Stability, enhanced gauge symmetry and suppressed cosmological constant in heterotic interpolating models

with S. Nakajima

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On nonsusy hetero

- arXiv: 1905.10745, PTEP INkjm1
- arXiv: 2003.1121, NPB INkjm 2
- INkjm3, in preparation
- I-Koga-Nakajima, in progress

I) Some background materials

- Q : What is string theory?
  A : Nobody knows.
- began with the construction based on a single string:

cf. point particle vs. field (wave)

↓ quantization

many body problem

↓ 2nd quantization

(↓ quantization)

wavicle

⇧

- began as a candidate for unified theory that must include quantum gravity

1997 → decoupling of gravity by D brane 2015 → direct connection to Planck scale

→ : paradigm shift

- UV finite, but couplings run in real world, so need **a big help from QFT**.

- Ironically, more successful in stimulating LEEF construction at IR

  e.g. geometric engineering, solvable matrix models, …

- α' only to begin with, but all marginal deformations of worldsheet action need to be taken into account

  ⇒ translates into undetermined V.E.V. (moduli) of LEEF

  ⇒ energetic consideration needed & nonsusy hetero helps ⇒ today's talk

● "Conservative" view

- just do the two simple things:

  i) $\Lambda^{\text{1-loop}}_{\text{string}} \sim \int_{\mathcal{F}} \frac{\mathrm{d}^2\tau}{\tau_2^2} Z^{\text{1-loop}}(q)$ ⟵ partition function used for state counting

  ii) (tree) amplitude $\sim$ gaussian integration in the presence of "vertex operators"

- To elaborate little more

  i) using $\log \frac{a}{b} = -\int_0^\infty \frac{\mathrm{d}t}{t}\left(e^{-ta} - e^{-tb}\right)$, can derive $\Lambda^{\text{1-loop}}_{\text{string}}$

  by $F^{\text{1-loop}}_{\text{QFT}} \sim \int \frac{\mathrm{d}t}{t} \cdots$ up to $\mathcal{F}$ cf. Nambu 1950 @OCU

  ii) just one diagram/amplitude

- leave the rest to QFT

  and/or

  i) $\alpha' \to 0$ (∞ tension) limit, interesting in the presence of $B_{\mu\nu}$, WL

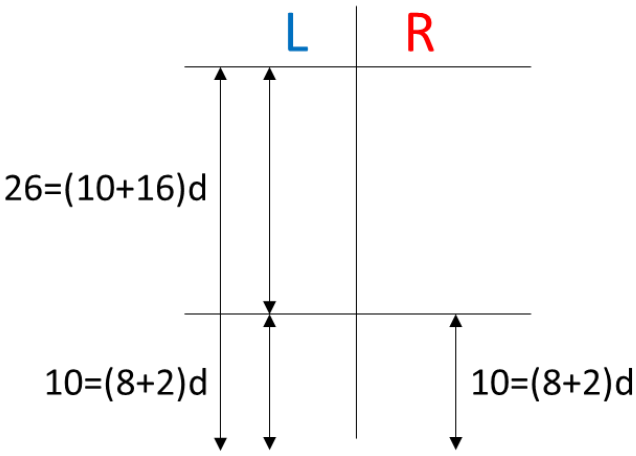
  ii) possibility of UV-IR separation in V_{eff} Abel, Dienes Newton Inst. talk…

● Heterotic string & history

GHMR '84

- Closed string only & inherently chiral with matter rep

Idea of Heterotic strings



adopt the lightcone coordinates

Right mover: 10d superstring $\bar{X}_R^i(\tau-\sigma),\ \bar{\psi}^i(\tau-\sigma)$

Left mover: 26d bosonic string out of which

internal 16d realize rank 16 current algebra

$X_L^i(\tau+\sigma),\ X_L^I(\tau+\sigma)$ (or fermions)

- fanatically investigated during '84 ～ '87
- got bored during '88 ～ '95
- got converted to D-braners or quit '96 ～
- In '86, modular inv (tachyon free) non SUSY hetero. found — Dixon-Harvey, Alvarez-Gaume et.al.
- SUSY-non SUSY interpolating model upon comp. found — H.I.-T.Taylor
- resurgence of interest since around 2015 as nothing found beyond Higgs at LHC so far

Cosmological constant in string with broken SUSY

- Due to the cosmological observation up until around 2000, the cosmological const Λ_{obs} **was** thought to be exactly zero.

  ⇒ should have $\Lambda_{\text{string}} = 0$ ∴ superstring

  The goal **was** to predict ~~SUSY~~ scale M_s in multi TeV region and superpartners : CY…

- Now in 2020, about 70% of the mass of our universe is attributed to something called dark energy whose leading candidate is Λ_{obs}.

  ⇒ nonSUSY hetero **is fine**, M_s can be as large as M_{Planck}.

  The issue is how to make Λ_{string} small $\sim \Lambda_{\text{obs}}$ upon compactification while keeping nonaberian gauge group.

- #(theories with SUSY) < #(theories without SUSY)

  - Type IIB
  - Type IIA
  - Type I
  - Heterotic $SO(32)$
  - Heterotic $E_8 \times E_8$

  call M_1

  - Type 0B
  - Type 0A
  - Heterotic $SO(32)$
  - Heterotic $SO(16) \times E_8$

  call M_2

  - Heterotic $SO(16) \times SO(16)$
  - Heterotic $E_7^2 \times SU(2)^2$
  - Heterotic $SO(24) \times SO(8)$
  - …

  today

  interpolation by a radius ($a = \sqrt{\alpha'/R}$) or in general radii interpolation of M_1 and M_2 upon compactification

  ’86: HI-Taylor

● gauge symmetry enhancement

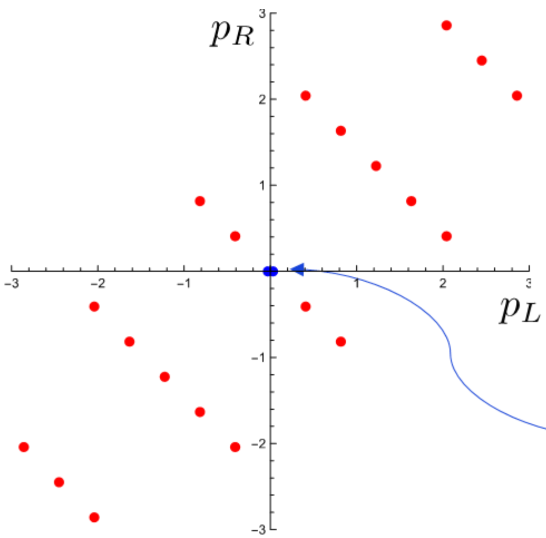
Simplest example: bosonic strings on S^1

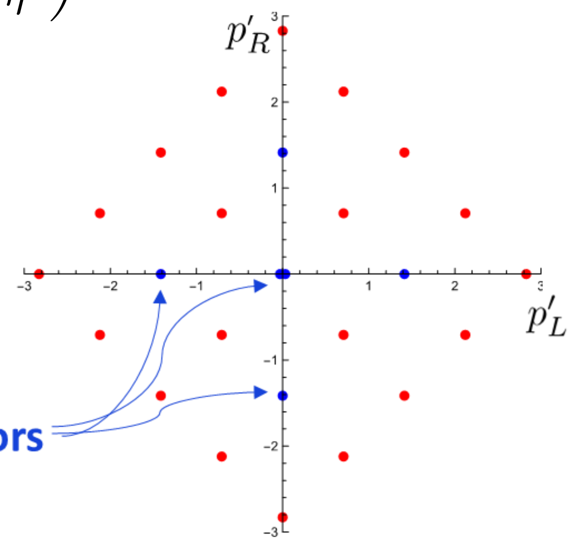
Mass formula : $\alpha' M^2 = 4(N-1) + 2p_L^2 = 4(\tilde{N}-1) + 2p_R^2$

(※) $\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} na + w/a \\ na - w/a \end{pmatrix}$ $\xrightarrow{\textbf{boost}}$ $\begin{pmatrix} p'_L \\ p'_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} n + w \\ n - w \end{pmatrix}$

$$\begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$$

$$e^{-\eta} = a$$





Massless vectors

gauge sym. is enhanced

$U(1)^2 \longrightarrow SU(2)\times SU(2)$

(※) forms SO(1,1) Lorentzian lattice: ∵ (momentum) ・ (winding) = a indep.

In QFT, we can regard more or less

⟵ Higgsing

⟶ unHiggsing

Our choice

M_2 = SO(16) x SO(16) tachyon free I-Taylor '86 & INkjm 1, 2

Dixon-Harvey '86, Alvarez-Gaume et al. '86

- warning: consider all marginal deformation of the world sheet action

$\Longrightarrow$
- full set of Wilson lines should be turned on
- generally spoil the nonabelian gauge group

extrema $\leftrightarrow$ points of sym. enhancement &
the stable 9D perturbative vacuum can be determined

Our results and ongoing work

- completed the above analysis in the case of $D = 9, d = 1$ in susy restoring region
- a few $n_F = n_B$ models found
- the minimum is SO(32)/$E_8 \times E_8$ gauge sym., massless bosons only $\Rightarrow$ AdS spacetime
- $\frac{\partial}{\partial \alpha'} \Lambda_{\text{string}} =$ dilaton tadpole is small to this order & will be made harmless
- a general analysis at d and study of interactions are in progress

The rest of my talk

II) 9D interpolating models

III) 9D interpolating models with WL

II)

- **formula for one-loop cosm. const in SUSY res. region:**

$$\Lambda^{(D)} = \xi(n_F - n_B)a^D + O(e^{-1/a})$$ H.I.-Taylor ('86)

M_1 0 ←—— a ——→ ∞ M_2

n_B, n_F; # of massless bosons & fermions in D dim.

sketch of the proof:

- The integrand of the τ_2 integration involves

$$(*) = \tau_2^{\#}(\Lambda_{0,0} - \Lambda_{1/2,0})e^{-m\pi\tau_2}$$

SUSY restoring factor

- apply the Jacobi imaginary transf. $m = 0$ 1st term & $m \neq 0$ 2nd term

- $n_B = n_F$ models (by now more than several existing) enjoy exponential suppression of $\Lambda^{(D)}$ e.g. Abel, Dienes ...

- In this setup, mass splitting due to broken SUSY is $\alpha' M_s^2 = a^2$.

e.g. $a \approx 0.01$ interesting possibility

State counting & characters

- $\mathrm{Tr} q^{L_0} \bar{q}^{\bar{L}_0}$ counts #(states) at level m as coeff. in $q(\bar{q})$ expansion
- It takes the form of $\sum_{i,j} \bar{\chi}_i^{\mathrm{Vir}}(\bar{q}) X_{ij} \chi_j^{\mathrm{Vir}}(q)$ and involves spacetime & internal SO(2n), n=4, 8 characters $\mathrm{ch(rep)} = O_{2n}, V_{2n}, S_{2n}, C_{2n}$ expressible in terms of the four theta constants and the Dedekind eta fn

$$\eta(\tau) = q^{-1/24} \prod_{n=1}^{\infty} (1-q^n)$$

- SO(32) hetero $\quad Z_B^{(8)} \left(\bar{V}_8 - \bar{S}_8\right) \left(O_{16}O_{16} + V_{16}V_{16} + S_{16}S_{16} + C_{16}C_{16}\right)$

  E_8 x E_8 hetero $\quad Z_B^{(8)} (\bar{V}_8 - \bar{S}_8)(O_{16} + S_{16})(O_{16} + S_{16})$

Construction

① **The starting point is a supersymmetric heterotic strings on S^1:**

$$Z_B^{(7)}\left(\bar{V}_8-\bar{S}_8\right)\left(\Lambda\left[\Gamma_{16},0,0\right]+\Lambda\left[\Gamma_{16},1/2,0\right]\right)$$

$$\Lambda\left[\Gamma,\alpha,\beta\right]=\eta^{-16}\left(\eta\bar{\eta}\right)^{-1}\sum_{\pi^I\in\Gamma}\sum_{n\in 2(\boldsymbol{Z}+\alpha)}\sum_{w\in\boldsymbol{Z}+\beta} q^{\frac{1}{2}\left(\pi^2+p_L^2\right)}\bar{q}^{\frac{1}{2}p_R^2}$$

with $p_L=\frac{1}{\sqrt{2}}\left(na+wa^{-1}\right)$, $p_R=\frac{1}{\sqrt{2}}\left(na-wa^{-1}\right)$

Γ_{16}: 16D Euclidean even self-dual lattice

② **Projection by** $\frac{1+(-1)^F\mathcal{T}Q}{2}$

- F: spacetime fermion number
- $\mathcal{T}$: half translation acting on X^9
- Q: half translation acting on Γ_{16}

$$\mathcal{T}\left|n\right\rangle=\begin{cases}\left|n\right\rangle & n\in 2\boldsymbol{Z}\\ -\left|n\right\rangle & n\in 2\boldsymbol{Z}+1\end{cases}$$

$$Q\left|\pi^I\right\rangle=\begin{cases}\left|\pi^I\right\rangle & \pi^I\in\Gamma_{16}^+\\ -\left|\pi^I\right\rangle & \pi^I\in\Gamma_{16}^-\end{cases}\qquad \Gamma_{16}=\Gamma_{16}^++\Gamma_{16}^-$$

③ **Adding twisted sectors**

$$Z_B^{(7)}\left\{\bar{V}_8\left(\Lambda\left[\Gamma_{16}^+,0,0\right]+\Lambda\left[\Gamma_{16}^-,1/2,0\right]\right)-\bar{S}_8\left(\Lambda\left[\Gamma_{16}^+,1/2,0\right]+\Lambda\left[\Gamma_{16}^-,0,0\right]\right)\right\}$$

+ (twisted sectors) → consist of $\Lambda\left[\Gamma_{16}^{\pm}+\delta,\alpha,1/2\right]$ → suppressed in the $a\approx 0$ region

nonzero winding

Example: SUSY $SO(32) \leftrightarrow SO(16)\times SO(16)$

● Partition function

- Contribution from X_L^I

$$\eta^{-16}\sum_{\pi^I\in\Gamma_{16}} q^{\frac{1}{2}\pi^2} = O_{16}O_{16}+V_{16}V_{16}+S_{16}S_{16}+C_{16}C_{16}$$

splitting by Q

$$\begin{cases} \eta^{-16}\sum_{\pi^I\in\Gamma_{16}^+} q^{\frac{1}{2}\pi^2} = O_{16}O_{16}+S_{16}S_{16} \\ \eta^{-16}\sum_{\pi^I\in\Gamma_{16}^-} q^{\frac{1}{2}\pi^2} = V_{16}V_{16}+C_{16}C_{16} \end{cases}$$

- Contribution from X^9

$$\Lambda_{\alpha,\beta} = (\eta\bar{\eta})^{-1}\sum_{n\in 2(\boldsymbol{Z}+\alpha)}\sum_{w\in\boldsymbol{Z}+\beta} q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2}$$

The contribution from the internal directions is

$$\Lambda\left[\Gamma_{16}^{\pm},\alpha,\beta\right] = \Lambda_{\alpha,\beta}\left(\eta^{-16}\sum_{\pi^I\in\Gamma_{16}^{\pm}} q^{\frac{1}{2}\pi^2}\right)$$

The one-loop partition function is

$$\begin{aligned} Z_{\text{int}}^{(9)} = Z_B^{(7)}\{&\Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right] \\ &+\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right] \\ &+\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right] \\ &+\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right]\} \end{aligned}$$

twisted sectors

Endpoint limits

- $R \to \infty$: contribution from the states with $w = 0$ only

$$\to \quad \Lambda_{\alpha,0} \to (2a)^{-1} Z_B^{(1)}, \quad \Lambda_{\alpha,1/2} \to 0$$

$$Z_{\rm int}^{(9)} \to (2a)^{-1} Z_B^{(8)} \left(\bar{V}_8 - \bar{S}_8\right)\left(O_{16}O_{16} + V_{16}V_{16} + S_{16}S_{16} + C_{16}C_{16}\right)$$

The partition function of SUSY $SO(32)$

- $R \to 0$: contribution from the states with $n = 0$ only

$$\to \quad \Lambda_{0,\beta} \to a Z_B^{(1)}, \quad \Lambda_{1/2,\beta} \to 0$$

$$Z_{\rm int}^{(9)} \to a Z_B^{(8)} \left\{\bar{O}_8\left(V_{16}C_{16} + C_{16}V_{16}\right) + \bar{V}_8\left(O_{16}O_{16} + S_{16}S_{16}\right) - \bar{S}_8\left(V_{16}V_{16} + C_{16}C_{16}\right) - \bar{C}_8\left(O_{16}S_{16} + S_{16}O_{16}\right)\right\}$$

The partition function of $SO(16) \times SO(16)$ model

III) Marginal deformations

● **The marginal deformations of the world-sheet action:**

$$A_{Ii}\int d^2z\partial X_L^I\bar{\partial}X_R^i + C_{ji}\int d^2z\partial X_L^j\bar{\partial}X_R^i,$$

$$\left[I=1,\cdots,16,\ i,j=10-d,\cdots,9 \right]$$

A_{Ii} : Wilson lines
C_{ji} : metric, anti-sym tensor

➡ The boosts of the momentum lattice forming the coset

$$\frac{SO(16+d,d)}{SO(16+d)\times SO(d)}$$

Narain, Sarmadi, Witten, (1986)

In the $d=1$ case,

$$\begin{cases} \pi^I = \pi^I \\ p_L = \dfrac{1}{\sqrt{2}}\left(na+wa^{-1}\right) \\ p_R = \dfrac{1}{\sqrt{2}}\left(na-wa^{-1}\right) \end{cases}$$

Deformed by Wilson lines ➡

$$\begin{cases} \pi'^I = \pi^I - wA^I \\ p'_L = \dfrac{a}{\sqrt{2}}\left(\pi\cdot A + n + w\left(a^{-2}-\dfrac{1}{2}A\cdot A\right)\right) \\ p'_R = \dfrac{a}{\sqrt{2}}\left(\pi\cdot A + n - w\left(a^{-2}+\dfrac{1}{2}A\cdot A\right)\right) \end{cases}$$

Let us consider these deformations for interpolating models and study the massless spectra!

Enhanced gauge symmetry

- **Mass formula:**

$$\alpha' M^2 = 4(N-1) + 2(\pi^2 + p_L^2) = 4(\tilde{N} - a) + 2p_R^2 \quad \left(a = \begin{cases} \frac{1}{2} & \text{(NS sector)} \\ 0 & \text{(R sector)} \end{cases} \right)$$

- **Massless states (NS)**

① Sector1: $N = 1, \quad \tilde{N} = \frac{1}{2}, \quad \pi^I = p_L = p_R = 0 \; (\pi^I = n = w = 0)$

NOT depend on moduli

D.O.F (sector1) = 8 × (8 + 16)

10D gravity multiplet ↑ ↑ Gauge bosons of $U(1)^{16}$

② Sector2: $N = 0, \quad \tilde{N} = \frac{1}{2}, \quad \pi^2 + p_L^2 = 2, \quad p_R = 0$ depend on moduli

$w = 0$ → $\pi^2 = 2, \quad \pi \cdot A = -n$

nonzero roots of a simply laced Lie algebra

At generic points in moduli space, the gauge symmetry is $U(1)^{16+2}$.

However, there are special points satisfying $\pi \cdot A = -n$ for $\pi^2 = 2$, where the gauge symmetry is enhanced to a simply laced Lie group.

Example: SUSY $\boldsymbol{SO(32)} \leftrightarrow \boldsymbol{SO(16)} \times \boldsymbol{SO(16)}$

● Enhanced gauge symmetry

- $A^I = ((0)^8; (1)^8)$ $\longrightarrow$ The gauge symmetry is enhanced to $SO(32)$. There is no massless fermion.
- $A^I = \left((0)^p, (1)^q; (0)^{p'}, (1)^{q'}\right)$ $\quad p + q = p' + q' = 8$

The massless spectrum at this point is

- The gauge bosons of $SO(2(p+q')) \times SO(2(q+p'))$
- The spinors in the bi-fund rep of $SO(2(p+q')) \times SO(2(q+p'))$

We can find a variety of symmetry enhancements depending on the Wilson lines.

➡ **Are there points where $\boldsymbol{n_F = n_B}$?**

In this model, the cosmological constant is suppressed when the gauge symmetry is enhanced to

$$\boldsymbol{SO(18) \times SO(14)} \text{ or } \boldsymbol{SO(12) \times SO(12) \times U(4)}\ .$$

Example: SUSY $SO(32) \leftrightarrow SO(16) \times SO(16)$

● Stability

- 1-loop effective potential: $\Lambda^{(9)} = -\frac{1}{2}\left(4\pi^2\alpha'\right)^{-9/2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}Z^{(9)}$

  Up to exponentially suppressed terms (in the $a \approx 0$ region),

$$\Lambda^{(9)}(a, A^I) \simeq C_0\left(\frac{a}{\sqrt{\alpha'}}\right)^9 8\Bigg\{-24 + 4\sum_{I_1=1}^{8}\sum_{I_2=9}^{16}\cos\left(\pi A^{I_1}\right)\cos\left(\pi A^{I_2}\right)$$
$$-4\sum_{\substack{I_1,I_1'=1\\ I_1>I_1'}}^{8}\cos\left(\pi A^{I_1}\right)\cos\left(\pi A^{I_1'}\right) - 4\sum_{\substack{I_2,I_2'=9\\ I_2>I_2'}}^{16}\cos\left(\pi A^{I_2}\right)\cos\left(\pi A^{I_2'}\right)\Bigg\}$$

- Stability analysis of Wilson lines

$$\frac{\partial\Lambda^{(9)}}{\partial A^I} = 0, \quad \frac{\partial^2\Lambda^{(9)}}{\partial A^I\partial A^J} \geq 0 \quad \xrightarrow{\text{solve}} \quad A^I = \left((0)^8;(1)^8\right)$$

Maximally enhanced gauge symmetry

↳ Negative cosmological constant

How about the points with $n_F = n_B$? ➡ Saddle points (Extrema with some unstable directions)

Our choice

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