超弦理論と宇宙定数問題

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Contents

Introduction : Cosmological Constant in Superstring

What's cosmological constant?

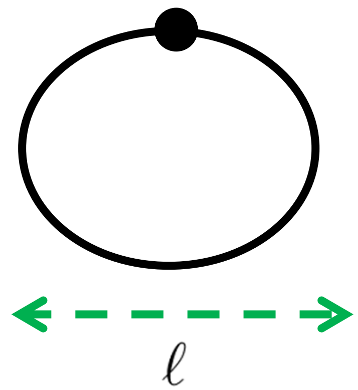
Cosmological const. $=$ vacuum energy density

$$\Lambda_{\text{1-loop}} \sim \sum \left[\text{1-loop vacuum diagram}\right]$$

no interaction

What's cosmological constant?

Point particle



$$\Lambda = \frac{1}{V_d}\left[\sum_{i\in\mathcal{H}_{\rm B}} - \sum_{i\in\mathcal{H}_{\rm F}}\right] Z_{S^1}(m_i^2)$$

$$\equiv \left[\sum_{i\in\mathcal{H}_{\rm B}} - \sum_{i\in\mathcal{H}_{\rm F}}\right] \int_\epsilon^\infty \frac{d\ell}{\ell} \int \frac{d^d p_i}{(2\pi)^d}\, e^{-\ell\frac{\alpha'}{2}\left[p_i^2+m_i^2\right]}$$

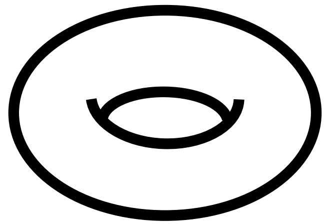
UV-cut off

Schwinger parameter (moduli)

What's cosmological constant?

String theory

Torus partition function



Torus, moduli τ

$$\Lambda = \frac{1}{V_d} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \underline{Z_{\text{torus}}(\tau)}$$

$$= \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{\tau_2^{\frac{d-2}{2}}} \sum_{(i,\tilde{i})\in\mathcal{H}_\perp} D(h_i, \tilde{h}_{\tilde{i}})\, q^{h_i-\frac{1}{2}} \overline{q^{\tilde{h}_{\tilde{i}}-\frac{1}{2}}}$$

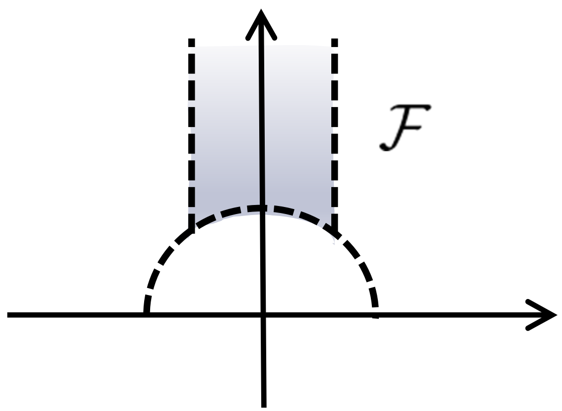
$$(q \equiv e^{2\pi i\tau})$$

What's cosmological constant?

Modular invariance

$$Z_{\rm torus}\left(-\frac{1}{\tau}\right) = Z_{\rm torus}(\tau), \qquad Z_{\rm torus}(\tau+1) = Z_{\rm torus}(\tau)$$

→ integration region :



What's cosmological constant?

※CC for string is formally identified as that of a particle theory with an infinite number of mass spectrum:

$$\left(\tau = \frac{\theta + i\ell}{2\pi}, \quad \frac{\alpha'}{4} m_i^2 = h_i - \frac{1}{2} = \tilde{h}_{\tilde{i}} - \frac{1}{2} \right)$$

$$\Lambda = \int_0^\infty \frac{d\ell}{\ell} \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} (2\pi\alpha'\ell)^{-\frac{d}{2}}$$

$$\times \sum_{i,\tilde{i}} D(h_i, \tilde{h}_{\tilde{i}}) \exp\left[-\left(h_i + \tilde{h}_{\tilde{i}} - 1 \right) \ell + i\theta \left(h_i - \tilde{h}_{\tilde{i}} \right) \right]$$

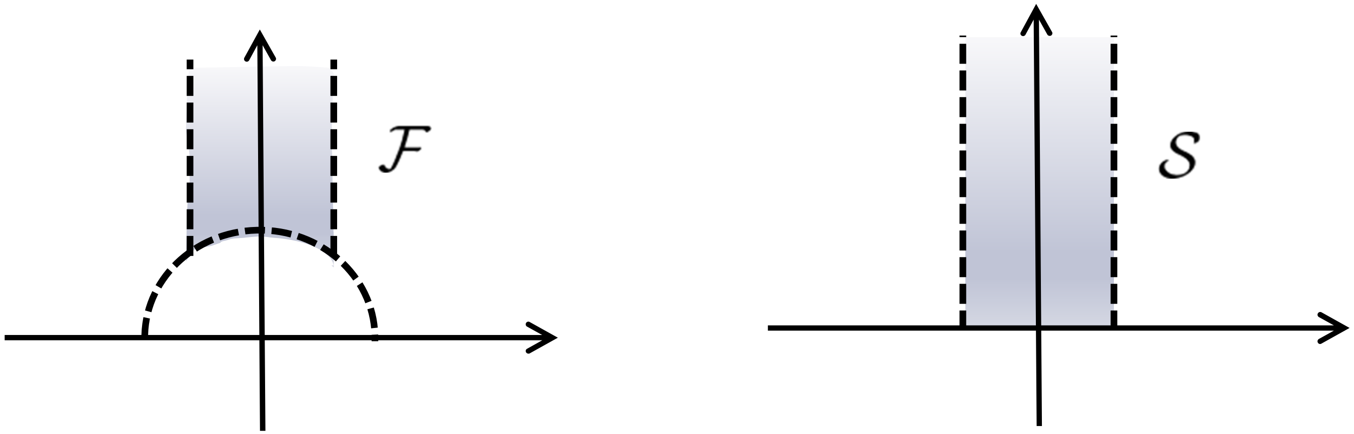
$$\propto \int_{\mathcal{S}} \frac{d^2\tau}{\tau_2^2} \frac{1}{\tau_2^{\frac{d-2}{2}}} \sum_{i,\tilde{i}} D(h_i, \tilde{h}_{\tilde{i}})\, q^{h_i - \frac{1}{2}} \overline{q^{\tilde{h}_{\tilde{i}} - \frac{1}{2}}},$$

But, the integration region is different.

Integration region of moduli
(Schwinger parameter)

String

Particle



‘cosmological constant problem’

$$\Lambda_{\text{observation}} \ll M^4_{\text{SUSY breaking}}$$

⇒ suggests a vacuum with (nearly) vanishing cosmological constant without SUSY?

Non-SUSY with $\Lambda = 0$?

Two possibilities

(1) $Z(\tau) \equiv 0$, but no supercharges exist.

(2) $Z(\tau) \not\equiv 0$, but $\Lambda = 0$

Asymmetric Orbifolds
as
Non-SUSY String Vacua with Vanishing Cosmological Constant

How to break SUSY?

Orbifolding by $(-1)^{F_L}$

(F_L : left-moving space-time fermion number)

➡ Removes massless Ramond states in the untwisted sector. (break SUSY?)

➡ But, new massless Ramond states emerge in the twisted sector. (eventually, SUSY)

How to break SUSY?

Orbifolding by $(-1)^{F_L} \otimes \mathcal{T}_{2\pi R}$

shift operator

'Scherk-Schwarz Compactification'

(see also Prof. Itoyama's talk)

→ Lightest Ramond states in the twisted sectors get massive. (No supercharge can appear.)

→ SUSY breaking

Non-SUSY Model with $\Lambda = 0$

Non-SUSY string vacua (type II)
with vanishing
1-loop cosmological constant

[Kachru, Kumar, Silverstein 1998], [Kachru, Silverstein 1998],
[Kachru, Silverstein 1998]

Asymmetric orbifold defined by the orbifold group:

$$G = \langle f, g \rangle$$

f, g do not commute. → Non-abelian orbifold

f → includes $(-1)^{F_R}$ and (chiral) shift op.

⇒ break all the right-moving SUSY

g → includes $(-1)^{F_L}$ and (chiral) shift op.

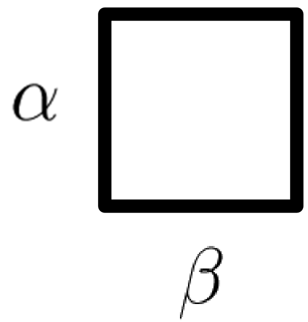
⇒ break all the left-moving SUSY

⇒ **Totally Non-SUSY**

Nevertheless, the 1-loop cosmological constant (torus partition function) strictly vanishes.

Bose-fermi cancelation without SUSY

Crucial point :



$$(\alpha, \beta \in G)$$

is well-defined, only when α commutes with β

Clever, but looks complicated…

Closely related studies :

[Harvey 1998]

[Shiu, Tye 1998]

[Blumenhagen, Gorlich 1998]

[Angelantonj, Antoniadis, Forger 1999]

[Aoki, D'Hoker, Phong 2003]

....

Non-SUSY Model with $\Lambda = 0$

My studies

[Satoh, Y.S, Wada 2015]

[Y.S, Wada 2016]

[Satoh, Y.S, Uetoko 2017]

[Y.S, Uetoko 2018]

[Aoyama, Y.S 2020]

[Aoyama, Y.S to appear]

Non-SUSY Model with $\Lambda = 0$

[Satoh, Y.S, Wada 2015]

- Realized by simpler (asymmetric) orbifold actions.

  (only single generator in the orbifold group)

- Each building block $Z_{(a,b)}(\tau)$ separately vanishes.

  (bose-fermi cancellation in each sector)

- No supercharges (globally defined on the total Hilbert space) exist.

Non-SUSY Model with $\Lambda = 0$

$$Z(\tau) = \sum_{a,b} \underline{Z_{(a,b)}(\tau)} \equiv \sum_{a,b} {}_{b}\square_{a}$$

$$Z_{(a,b)}\left(-\frac{1}{\tau}\right) = Z_{(b,-a)}(\tau)$$

$$Z_{(a,b)}\left(\tau + 1\right) = Z_{(a,a+b)}(\tau)$$

'modular covariance'

Non-SUSY Model with $\Lambda = 0$

[Aoyama, Y.S 2020]

[Aoyama, Y.S to appear]

- Asymmetric orbifolds of Gepner models
- Each building block $Z_{(a,b)}(\tau)$ does not necessarily vanish.
  (bose-fermi cancellation among different twisted sectors)

Non-SUSY Vacua with Vanishing Cosmological Constant : part 2

[Satoh, Y.S work in progress]

Non-SUSY Model with $\Lambda = 0$: part 2

Two possibilities

(1) $Z(\tau) \equiv 0$, but no supercharges exist.

(2) $Z(\tau) \not\equiv 0$, but $\Lambda = 0$

no bose-fermi cancellation

Non-SUSY Model with $\Lambda = 0$: part 2

[Moore 1987] 'Atkin-Lehner Symmetry'

$$\Lambda = \langle \psi_1 | \psi_2 \rangle$$

If $\psi_1, \ \psi_2$ have opposite 'parity',

Λ should vanish.

→ Beautiful idea, but difficulty in concrete model building . . .

Non-SUSY Model with $\Lambda = 0$: part 2

Our idea :

$$Z_{\text{total}}(\tau) = \sum_{w,m\in\mathbb{Z}} Z_{(w,m)}(\tau)$$

⇨ Modular invariant

(w, m) : 'winding numbers'

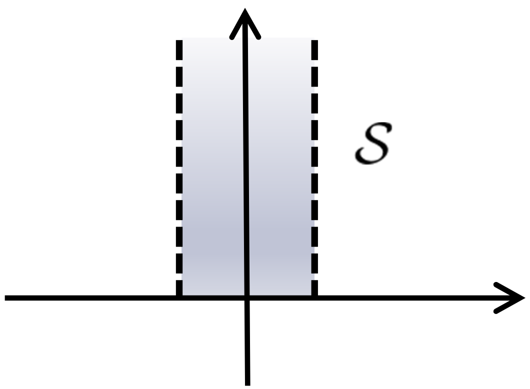
($SL(2,\mathbb{Z})$ -doublet)

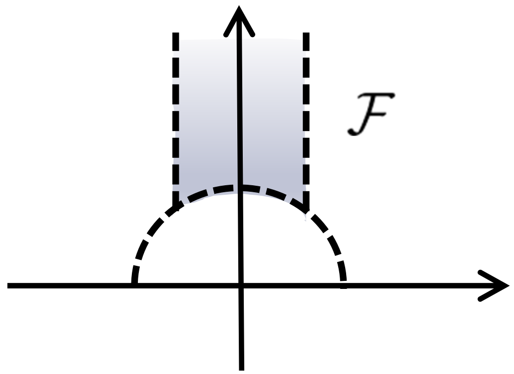
'Polchinski's trick'

[Polchinski 1986]

$$\int_{\mathcal{S}} \frac{d^2\tau}{\tau_2^2} \sum_{m\in\mathbb{Z}} Z_{(0,m)}(\tau) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \sum_{w,m\in\mathbb{Z}} Z_{(w,m)}(\tau)$$

(assume $Z_{(0,0)}(\tau) \equiv 0$)





Assume the situation :

$$\sum_{m\in\mathbb{Z}} Z_{(0,m)}(\tau) \not\equiv 0 \text{ , but } \sum_{w,m\in\mathbb{Z}} Z_{(w,m)}(\tau) \equiv 0$$

$$\Rightarrow \quad \text{`}Z_{\text{particle}}(\tau)\text{'} \left(\equiv \sum_{m\in\mathbb{Z}} Z_{(0,m)}(\tau) \right) \not\equiv 0$$

$$\text{but } \Lambda \equiv \int_{\mathcal{S}} \frac{d^2\tau}{\tau_2^2} \text{`}Z_{\text{particle}}(\tau)\text{'} = 0$$

Correct integration region for the particle theory !

interpretable as a particle theory with $\Lambda = 0$,
without the bose-fermi cancellation ?
(including infinite number of mass spectrum)

Starting with various SUSY string vacua,
we can realize this situation by implementing
asymmetric orbifolding

$Z_{(0,m)}(\tau)$ ~ 'untwisted sector'

$Z_{(w,m)}(\tau)\ (w \neq 0)$ ~ 'twisted sector'

Summary

- Non-SUSY stirng vacua with $Z(\tau) \equiv 0$

  → can be constructed by asymmetric ('non-geometric') orbifold

- Vanishing CC with $Z(\tau) \neq 0$

  → very difficult in string theory,

  can be realized as particle vacua ？

ご清聴ありがとうございました。