ at 1σ confidence level and with a small positive correlation between the amplitude and the exponent.

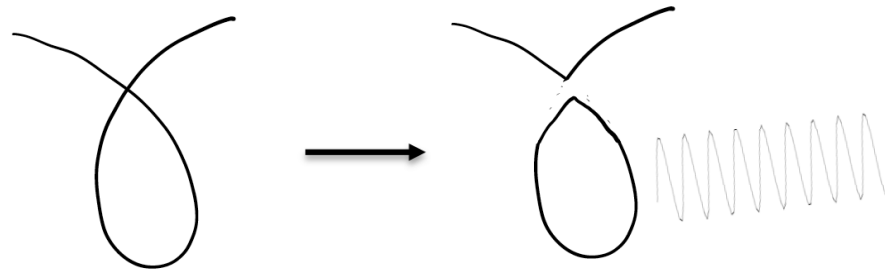


Note added: – Soon after the first version of this work, Ref. [74] claimed that the NANOGrav signal may be consistent with the LIGO/Virgo PBH scenario attributing the discrepancy between our conclusions to the choice of window function used in Eq. (9). We improved our PBH abundance estimate following [54] and found that our conclusions remain in tact. The differences between our conclusions and Ref. [74] can be resolved when we omit the non-linear relation between the density contrast and curvature perturbations. Additionally, Ref. [75] appeared pointing out a potential scenario for light PBH DM consistent with NANOGrav, which was not considered here.

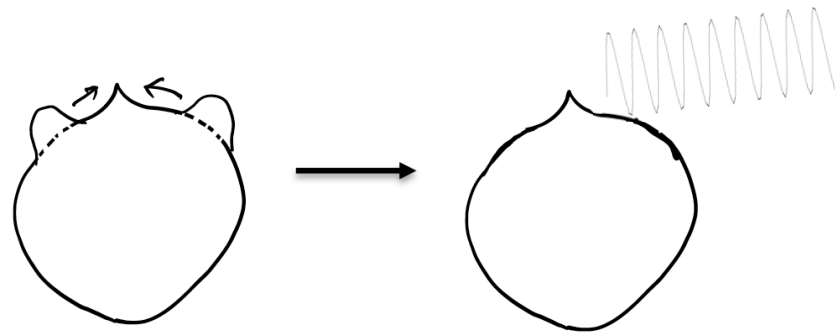
GW emitted from Cosmic string which was produced after GUT phase transition

Jeff A. Dror, Takashi Hiramatsu, Kazunori Kohri, Hitoshi Murayama, Graham White,
arXiv:1908.03227 [hep-ph]

kink

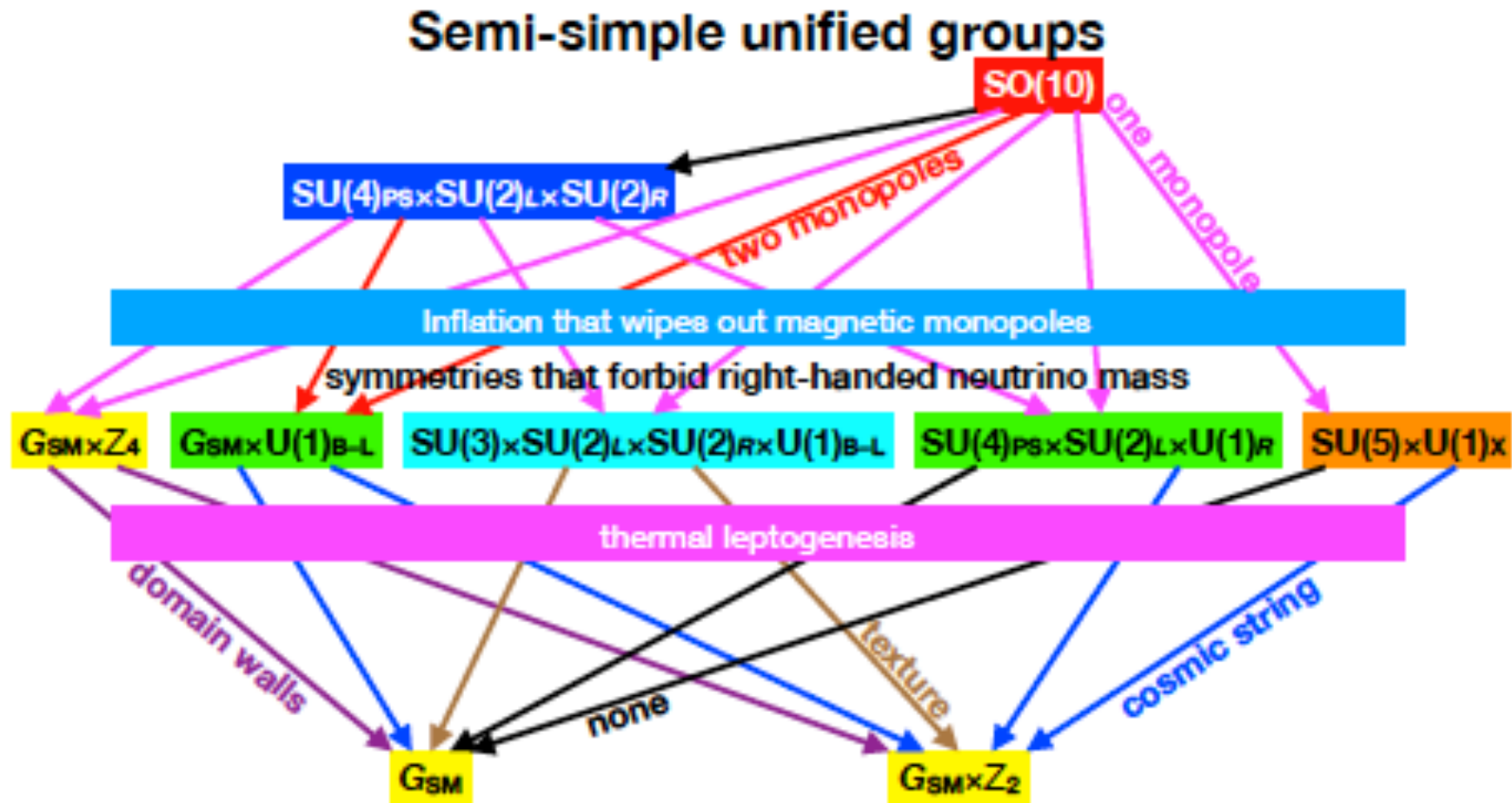


cusp



Seesaw and Leptogenesis searched by future GW

Jeff A. Dror, Takashi Hiramatsu, Kazunori Kohri, Hitoshi Murayama, Graham White,
arXiv:1908.03227 [hep-ph]



Seesaw and Leptogenesis searched by future GW

Jeff A. Dror, Takashi Hiramatsu, Kazunori Kohri, Hitoshi Murayama, Graham White,
arXiv:1908.03227 [hep-ph]

G	$H = G_{\text{SM}}$		$H = G_{\text{SM}} \times \mathbb{Z}_2$	
	defects	Higgs	defects	Higgs
G_{disc}	domain wall*	$B - L = 1$	domain wall*	$B - L = 2$
G_{B-L}	abelian string*	$B - L = 1$	\mathbb{Z}_2 string [†]	$B - L = 2$
G_{LR}	texture*	$(1, 1, 2, \frac{1}{2})$	\mathbb{Z}_2 string	$(1, 1, 3, 1)$
G_{421}	none	$(4, 1, 1)$	\mathbb{Z}_2 string	$(15, 1, 2)$
G_{flip}	none	$(10, 1)$	\mathbb{Z}_2 string	$(50, 2)$

$$G_{\text{disc}} = G_{\text{SM}} \times \mathbb{Z}_N,$$

$$G_{B-L} = G_{\text{SM}} \times U(1)_{B-L},$$

$$G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_{\text{PS}} \times SU(2)_L \times U(1)_Y,$$

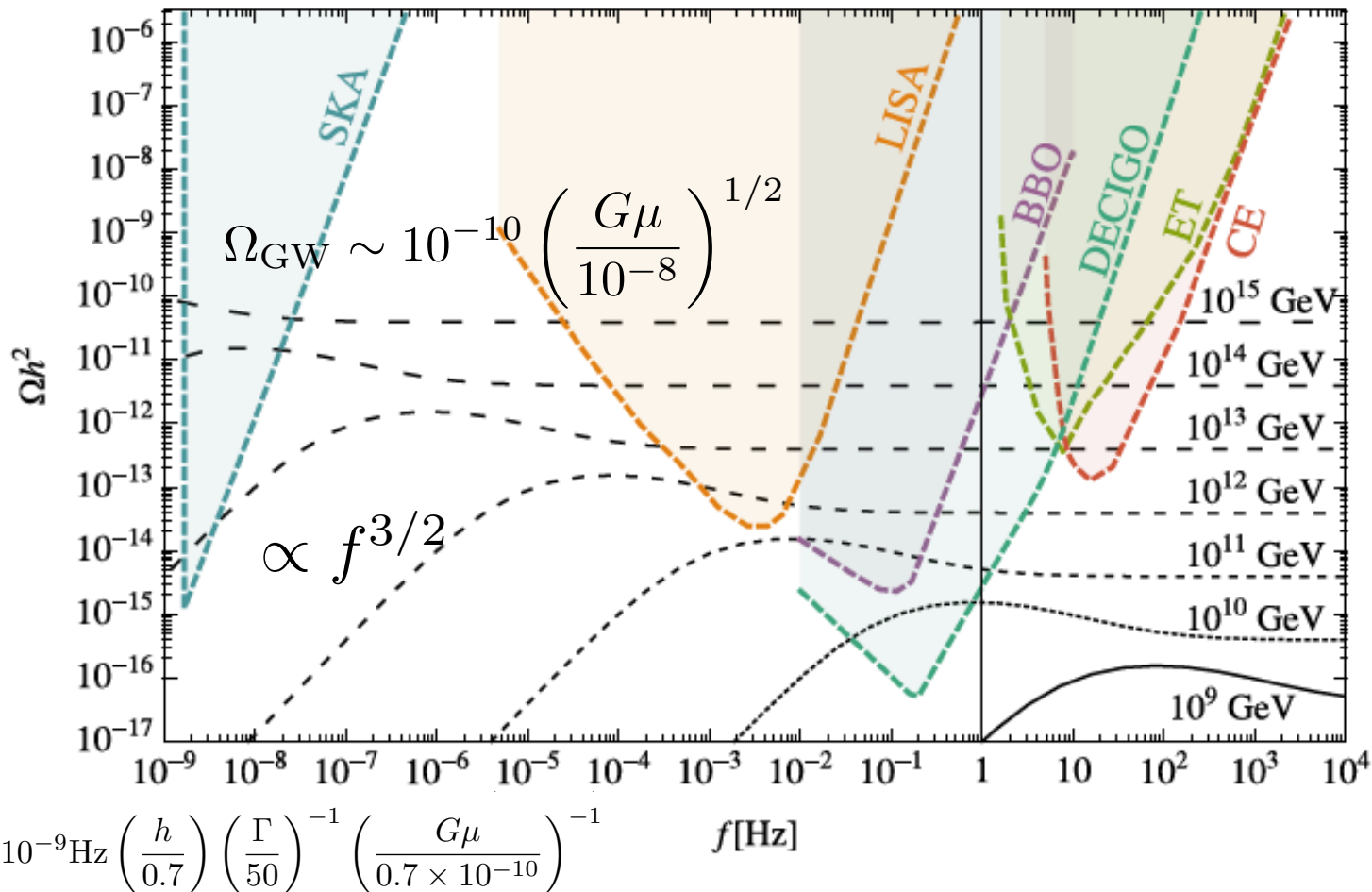
$$G_{\text{flip}} = SU(5) \times U(1).$$

Monopole production rate is small

$$\frac{\Gamma}{L} = \frac{\mu}{2\pi} \frac{g}{4\pi} e^{-\pi m^2/\mu}$$

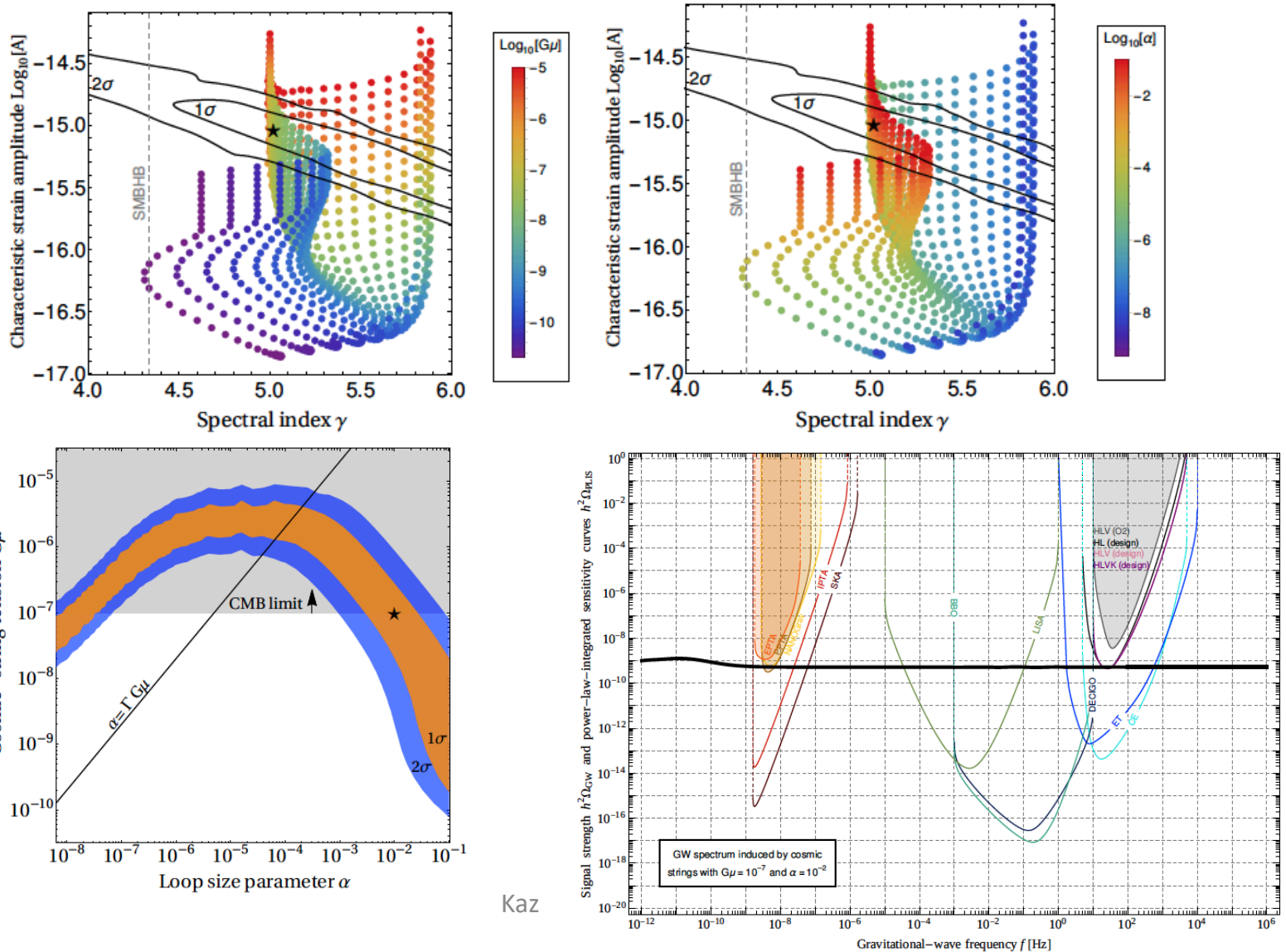
GW emitted from Cosmic string which was produced after GUT phase transition

Jeff A. Dror, Takashi Hiramatsu, Kazunori Kohri, Hitoshi Murayama, Graham White, arXiv:1908.03227 [hep-ph]



NanoGRAV found GWs from cosmic string

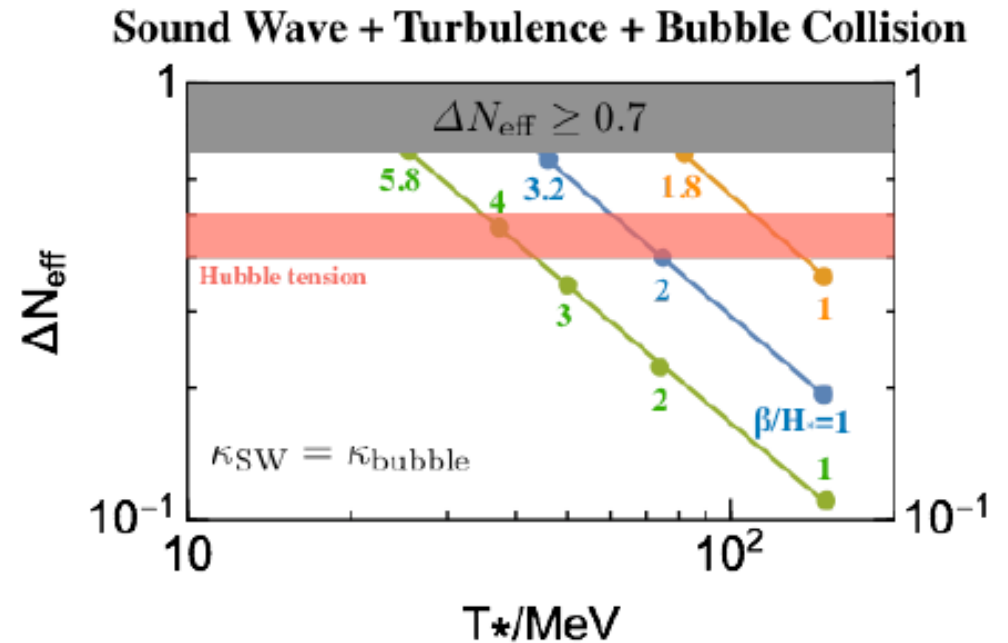
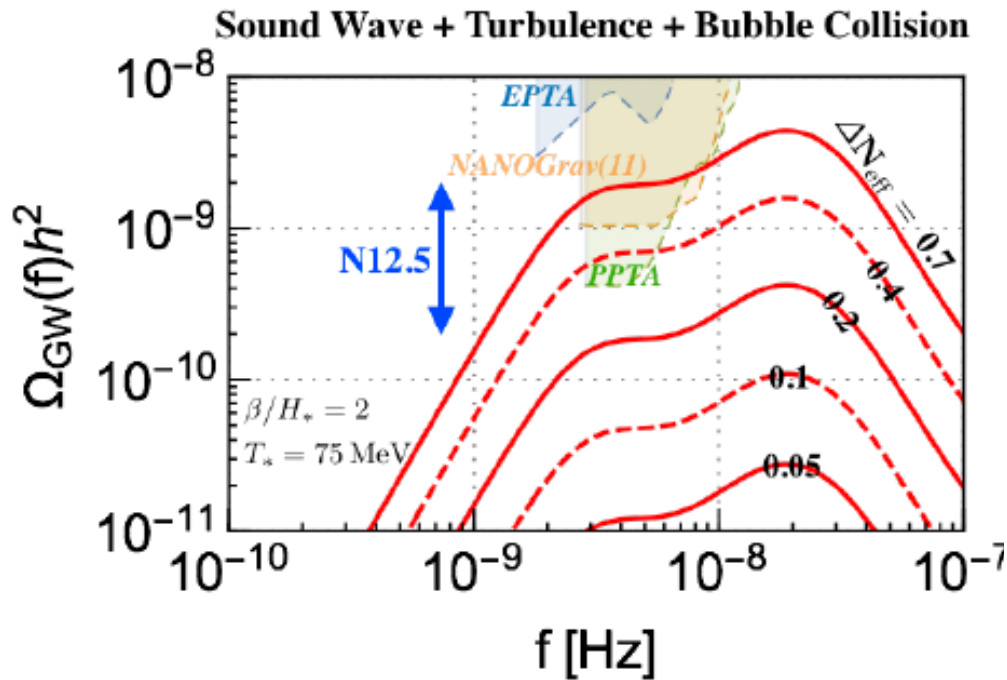
Simone Blasi, Vedran Brdar, Kai Schmitz, arXiv:2009.06607 [astro-ph.CO]



1st order phase transition of unknown scalar

Gravitational Waves and Dark Radiation from Dark Phase Transition: Connecting NANOGrav Pulsar Timing Data and Hubble Tension

Yuichiro Nakai, Motoo Suzuki, Fuminobu Takahashi, Masaki Yamada,
arXiv:2009.09754 [astro-ph.CO]



Summary

- Binary PBHs are good candidates for [LIGO/Virgo](#) events
- We can probe high-energy physics, the early Universe, and gravity with PBHs
- The [NANOGrav 12.5yr](#) data can be fitted by secondary GWs induced by large curvature perturbation, which could have produced PBHs with $O(1) M_{\odot}$ simultaneously
- We will be able to distinguish a model of PBH formations from others by future observations such as [LIGO/Virgo/KAGRA](#) or [DECIGO/BBO](#) and so on.