From 3d dualities to hadron physics

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Introduction and Motivation

- Recently, there are exciting developments in QCD3 and its dualities.

[Aharony, Benini, Karch, Komargodski, Seiberg, Tong,…]

- For example,

[Komargodski, Seiberg ('17)]

$$SU(N)_k \text{ with } N_f \text{ fermions}$$

$$\Big\Updownarrow \quad 2|k| < N_f < N_*$$

$$U(N_f/2 \pm k)_{\mp N} \text{ with } N_f \text{ scalars}$$

w/ some conditions

- In the phase of QCD3 w/ small m_f, the flavor sym is broken, which looks similar to QCD4.

- Moreover, the sym breaking is described by the Higgs phenomena in dual scalar theory.

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➡ **Dual theory of hadrons?**

Dualities in 3d

[Komargodski, Seiberg ('17)]

$SU(N)_k$ w/ N_f ψ

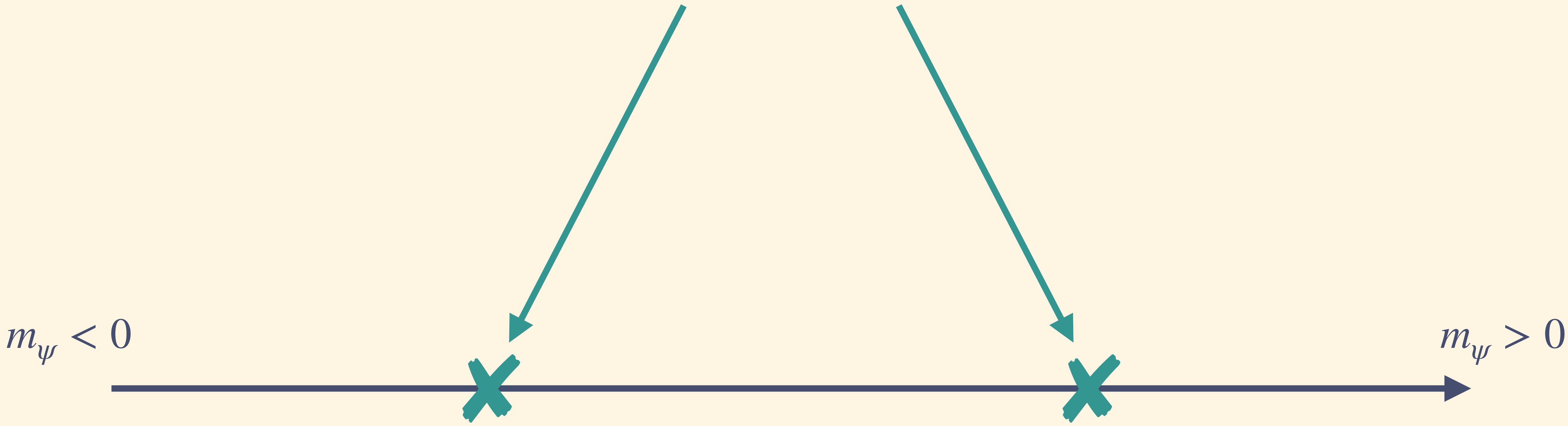
$2|k| < N_f < \mathcal{N}_*,\ N > 2$

$m_\psi < 0$ $m_\psi > 0$

[Komargodski, Seiberg ('17)]

$SU(N)_k$ w/ $N_f\ \psi$

$2|k| < N_f < \mathcal{N}_*,\ N > 2$



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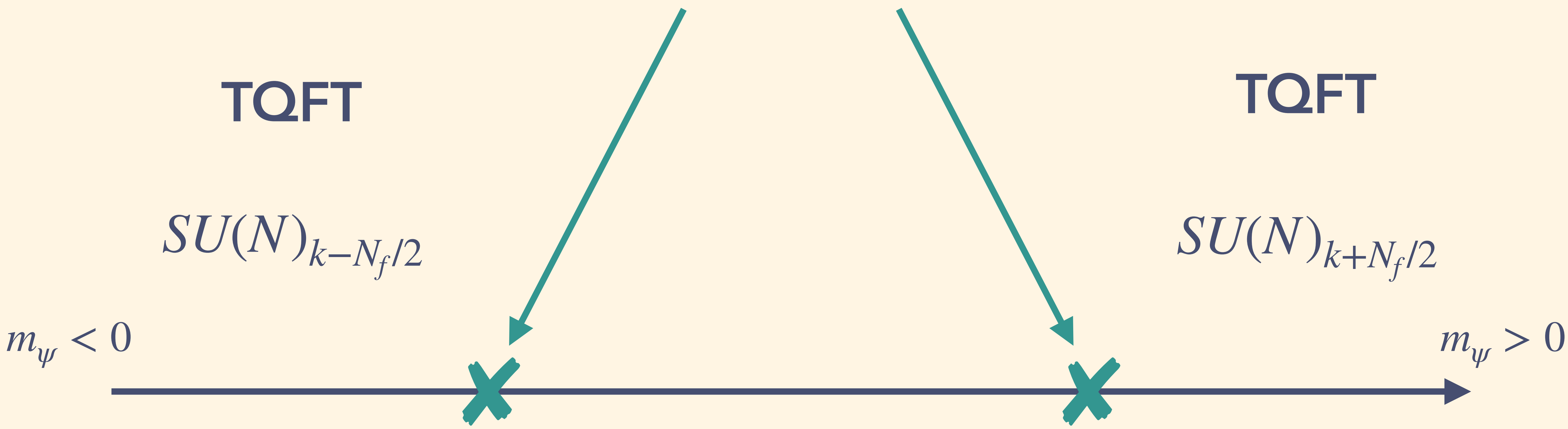

[Komargodski, Seiberg ('17)]

$SU(N)_k$ w/ N_f ψ

$2\,|k| < N_f < \mathcal{N}_*,\ N > 2$

TQFT

$SU(N)_{k-N_f/2}$

NLSM

TQFT

$SU(N)_{k+N_f/2}$

$m_\psi < 0$

$m_\psi > 0$

$$\frac{U(N_f)}{U(N_f/2+k)\times U(N_f/2-k)}$$

with $N\Gamma$

[Komargodski, Seiberg ('17)]

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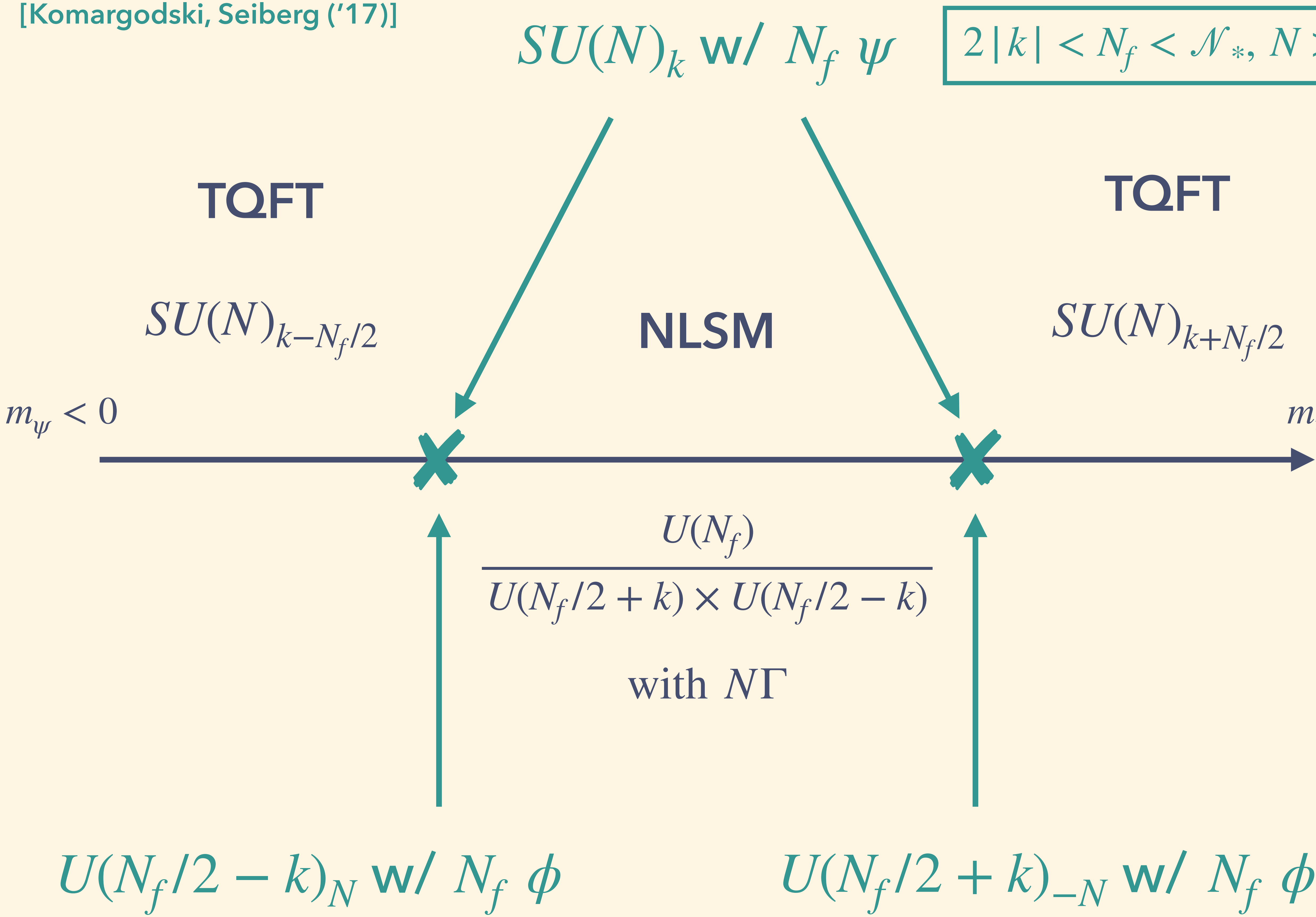

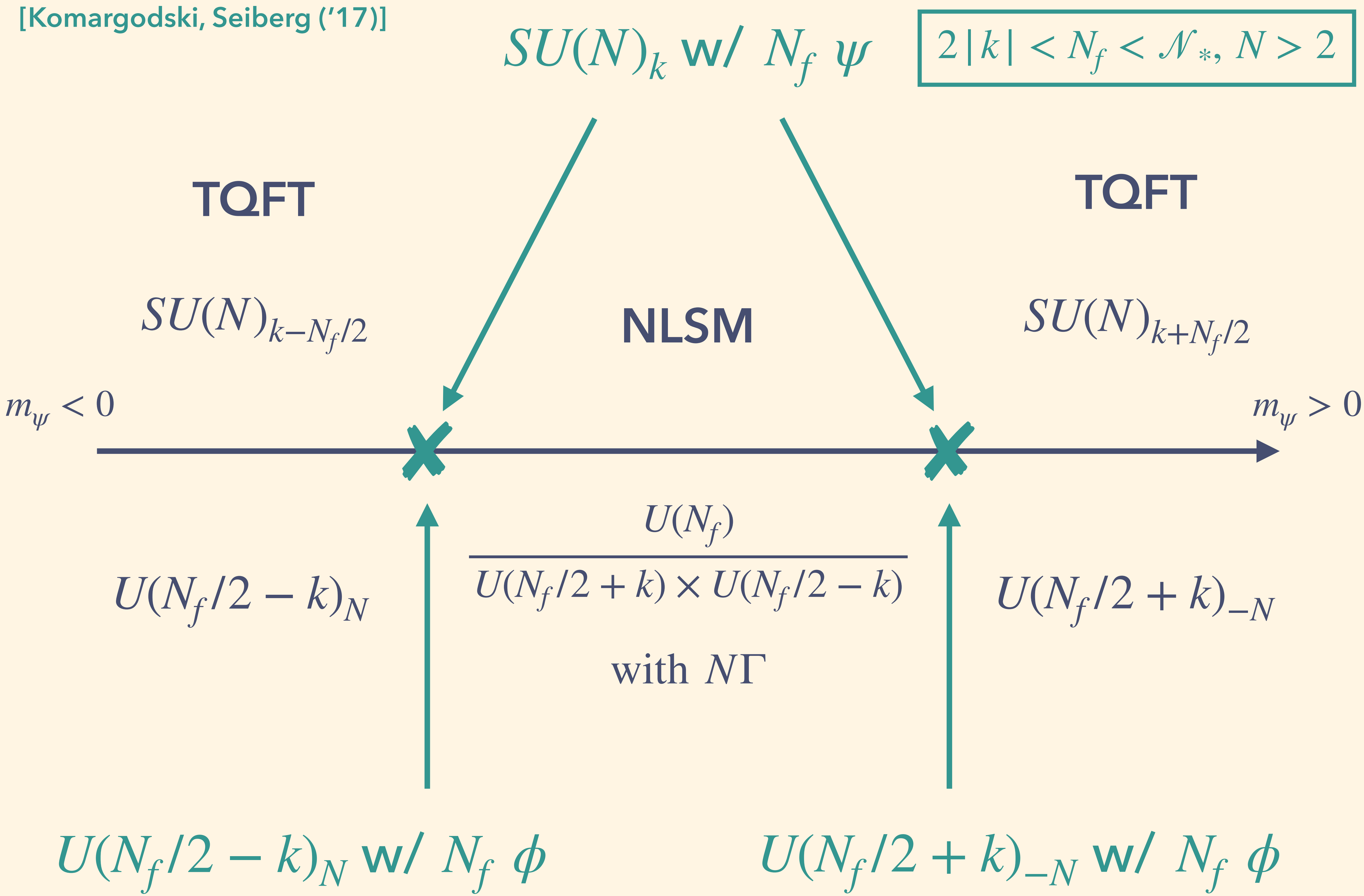
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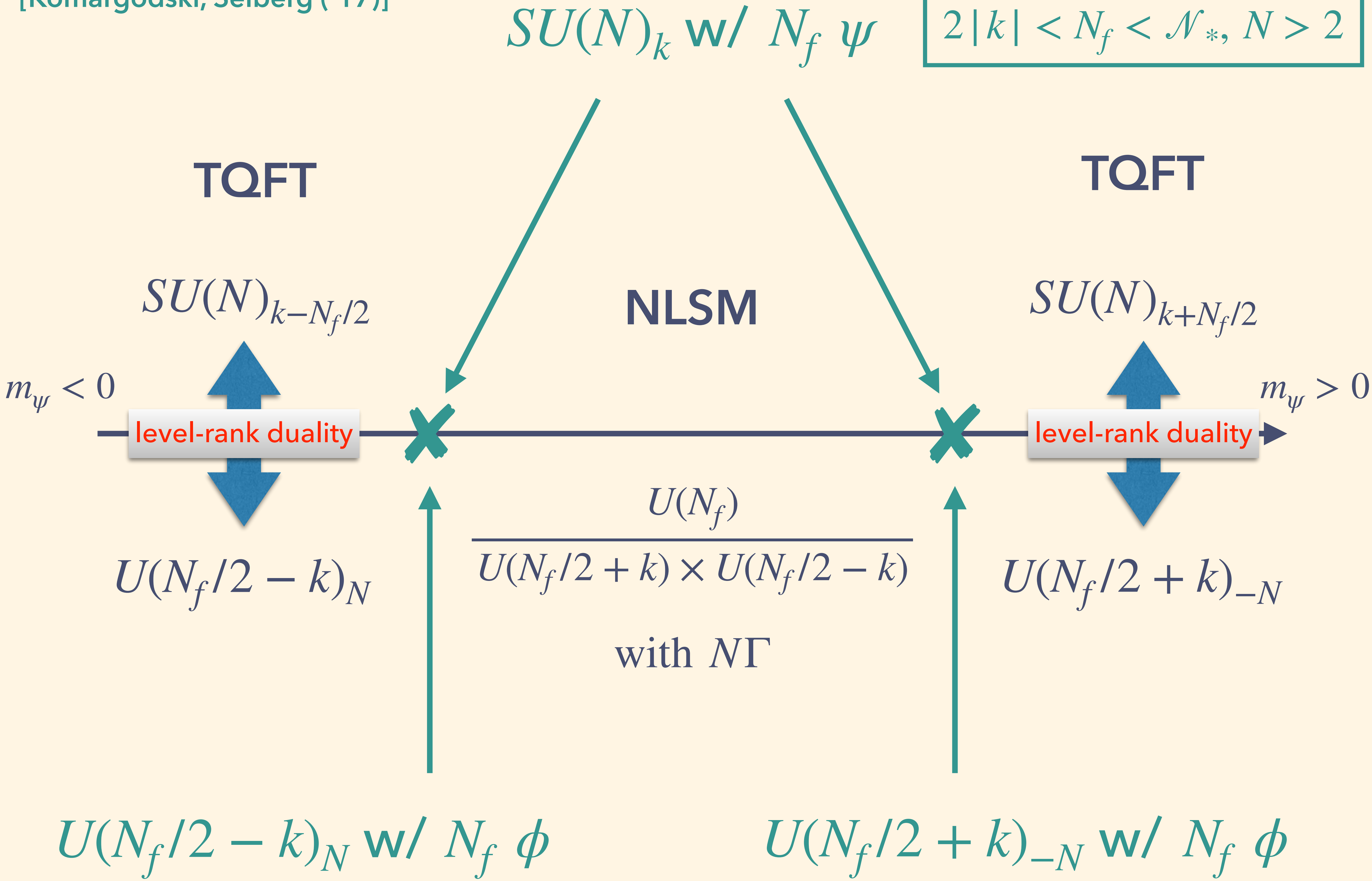
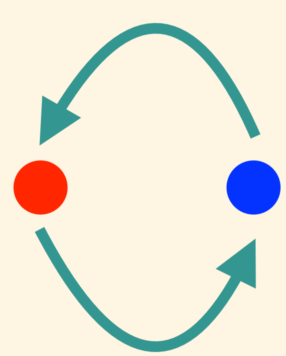


- In 3d, we can write the Chern-Simons term:

$$S_{\rm CS} = \frac{k}{4\pi}\int d^3x\ \epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho,$$

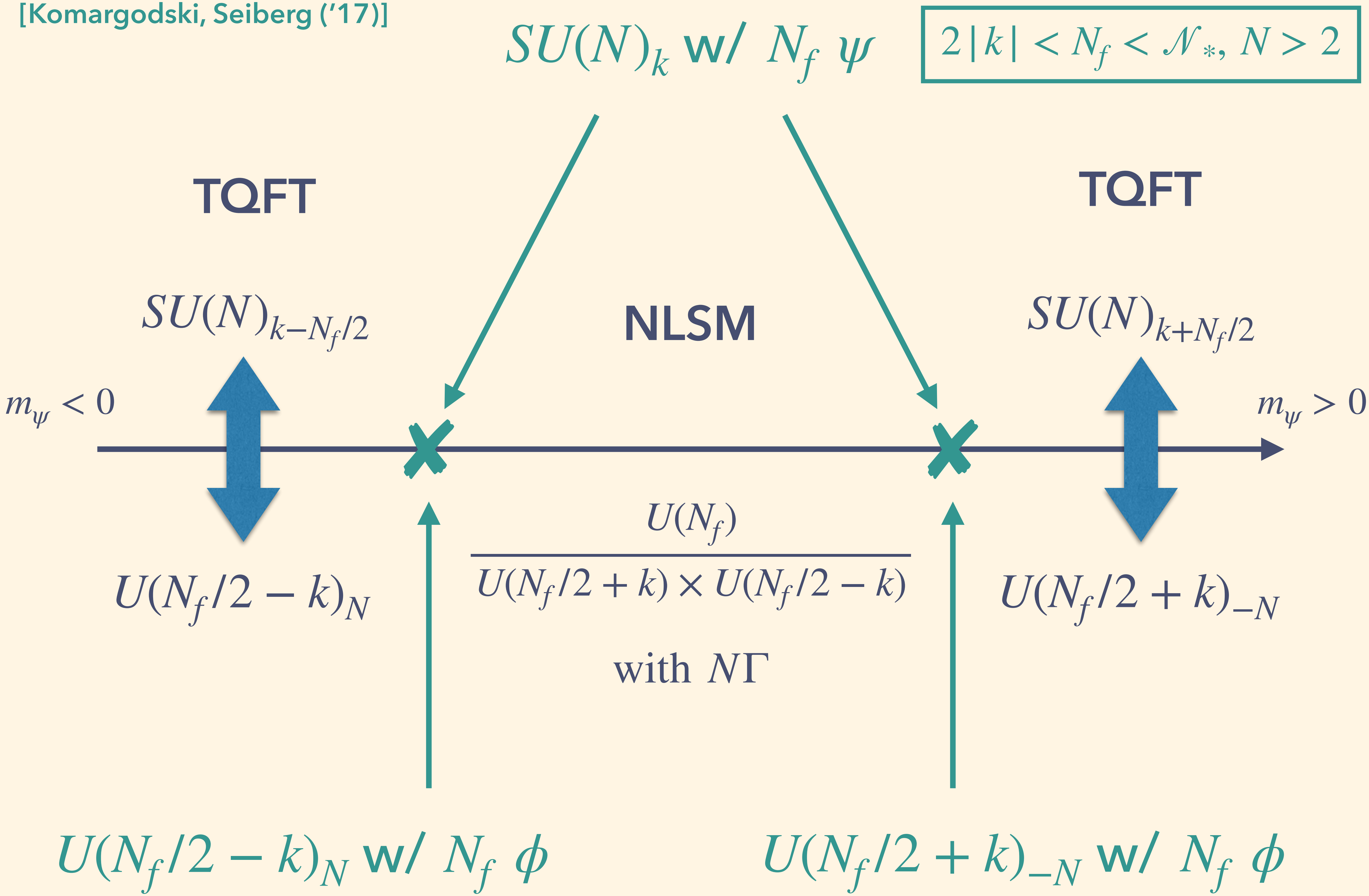
where $k \in \mathbb{Z}$.

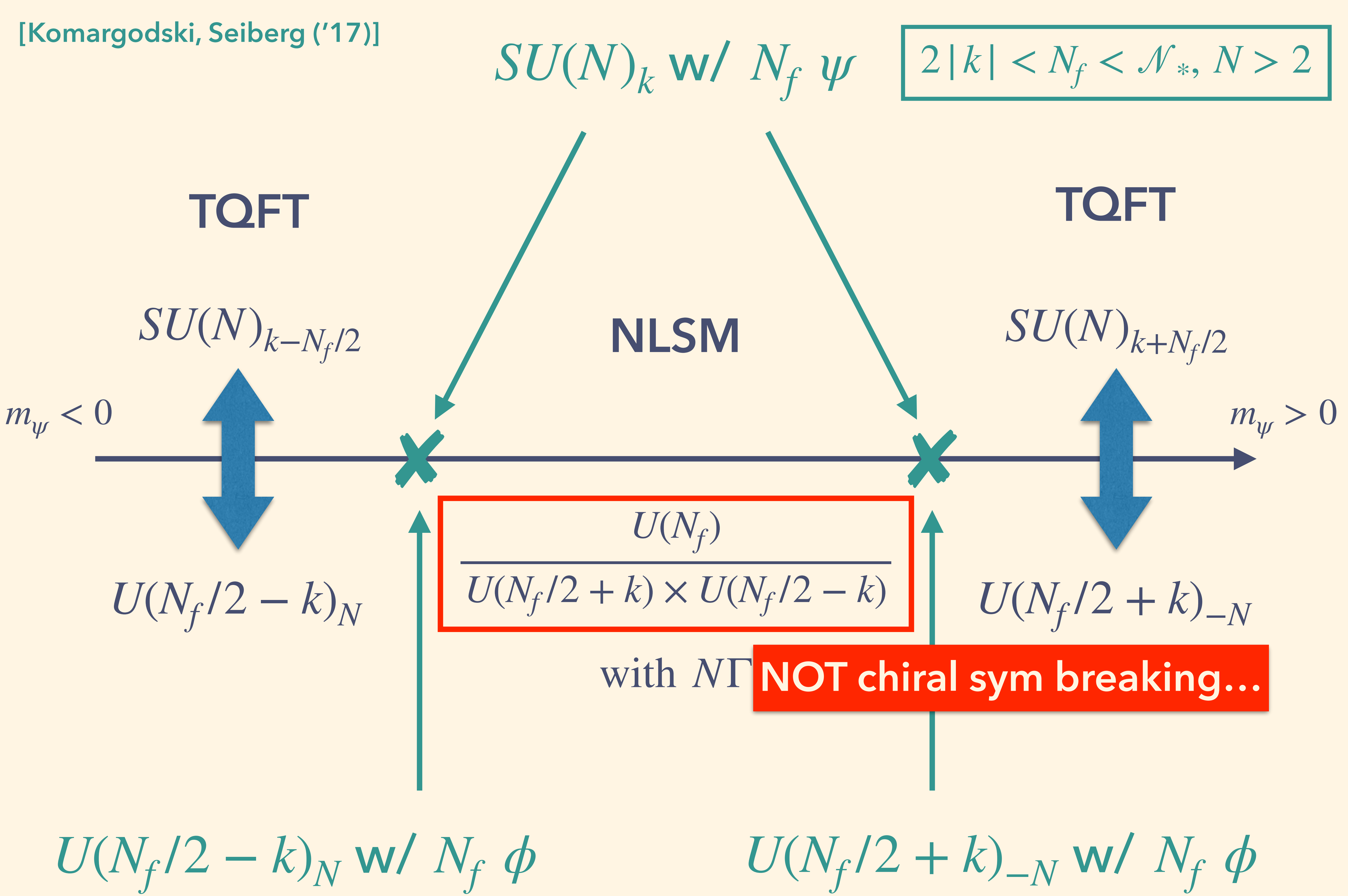
- CS term is topological.
- CS term contributes to the statistical phase:



$= \pm e^{i\pi/k}$

[Komargodski, Seiberg ('17)]

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with $N\Gamma$
NOT chiral sym breaking…
$U(N_f/2-k)_N$ w/ $N_f\ \phi$
$U(N_f/2+k)_{-N}$ w/ $N_f\ \phi$

- **We add the explicit breaking terms**

$$\mathscr{L}_{\text{ex}} = -\bar{\psi} a_3 \psi + \bar{\tilde{\psi}} a_3 \tilde{\psi},$$

to $SU(N)_0$ **with** $(N_f + N_f)$ **fermions.**

- **The theory in the broken phase is described by** $(U(N_f) \times U(N_f))/U(N_f)$ **.**

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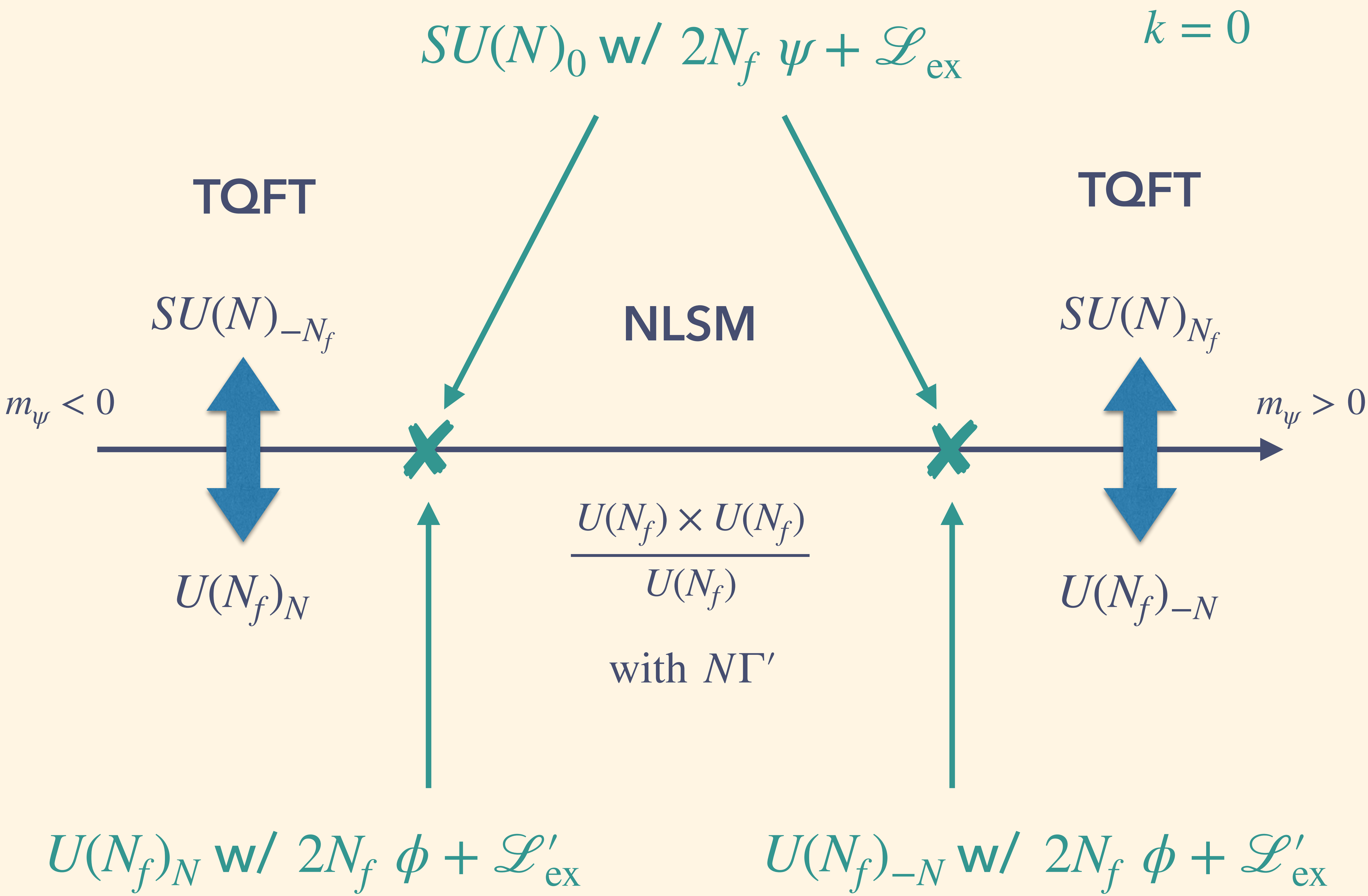
➡ coincide with chiral symmetry breaking

[NK, Kitano, Yankielowicz, Yokokura ('19)]



QCD_4

- **We start with QCD4 on $M_3 \times S^1$,**

$$S = \int_{M_3 \times S^1} \left[-\frac{1}{2g_4^2} \mathrm{Tr}\, |f|^2 + \frac{\theta(x_3)}{8\pi^2} \mathrm{Tr} \left(f^2 \right) + i \sum_{i=1}^{N_f} \bar{\Psi}_i \, \not{D}_a \Psi_i \right],$$

where the θ winds around S^1,

$$\int_{S^1} d\theta = 2\pi k$$

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$$\int_{S^1} d\theta = 2\pi k = 2\pi N_f \,.$$

Effective theory
for small and large radius

- For small radius, $\Lambda_4 R \ll 1$, we can perform the KK decomposition.
- From the θ-term, we find the CS term,

$$\frac{1}{8\pi^2}\int_{M_3\times S^1} \theta \mathrm{Tr}(ff) = \frac{1}{8\pi^2}\int_{M_3\times S^1} \mathrm{Tr}\left(ada + \frac{2}{3}a^3\right) d\theta, \quad \mathrm{mod}\ 2\pi\,.$$

- There is a mass gap, but the low energy limit is the CS theory, $SU(N)_{N_f}$.

- **For large radius, $\Lambda_4 R \gg 1$, the low effective theory is given by**

$$S_{\text{eff}} = \int_{M_3 \times S^1} d^4x \left[f_\pi^2 \text{Tr} \, | \, \partial_M U \, |^2 - \frac{m_\eta^2 f_\pi^2}{N_f} \left| \log(e^{-i\theta} \det U) \right|^2 + \cdots \right],$$

where $U = \exp(i\pi^a T^a + i\eta)$ **.**

- The EoM for η is

$$\frac{\partial^2}{\partial x_3^2}\eta = m_\eta^2\left(\eta - \frac{\theta}{N_f}\right),$$

➡ the η gets a winding,

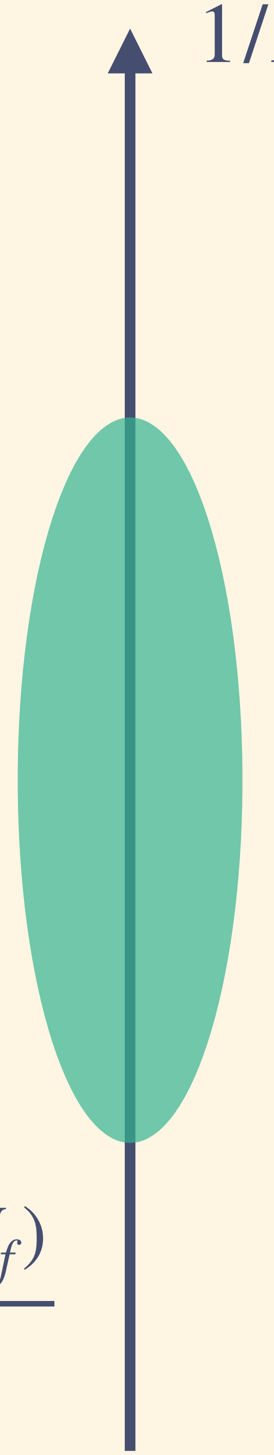
$$\eta(x_3 + 2\pi R) = \eta(x_3) + 2\pi\,.$$

- Under the background where the η has winding, the 4d WZW term which couple to the external gauge fields includes 3d WZ term,

$$S_{\mathrm{WZW}} \supset -\frac{N}{8\pi^2}\int_{M_3\times S^1} \mathrm{Tr}\left(AdA + \frac{2}{3}A^3\right) d\eta \,.$$

4d on S^1

QCD4 on S^1

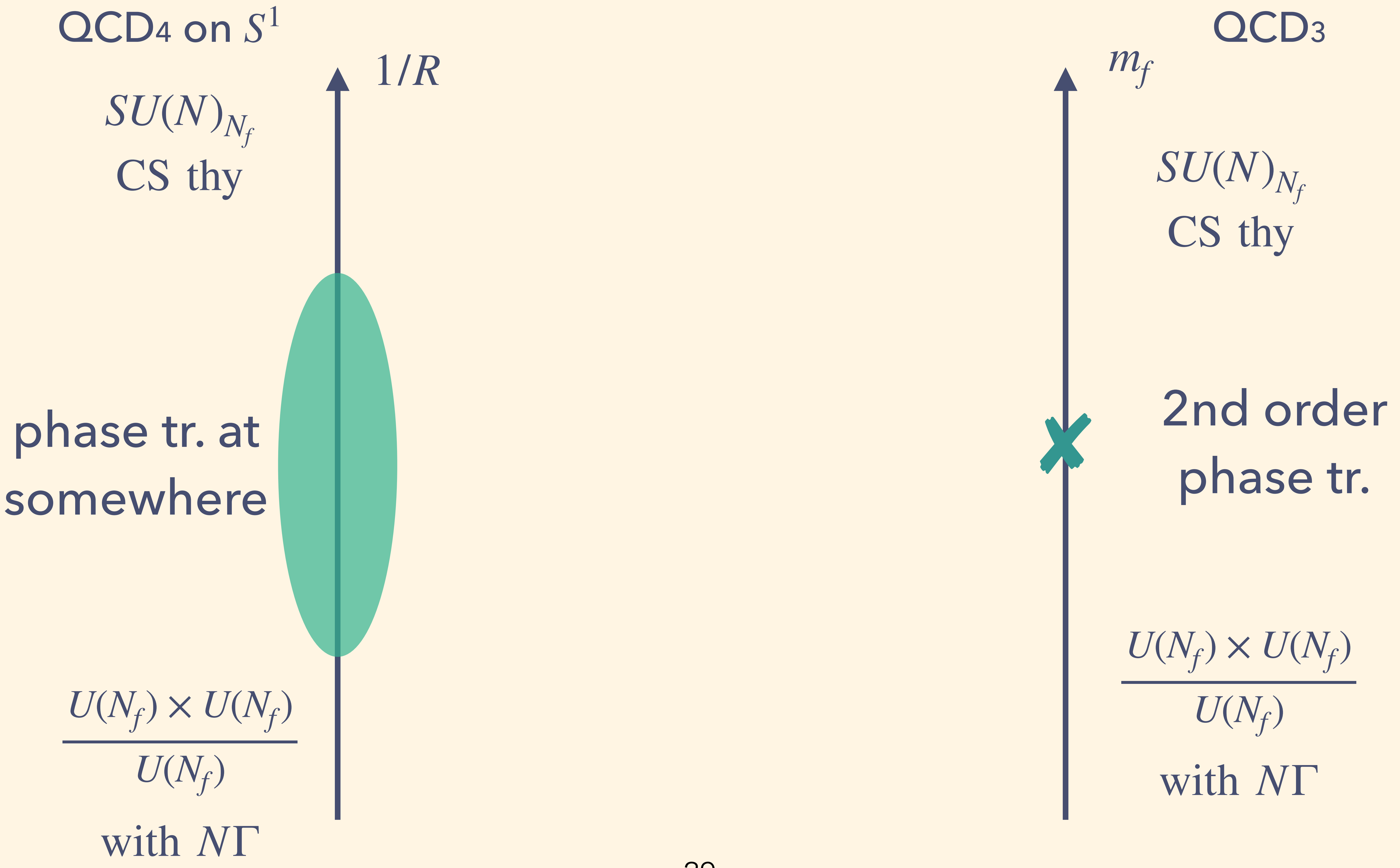


$SU(N)_{N_f}$ CS thy

phase tr. at somewhere

$\frac{U(N_f)\times U(N_f)}{U(N_f)}$ with $N\Gamma$

4d on S^1 | 3d w/ $\mathscr{L}_{\text{ex.}}$



4d on S^1

QCD4 on S^1

$1/R$

$SU(N)_{N_f}$ CS thy

phase tr. at somewhere

$\frac{U(N_f)\times U(N_f)}{U(N_f)}$ with $N\Gamma$

3d w/ $\mathscr{L}_{\text{ex.}}$

QCD3

m_f

$U(N_f)_{-N}$ CS thy

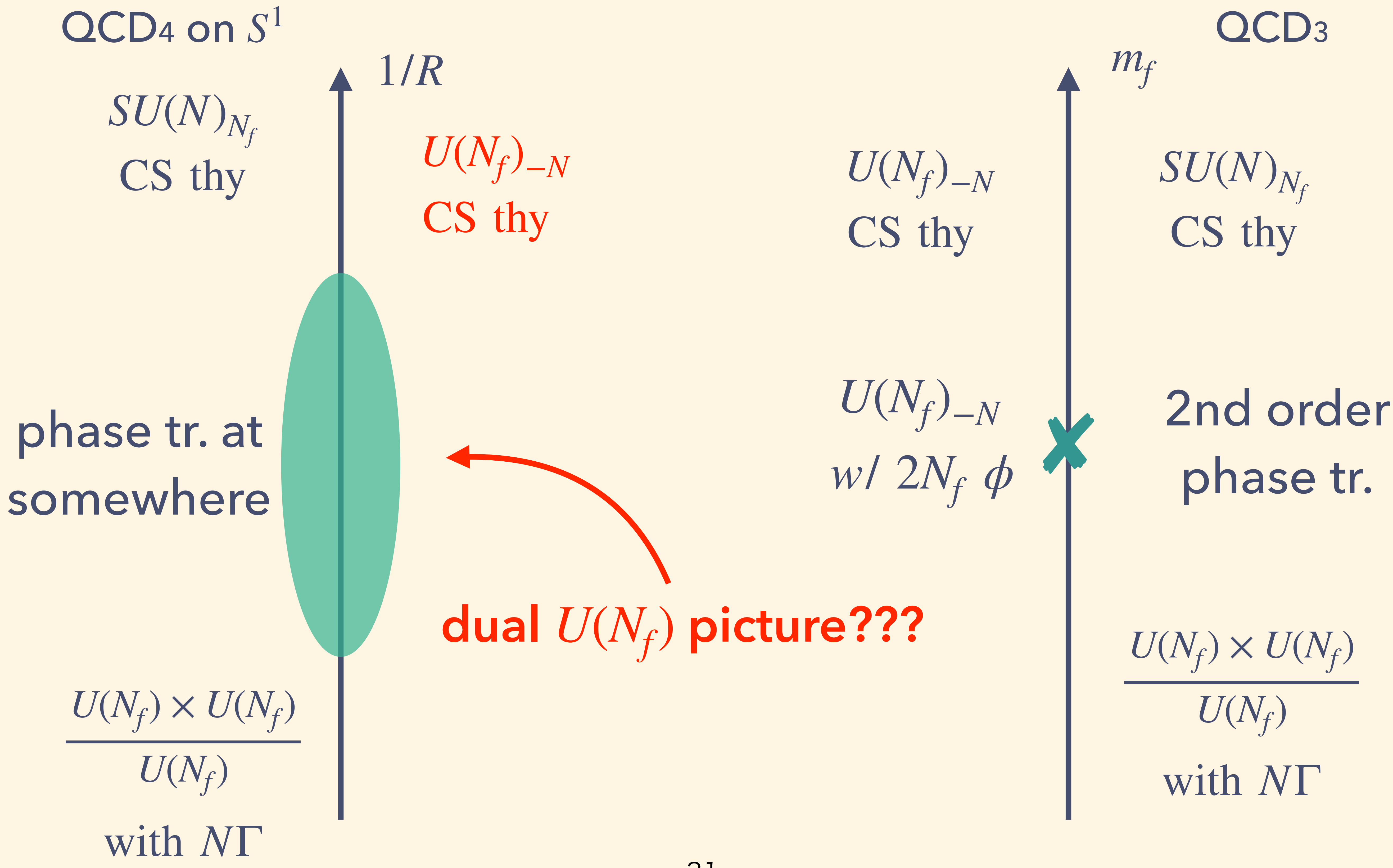
$SU(N)_{N_f}$ CS thy

$U(N_f)_{-N}$ w/ $2N_f$ ϕ

2nd order phase tr.

$\frac{U(N_f)\times U(N_f)}{U(N_f)}$ with $N\Gamma$

4d on S^1 | 3d w/ $\mathscr{L}_{\text{ex.}}$



- **The extension of the chiral Lagrangian to $U(N_f)$ gauge theory is known to give a great success to describe the phenomenology of the vector mesons ρ and ω.** [Bando, Kugo Uehara, Yamawaki, Yanagida ('85)]

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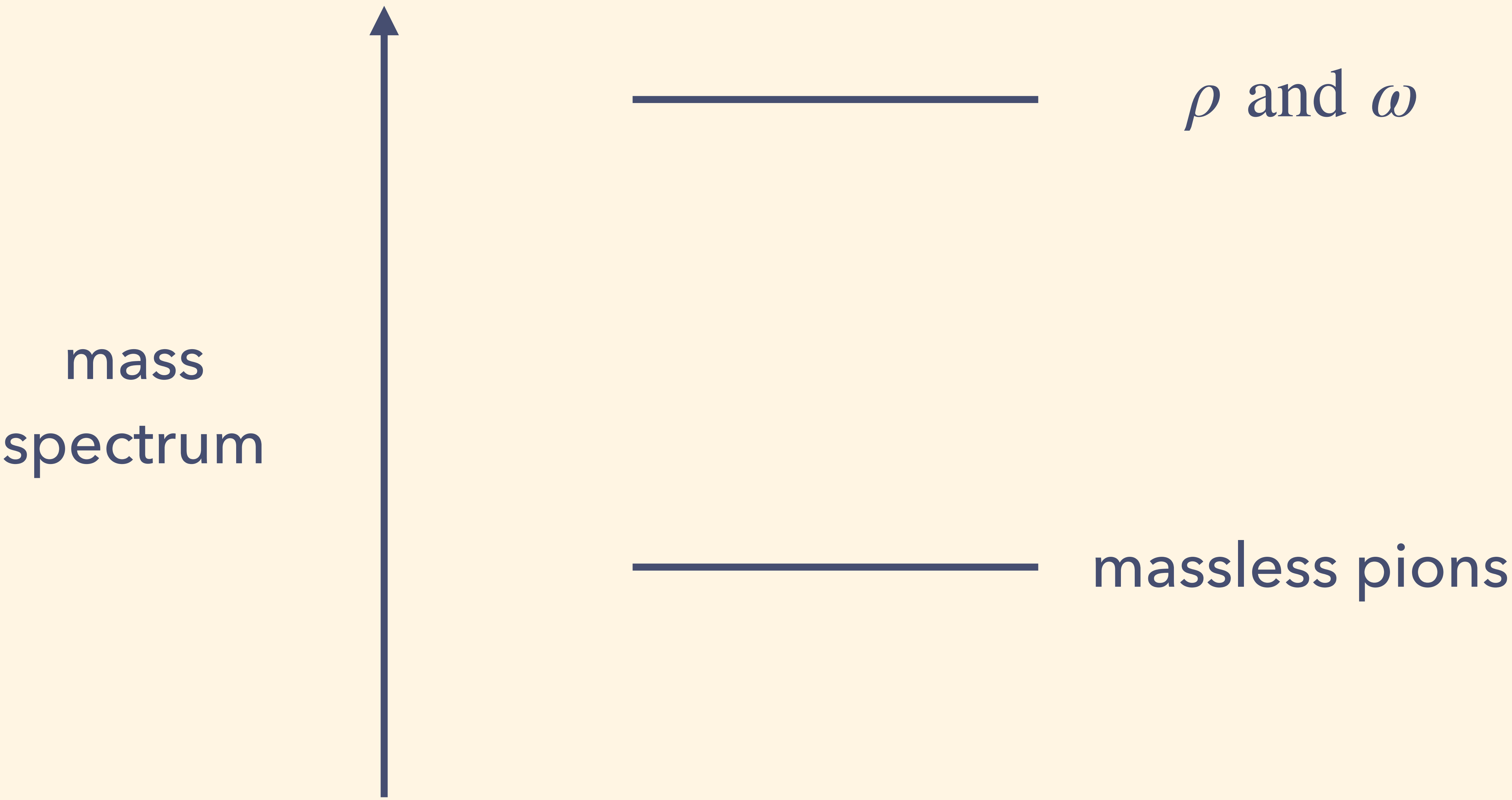
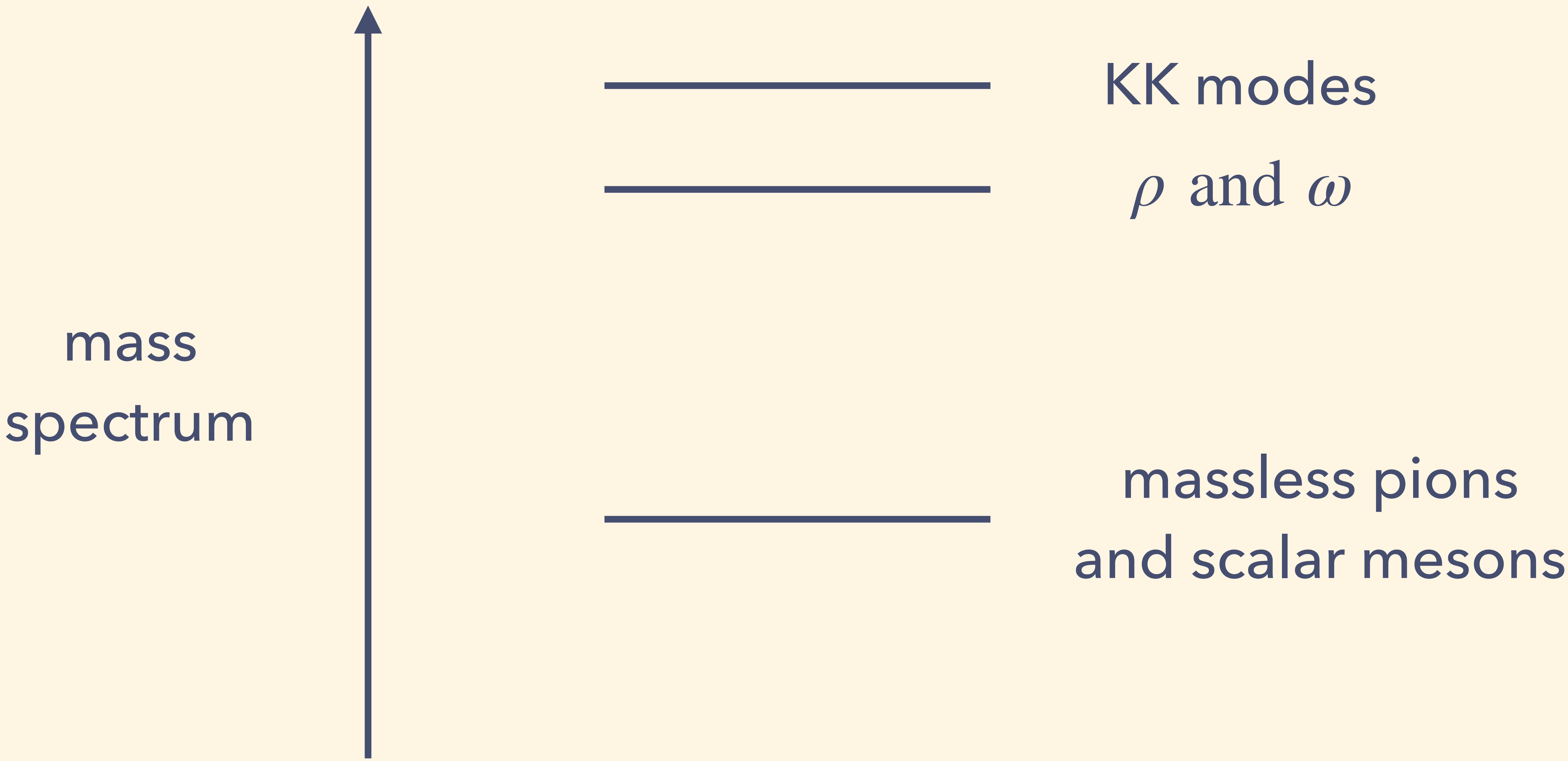
natural candidates for $U(N_f)$ gauge boson

A scenario of the dual theory

- **At large radius,** $\Lambda_4 R \gg 1$,



- **Close to critical radius,** $\Lambda_4 R \sim 1$,



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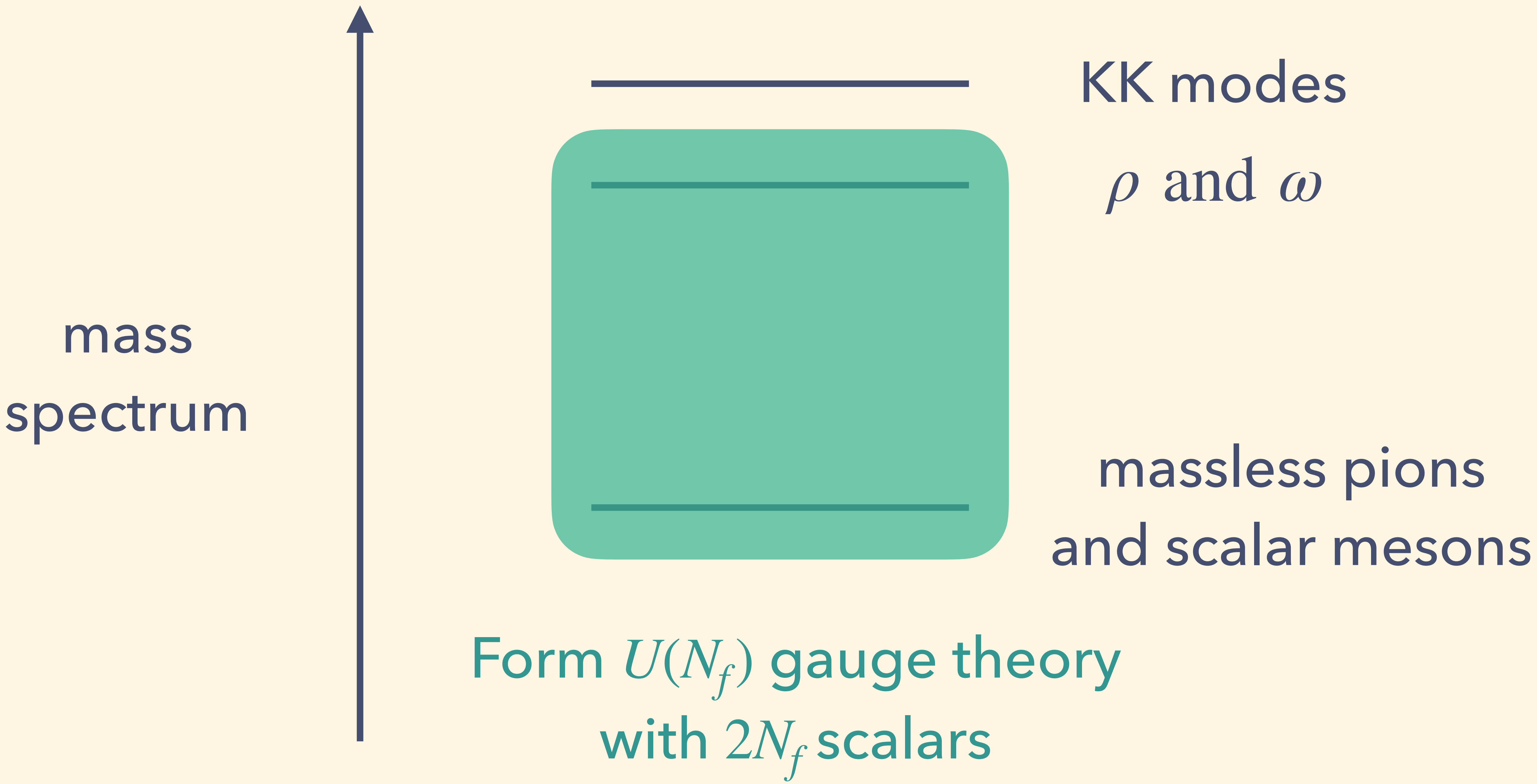
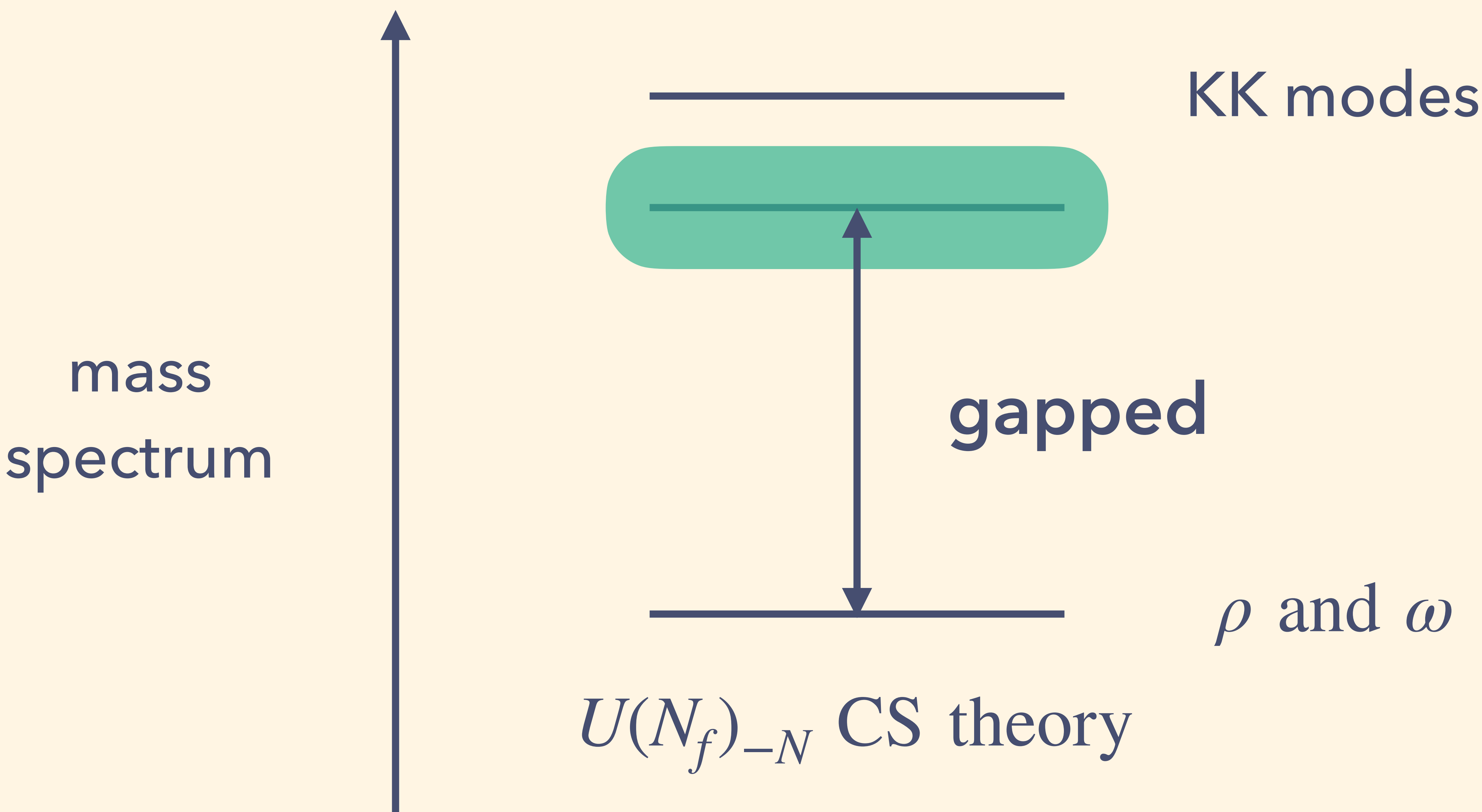


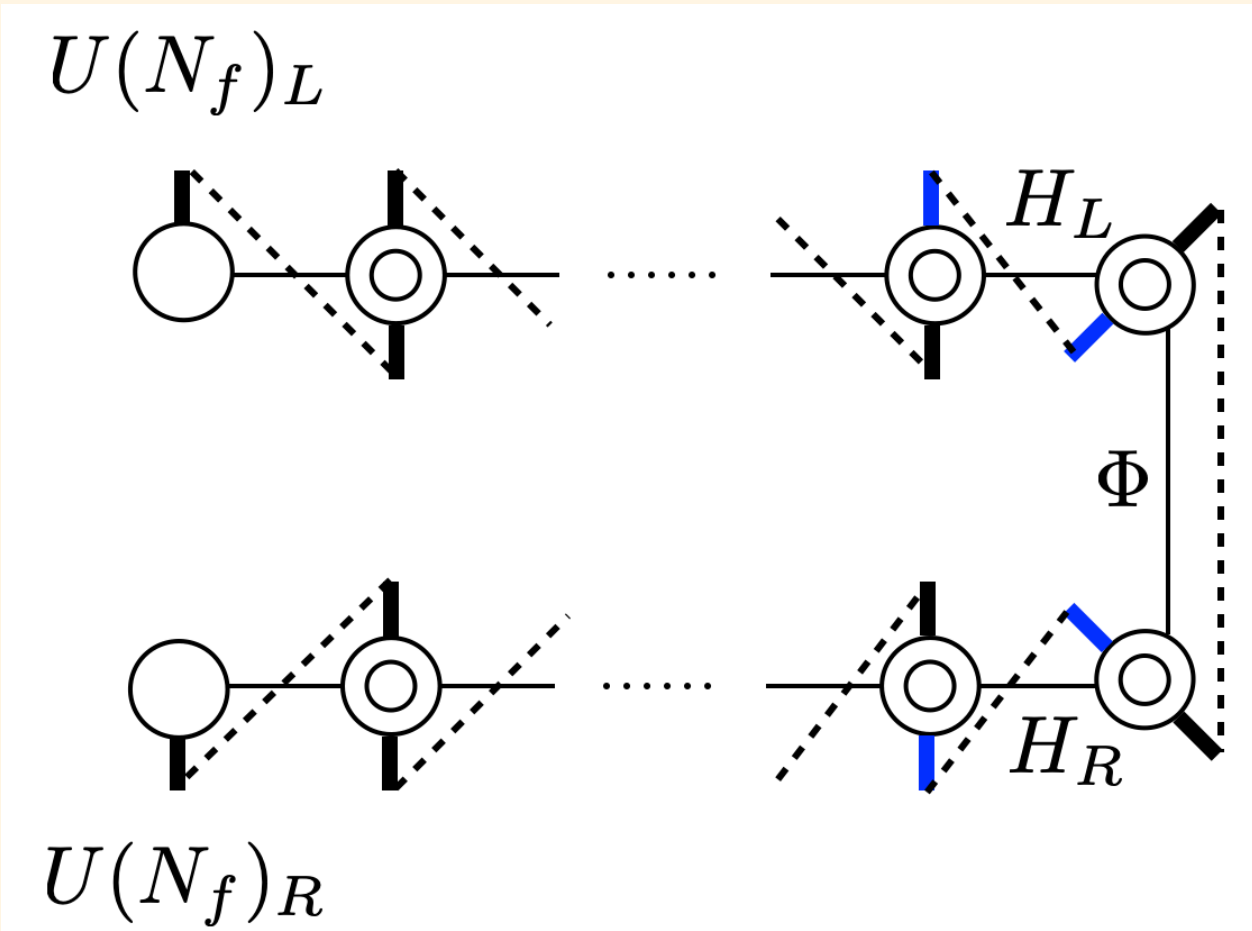
- At small radius, $\Lambda_4 R \ll 1$,



(chiral sym is restored)

Holographic model

The quiver diagram of our model



: $U(N_f)$ gauge field

: bifund. link field

: fund. chiral fermion

: fermion mass term

- For $\langle H_{L,R} \rangle \neq 0$, describing π's and η.
- When $\langle H_{L,R} \rangle = 0$, the model becomes $U(N_f)_{-N}$.

Summary

In QCD_4 with winding θ, there is a phase transition between the large and small radius.

We conjectured the dual $U(N_f)$ description near the critical pt from 3d duality, and suggested the new picture of hadrons!!!

We proposed the holographic model of the dual theory that realizes our picture.

ばっくあっぷ

Conventions

- Dynamical gauge fields are given by lowercase letters $a_\mu, b_\mu, \dots$; $A_\mu, B_\mu, \dots$ represent non-dynamical fields.

- We represent the Lagrangian of QED with a CS term as

$$\mathscr{L} = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - i a_\mu)\psi - m\bar{\psi}\psi + \frac{k_{bare}}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

$$\equiv i\bar{\psi}\not{D}_a\psi + \frac{k_{bare}}{4\pi} ada,$$

where $k_{bare} \in \mathbb{Z}$.

- **When $|m| \gg e^2$, we can integrate out a fermion, and this shifts the bare CS level by $\text{sgn}(m)/2$.**

- **In addition, the theory is regularized to preserve gauge invariance with a Pauli-Villars regulator, which shifts the bare CS level by $-1/2$.**

- For this reason, we define the CS level k as

$$k = k_{bare} - N_f/2,$$

where N_f is the number of the fermions.

- For example, QED with $k_{bare} = 0$

$$\mathscr{L} = i\bar{\psi}\not{D}_a\psi$$

is expressed as $U(1)_{-1/2} + \psi$.

- When the fermions have masses and they are integrated out, the CS level in the low energy theory is

$$k_{IR} = k + N_f \cdot \mathrm{sgn}(m)/2.$$

- Our labeling of theories with non-Abelian gauge groups is analogous.