Anomaly of subsystem symmetry and anomaly inflow

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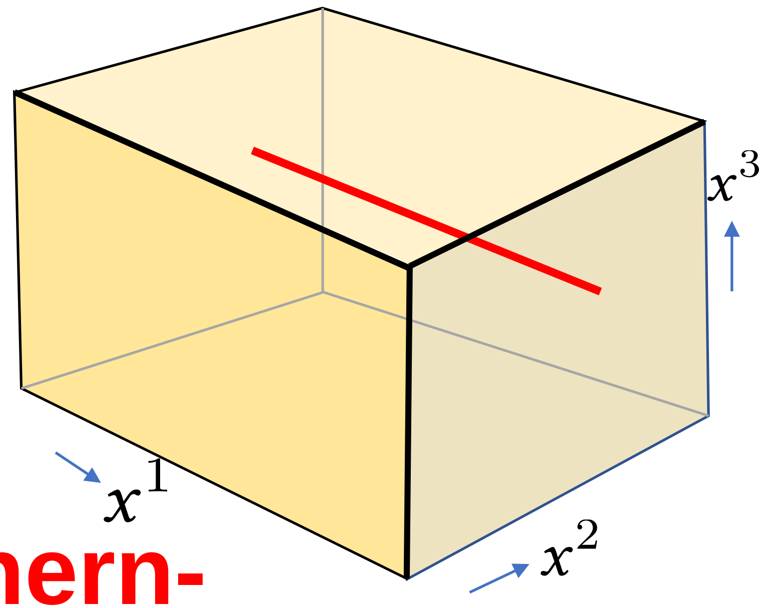
Based on [SY, arXiv:2110.12861]

key word

Subsystem symmetry

Subsystem symmetry

- An exotic symmetry in non-relativistic quantum system
- Charges are conserved within a certain subsystem
- It plays an important role in the study of fracton phases
- ≠ (higher form symmetry)



I would like to explore **gauge theory, Chern-Simons theory, anomaly inflow, ...** for **subsystem symmetry.**

Today's main setup: 4+1 dimensions

Lorentz symmetry

$$x^0, x^1, x^2, x^3, x^5$$

time

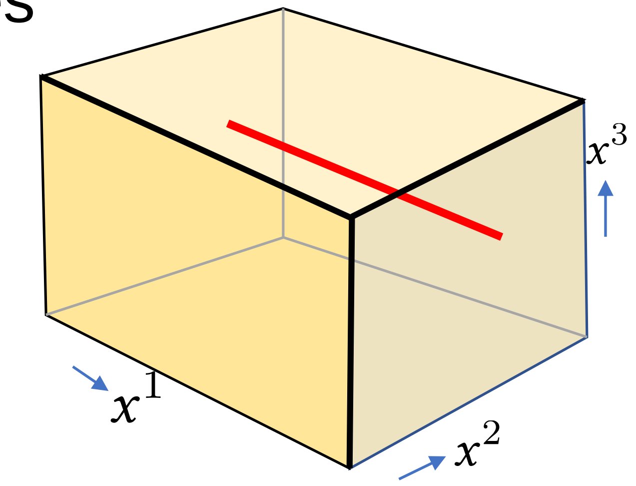
exotic space
(no continuous rotation)

normal space

Periodic boundary condition is imposed for all space directions.

Suppose three component field $(J^0,\ J^\#,\ J^5)$ satisfies

$$\partial_0 J^0 + \partial_1\partial_2\partial_3 J^\# + \partial_5 J^5 = 0$$



$Q^{23}(x^2, x^3) := \int dx^1 dx^5 J^0$ is conserved for each (x^2, x^3) independently.

so are Q^{12}, Q^{31}

$$\because\ \partial_0 Q^{23} = \int dx^1 dx^5 \partial_0 J^0 = -\int dx^1 dx^5 (\partial_1\partial_2\partial_3 J^\# + \partial_5 J^5) = 0$$

Symmetry associated with these charges is an example of

subsystem symmetry

Gauge field
for subsystem symmetry

Gauge field $(C_0, C_\#, C_5)$

(coupling) $\sim \int d^5x(C_0J^0 + C_\#J^\# + C_5J^5) = \int d^5x C_A J^A$

$A = 0, \#, 5$

$\partial_\# := \partial_1\partial_2\partial_3$

Gauge transformation

$$C_A \longrightarrow C'_A = C_A + \partial_A\lambda(x) \qquad \lambda \sim \lambda + 2\pi$$

➡ (coupling) is gauge invariant.

※Notice $(\partial_\# f)g + f(\partial_\# g) = (\text{total derivative}) \neq \partial_\#(fg)$ ➡ Integration by parts without boundary is possible

Gauge transformation $C_A \to C'_A = C_A + \partial_A \lambda(x)$ $\partial_\# := \partial_1 \partial_2 \partial_3$

Gauge invariant field strength

$$G_{AB} := \partial_A C_B - \partial_B C_A \qquad A, B = 0, \#, 5$$

Exotic Maxwell theory

$$S_M = \int d^5x \left[\frac{1}{2h^2} G_{05}^2 + \frac{1}{2g^2} G_{\#0}^2 - \frac{1}{2g^2} G_{\#5}^2 \right]$$

g, h : coupling constants

The gauge field has three components, which resembles to 2+1-dimensional gauge field

Exotic Chern-Simons term

$$S_{CS} = \frac{k}{4\pi} \int d^5x \, \epsilon^{ABC} C_A \partial_B C_C$$

totally anti-symmetric symbol

$\epsilon^{0\#5} = 1$

$A, B, C = 0, \#, 5$

$\partial_{\#} := \partial_1 \partial_2 \partial_3$

$k \in \mathbb{Z}$ parameter

Gauge invariant!

We want to consider

Exotic Maxwell-Chern-Simons theory

$$S = \int d^5x \left[\frac{1}{2h^2} G_{05}^2 + \frac{1}{2g^2} G_{\#0}^2 - \frac{1}{2g^2} G_{\#5}^2 + \frac{1}{4\pi} \epsilon^{ABC} C_A \partial_B C_C \right]$$

An analog of the topologically massive gauge theory in 2+1 dimensions.

[Deser, Jackiw, Templeton 82]

The topologically massive gauge theory in 2+1 dimensions. (2+1-dimensional Maxwell-Chern-Simons theory)

Boundary ⇒ localized gapless mode = 1+1-dimensional chiral boson

Protected by anomaly inflow mechanism

…[Hsieh, Tachikawa, Yonekura 20]

Does similar phenomenon happen in our exotic Maxwell-Chern-Simons theory?

Yes!

One of the simplest QFT with subsystem symmetry

[Gorantla, Lam, Seiberg, Shao 20]

[Burnell, Devakul, Gorantla, Lam, Shao 21]

Exotic Maxwell-Chern-Simons theory

Boundary ⇒ localized gapless mode = 3+1-dimensional chiral φ theory

Protected by anomaly inflow mechanism for **subsystem symmetry**

Introducing boundary

$$x^5 > 0$$

$x^5 = 0$ is the boundary

Boundary condition

$(C_0, C_\#)$ is pure gauge at $x^5 = 0$

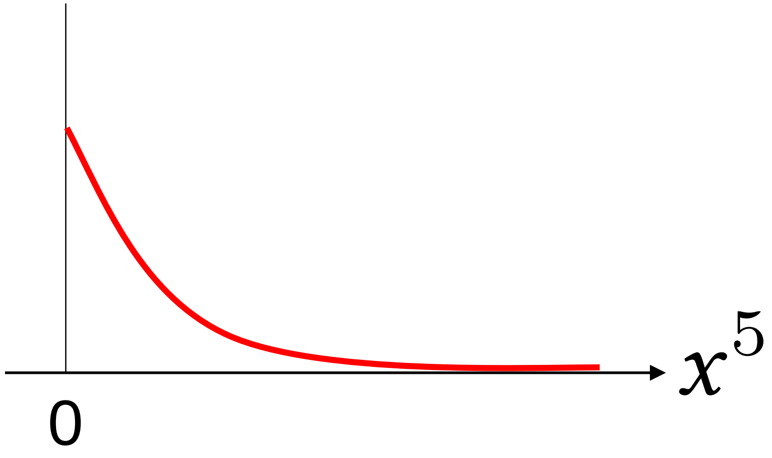
↓

Gauge symmetry is preserved.

Boundary localized mode

$$\partial_\# := \partial_1\partial_2\partial_3$$

$$\exists\varphi(x^0,x^1,x^2,x^3) \qquad \varphi \sim \varphi + 2\pi$$



$$C_0 = \partial_0\varphi e^{-mx^5},$$

$$C_\# = \partial_\#\varphi e^{-mx^5},$$

$$C_5 = 0 \qquad m := \frac{hg}{2\pi}$$

φ satisfies

$$(\partial_0 - \frac{h}{g}\partial_\#)\varphi = 0$$

"chiral φ theory"

(cf 1+1 dim chiral boson $(\partial_0 - \partial_1)\varphi = 0$)

Robust?

Global subsystem symmetry "magnetic center symmetry"

Do not confuse with the gauge subsystem symmetry

Current

$$M^A := \frac{1}{2\pi}\epsilon^{ABC}\partial_B C_C \quad \Rightarrow \quad \partial_A M^A = 0$$

conservation law for subsystem symmetry

Our localized mode is protected by the anomaly inflow mechanism for this magnetic center symmetry.

Introduce background gauge field A_A (Do not confuse with the dynamical gauge field C_A)

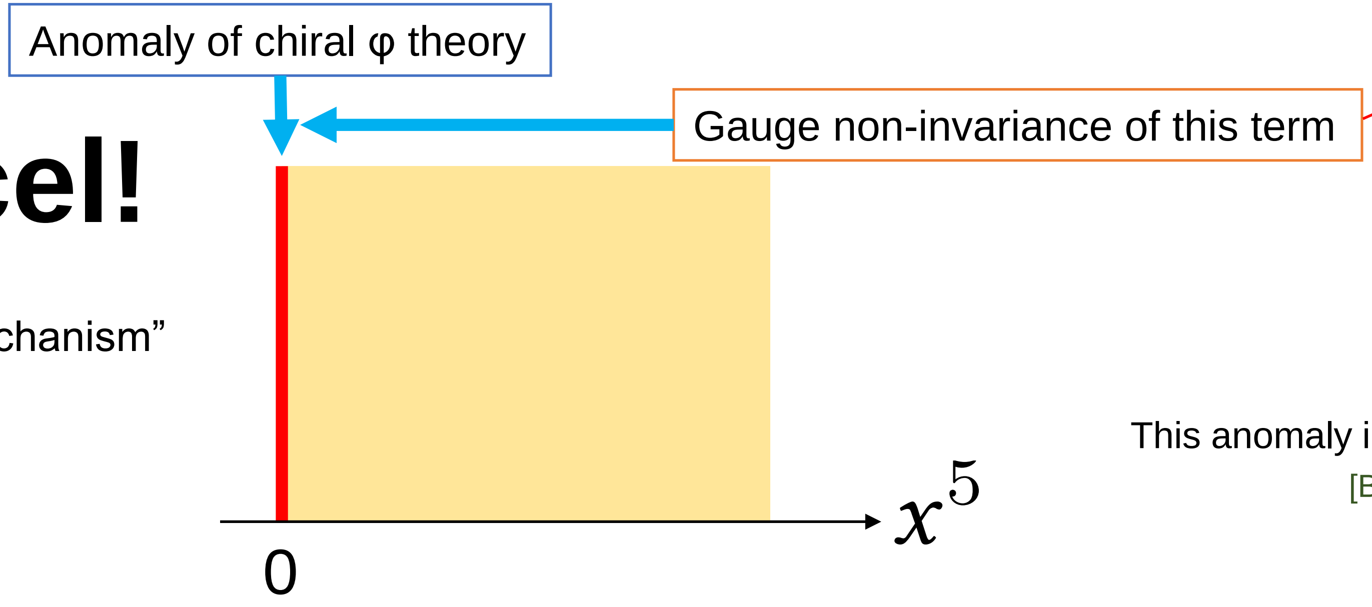
$$Z[A] = \int DC \exp(-S[C] - \int d^5x A_A M^A) = |Z[A]| \exp(-\frac{i}{4\pi}\int d^5x \epsilon^{ABC} A_A \partial_B A_C)$$

This part does not change by continuous perturbation without closing the gap.

Boundary

Cancel!

"Anomaly inflow mechanism"



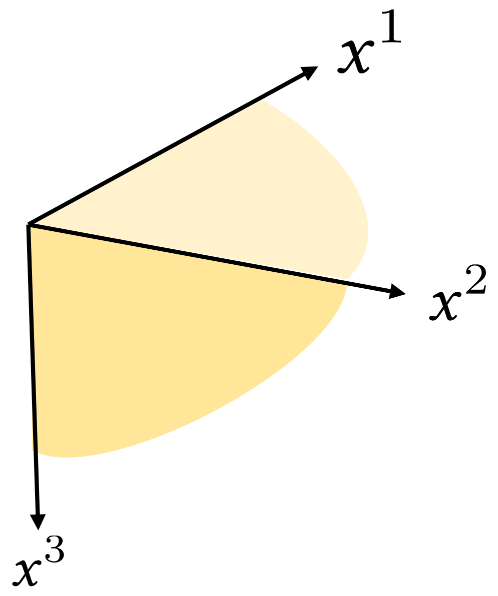
This anomaly inflow mechanism is also discussed in [Burnell, Devakul, Gorantla, Lam, Shao 21]

Chiral φ theory cannot disappear due to the gauge invariance.

Corner state

Consider space with corner

$$x^1, x^2, x^3 > 0$$



$$\phi = \phi(x^0, x^5), \quad \phi \sim \phi + 2\pi$$

$$C_0 = \partial_0 \phi \exp(-m_1 x^1 - m_2 x^2 - m_3 x^3)$$

$$C_5 = \partial_5 \phi \exp(-m_1 x^1 - m_2 x^2 - m_3 x^3)$$

$$C_\# = 0$$

$$m_1, m_2, m_3 > 0, \quad m_1 m_2 m_3 = \frac{g^2}{2\pi}$$

Infinite number of such solution.

$$(\partial_0 + \partial_5)\phi = 0$$ ⇨ 1+1 dim chiral boson

Robust? Anomaly inflow? Future problem

Summary and discussion

Summary

Subsystem symmetry

Exotic Maxwell-Chern-Simons theory

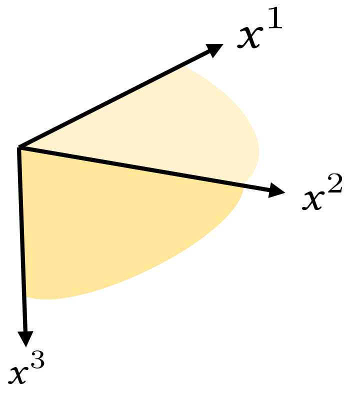
Global subsystem symmetry
(Magnetic center symmetry)

Boundary

Boundary localized chiral φ theory protected by anomaly inflow.

Corner

Infinite number of chiral bosons.



Discussion

- Nice mathematical formulation of a gauge field for subsystem symmetry?

  ↓ (cf ordinary (higher-form) U(1) gauge theory = differential cohomology)

  [Hsieh, Tachikawa, Yonekura 20]

  Curved space?

- Anomaly inflow for the corner states?

- Applications to high-energy physics?

  [Razamat 21]
  [Geng, Kachru, Karch, Nally, Rayhaun21]