Heavy-Heavy-Light Three-Point Functions from D-branes Revisited

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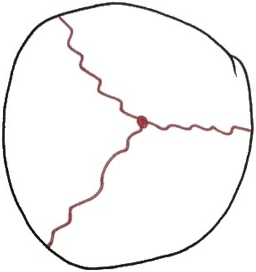
  - BPS SUGRA solutions (bubbling geometry) *[Lin, Lunin, Maldacena, 04]*.
- Some non-BPS local operators with large conformal weights are dual to semi-classical string solutions. *[GKP, 02, for $\mathcal{N}=4$ SYM][Bin Chen,* ***JW*** *08, for ABJM]*

Witten diagrams

- The three point function of single trace light operators (dual to supergravitons) are computed holographically using Witten diagrams. *[GKP, 98][Witten, 98]*.



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- In ABJM theory, the 3pt functions of $1/3$-BPS operators do not enjoy such a non-renormalization theorem *[Hirano, Kristjansen, Young, 12]*.
- Computation of such functions for most general case is still great challenge for supersymmetric localization and integrability method.

Two-point functions: integrability and holography

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- In planar limit, this problem is essentially solved by integrability. (Review: *[Beisert etal, 10]*)
- The holographic computation of the conformal weight is just compute the energy of the dual string solutions.

Holographic Heavy-Heavy-Light(HHL) 3pt functions

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- For two point functions, we stressed the role played by Rothian.
- An example of 3pt functions was computed to show the prescription.
- Contributions from open string attached on such D-branes was also computed.

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- Later on, more examples of 3pt HHL correlators for D-branes were computed, both for $\mathcal{N} = 4$ SYM *[Bissi, Kristjansen, Young, Zoubos, 11]* and ABJM theories *[Hirano, Kristjansen, Young, 12]*.

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- Regularization for extremal case was proposed. *[Lin, 12][Kristjansen, Mori, Young, 15]*
- But these regularization methods cannot be justified physically. And it does not resolve all the mismatches.

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- A. Their formulae did not reproduce the charge conservation we will discuss below.
- B. They did not take into account the effects of higher conversed charges in the orbit average.

More on ABJM case

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- The results at weak coupling and strong coupling are different, as expected.
- It is interesting to get wrapping corrects at strong coupling from the holographic result.

A toy model from QM

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- Here by light, we mean the quantum numbers of $\mathcal{O}$ are small.

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  and the path integral

$$\langle J|\mathcal{O}(t=0)|J\rangle = \int D\theta(t)\, e^{-iJ\theta(t=+\epsilon)}\mathcal{O}[\theta(t=0)]e^{iJ\theta(t=-\epsilon)}e^{\frac{i}{\hbar}S[\theta]}\,, \tag{5}$$

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- Here we have already assume that $\langle\theta|\mathcal{O}|\theta'\rangle = \mathcal{O}[\theta]\delta(\theta-\theta')$.

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- Note that the operator $\mathcal{O}$ does not affect the saddle-point equation since we assumed that it is light.
- Now, suppose we found one solution satisfying the equation (6), $\theta_0^*(t)$. Then, it immediately follows from the $U(1)$ invariance (1) that there should be a family of solutions, or equivalently a moduli of solutions, given by

$$\theta_c^*(t) \equiv \theta_0^*(t) + c\,, \qquad c \in [0, 2\pi]\,. \tag{7}$$

Orbit average

- Therefore, the correct saddle-point formula is given by

$$\langle J|\mathcal{O}(t=0)|J\rangle \overset{\mathrm{WKB}}{=} \int_0^{2\pi} \frac{\mathrm{d}c}{2\pi}\, e^{-iJ\theta_c^*(+\epsilon)}\mathcal{O}[\theta_c^*(0)]e^{iJ\theta_c^*(-\epsilon)}e^{\frac{i}{\hbar}S[\theta_c^*]}\,. \tag{8}$$

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- As we can see, the final result is given by an average over the parameter c and this is precisely the orbit average discussed in [Bajnok, Janik, Wereszczynski, 14].

Boundary term

- Let us now generalize the computation slightly and consider the situation in which the bra and ket states are not identical: $\langle J+q|\mathcal{O}|J\rangle$. We assume J is again large $(J\sim 1/\hbar \gg 1)$ while q is taken to be $O(1)$.

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$$\langle J+q|\mathcal{O}(t=0)|J\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]-iq\theta_0^*(0)}\int_0^{2\pi}\frac{\mathrm{d}c}{2\pi}\,e^{-iqc}\,\mathcal{O}[\theta_c^*(0)]\,. \quad (11)$$

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- Especially, for $\mathcal{O}$ being $\mathcal{O}_p \equiv e^{ip\theta}$, an operator with $U(1)$ charge p, we have

$$\langle J+q|\mathcal{O}_p(t=0)|J\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]}\delta_{p,q}, \quad (12)$$

where $\delta_{p,q}$ is manifestation of the $U(1)$ charge conservation.

Two lessons on boundary term

- First, when the bra and ket states are different, there is a nontrivial (boundary-term) contribution from the wave functions.
- Second, such contributions, together with the orbit average, are essential for reproducing a correct charge conservation $\delta_{p,q}$.

HHL $3-$point functions

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- The sub-determinant operator with charge M will be denoted by $\mathcal{D}_M$ and the single trace operator with charge L will be denoted by $\mathcal{O}_L$.
- Structure constant:

$$\langle\hat{\mathcal{D}}_{M+k}|\hat{\mathcal{O}}_L(t=0)|\hat{\mathcal{D}}_M\rangle = \int DX\,\Psi^*_{M+k}[X]\hat{\mathcal{O}}_L[X(t=0)]\,\Psi_M[X]e^{-S_{\mathrm{DBI+WZ}}[X]}\,.$$

Global charges

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- $U(1)$ R-charge J .
- $$X^*_{\tau_0,\phi_0} = X^*_0|_{t\to t-i\tau_0,\phi\to\phi+\phi_0}. \qquad (13)$$
- Shift in wave functions $\Psi \sim \exp(-i\Delta t + iJ\phi)$,

$$\Psi \mapsto e^{-\Delta\tau_0+iJ\phi_0}\Psi. \qquad (14)$$

Master equation

$$\langle \hat{\mathcal{D}}_{M+k} | \hat{\mathcal{O}}_L(t=0) | \hat{\mathcal{D}}_M \rangle = \underbrace{\int \mathrm{d}\tau_0 \int \frac{\mathrm{d}\phi_0}{2\pi}}_{\text{orbit average}} \hat{\mathcal{O}}_L[X^*_{\tau_0,\phi_0}(t=0)]$$
$$\underbrace{e^{(\Delta_{M+k}-\Delta_M)\tau_0} e^{-i(J_{M+k}-J_M)\phi_0}}_{\text{wave function}} . \tag{15}$$

Holographic dictionary

- $\mathcal{D}_M$ is dual to probe D-brane/M-brane.

Holographic dictionary

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  as in the old approach *[Bak, Chen, **JW**, 11][Bissi, Kristjansen, Young, Zoubos, 11]*.
- *Remark:* The last step is similar to the holographic computations of correlators of BPS Wilson loops (surfaces) and local BPS operators *[Berenstein, Corrado, Fischler, Maldacena, 98][Giombi, Ricci, Trancanelli, 06][Chen, Liu, **JW**, 07]*

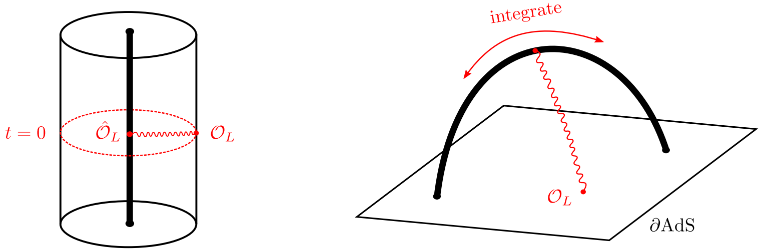

Figure: Comparison of new and old approaches.

- We consider the following sub-determinant operator in $\mathcal{N}=4$ super Yang-Mills

$$\mathcal{D}_M = \chi_M(Z) \equiv \frac{1}{M!}\delta^{[b_1 b_2 \cdots b_M]}_{[a_1 a_2 \cdots a_M]} Z^{a_1}_{b_1} \cdots Z^{a_M}_{b_M} \,, \tag{17}$$

$$\delta^{[b_1 \cdots b_M]}_{[a_1 \cdots a_M]} \equiv \sum_{\sigma \in S_M} (-1)^{|\sigma|} \delta^{b_1}_{a_{\sigma_1}} \cdots \delta^{b_M}_{b_{\sigma_M}} \,. \tag{18}$$

and the following single trace operator

$$\mathcal{O}_L \equiv \mathrm{tr}\tilde{Z}^L \,, \qquad \tilde{Z} = \frac{Z+\bar{Z}+Y-\bar{Y}}{2} \,, \tag{19}$$

The metric of $AdS_5 \times S^5$ with unit radius and in terms of the global coordinates,

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\text{AdS}} + \mathrm{d}s^2_{S^5} \,, \tag{20}$$

where

$$\begin{aligned} \mathrm{d}s^2_{\text{AdS}} &= -\cosh^2\rho\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\widetilde{\Omega}_3^2 \,, \\ \mathrm{d}s^2_{S^5} &= \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2 + \cos^2\theta\,\mathrm{d}\Omega_3^2 \,. \end{aligned} \tag{21}$$

where $\mathrm{d}\widetilde{\Omega}_3^2$ and $\mathrm{d}\Omega_3^2$ are the metric on S^3 which we parametrize as

$$\begin{aligned} \mathrm{d}\tilde{\Omega}_3^2 &= \mathrm{d}\tilde{\chi}_1^2 + \sin^2\tilde{\chi}_1\,\mathrm{d}\tilde{\chi}_2^2 + \cos^2\tilde{\chi}_1\mathrm{d}\tilde{\chi}_3^2 \,, \\ \mathrm{d}\Omega_3^2 &= \mathrm{d}\chi_1^2 + \sin^2\chi_1\,\mathrm{d}\chi_2^2 + \cos^2\chi_1\mathrm{d}\chi_3^2 \,. \end{aligned} \tag{22}$$

- The D-brane dual to $\mathcal{D}_M$ is localized at $\theta = \theta_0$ and extended along $\chi_{1,2,3}$ directions. It is rotating along the ϕ direction at the speed of light. The worldvolume coordinates of the D3 brane σ^μ $(\mu = 0, 1, 2, 3)$ are identified with the target space coordinates as follows:

$$\rho = 0, \qquad \sigma^0 = t, \quad \phi = t, \qquad \sigma^i = \chi_i, \quad i = 1, 2, 3. \tag{23}$$

- The value of θ_0 is related to the charge of the giant graviton as;

$$\cos^2\theta_0 = \frac{M}{N}\,, \tag{24}$$

- Note that the classical D3-brane equations of motion lead to $\phi = t$.
- The holographic dual of $\mathcal{O}_L$ is the fluctuation of the background fields (super-graviton). I omit the details here.

Results for $\mathcal{N} = 4$ SYM

- Diagonal structure constant

$$C_{\mathcal{D}_M\mathcal{D}_M\mathcal{O}_L} = -\frac{i^L + (-i)^L}{2\sqrt{L}} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0)\right) . \quad (25)$$

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- The holographic off-diagonal structure constant, with orbit average and contributions of wave functions included, matches the field theory results for non-extremal cases as well.

Results for $\mathcal{N}=4$ SYM

- Off-diagonal structure constant,

$$
\begin{aligned}
&C_{\mathcal{D}_{M+k}\mathcal{D}_M\mathcal{O}_L} = \\
&-\frac{1}{2}\sqrt{L}\left(i^{L-k}+(-i)^{L-k}\right)\frac{\Gamma(\frac{L+k}{2})\cos^2\theta_0\sin^k\theta_0}{\Gamma(1+k)\Gamma(1+\frac{L-k}{2})} \\
&{}_2F_1\left(1+\tfrac{k-L}{2},1+\tfrac{k+L}{2},1+k;\sin^2\theta_0\right)\ .
\end{aligned}
$$

for $L>k$.

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$${}_2F_1\left(1+\tfrac{k-L}{2}, 1+\tfrac{k+L}{2}, 1+k; \sin^2\theta_0\right) .$$

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Application to ABJM theory

Diagonal structure constant,

$$
\begin{aligned}
C_{\mathcal{D}_M \mathcal{D}_M \mathcal{O}_L} &= \left(\frac{\lambda}{2\pi^2}\right)^{1/4} \frac{\sqrt{2L+1}}{L}(1+(-1)^L) \\
& \frac{(-1)^{\frac{L}{2}+1} 2^L \sqrt{\pi}\Gamma(\frac{L}{2}+1)}{\Gamma(\frac{L+3}{2})}(1-4\alpha^4)^{\frac{1}{2}(L-1)} \\
\times & \left[(1-4\alpha^4)\,{}_2F_1\left(-\tfrac{1}{2}(L+1), -\tfrac{L}{2}; 1; \tfrac{4\alpha^4}{4\alpha^4-1}\right)\right. \\
& \left. +2\alpha^4 (L+1)\,{}_2F_1\left(-\tfrac{1}{2}(L-1), -\tfrac{L}{2}+1; 2; \tfrac{4\alpha^4}{4\alpha^4-1}\right)\right] .
\end{aligned} \tag{26}
$$

with the relation among M, N and α is

$$
\frac{M}{N} = \sqrt{1-4\alpha^4} - 4\alpha^4 \log\left(\frac{1+\sqrt{1-4\alpha^4}}{2\alpha^2}\right) . \tag{27}
$$

Application to ABJM theory

- The strong coupling results are different from the weak coupling ones.
- This is as expected, since there are no non-renormalization theorems for BPS $3-$pt functions in ABJM theory.
- The result is to be tested against integrability.

Conclusion

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- For off-diagonal case $\langle \mathcal{D}_{M+k} | \mathcal{O}_J | \mathcal{D}_M \rangle$, the holographic result is sensitive to k, though $k \ll M, N$.

Outlook

- Compute the HHL correlators involving light non-BPS operators at arbitrary coupling in planar limit using integrability. *[Jiang, Komatsu,* ***JW****, Yang, in progress]*

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- Compute the HHL correlators involving light non-BPS operators at arbitrary coupling in planar limit using integrability. *[Jiang, Komatsu,* ***JW****, Yang, in progress]*
- Revisit the holographic computations of HHL correlators for GKP strings.

Thanks for Your Attention !