23/11/2020@素粒子現象論研究会2020

電弱アクシオンストリング と 超伝導

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Ref. arXiv:2010.02834[hep-ph]

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電弱アクシオンストリング

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と

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超伝導

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強いCP問題

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_s^2}{\rm tr}(G_{\mu\nu})^2 + \frac{i\theta}{8\pi^2}{\rm tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \mathcal{L}_{\rm matter}$$

- θ-termはnon-zeroの中性子EDMを導く
- しかし、中性子EDMは実験で厳しく制限

[C. A. Baker et al. '06]

$$|d_n| < 2.9 \times 10^{-26}\, e\,{\rm cm}$$

[P. G. Harris et al. '99]

$$|\theta| \lesssim 10^{-10}$$

$0 \leq \theta \leq 2\pi$のパラメータθの値がなぜそんなに小さい？

PQ機構

[Peccei-Quinn '77]

- Peccei-Quinn(PQ)対称性と呼ばれるanomalous U(1)対称性を導入
- $U(1)_{\rm PQ}$のSSB → NGボソン(axion)

$$\theta_{\rm eff} = \frac{\langle a \rangle}{f_a} + \theta \to 0$$

パラメータθを場に格上げ

- 模型での実現
  - KSVZ model

[Kim '79, Shifman-Vainshtein-Zakhrov, '80]

Extra heavy quark + singlet PQ scalar

  - DFSZ model

[Zhitnitsky, '80, Dine-Fischler-Srednicki, '83]

Two Higgs doublets + singlet PQ scalar

アクシオンストリング

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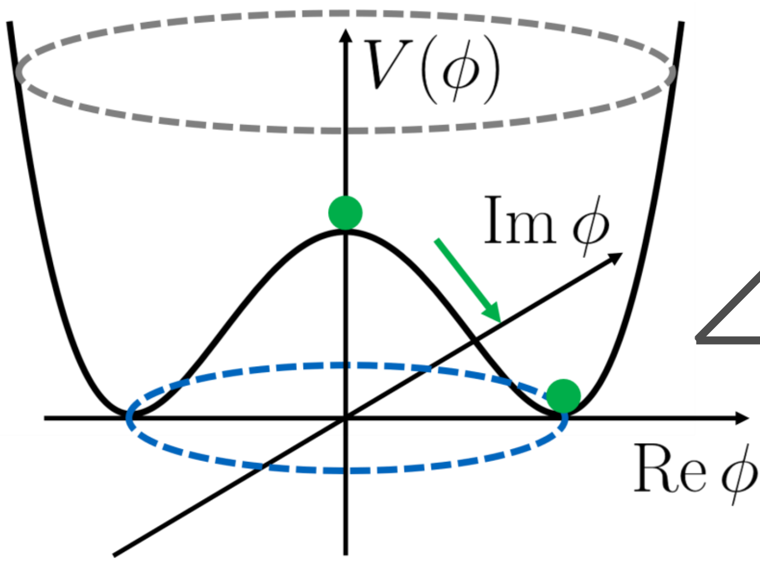
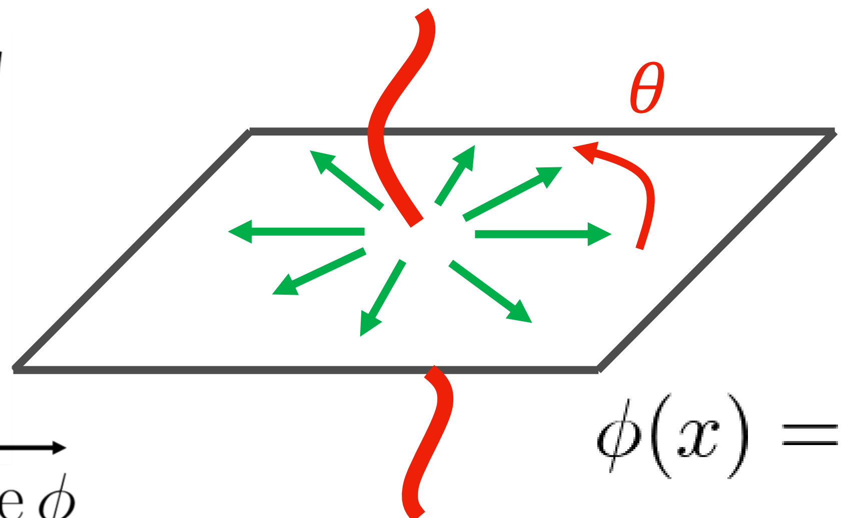
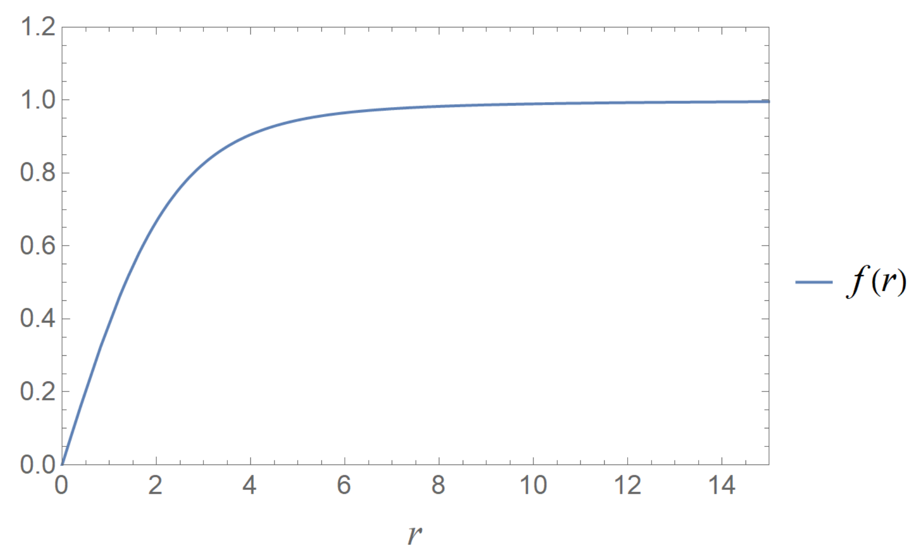
- $U(1)_{\mathrm{PQ}}$の破れに伴うglobal string





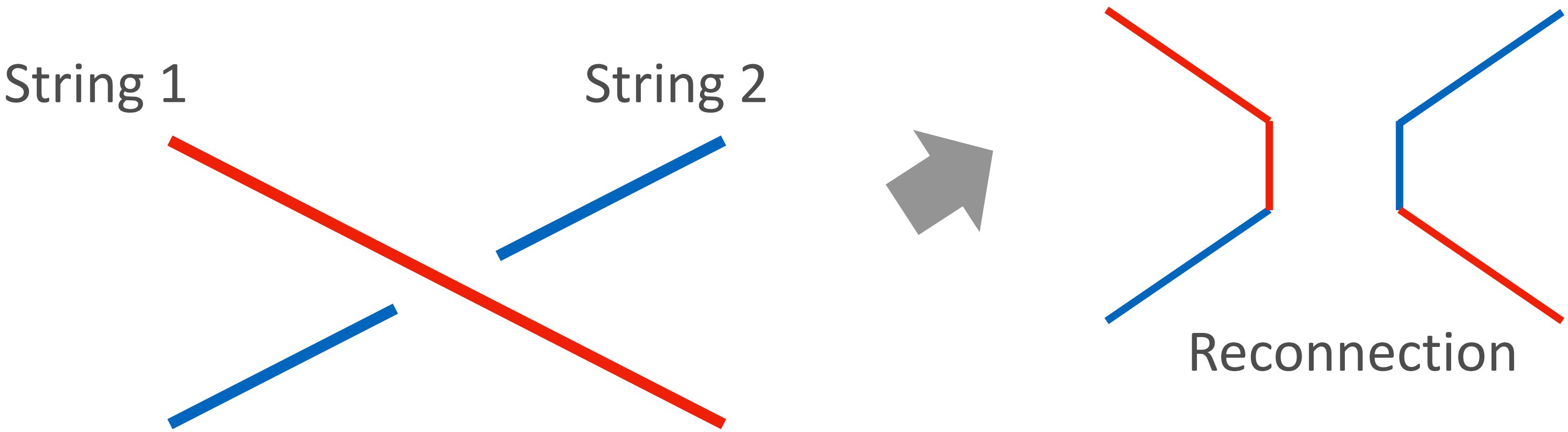
$$\phi(x) = v_\phi f(r) e^{i\theta}$$

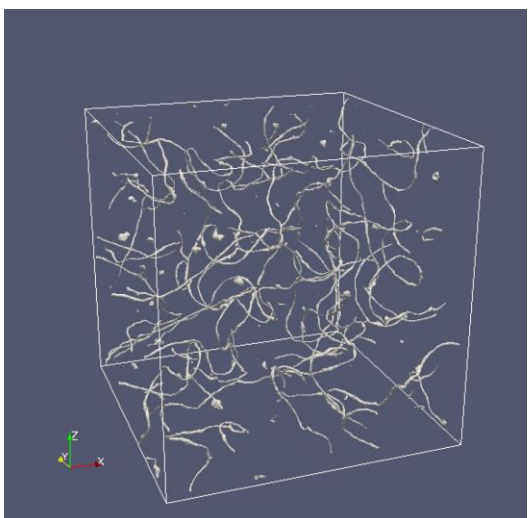
- Field profile



String Reconnection

- String – stringのdynamics





[Hiramatsu-san's talk at COSMO-17]

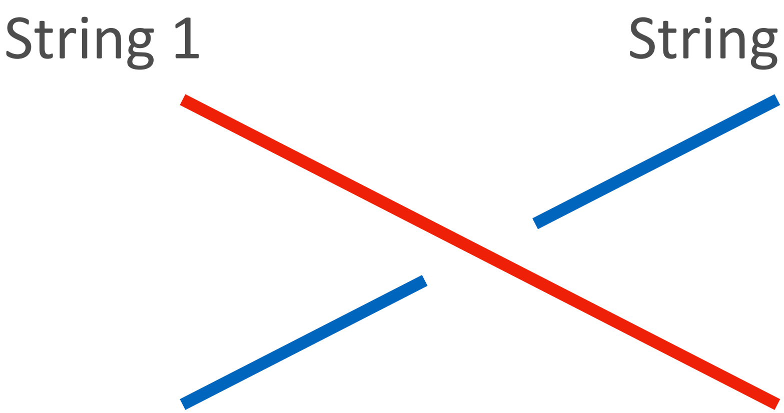
Scaling behavior

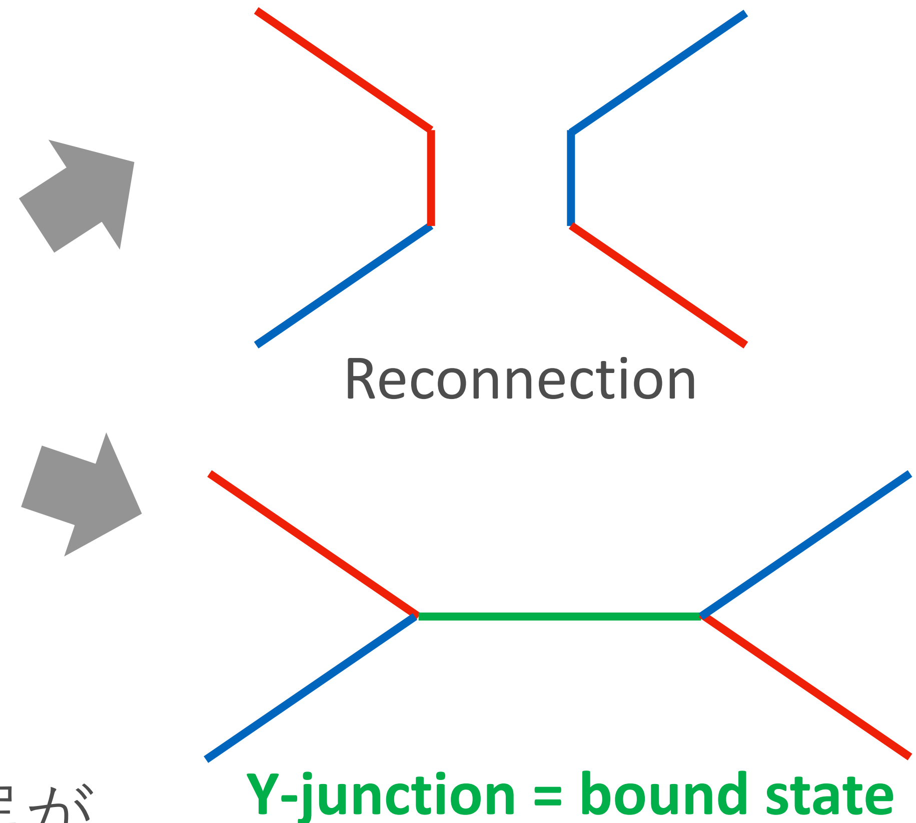
$$\frac{\rho_{\mathrm{string}}}{\rho_\gamma} \sim \mathrm{const.}$$

Y-(Shaped) Junction

[Betterncourt-Kibble ’94, Betterncourt-Laguna-Matzner ’96]

- String – stringのdynamics





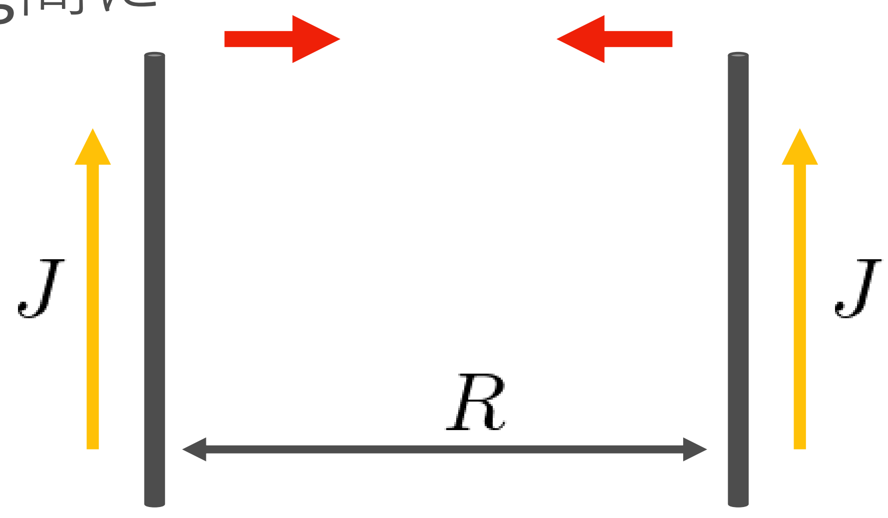
- String間に強い引力
  → bound stateの形成
- Reconnectionが機能しない
  →string networkの時間発展が変わってくる

Superconducting String

[Witten, '80]

- String内部で$U(1)_{\mathrm{EM}}$が破れる
  → superconducting string
- EM currentが散逸なくstring上を流れる
- String上のEM currentがstring間に磁場の引力相互作用を導く

  → Y-junctionの形成



- Cf. アノマリーインフローによるsuperconducting string

[Jakiw-Rossi, '81, Callan-Harvey, '85, Ganoulis-Lazarides, '89, Lazarides-Shafi, '85, Kim, '86, Lazarides et al, '88, Iwasaki, '97, Fukuda et al, '20]

メッセージ

- DFSZ modelにおけるaxion stringは、従来の予想よりも複雑で興味深い構造を持っている。

  → Electroweak axion string

- EW axion stringはsuperconducting stringになりえて、非常に大きなcurrentを運ぶことでY-junctionを形成する可能性がある

  → DFSZ modelではreconnectionが機能しない？

Model

DFSZ Model

[Zhitnitsky '80 ,Dine-Fischler-Srednicki '81]

- Particle contents

Two Higgs doublet model + SM singlet scalar

	H_1	H_2	S
$SU(2)_W$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)_Y$	1	1	0
$U(1)_{\mathrm{PQ}}$	X_1	X_2	X_s

- Scalar potential

$$V(H,S) = V_H + V_S + V_{\mathrm{mix}}$$

$$\begin{aligned} V_H =& m_{11}^2|H_1|^2 + m_{22}^2|H_2|^2 + \frac{\beta_1}{2}|H_1|^4 + \frac{\beta_2}{2}|H_2|^4 \\ &+ \beta_3|H_1|^2|H_2|^2 + \beta_4(H_1^\dagger H_2)(H_2^\dagger H_1) \end{aligned}$$

$$V_S = -m_S^2|S|^2 + \lambda_S|S|^4$$

$$V_{\mathrm{mix}} = \left(\kappa S^2 H_1^\dagger H_2 + \mathrm{h.c.}\right) + \kappa_{1S}|S|^2|H_1|^2 + \kappa_{2S}|S|^2|H_2|^2$$

- Charge relation

$$\boxed{2X_S - X_1 + X_2 = 0}$$

PQ Charge

- 擬スカラー場の質量行列

$$\kappa \begin{pmatrix} -\frac{v_2 v_s^2}{v_1} & v_s^2 & 2v_2 v_s \\ v_s^2 & -\frac{v_1 v_s^2}{v_2} & -2v_1 v_s \\ 2v_2 v_s & -2v_1 v_s & -4v_1 v_2 \end{pmatrix}$$

- $U(1)_{\mathrm{PQ}}$ currentがZ-bosonと結合しないようにPQ変換を次で定義：

$$H_1 \mapsto e^{2i\alpha \sin^2\beta} H_1, \quad H_2 \mapsto e^{-2i\alpha \cos^2\beta} H_2, \quad S \mapsto e^{i\alpha} S$$

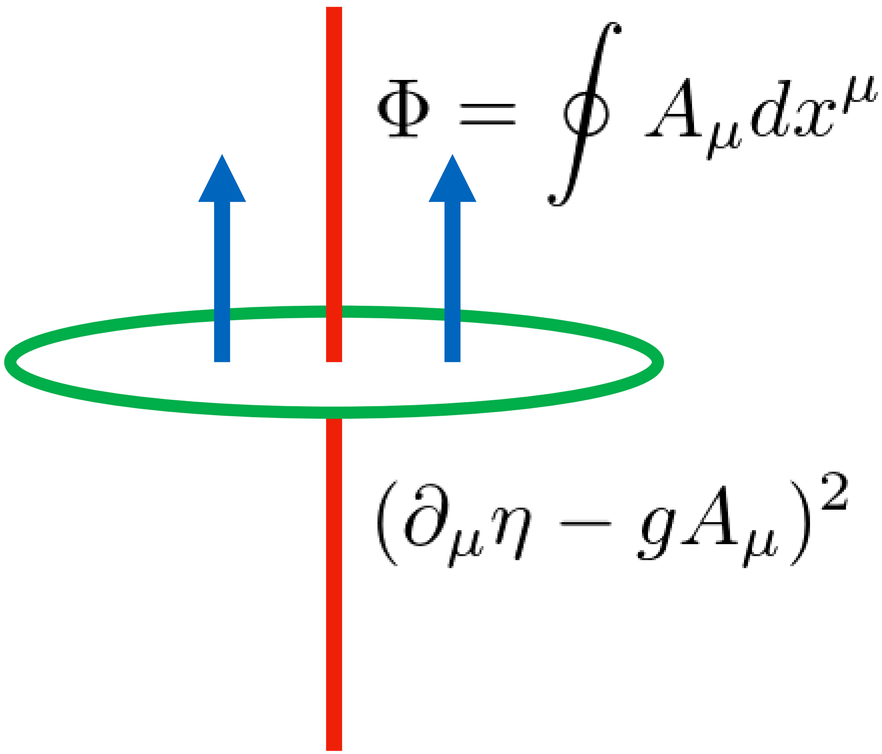
- PQ charge

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

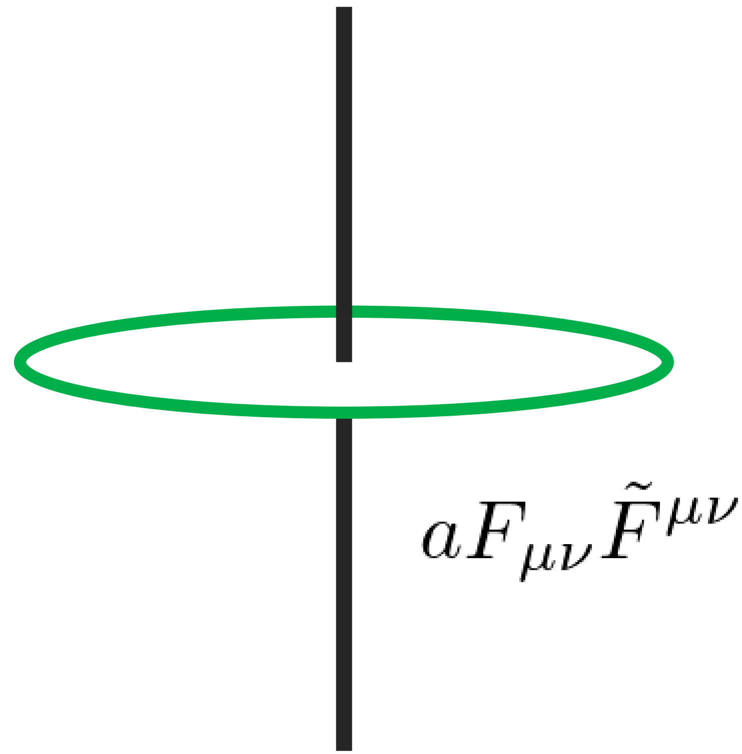
Electroweak Axion String

Strings

- 2種類のストリングの配位が存在



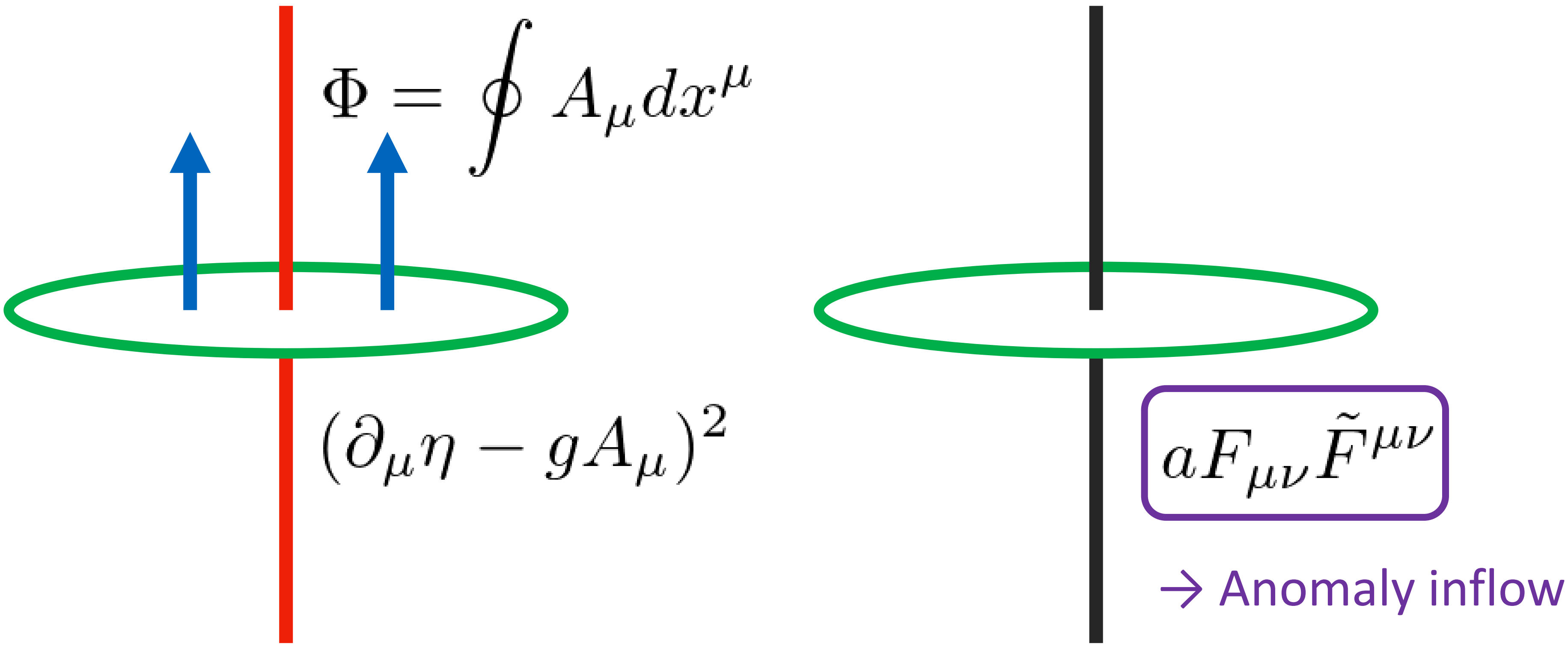
Characterized by NGB form Higgs boson



Characterized by axion (NGB)

Strings

- 2種類のストリングの配位が存在

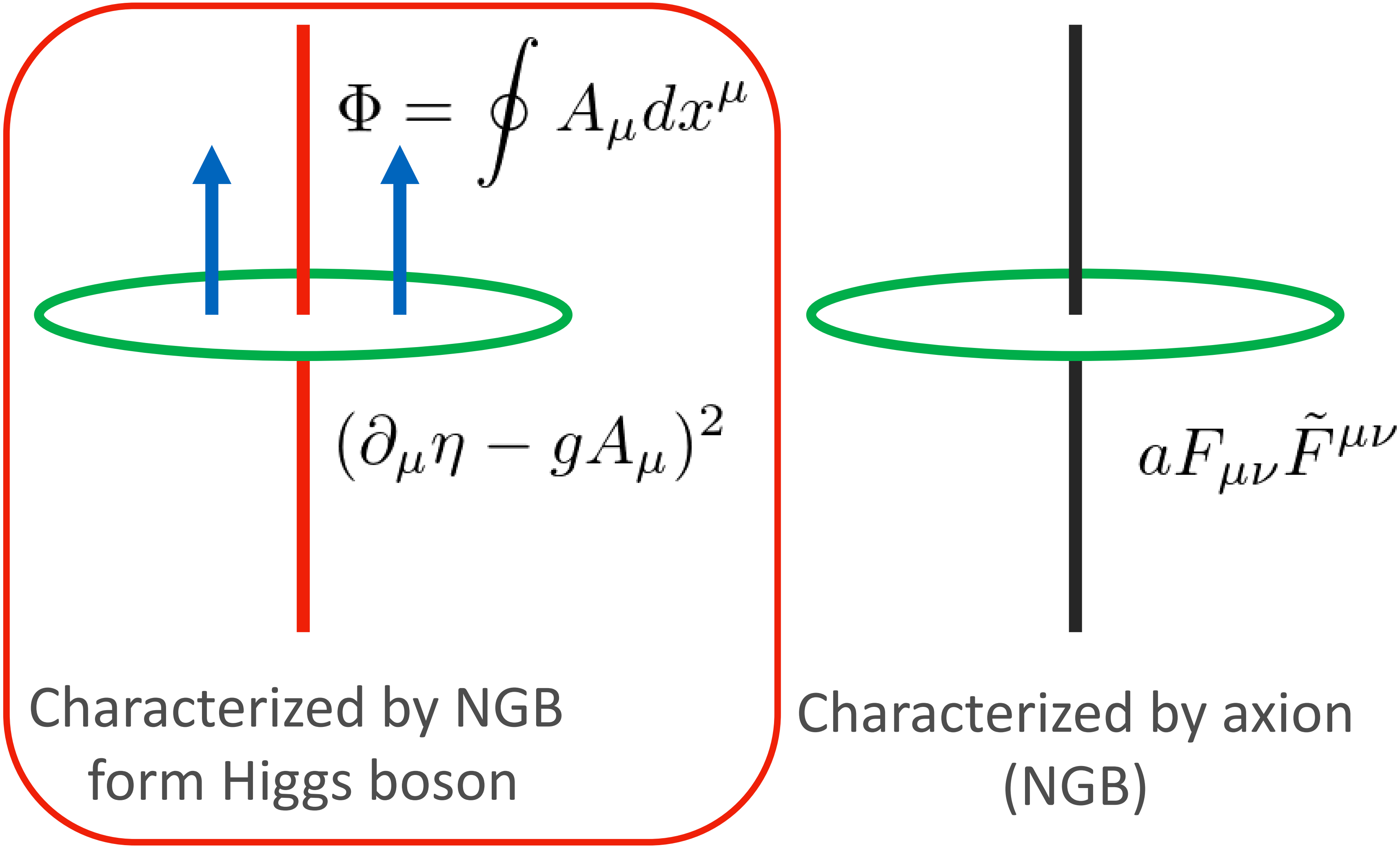


Characterized by NGB form Higgs boson

Characterized by axion (NGB)

Strings

- 2種類のストリングの配位が存在



Electroweak Axion String

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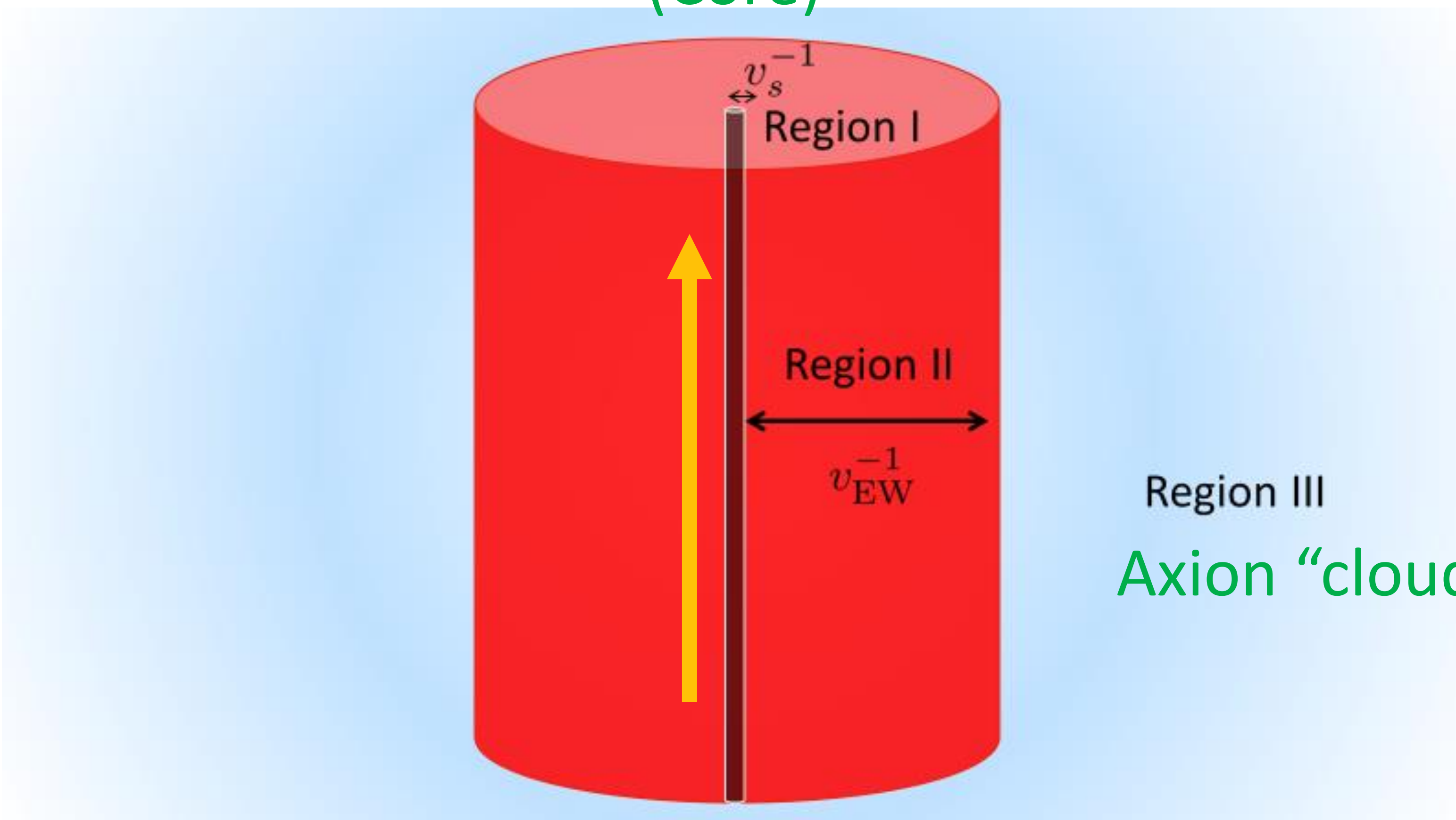
Axion string core
(Core)



Axion “cloud”

Electroweak flux tube

Electroweak Axion String

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Electroweak Axion String ~ Frankfurt

Electroweak flux tube
(Frankfurt)

Axion "cloud"
(Delicious smell)

Axion string core
(Stick)

Axion String in DFSZ Model

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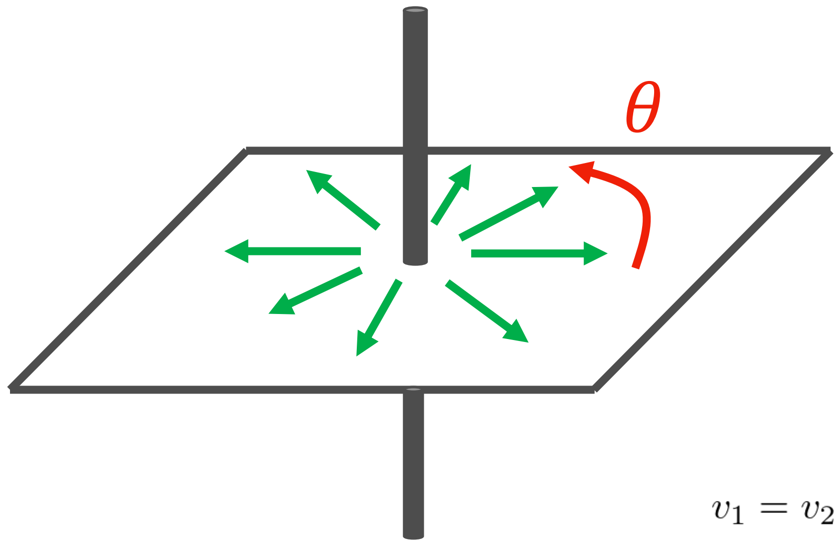
- 電弱対称性はunbroken phaseで、PQ対称性のみがbreakingしている状況を考える

場の配位

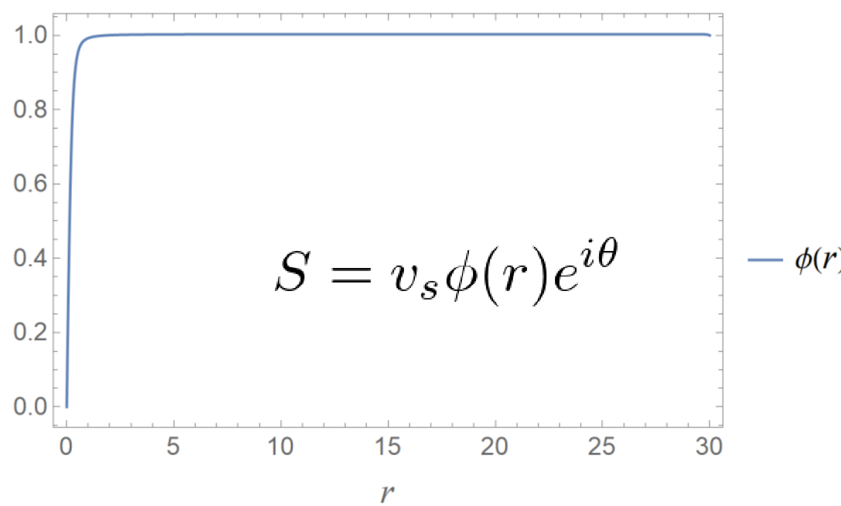
$$S \sim v_s e^{i\theta}$$

$$H_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

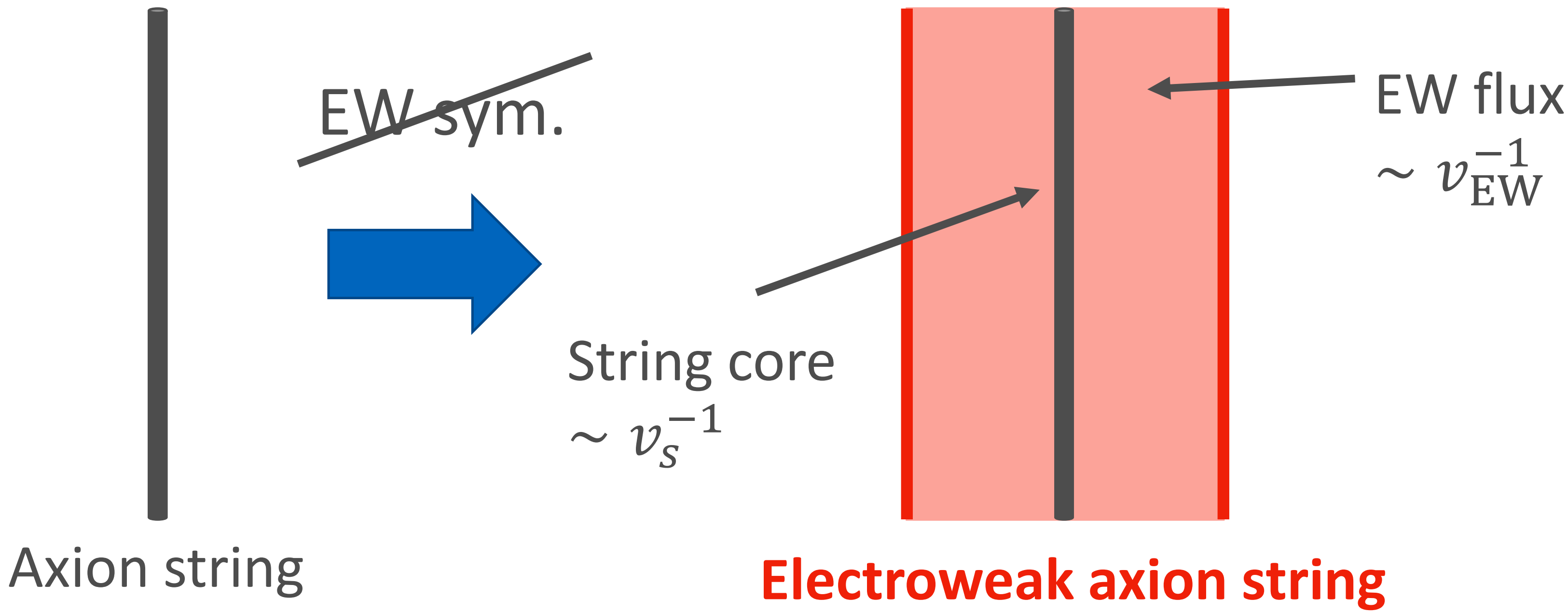


$$v_1 = v_2 = v = \frac{1}{10} v_s$$



Electroweak Axion Strings

- 電弱対称性も破れる
- 2つのHiggs doubletもnon-zero VEVを獲得



- HiggsのWinding patternに応じて、三種類のEW axion stringが出てくる　→ Type-A, B, C

Superconducting string

Electroweak Axion Strings ②

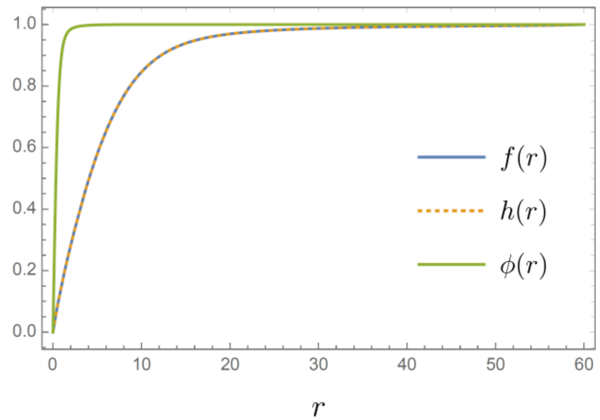
- Assumptions:

$$v_1 = v_2 = v = \frac{1}{10} v_s$$

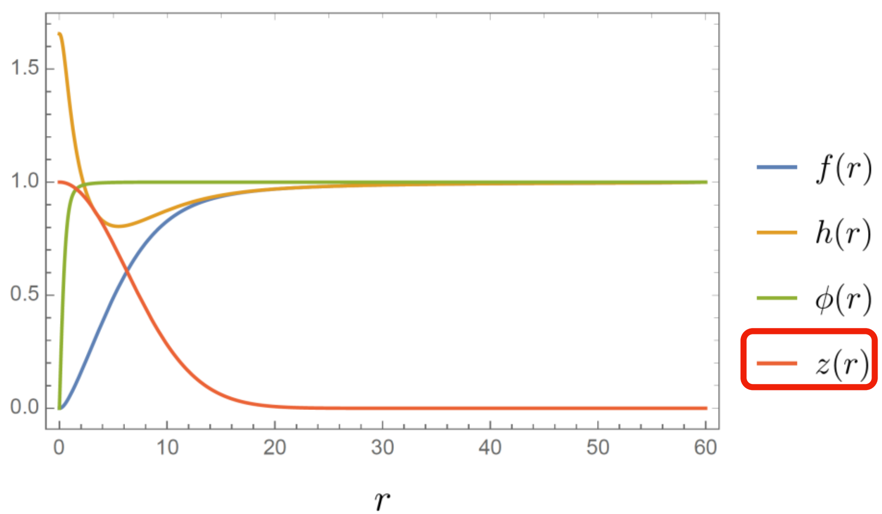
定性的な振る舞いは変わらない

- Field profiles

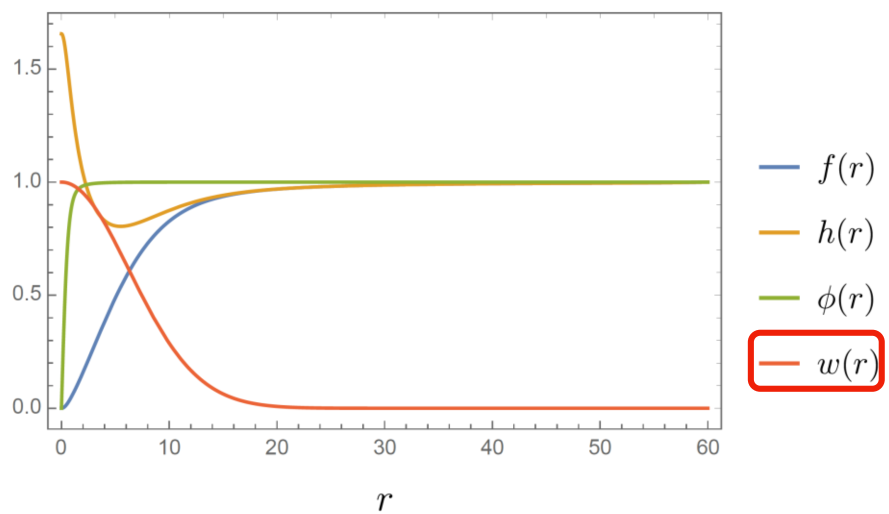
Type-A



Type-B



Type-C



Type-A EW Axion String

- Type-A EW axion stringは次で特徴づけられる

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$V_{\rm mix} \supset \kappa S^2 H_1^\dagger H_2 + \text{h.c.}$$

$$H_1 \sim e^{i\theta}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-i\theta}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

Type-A EW Axion String

- Type-A EW axion stringは次で特徴づけられる

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{i\theta}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} = \underline{e^{2i\theta\sin^2\beta}}\,\underline{e^{-i\theta\sigma_3\cos 2\beta}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-i\theta}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} = \underline{e^{-2i\theta\cos^2\beta}}\,\underline{e^{-i\theta\sigma_3\cos 2\beta}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$U(1)_{\mathrm{PQ}}$ winding　　$U(1)_Z$ winding

NB: $U(1)_Z$ のwindingは一価性のために必要

Type-A EW Axion String

- Non-zero Z-boson fieldが存在する

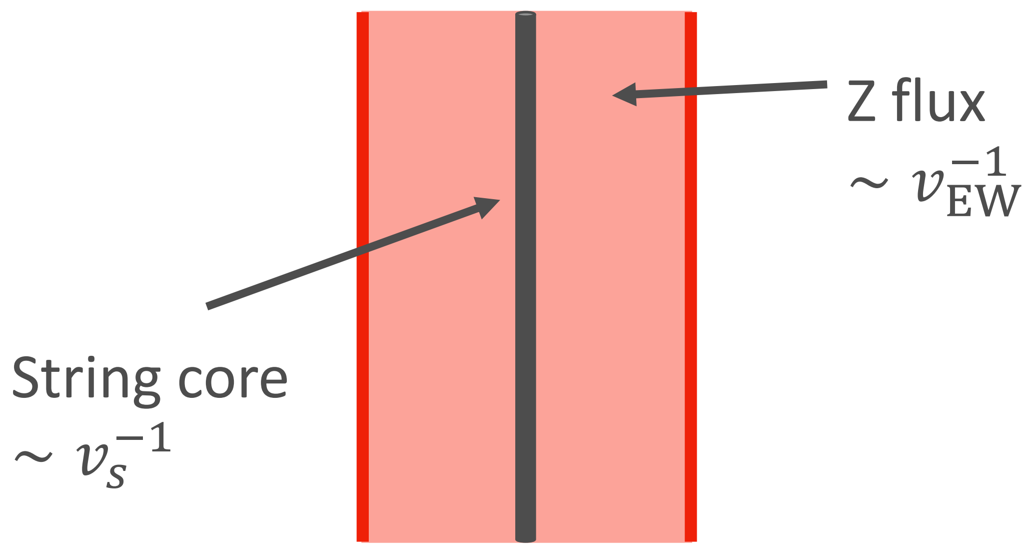
$$g_Z \equiv \sqrt{g^2 + g'^2}$$

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{2i\theta \sin^2 \beta} e^{-i\theta\sigma_3 \cos 2\beta} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim e^{-2i\theta \cos^2 \beta} e^{-i\theta\sigma_3 \cos 2\beta} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$Z_\theta \sim \frac{-2\cos 2\beta}{g_Z}$$



Z-flux

$$\Phi_Z = \int F^Z = \frac{-4\pi \cos\beta}{g_Z}$$

NB: $\tan\beta = 1 \Rightarrow \Phi_Z = 0$

Type-B EW Axion String

- Type-A EW axion stringは次で特徴づけられる

H_1	H_2	S
$2\sin^2\beta$	$-2\cos^2\beta$	1

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim e^{2i\theta}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} = \underline{e^{2i\theta\sin^2\beta}}\,\underline{e^{-2i\theta\sigma_3\cos^2\beta}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

$$H_2 \sim \begin{pmatrix} 0 \\ v_2 \end{pmatrix} = \underline{e^{-2i\theta\cos^2\beta}}\,\underline{e^{-2i\theta\sigma_3\cos^2\beta}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$U(1)_{\mathrm{PQ}}$ winding

$U(1)_Z$ winding

- Z-boson and Z-flux

$$Z_\theta \sim \frac{-4\cos^2\beta}{g_Z}$$

$$\Phi_Z = \frac{-8\pi\cos^2\beta}{g_Z}$$

Type-B EW Axion String

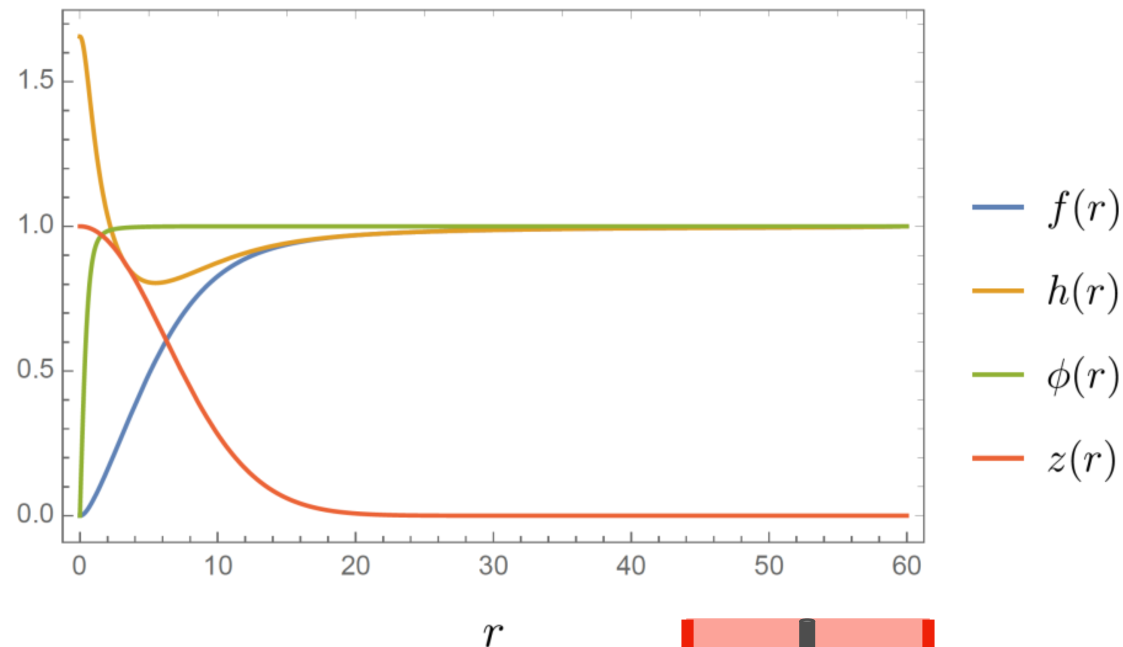
- Type-B EW axion stringのprofile

$$S = v_s e^{i\theta}\phi(r),$$

$$H_1 = v_1 e^{2i\theta}\begin{pmatrix} 0 \\ f(r) \end{pmatrix},$$

$$H_2 = v_2 \begin{pmatrix} 0 \\ h(r) \end{pmatrix},$$

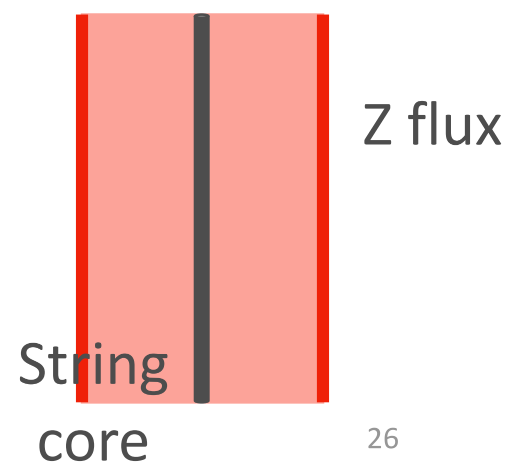
$$Z_\theta = \frac{-4\cos^2\beta}{g_Z}(1 - z(r)).$$



- Boundary condition

$$f(0) = \phi(0) = 0,\ \partial_r h|_{r=0} = 0,\ z(0) = 0$$

$$\phi(\infty) = f(\infty) = h(\infty) = z(\infty) = 1$$



Type-C EW Axion String

- Type-C EW axion stringの配位

$$S \sim v_s e^{i\theta}$$

$$H_1 \sim \frac{v_1}{2}\begin{pmatrix} e^{2i\theta}-1 \\ e^{2i\theta}+1 \end{pmatrix} = \underline{e^{2i\theta \sin^2\beta}}\,\underline{e^{-i\theta\cos 2\beta\sigma_Z}}\,\underline{e^{i\theta\sigma_1}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}$$

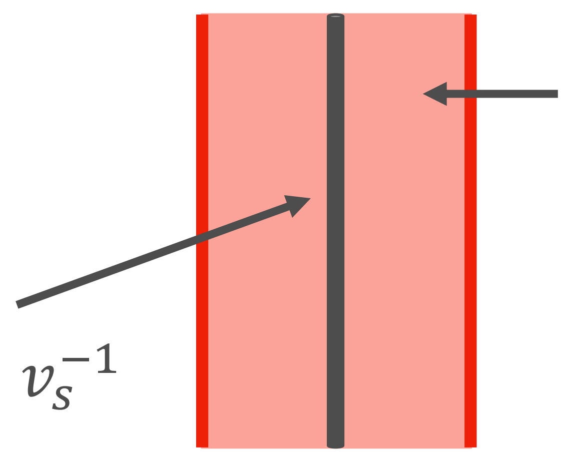
$$H_2 \sim \frac{v_2}{2}\begin{pmatrix} 1-e^{-2i\theta} \\ 1+e^{-2i\theta} \end{pmatrix} = \underline{e^{-2i\theta \cos^2\beta}}\,\underline{e^{-i\theta\cos 2\beta\sigma_Z}}\,\underline{e^{i\theta\sigma_1}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$U(1)_{\mathrm{PQ}}$ winding　$U(1)_Z$ winding　$U(1)_{W^1}$ winding

- Z-flux and W-flux

$$\Phi_Z = \frac{-4\pi\cos 2\beta}{g_Z}$$

$$\Phi_{W^1} = \frac{-4\pi}{g}$$



Z and W flux $\sim v_{\mathrm{EW}}^{-1}$

Core $\sim v_s^{-1}$

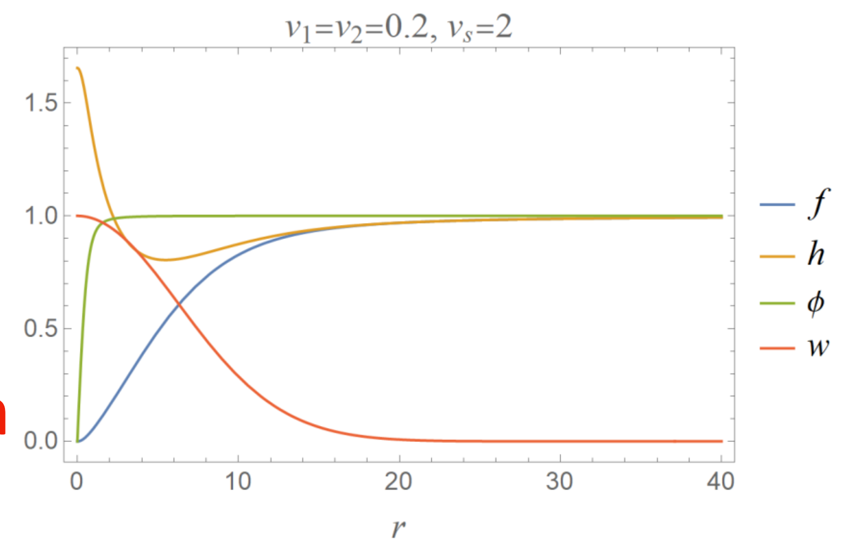
Type-C EW Axion String

- String内部では、場の配位は“smeared ansatz”で記述される:

$$H_1 = \frac{1}{2} v_1 e^{i\theta} \begin{pmatrix} f(r) e^{2i\theta} - h(r) \\ f(r) e^{2i\theta} + h(r) \end{pmatrix}, \quad H_2 = \frac{1}{2} v_2 \begin{pmatrix} h(r) - f(r) e^{-2i\theta} \\ h(r) + f(r) e^{-2i\theta} \end{pmatrix}$$

- EW sym.

$$\hat{Q}_{\rm EM} H_i \propto f - h \begin{cases} \to 0 & (r \to \infty) \\ \neq 0 & (r \lesssim v_{\rm EW}^{-1}) \end{cases}$$



- $U(1)_{EM}$→spontaneously broken

  → **Superconducting**

- Current carrier: Higgsのcharged componentsとW-boson

String Tension

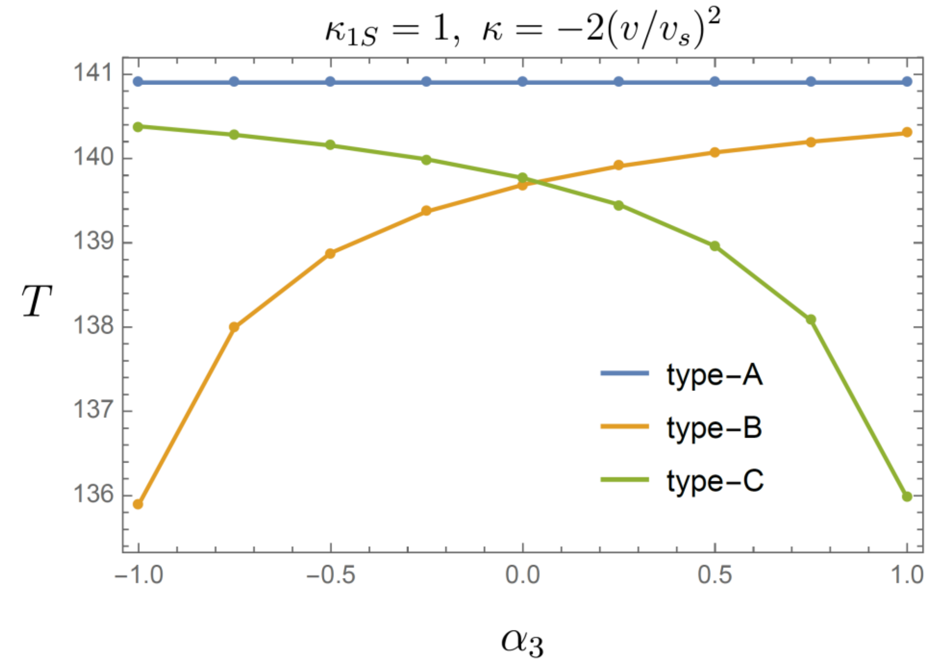
- 3種類のEW axion stringsのtensionを比較

$$\alpha_3 = \frac{1}{8}(\beta_1 + \beta_2 - 2\beta_3)$$

$$v_s = 10 v_1,\ m_h^2 = (125\ \mathrm{GeV})^2,\ \tan\beta = 1,$$

$$\kappa_{1S} = \kappa_{2S},\ \kappa = -2(v/v_s)^2,\ \lambda_S = 1$$

Length unit: $v_1 = 0.2$



- Type-C stringが最も安定になりうる

  → パラメータ空間によってはfavored

- EW phase transitionの後、axion stringがtype-C stringになる

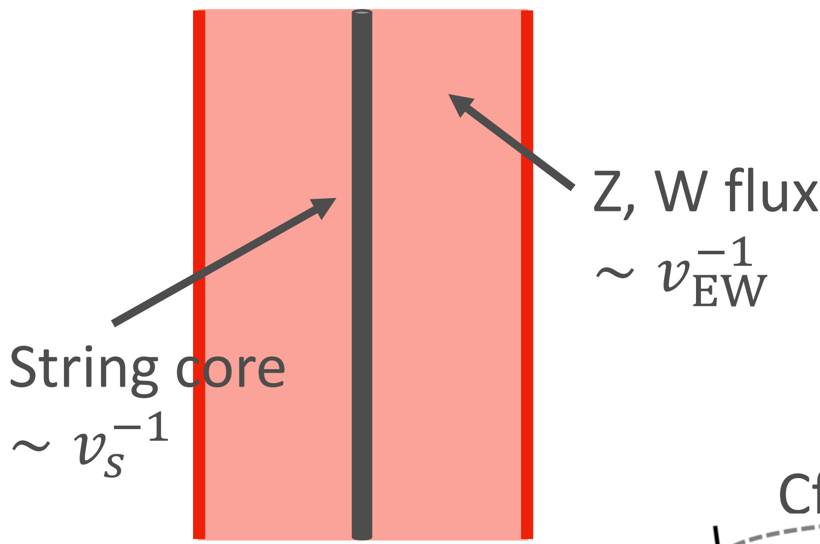
③

Superconductivity

Type-C Stringに流れるSupercurrent

- EW axion stringに沿って流れるcurrentをどのように評価するか？

[Alford-Benson-Coleman-March-Russell-Wilczek '91]



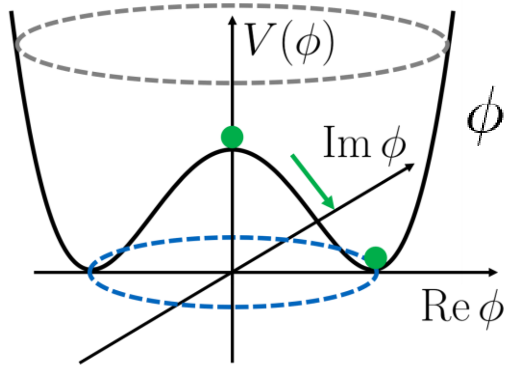
Type-C
electroweak axion string

Background

$$\tilde{S}(r,\theta), \widetilde{H}_{1,2}, (r,\theta), \widetilde{W}_\mu(r,\theta)$$

$U(1)_{\mathrm{EM}}$がaxion string中で破れる
→ zero modeの存在

Cf.



$$\phi = (v_\phi + \rho)\exp(i\pi/v_\phi)$$

Type-C StringのSupercurrent

[YA-Hamada-Yoshioka ’20]

- (z,t) dependent zero mode $\eta(z,t)$を考える
  $\tilde{S}, \widetilde{H}_i, \widetilde{W}_\mu, \tilde{Y}_\mu$ はtype-C backgroundsを表す

$$S = \tilde{S}$$

$$H_i = \exp\left[i\hat{Q}_{\rm EM}\eta(z,t)\xi(r)\right]\tilde{H}_i$$

$$W_\mu = \tilde{W}_\mu - \frac{\eta(z,t)}{g}D_\mu(\xi(r)n)$$

$$Y_\mu = \tilde{Y}_\mu + \frac{\eta(z,t)}{g'}\partial_\mu\xi(r)$$

$$n^a \equiv \frac{H^\dagger\sigma_a H_1}{H_1^\dagger H_1} = \frac{H_2^\dagger\sigma_a H_2}{H_2^\dagger H_2}$$

- EOMs

$$\left(D_\nu W^{\nu\mu}\right)^a = -j_W^{\mu,a}, \quad \partial_\nu Y^{\nu\mu} = -j_Y^\mu, \quad D_\mu D^\mu H_i = -\frac{\delta V}{\delta H_i^\dagger}$$

Type-C Stringに流れるSupercurrent

- Linearized EOM at large r

$$(\partial_z^2 - \partial_t^2)\eta(z,t) = 0, \quad \xi(r) \sim \log r$$

$\eta(z,t)$ is massless excitation along the string

- Static solution → $\eta(z,t) = \omega z$ with a constant ω

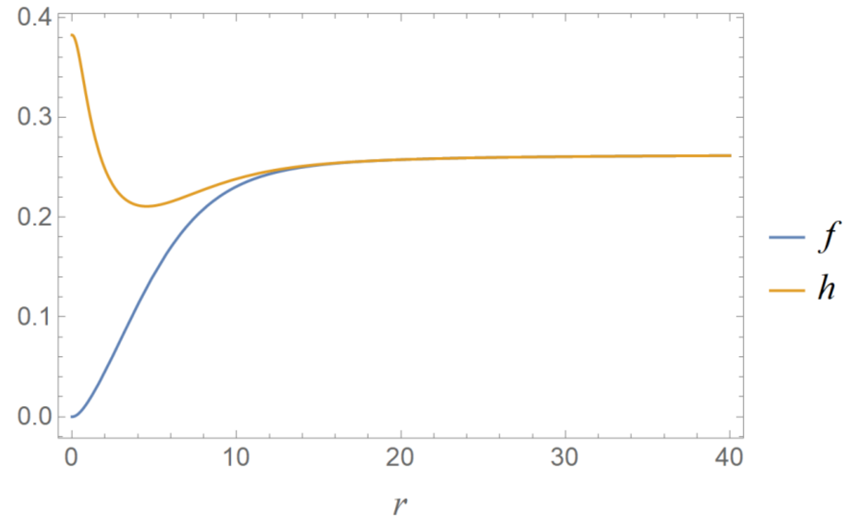
- EM field strength and electric current

$$F_{rz}^{\mathrm{EM}} \sim -\frac{\omega}{er}, \quad J_{\mathrm{EM}} \equiv -2\pi r F_{rz}^{\mathrm{EM}} \sim \frac{2\pi\omega}{e}$$

Type-C StringにおけるCurrentの最大値

- Zero mode η はHiggs場に "mass" term を供給:

$$\mathcal{L} \supset -(\omega\xi)^2 \frac{v^2}{2}(f-h)^2$$



- $U(1)_{\mathrm{EM}}$が回復して superconducting stateが壊れる　(current quenching)
- Currentの最大値は、この寄与がHiggs potentialの negative mass termsとバランスするところ

$$m_1^2 \sim C_1 v^2 + C_2 v_s^2$$

- 非常に大きなcurrentが流れうる

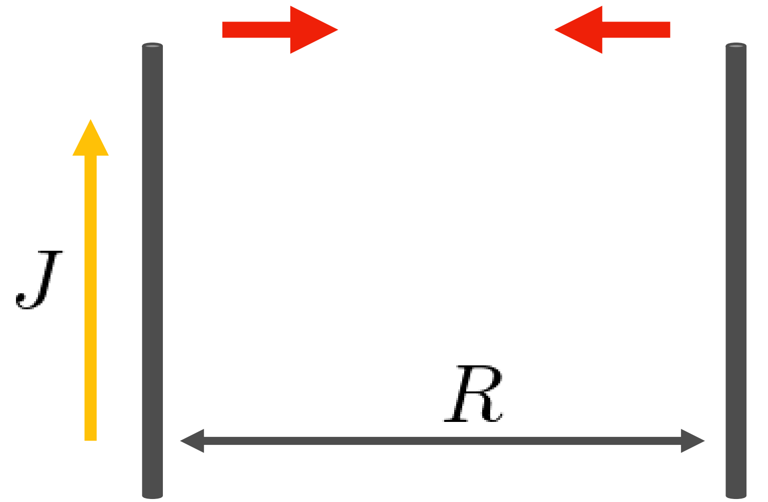
$$J_{\max} \sim v_s$$

String間の相互作用

- String間にかかる、currentが誘導する力：

引力

$$F_{\rm mag} \sim \frac{v_s^2}{e^2 R^2}$$



- 一方、axionは斥力をstring間にもたらす

$$F_{\rm axion} \sim \frac{v_s^2}{R^2}$$

- Stringの間にnetな引力が生じる可能性

  → **Y-junction?**

まとめ

- EW phase transitionのあと、DFSZ axion stringはelectroweak gauge fluxをまとった配位になる

  → Electroweak axion string ~ フランクフルト

- Type-C EW axion stringは$U(1)_{\mathrm{EM}}$を破ることで、bosonic carrierを伴うsuperconducting stringになりうる

- String上に$J_{\mathrm{max}} \sim v_s$程度のcurrentを流す可能性がある

- Currentが誘導する引力によって、DFSZ modelにおけるaxion stringはY-junctionを形成する可能性がある

Backup

Superconducting String

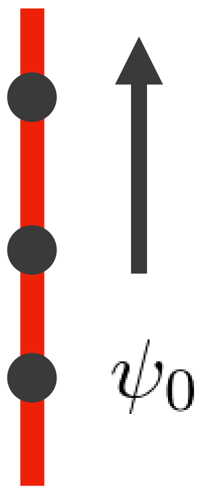
- Axion stringは内部にfermion zero modesを持っている

[Lazarides-Shafi '85, Lazarides-Panagiotakopoulos-Sfafi '88, Ganolis-Lazarides '89, Iwazaki '97]

$$\mathcal{L} \supset -y\phi\overline{\Psi_0}\Psi_0, \quad \phi(x) \sim v_\phi e^{i\theta}$$

$$\mathrm{Ind}(i\not{D} + y\phi(x)) = 1$$

[Witten '80
Jackiw-Rossi '81,
Callan-Harvey '85]



- zero modeがEM currentを運ぶ

→ superconducting string

[Witten '85]

$$\partial_\mu J^\mu = \frac{e^2}{16\pi} E_z$$

- currentの最大値

$$J_{\max} \sim M_{\mathrm{fermion}}$$

Phenomenology, cosmology
see
Fukuda-Manohar-Murayama-Telem
arXiv:2010.02763 [hep-ph]

Higgs Bi-linear Formalism

[Grzadkowki-Maniastis-Wudka, '11]

- Higgs matrixを導入:

$$H = (i\sigma_2 H_1^*, H_2)$$

- Covariant derivative

$$D_\mu H = \partial_\mu H - i\frac{g}{2}\sigma_a W_\mu^a H + i\frac{g'}{2} H \sigma_3 Y_\mu$$

- Higgs potential

$$V_H = -m_1^2 \mathrm{tr}|H|^2 - m_2^2 \mathrm{tr}(|H|^2\sigma_3) + \alpha_1 \mathrm{tr}|H|^4 + \alpha_2(\mathrm{tr}|H|^2)^2$$
$$+ \alpha_3 \mathrm{tr}(|H|^2\sigma_3|H|^2\sigma_3) + \alpha_4 \mathrm{tr}(|H|^2\sigma_3|H|^2)$$

Higgs Bi-linear Formalism

[Grzadkowki-Maniastis-Wudka, '11]

- Scalar potential

$$V_H = - m_1^2 \mathrm{tr}|H|^2 - m_2^2 \mathrm{tr}(|H|^2\sigma_3) + \alpha_1 \mathrm{tr}|H|^4 + \alpha_2(\mathrm{tr}|H|^2)^2 + \alpha_3 \mathrm{tr}(|H|^2\sigma_3|H|^2\sigma_3) + \alpha_4 \mathrm{tr}(|H|^2\sigma_3|H|^2),$$

$$V_{\mathrm{mix}} = (\kappa S^2 \det H + \mathrm{h.c.}) + \frac{1}{2}(\kappa_{1S} + \kappa_{2S})|S|^2 \mathrm{tr}|H|^2 + \frac{1}{2}(\kappa_{1S} - \kappa_{2S})|S|^2 \mathrm{tr}(|H|^2\sigma_3)$$

- Parameter relations

$$m_{11}^2 = m_1^2 + m_2^2, \quad m_{22}^2 = m_1^2 - m_2^2,$$

$$\beta_1 = 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4), \quad \beta_2 = 2(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4),$$

$$\beta_3 = 2(\alpha_1 + \alpha_2 - \alpha_3), \quad \beta_3 = 2(\alpha_3 - \alpha_1)$$

$U(1)_{\mathrm{EM}}$群

[Eto-Hamada-Nitta, '20]

- Unbroken $U(1)_{\mathrm{EM}}$ を次で定義する

$$\hat{Q}H \equiv -n^a \frac{\sigma_a}{2} H - H \frac{\sigma_3}{2}$$

where

$$n^a \equiv \frac{\sum_{i=1,2} |H_i|^2 n_i^a}{C} \qquad n_1^a \equiv \frac{H_1^\dagger{}_a H_1}{|H_1|^2}, \; n_2^a \equiv \frac{H_2^\dagger \sigma_a H_2}{|H_2|^2}$$

Positive normalization factor C defined to satisfy $n^a n^a = 1$

- $SU(2)_W \times U(1)_Y$ のsubgroupの$U(1)_Z$は次で定義

$$\hat{T}_Z H \equiv -{}^a \frac{\sigma_a}{2} H - \sin^2\theta_W \hat{Q} H$$

- NB: H はHiggs matrix.

$U(1)_{\mathrm{EM}}$群

[Eto-Hamada-Nitta, ’20]

- Z-bosonとphoton

$$\begin{cases} Z_\mu \equiv -n^a W_\mu^a \cos\theta_W - Y_\mu \sin\theta_W \\ A_\mu \equiv -n^a W_\mu^a \sin\theta_W + Y_\mu \cos\theta_W \end{cases}$$

- $\hat{Q}$ のHiggs doubletに対する作用

$$\hat{Q} H_i = \left(-n^a \frac{\sigma_a}{2} + \frac{1}{2}\mathbf{1}\right) H_i,$$

$$\hat{T}_Z H_i = \left(-n^a \frac{\sigma_a}{2} - \sin^2\theta_W \hat{Q}\right) H_i$$

EOMs of Type-A String

$$f''(r) + \frac{f'(r)}{r} - \frac{f(r)}{r^2}$$
$$- \left(2\alpha_{123}\, v^2 f(r)^2 + 2\alpha_2 v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) f(r) - \kappa v_s^2 h(r)\phi(r)^2 = 0,$$

$$h''(r) + \frac{h'(r)}{r} - \frac{h(r)}{r^2}$$
$$- \left(2\alpha_{123}\, v^2 h(r)^2 + 2\alpha_2 v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) h(r) - \kappa v_s^2 f(r)\phi(r)^2 = 0,$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2}$$
$$- \left(2\lambda_S v_s^2 \phi(r)^2 + 2\kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) - m_S^2\right)\phi(r) = 0.$$

- Boundary conditions

$$f(0) = h(0) = \phi(0) = 0 \qquad f(\infty) = h(\infty) = \phi(\infty) = 1$$

EOMs of Type-B String

$$f''(r) + \frac{f'(r)}{r} - \frac{(1+z(r))^2}{r^2} f(r)$$
$$- \left(2\alpha_{123}\, v^2 f(r)^2 + 2\alpha_2 v^2 h(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) f(r) - \kappa v_s^2 h(r)\phi(r)^2 = 0,$$

$$h''(r) + \frac{h'(r)}{r} - \frac{(-1+z(r))^2}{r^2} h(r)$$
$$- \left(2\alpha_{123}\, v^2 h(r)^2 + 2\alpha_2 v^2 f(r)^2 + \kappa_{1S} v_s^2 \phi(r)^2 - m_1^2\right) h(r) - \kappa v_s^2 f(r)\phi(r)^2 = 0,$$

$$\phi''(r) + \frac{\phi'(r)}{r} - \frac{\phi(r)}{r^2}$$
$$- \left(2\lambda_S v_s^2 \phi(r)^2 + \kappa_{1S} v^2 (f(r)^2 + h(r)^2) + 2\kappa v^2 f(r) h(r) - m_S^2\right) \phi(r) = 0,$$

$$z''(r) - \frac{z'(r)}{r} - \frac{g_Z^2 v^2}{2} f(r)^2 (1 + z(r)) - \frac{g_Z^2 v^2}{2} h(r)^2 (-1 + z(r)) = 0.$$

- Boundary conditions

$$f(0) = \phi(0) = 0,\ \ \partial_r h|_{r=0} = 0,\ \ z(0) = 0$$

$$f(\infty) = h(\infty) = \phi(\infty) = z(\infty) = 1$$

EOMs of Type-C String

$$f''(r)+\frac{f'(r)}{r}-\frac{(1+w(r))^2}{r^2}f(r)$$
$$-\Big(2(\alpha_1+\alpha_2)v^2f(r)^2+2(\alpha_2+\alpha_3)v^2h(r)^2+\kappa_{1S}v_s^2\phi(r)^2-m_1^2\Big)f(r)-\kappa v_s^2h(r)\phi(r)^2=0,$$
$$h''(r)+\frac{h'(r)}{r}-\frac{(-1+w(r))^2}{r^2}h(r)$$
$$-\Big(2(\alpha_1+\alpha_2)v^2h(r)^2+2(\alpha_2+\alpha_3)v^2f(r)^2+\kappa_{1S}v_s^2\phi(r)^2-m_1^2\Big)h(r)-\kappa v_s^2f(r)\phi(r)^2=0,$$
$$\phi''(r)+\frac{\phi'(r)}{r}-\frac{\phi(r)}{r^2}$$
$$-\Big(2\lambda_Sv_s^2\phi(r)^2+\kappa_{1S}v^2(f(r)^2+h(r)^2)+2\kappa v^2f(r)h(r)\Big)\phi(r)=0,$$
$$w''(r)-\frac{w'(r)}{r}-\frac{g^2v^2}{2}f(r)^2(1+w(r))-\frac{g^2v^2}{2}h(r)^2(-1+w(r))=0.$$

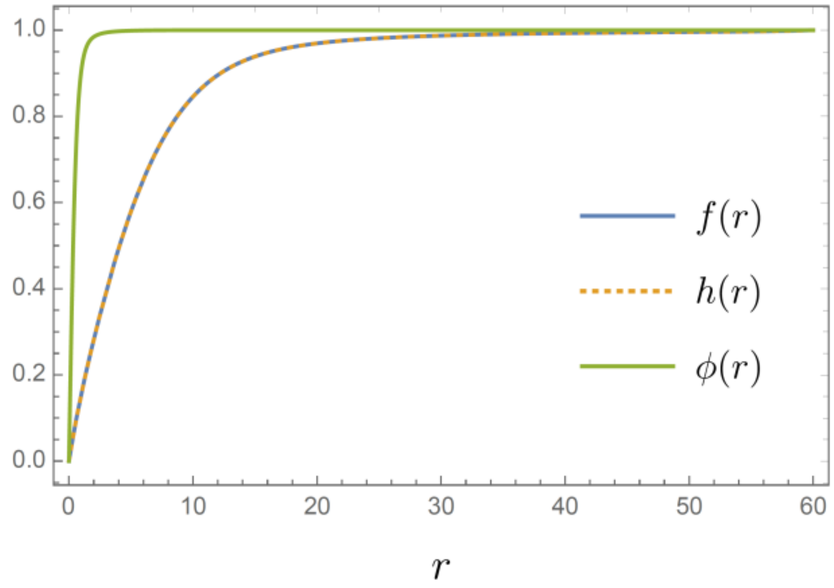
- Boundary conditions

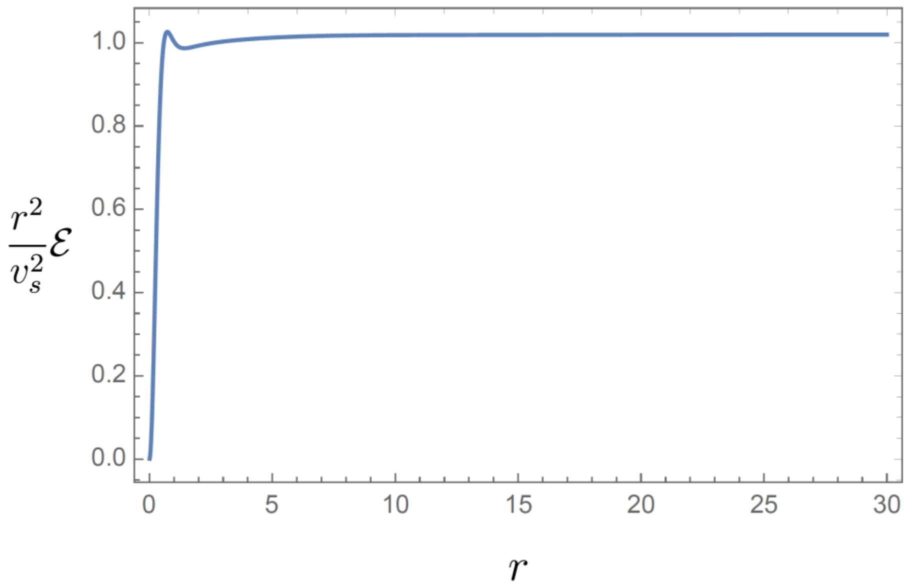
$$f(0)=\phi(0)=0,\ \ \partial_rh|_{r=0}=0,\ \ w(0)=z(0)=0$$

$$f(\infty)=h(\infty)=\phi(\infty)=w(\infty)=z(\infty)=1$$

Profile of Strings

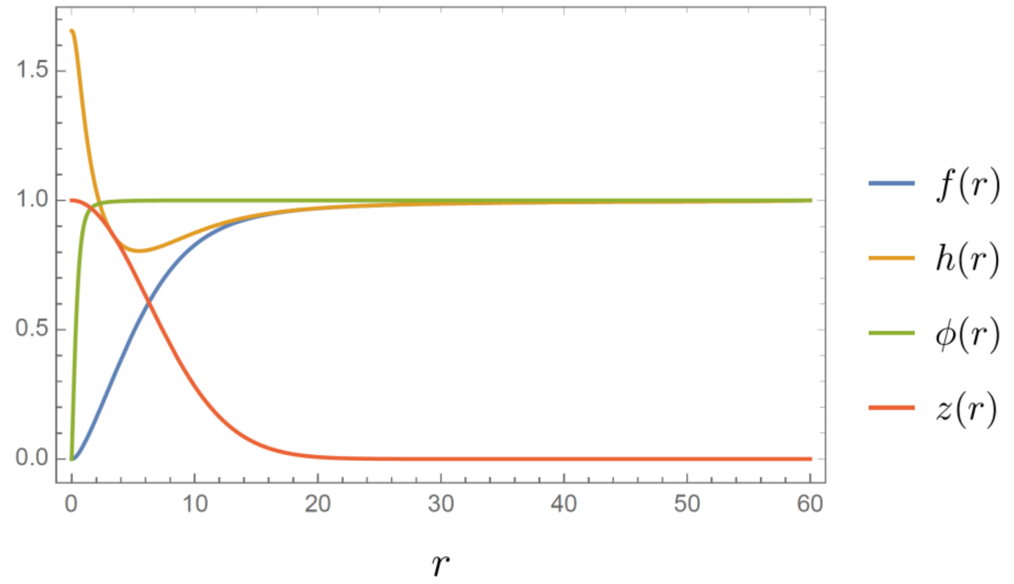
- Type-A

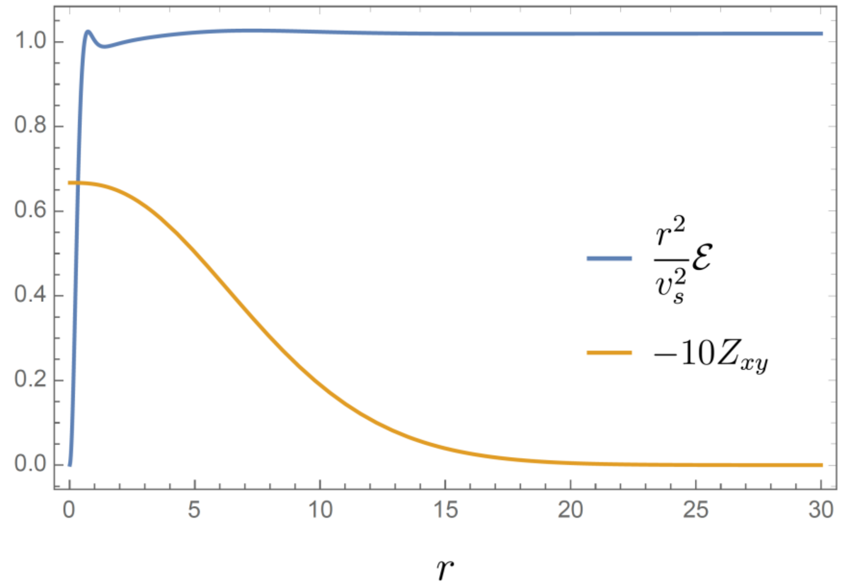




Profile of Strings

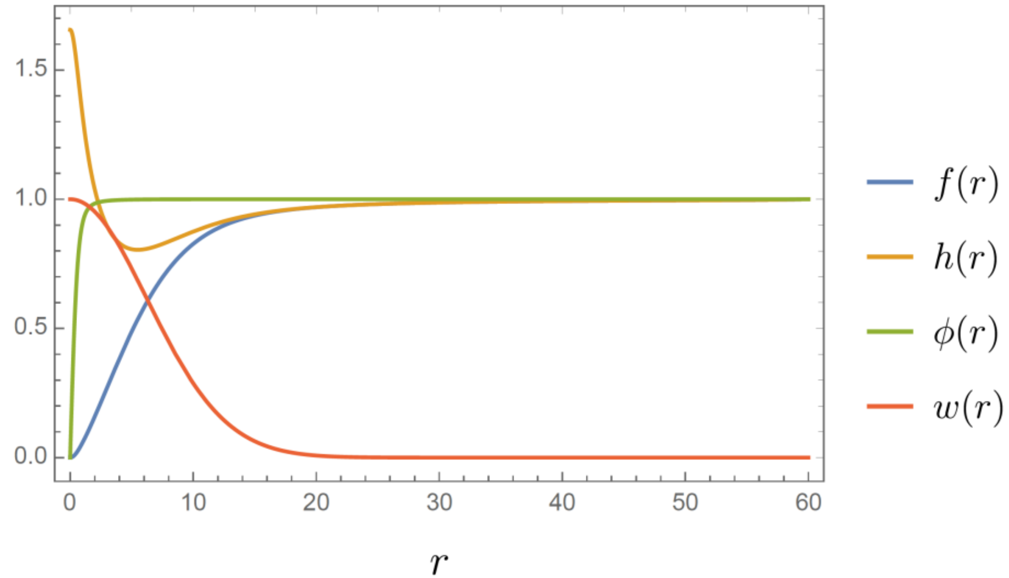
- Type-B

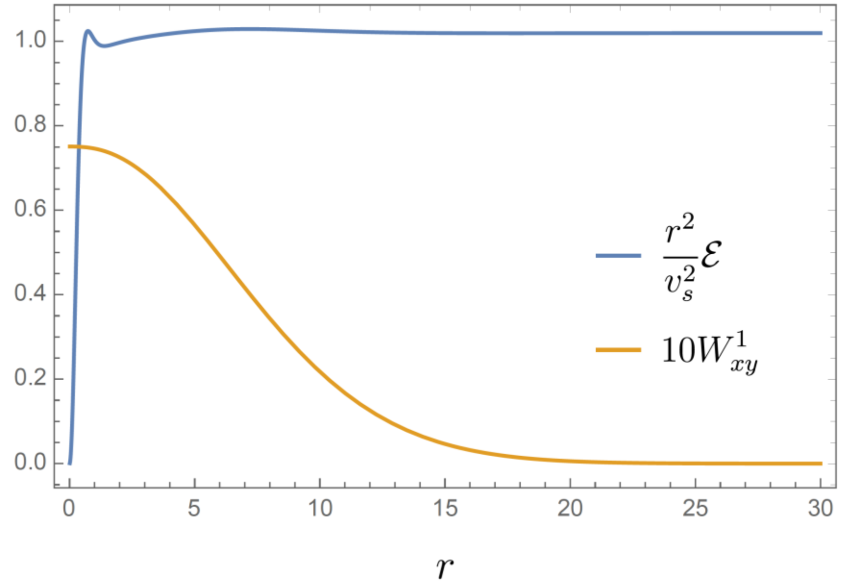




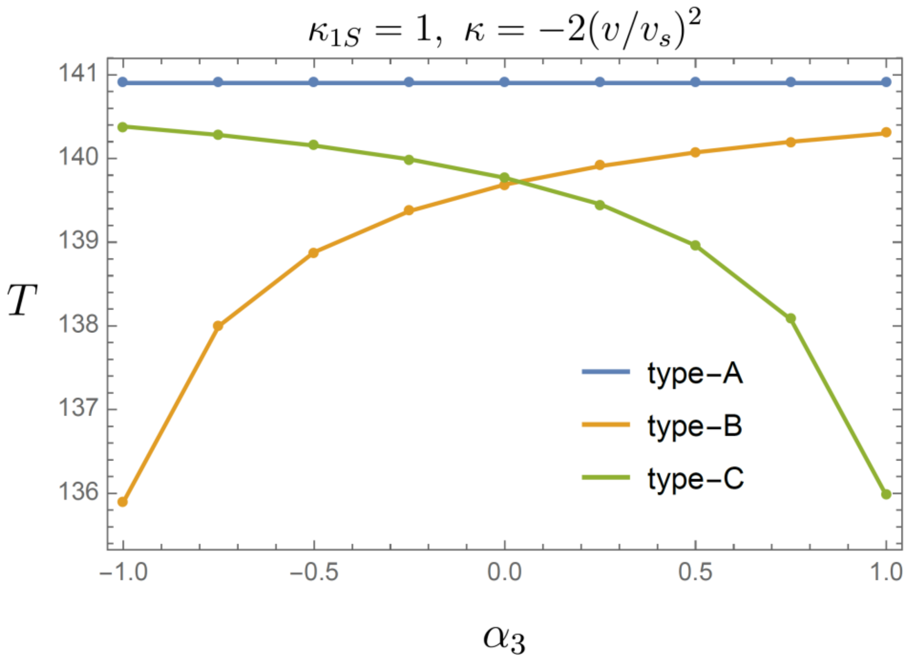
Profile of Strings

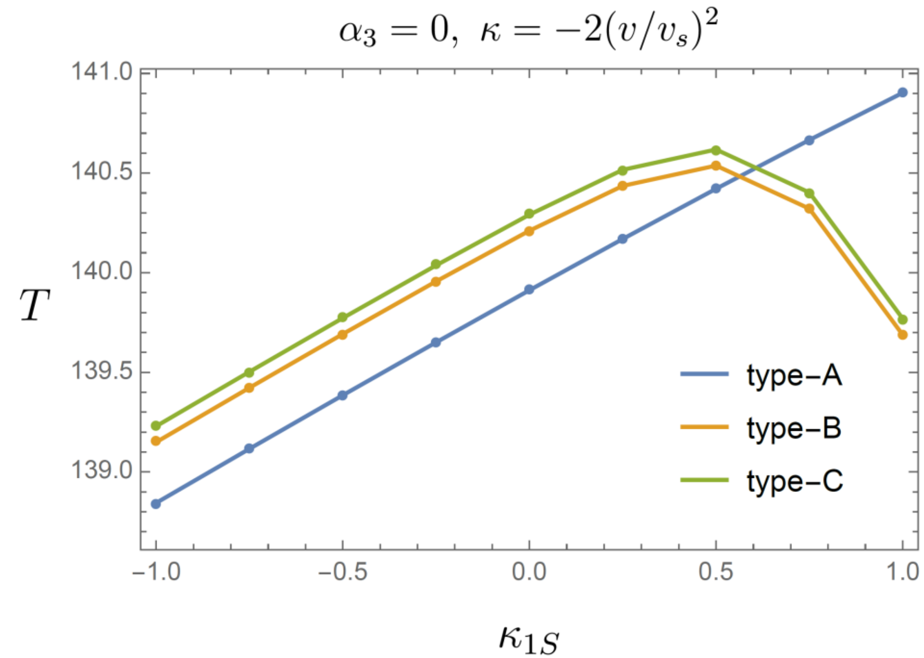
- Type-C





String Tension





Parameters

$$v_s = 10v_1,\ m_h^2 = (125\ \mathrm{GeV})^2,\ \tan\beta = 1, \qquad \alpha_3 = \frac{1}{8}(\beta_1 + \beta_2 - 2\beta_3)$$

$$\kappa_{1S} = \kappa_{2S},\ \kappa = -2(v/v_s)^2,\ \lambda_S = 1$$

Length unit: $v_1 = 0.2$

Supercurrent of Type-C String

- EOMs

$$\partial^\alpha\eta\left(\tilde{D}_j\tilde{D}^j\chi\right)^a = \frac{-g^2}{2}\partial^\alpha\eta\left(\chi^a\mathrm{Tr}|\tilde{H}|^2 + \xi\mathrm{Tr}\left[\tilde{H}^\dagger\sigma_a\tilde{H}\sigma_3\right]\right),$$

$$\partial^\alpha\eta\partial_j\partial^j\xi = \frac{-g'^2}{2}\partial^\alpha\eta\left(\xi\mathrm{Tr}|\tilde{H}|^2 + 2\mathrm{Tr}\left[\tilde{H}^\dagger\chi\tilde{H}\sigma_3\right]\right),$$

$$\tilde{D}^j\chi\ \partial^\alpha\partial_\alpha\eta = 0,$$

$$\partial^j\xi\ \partial^\alpha\partial_\alpha\eta = 0,$$

$$\partial^\alpha\partial_\alpha\eta(2\chi\tilde{H} + \xi\tilde{H}\sigma_3) = 0.$$

- Linearized EOM for η

$$\partial^\alpha\partial_\alpha\eta = (\partial_t^2 - \partial_z^2)\eta = 0$$

- Zero mode solutions:

$$\eta(z,t) = \eta^\pm(z \pm t)$$

Supercurrent of Type-C String

- Static zero mode solution

$$\eta(z) = \omega z$$

- $r \to \infty$で $U(1)_{\mathrm{EM}}$が回復しbackgroundは次を満たす

$$\tilde{H}\sigma_3 + n^a \sigma_a \tilde{H} = 0, \quad n^a = -\frac{\mathrm{tr}(\sigma_3 \tilde{H}^\dagger \sigma_a \tilde{H})}{\mathrm{tr}|\tilde{H}|^2}, \quad \left(\tilde{D}_\mu n\right)^a = 0, \quad \mathrm{tr}|\tilde{H}|^2 = 2v^2$$

- EOMsから次の方程式が得られる

$$\left(\tilde{D}_j \tilde{D}^j \chi\right)^a = -g^2 v^2 (\chi^a - n^a \xi), \quad \partial_j \partial^j \xi = -g'^2 v^2 (\xi - \chi^a n^a)$$

- Long-range force → $\chi^a - n^a \xi = 0$

$$\frac{1}{r}\partial_r(r\partial_r \xi) = 0$$

Current Quenching

- Stringの内部 $r \to 0$ では f と h は次のようなmass termsをLagrangianの中で感じる:

$$-\mathcal{L} \supset \frac{4v^2}{r^2} f^2 + 2m_{11}^2 v^2 (f^2 + h^2) + \omega^2 \frac{v^2}{2} (f - h)^2$$

$$= v^2 \begin{pmatrix} f & h \end{pmatrix} \begin{pmatrix} \frac{4}{r^2} + 2m_{11}^2 + \frac{\omega^2}{2} & -\frac{\omega^2}{2} \\ -\frac{\omega^2}{2} & 2m_{11}^2 + \frac{\omega^2}{2} \end{pmatrix} \begin{pmatrix} f \\ h \end{pmatrix}$$

- Determinant of the mass matrix:

$$\det M^2 = (2m_{11}^2)^2 + \frac{4v^2}{r^2} \left(2m_{11}^2 + \frac{\omega^2}{2} \right) + \omega^2 m_{11}^2$$

- Quenchingを避ける ⇒ ∃ region where $\det M^2 < 0$

$$\boxed{|\omega| \lesssim |m_{11}| \sim v_s}$$