Conformal anomalies a vs c

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Based on [Agarwal, JS 1912.12881][Agarwal, KH Lee, JS 2007.16165]
[Kang, Lawrie, JS 2106.12579][Kang, Lawrie, KH Lee, JS 2111.xxxxx]

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Central charges of 4d CFT

- **Conformal anomalies** of a 4d CFT are parametrized by two parameters (central charges) *a* & *c*:

$$\langle T^{\mu}_{\mu} \rangle = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$$

- It is now well-established that *a*-function is a monotonically decreasing function along the RG flow (*a*-theorem): [Komargodski-Schwimmer]

$$a_{IR} < a_{UV}$$

- One can think of the *a*-function as a quantity that measures degrees of freedom.
- The *c*-function, on the other-hand, does ***not*** always decrease along the RG flow.

Hofman-Maldacena bound on central charges

- The ratio a/c of central charges is bounded by **unitarity**: [Hofman-Maldacena]

$$\frac{1}{2} \leq \frac{a}{c} \leq \frac{31}{18}$$ (lower/upper bound saturated by free scalar/free vector)

- For superconformal theory:
  - $\mathcal{N}$=1 SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$ (lower/upper bound saturated by free chiral/free vector)
  - $\mathcal{N}$=2 SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{5}{4}$ (lower/upper bound saturated by free hyper/free vector)
  - $\mathcal{N}$=3 or $\mathcal{N}$=4 SCFT: $a = c$ [Aharony-Evtikhiev]

The role of *a* and *c*

- Any **holographic** theories have $a = c$ (for **large *N***). [Henningson-Skenderis]

- When $a \neq c$, there is a correction to the celebrated **entropy-viscosity ratio bound** of [Kovtun-Son-Starinet] to [Katz-Petrov][Buchel-Myers-Sinha]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \cdots\right)$$

- Also appears in the universal part of **entanglement entropy**. [Perlmutter-Rangamani-Rota]

- The '**high-temperature limit**' of the supersymmetric index is governed by ***a*** & ***c***:
  [J. Kim, S. Kim, JS] [Cabo-Bizet, Cassani, Martelli, Murthy]

$$I(p = q = e^{-\beta}) \to \exp\left(\#\frac{3c - 2a}{\beta^2}\right)$$

  This formula accounts for the **entropy of supersymmetric black holes** in AdS_5.
  [Choi, Kim, Kim, Nahmgoong][Benini-Milan]

Large N scaling behavior of *a* and *c*

- Typical 4d gauge theories (of rank *N*) have

$$a \sim c \sim \mathcal{O}(N^2), \quad \text{and} \quad c - a \sim \mathcal{O}(N)$$

  so that $a = c$ in the **large *N*** limit, but **not for a finite *N***. (satisfying the necessary condition for it to be holographic)

- Is this true in general?
  - Is the above **scaling behavior** for *a* and *c* true in general?
  - Any **universality** for the sign of $c - a$?
  - Is it possible to have $a = c$ for **finite *N***? (for $\mathcal{N}$=0, 1, 2 SUSY)

Non-universal of scaling behavior of central charges a & c

Example: 'Simplest' Large N SCFT

[Agarwal, JS 1912]

Matter contents:

	$SU(N)$	$U(1)_B$	$U(1)_A$	R
Q	N	1	N	$1 - NR_\Phi$
$\tilde{Q}$	$\bar{N}$	-1	N	$1 - NR_\Phi$
Φ	adj	0	-1	R_Φ

Gauge invariant operators:

- Coulomb branch operators: $\Phi^n, \; 2 \leq n \leq N$
- dressed mesons: $Q\Phi^n\widetilde{Q}, \; 0 \leq n \leq N-1$
- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\ldots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\ldots(\Phi^{N-1}\widetilde{Q})$

It looks like any other gauge theories with a sparse low-lying spectrum.

This theory flows to a superconformal fixed point in the IR.

	$SU(N)$	$U(1)_B$	$U(1)_A$	R
Q	N	1	N	$1 - NR_\Phi$
$\tilde{Q}$	$\bar{N}$	-1	N	$1 - NR_\Phi$
Φ	adj	0	-1	R_Φ

- Coulomb branch operators: $\Phi^n,\ 2 \leq n \leq N$
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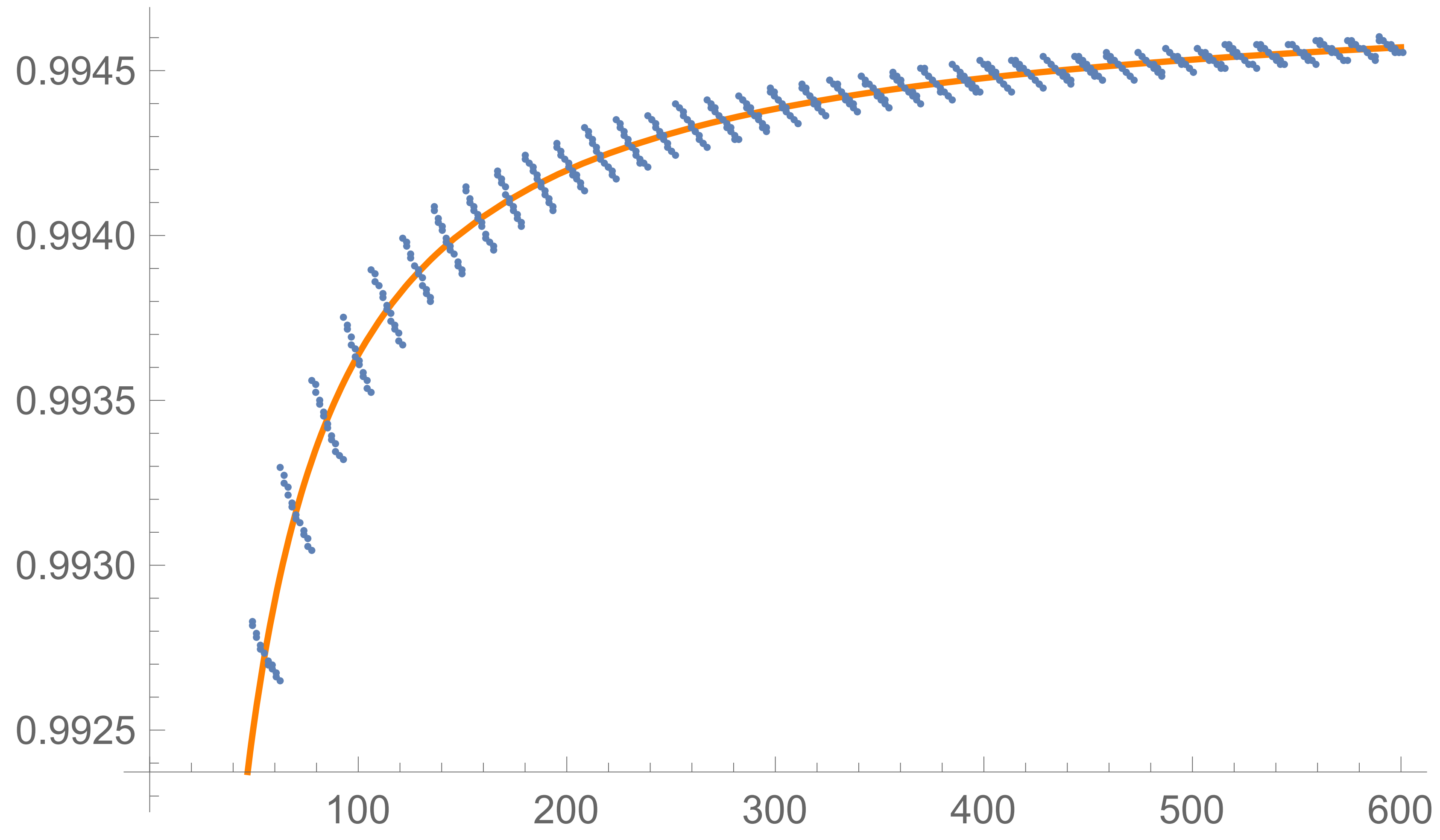
- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\ldots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\ldots(\Phi^{N-1}\widetilde{Q})$

- This simple theory flows to a superconformal fixed point with a number of decoupled free fields.
- Some of the Coulomb branch operators $\mathrm{Tr}\Phi^i$ and the dressed mesons $\tilde{Q}\Phi^i Q$ decouple for low *i*.
- None of the 'baryons' decouple. $\Delta_B \sim O(N)$
- The decoupled field can be removed by introducing flip field and the superpotential coupling $W = X\mathcal{O}$. "$\mathcal{O} \leftrightarrow X$"

Feature 1: The O(N) degrees of freedom



$$a \simeq 0.500819\, N - 0.692539$$

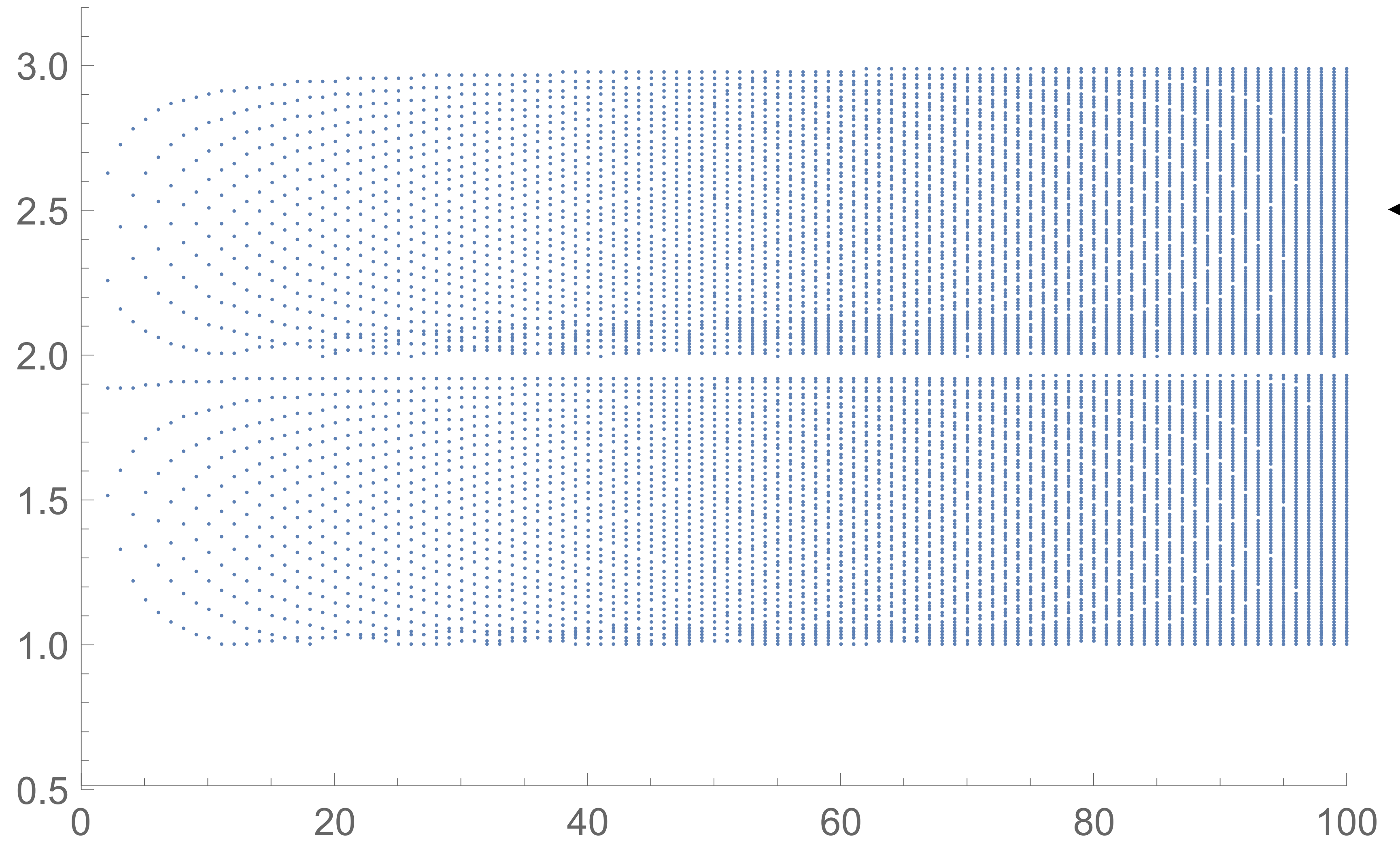
$$c \simeq 0.503462\, N - 0.640935$$

$$a/c \sim 0.994757 \, - 0.111888/N$$

The degrees of freedom grows as $O(N^1)$ instead of
the natural matrix degrees of freedom $O(N^2)$!

The ratio *a/c* asymptotes to a value close to 1, but *not exactly*.

Feature 2: Dense spectrum



Flip fields for $\mathrm{Tr}\Phi^i$ small *i*

$\mathrm{Tr}\Phi^i,\ Q\Phi^i\widetilde{Q}$

$R_\Phi \simeq 0.712086/N$

$R_Q \simeq 0.284372 + 0.609971/N$

The spectrum of chiral operators form a **dense band**, instead of being sparse!
(analog of the Liouville theory? Decompactification?)

It does not seem to exhibit confinement/deconfinement transition.

Classifying SUSY large *N* theories

[Agarwal, Lee, JS]

- Let us classify all possible supersymmetric **large *N* *gauge theories*** in 4d with the following conditions:
  - The gauge group is **simple**: G=SU(*N*), SO(*N*), Sp(*N*)
  - The **flavor symmetry is fixed** as we take large *N* limit.
  - No superpotential except the flip for the decoupled ops (at the moment).
- In the context of AdS/CFT:
  flavor symmetry of the boundary CFT = **gauge symmetry in the bulk**.

See [Bhardwaj, Tachikawa] for the classification of $\mathcal{N}$=2 gauge theories.

The full list of SU(N) theories with large N limit. (4+16 classes of theories)

Theory	β_{matter}	chiral	dense	N_f
$1\ \mathbf{Adj} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$N_f \geq 1$
$1\ \ydiagram{2} + 1\ \overline{\ydiagram{2}} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$N_f \geq 0$
$1\ \ydiagram{1,1} + 1\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$N_f \geq 4$
$1\ \ydiagram{2} + 1\ \overline{\ydiagram{1,1}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim N$	Y	Y	$N_f \geq 0$
$2\ \ydiagram{2} + 2\ \overline{\ydiagram{2}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$1\ \ydiagram{2} + 2\ \overline{\ydiagram{2}} + 1\ \ydiagram{1,1} + 8\ \square + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$N_f \geq 0$
$1\ \ydiagram{2} + 1\ \overline{\ydiagram{2}} + 1\ \ydiagram{1,1} + 1\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$1\ \ydiagram{2} + 1\ \ydiagram{1,1} + 2\ \overline{\ydiagram{1,1}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$N_f \geq 0$
$2\ \ydiagram{2} + 2\ \overline{\ydiagram{1,1}} + 16\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$N_f \geq 0$
$1\ \mathbf{Adj} + 1\ \ydiagram{2} + 1\ \overline{\ydiagram{2}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$2\ \ydiagram{1,1} + 2\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$1\ \mathbf{Adj} + 1\ \ydiagram{2} + 1\ \overline{\ydiagram{1,1}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$N_f \geq 0$
$1\ \mathbf{Adj} + 1\ \ydiagram{1,1} + 1\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$2\ \mathbf{Adj} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$N_f \geq 0$
$1\ (\ydiagram{2} + \overline{\ydiagram{2}}) + 2\ (\ydiagram{1,1} + \overline{\ydiagram{1,1}}) + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \leq N_f \leq 2$
$3\ \ydiagram{1,1} + 3\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \leq N_f \leq 6$
$1\ \mathbf{Adj} + 2\ \ydiagram{1,1} + 2\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \leq N_f \leq 4$
$1\ \mathbf{Adj} + 1\ (\ydiagram{2} + \overline{\ydiagram{2}}) + 1\ (\ydiagram{1,1} + \overline{\ydiagram{1,1}})$	$\sim 3N$	N	N	$\cdot$
$2\ \mathbf{Adj} + 1\ \ydiagram{1,1} + 1\ \overline{\ydiagram{1,1}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \leq N_f \leq 2$
$3\ \mathbf{Adj}$	$\sim 3N$	N	N	$\cdot$

$\mathcal{N}$=2 SCFT (for N_f = 4)

$\mathcal{N}$=4 SYM

Theory	β_{matter}	dense spectrum	N_f
$1\,\square\square + N_f\,\square$	$\sim\, N$	Y	$N_f \geq 0$
$1\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + N_f\,\square$	$\sim\, N$	Y	$N_f \geq 1$
$2\,\square\square + N_f\,\square$	$\sim 2\,N$	N	$N_f \geq 0$
$1\,\square\square + 1\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + N_f\,\square$	$\sim 2\,N$	N	$N_f \geq 0$
$2\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + N_f\,\square$	$\sim 2\,N$	N	$N_f \geq 0$
$3\,\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\sim 3\,N$	N	.

SO(*N*) theories

Theory	β_{matter}	dense spectrum	N_f
$1\,\square\square + 2N_f\,\square$	$\sim N$	Y	$N_f \geq 1$
$1\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2N_f\,\square$	$\sim N$	Y	$N_f \geq 4$
$2\,\square\square + 2N_f\,\square$	$\sim 2N$	N	$N_f \geq 0$
$1\,\square\square + 1\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$N_f \geq 0$
$2\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2\,N_f\,\square$	$\sim 2N$	N	$N_f \geq 0$
$2\,\square\square + 1\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f \leq 2$
$1\,\square\square + 2\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f \leq 4$
$3\,\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2N_f\,\square$	$\sim 3N$	N	$N_f \leq 6$
$3\,\square\square$	$\sim 3N$	N	.

Sp(*N*) theories

$\longleftarrow$ $\mathcal{N}$=2 SCFT for N_f = 0

Feature 3: Multiple bands

eg) SU(N) + 1 adj + N_f=2

[Agarwal, Lee, JS]

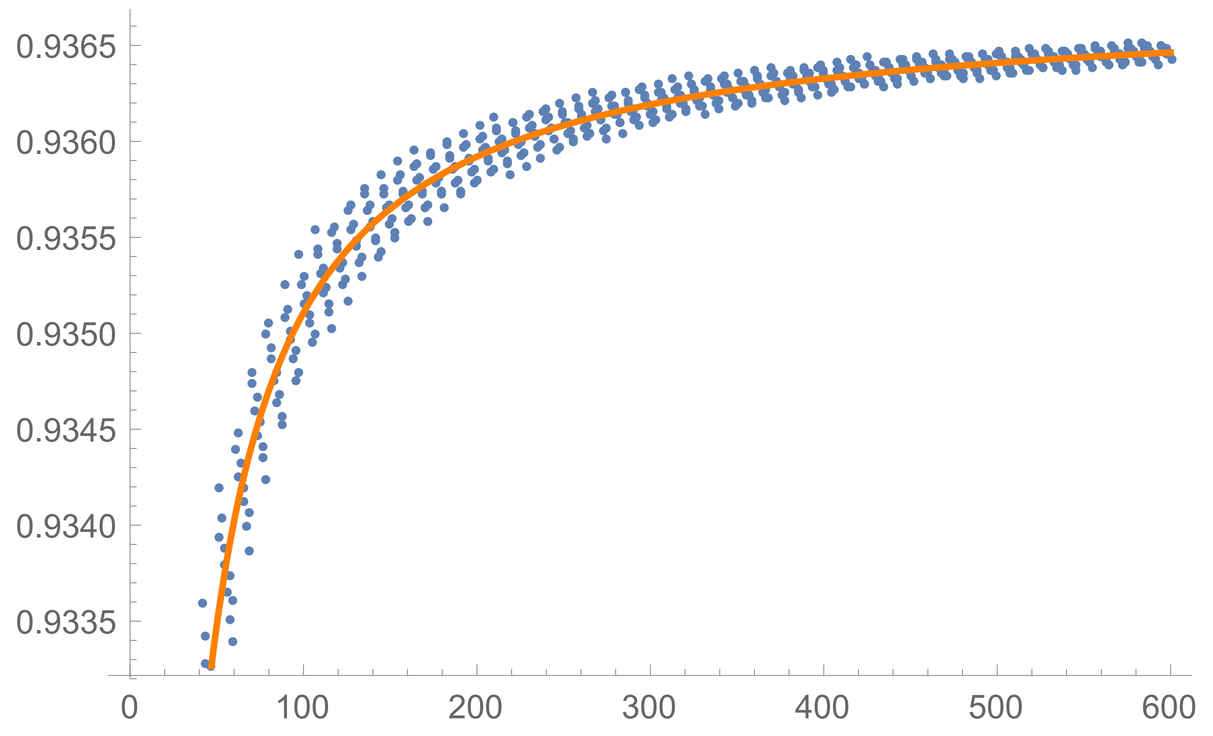


Figure 6: Plot of a/c vs N for the $SU(N)$ theory with 1 adjoint and $N_f = 2$. The orange curve fits the plot with $a/c \sim 0.936734 - 0.162684/N$.

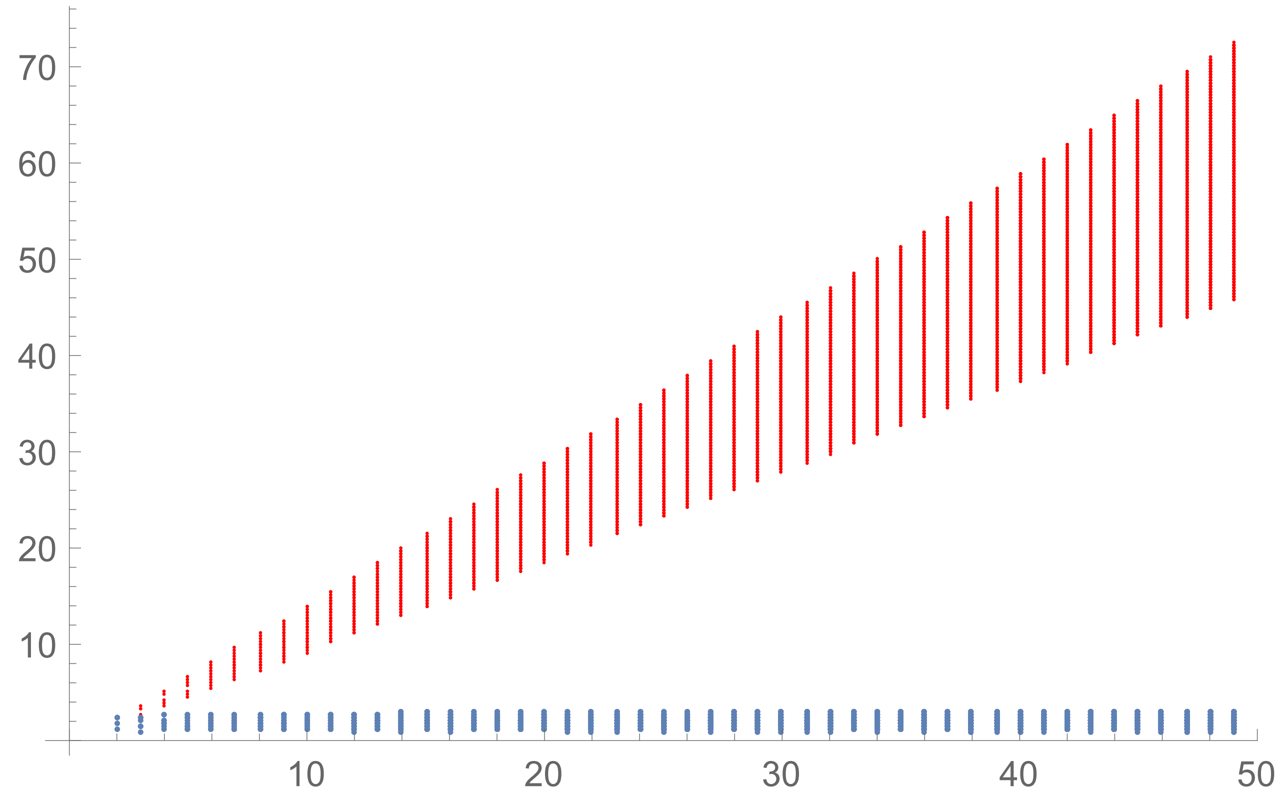


Figure 8: Dimensions of single-trace gauge-invariant operators including baryons in $SU(N)$ + 1 **Adj** + 2 ($\square + \overline{\square}$) theory. The baryons(red) form another band above the band of Coulomb branch operators and mesons.

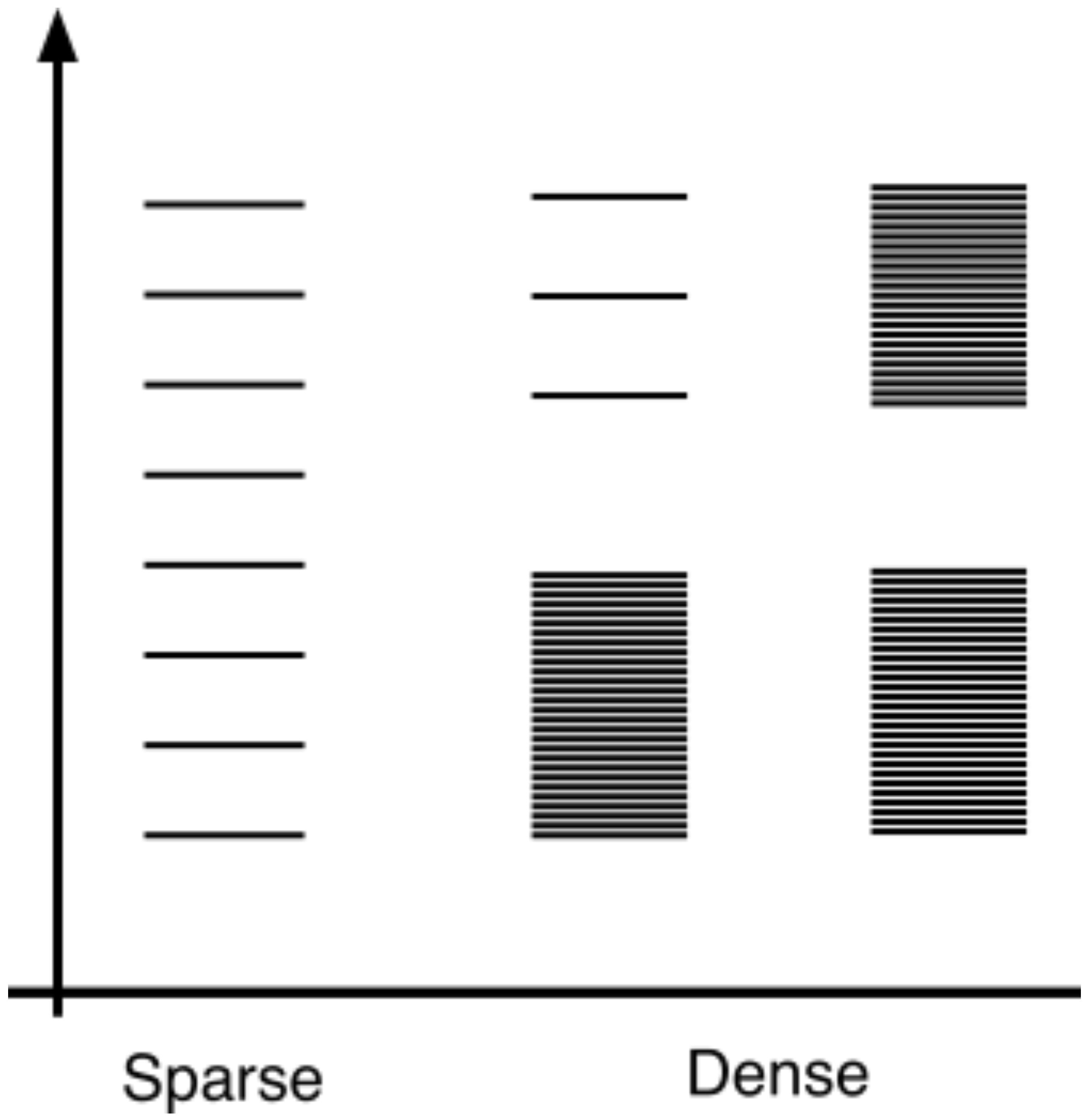
The ratio of central charges a/c does not go to 1.

We see the **secondary band** of size O(*N*). They are formed by 'baryons'.

- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\ldots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\ldots(\Phi^{N-1}\widetilde{Q})$

Supersymmetric analog of 'band' theory?

Sparse vs Dense spectrum



Out of 35 classes of all possible large *N* gauge theories, 8 of them have **dense spectrum** and the rest have sparse spectrum.

Sparse: The gap is O(1). $a = c$ at large N.

Dense: The gap is O(1/*N*). $a \neq c$ at large N.

$c - a$ can have either sign.
No universality!

Can we have 4d CFTs with $a = c$ even at finite N? (with $\mathcal{N}$=0, 1, 2 SUSY)

*$\mathcal{N}$=3, 4 SCFTs ***must*** have a=c.

$\mathcal{N}$=2 SCFTs with $a = c$ (and beyond)

[Kang-Lawrie-JS]

- There exists **genuinely $\mathcal{N}$=2 SCFTs** with $a = c$ (exact in N)!
- $\hat{\Gamma}(G)$ theory labelled by two ADE Lie algebras Γ, G.
  - G labels the 'gauge group' and Γ labels the shape of the 'quiver'.
  - Ingredients:
    - $\mathcal{D}_p[G]$ Argyres-Douglas type theories. [Cecotti-Del Zotto]
    - (G, G) conformal matter theories. [Del Zotto-Heckman-Tomasiello-Vafa] [Ohmori-Shimizu-Tachikawa-Yonekura]
    - Gauge the diagonal G. It is a non-Lagrangian theory in general.
- For $\Gamma = D_4, E_6, E_7, E_8$ and some special choice of G, $\hat{\Gamma}(G)$ theory has $a = c$.
  These choices do not involve conformal matter. ($a \neq c$ for other choices)

$\mathscr{D}_p[G]$ theory

[Cecotti-Del Zotto]
[Cecotti-Del Zotto-Giacomelli]
[Xie][Wang-Xie]

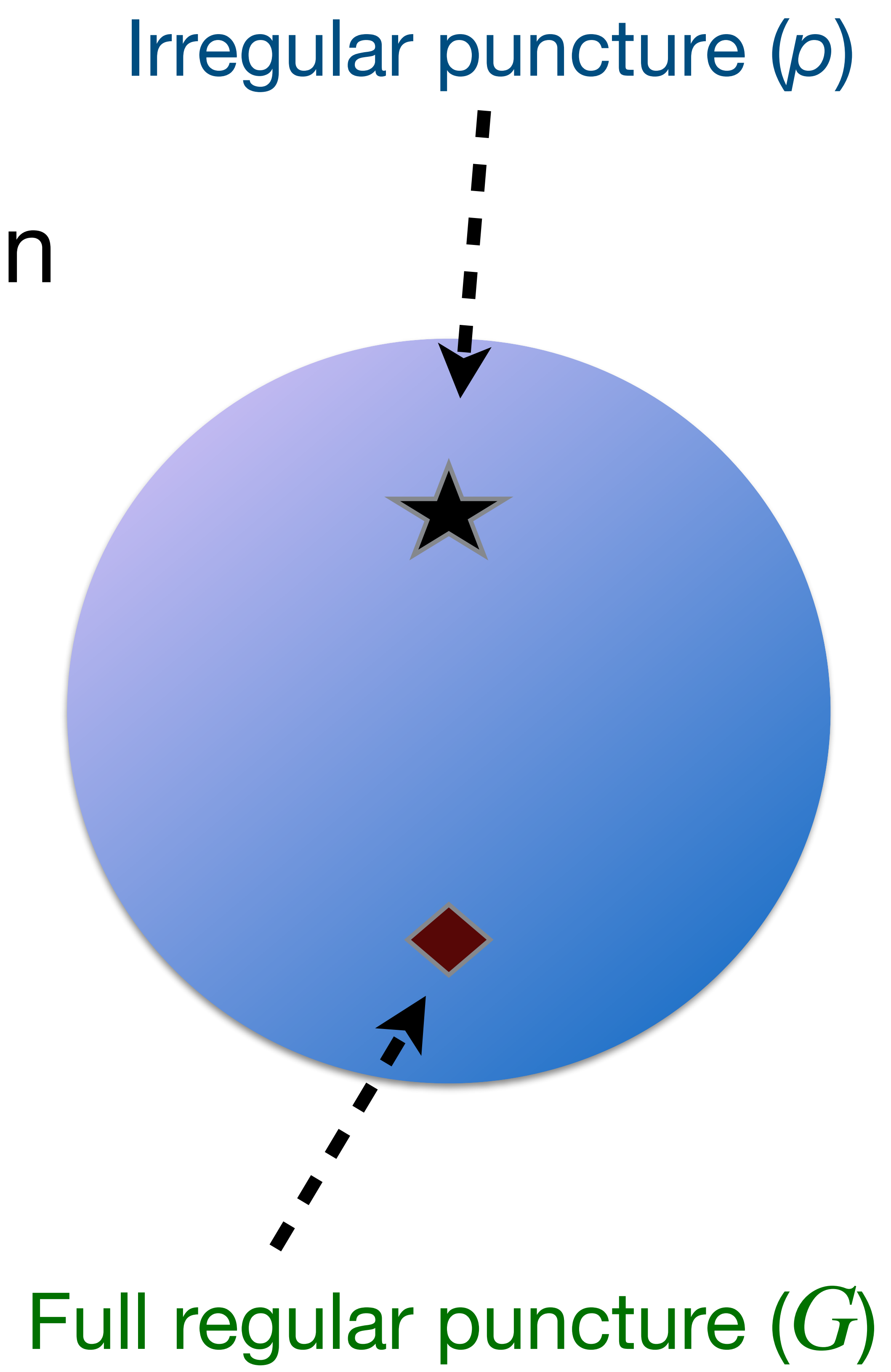
- It is a 4d $\mathcal{N}$=2 SCFT (Argyres-Douglas type) with **flavor symmetry G** (or larger).

- It can be realized as the 6d $\mathcal{N}$=(2, 0) theory of type G compactified on a **sphere** with one **irregular puncture** (p) and one **full regular puncture** (flavor G).

- The flavor symmetry is **exactly** G for *some choice* of *p*, when the irregular puncture does not possess extra flavor symmetry.

- The flavor central charge for G: $$k_G = \frac{2(p-1)}{p} h_G^\vee$$

G	$SU(N)$	$SO(2N)$	E_6	E_7	E_8
No additional symmetry	$(p,N)=1$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 3\mathbb{Z}_{>0}$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 30\mathbb{Z}_{>0}$



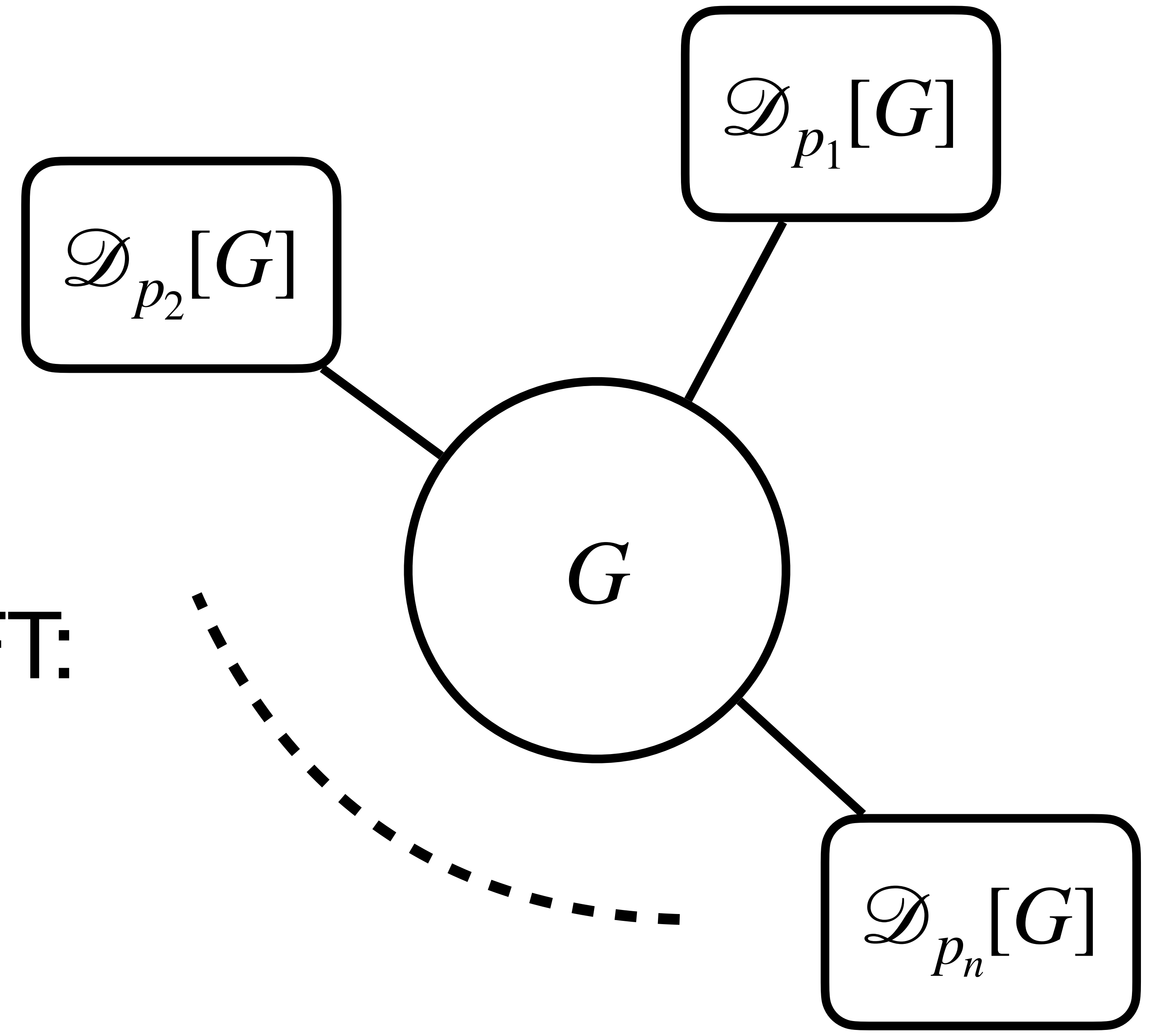
Gauging $\mathscr{D}_p[G]$ theories

[Cecotti,Vafa]
[Cecotti,Del Zotto, Giacomelli]
[Closset,Giacomelli,Schafer-Nameki,Wang]
[Kang-Lawrie-JS]

- In order to gauge the flavor and obtain SCFT, the 1-loop beta function for the gauge group should vanish:

$$\beta_G = 0 \quad \leftrightarrow \quad \sum_i k_i = 4h_G^\vee$$

flavor central charges k_i : "matter" contribution to the beta function.



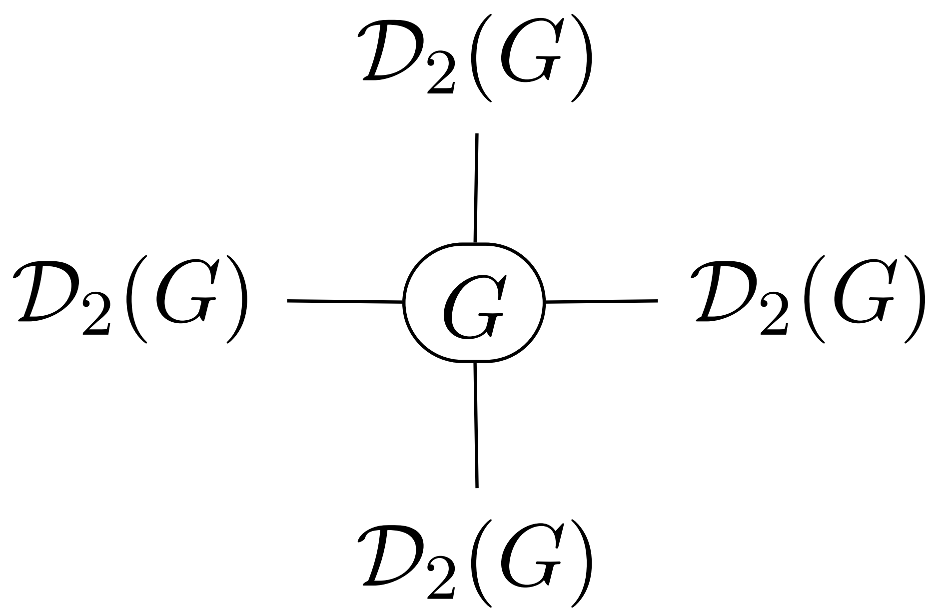
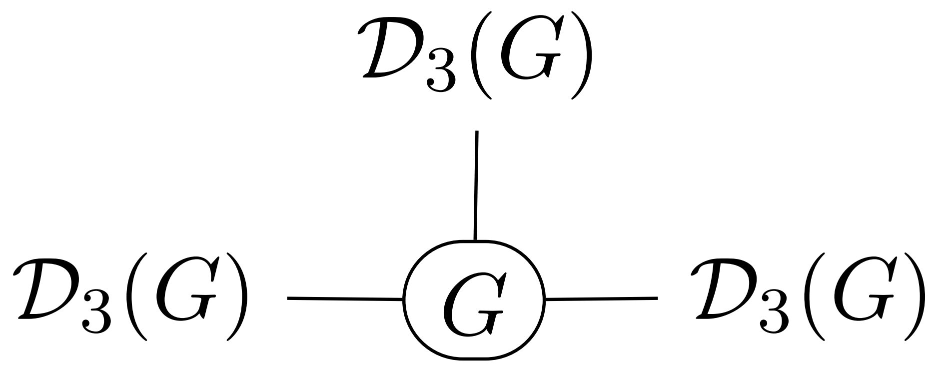
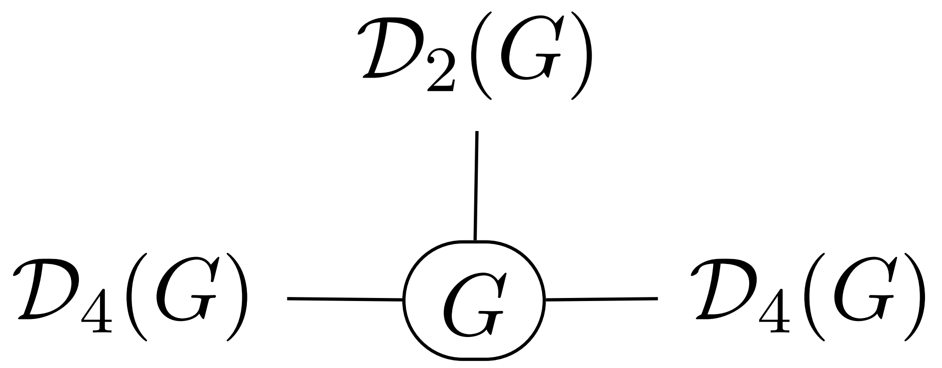
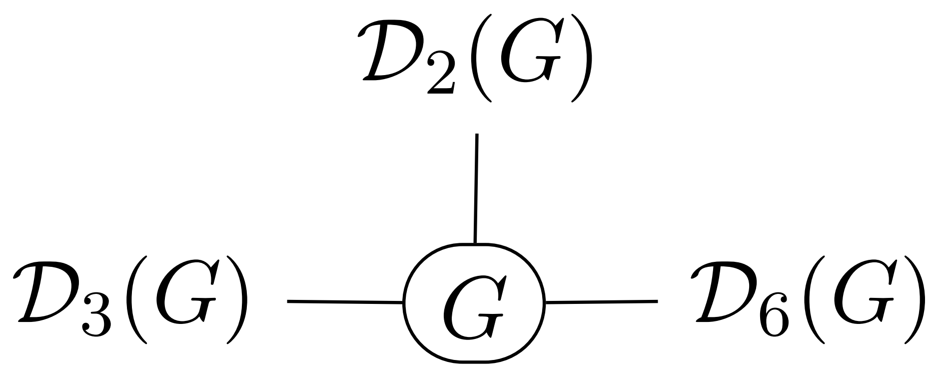
- Consider gluing a number of $\mathscr{D}_p[G]$ theories to form $\mathcal{N}$=2 SCFT:

$$\sum_{i=1}^{n} \frac{2(p_i - 1)}{p_i} h_G^\vee = 4h_G^\vee \quad \rightarrow \quad \sum_{i=1}^{n} \frac{1}{p_i} = n - 2$$

- **Only 4 non-trivial solutions**: (2, 2, 2, 2), (3, 3, 3), (2, 4, 4), (2, 3, 6)

$\hat{\Gamma}(G)$ theory with $\Gamma = D_4, E_6, E_7, E_8$

[Kang-Lawrie-JS]

(p_1, p_2, p_3, p_4)	$\widehat{\Gamma}(G)$	Quivers via gauging $\mathcal{D}_p(G)$s	$a = c$
$(2,2,2,2)$	$\widehat{D}_4(G)$		$\frac{1}{2}\dim(G)$
$(1,3,3,3)$	$\widehat{E}_6(G)$		$\frac{2}{3}\dim(G)$
$(1,2,4,4)$	$\widehat{E}_7(G)$		$\frac{3}{4}\dim(G)$
$(1,2,3,6)$	$\widehat{E}_8(G)$		$\frac{5}{6}\dim(G)$

$a = c$ is obtained when the largest comark α_Γ of Γ satisfies

$$\gcd(h^\vee_G, \alpha_\Gamma) = 1 \quad \Longrightarrow \quad a = c.$$

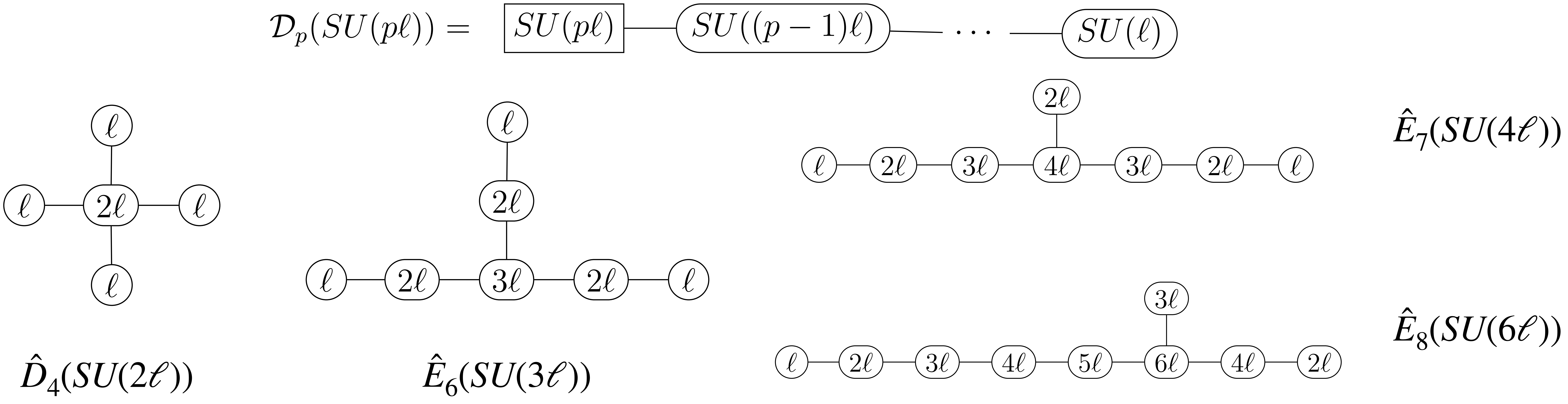
$$\alpha_{D_4} = 2, \ \alpha_{E_6} = 3, \ \alpha_{E_7} = 4, \ \alpha_{E_8} = 6.$$

The $\hat{\Gamma}(G)$ theory with $a = c$ has **no flavor symmetry**.

Lagrangian $\hat{\Gamma}(G)$ theory with $\Gamma = D_4, E_6, E_7, E_8$

- When $G = SU(N)$ with $N = \alpha_\Gamma \ell$, we recover **affine quiver gauge theory** obtained via ℓ D3-branes probing ALE singularity $\mathbb{C}^2/\Gamma$.

- Our theory is a natural generalization of the affine quiver gauge theory. For $N \neq \alpha_\Gamma \ell$, we 'fractionalize' the D3-brane charges.



$\mathcal{N}$=2 SCFTs with $a = c$

- Some of these theories have class-$\mathcal{S}$ realization
  - $\hat{E}_6(SU(2)) = (A_2, D_4)$:
    - Coulomb branch op: {4/3, 4/3, 4/3, 2}
  - $\hat{E}_7(SU(3)) = E_6^{12}[4]$
  - $\hat{E}_8(SU(5)) = E_8^{30}[6]$
  - Most of $\hat{\Gamma}(G)$ theories are not found in class-$\mathcal{S}$.
- They all have 1 exactly marginal coupling.
- They all have center 1-form symmetry $Z(G)$.

$\widehat{\Gamma}(G)$	$a = c$
$\widehat{D}_4(SU(2\ell+1))$	$2\ell(\ell+1)$
$\widehat{E}_6(SU(3\ell \pm 1))$	$2\ell(3\ell \pm 2)$
$\widehat{E}_6(SO(6\ell))$	$2\ell(6\ell+1)$
$\widehat{E}_6(SO(6\ell+4))$	$2(2\ell+1)(3\ell+2)$
$\widehat{E}_7(SU(4\ell \pm 1))$	$6\ell(2\ell \pm 1)$
$\widehat{E}_8(SU(6\ell \pm 1))$	$10\ell(3\ell \pm 1)$

The full list of $a = c$ theories in $\hat{\Gamma}(G)$

Schur index for $\Gamma = D_4, E_6, E_7, E_8$

- For the $a = c$ theories we consider, the relevant $\mathscr{D}_p[G]$ theories **do not have additional flavor** symmetry besides G. For such case, **a concise expression for the Schur index** is known:

$$I_{\mathscr{D}_p(G)}(q, \vec{z}) = \mathrm{PE}\left[\frac{q - q^p}{(1-q)(1-q^p)} \chi^G_{\mathrm{adj}}(\vec{z})\right]$$

[JS-Xie-Yan]
[Kac-Wakimoto]

- From this, we obtain a neat expression of the Schur index for the $\hat{\Gamma}(G)$ theory as:

$$I_{\hat{\Gamma}(G)}(q) = \int [d\vec{z}]\mathrm{PE}\left[\frac{q + q^{\alpha_\Gamma - 1} - 2q^{\alpha_\Gamma}}{(1-q)(1-q^{\alpha_\Gamma})} \chi^G_{\mathrm{adj}}(\vec{z})\right]$$

- For the $\hat{D}_4(SU(2\ell+1))$ theory, we find the index can be written in terms of MacMahon's generalized 'sum-of-divisor' function which is **quasi-modular**:

$$I_{\widehat{D}_4(SU(2k+1))}(q) = q^{-k(k+1)} A_k(q^2)$$

$$I^{\mathcal{N}=4}_{SU(2k+1)}(q) = q^{-\frac{k(k+1)}{2}} A_k(q)$$

$$A_k(q) = \sum_{0<m_1<m_2 \cdots <m_k} \frac{q^{m_1 + \cdots m_k}}{(1-q^{m_1})^2 \cdots (1-q^{m_k})^2}$$

$\mathcal{N}$=4 SYM and $\hat{\Gamma}(G)$ theory

- The Schur index of $\hat{\Gamma}(G)$ theory is **identical to that of the $\mathcal{N}$=4 SYM** upon rescaling!

$$I_{\hat{\Gamma}(G)}(q) = I_G^{\mathcal{N}=4}(q^{\alpha_\Gamma}; q^{\alpha_\Gamma/2-1})$$

- This relation **holds beyond $a = c$ theories:**

$$\widehat{\Gamma}(SU(N)) \quad \text{with } \gcd(\alpha_\Gamma, N) = 1\,,$$
$$\widehat{E}_6(SO(2N))\,, \quad \widehat{D}_4(E_6)\,, \quad \widehat{E}_6(E_7)\,, \quad \widehat{E}_7(E_6)\,,$$
$$\widehat{D}_4(E_8)\,, \quad \widehat{E}_6(E_8)\,, \quad \widehat{E}_7(E_8)\,, \quad \widehat{E}_8(E_8)\,.$$

- The SU(N) case was found earlier by [Buican-Nishinaka] and showed that there is an isomorphism between **associated VOA**s as a graded vector space.

- More connections to $\mathcal{N}$=4 SYM:
  - 1 exactly marginal gauge coupling (S-duality?)
  - 1-form center symmetry $Z(G)$.

Generalization to $\mathcal{N}$=1 SCFTs

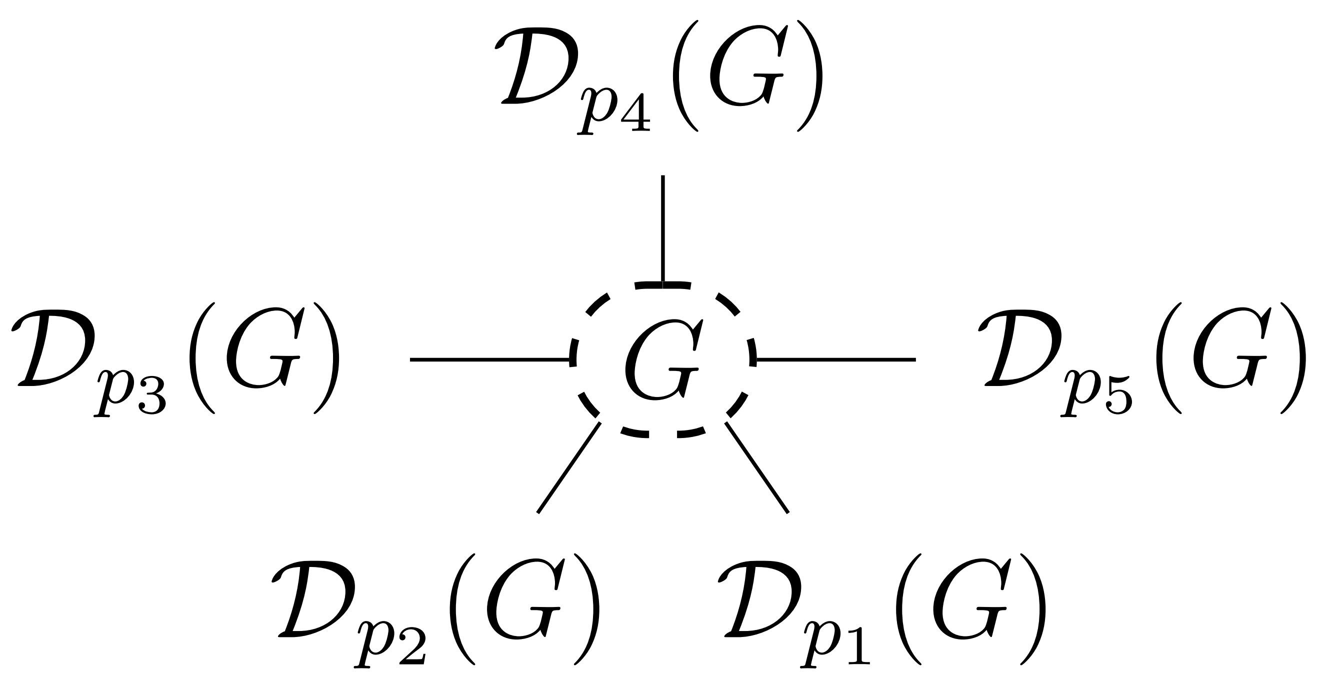
[Kang-Lawrie-Lee-JS, to appear]

- Consider a number of $\mathscr{D}_p[G]$ theories gauged via **$\mathcal{N}$=1 vector** multiplet.

- It modifies the condition to be a CFT in the IR, since the theory now **RG flows**. From **asymptotic freedom** bound:

$$\sum_{i=1}^{N} \frac{2(p_i-1)}{p_i} h_G^\vee < 6h_G^\vee \qquad \sum_{i=1}^{N} \frac{1}{p_i} > N-3$$

- The IR SCFT has a number of U(1) **flavor symmetry** originates from broken R-symmetry of each block.



p_1	p_2	p_3	p_4	p_5
1	1	1	1	p_5
1	1	1	p_4	p_5
1	1	p_3	p_4	p_5
1	2	2	p_4	p_5
1	2	3	≤ 6	p_5
1	2	3	7	≤ 41
1	2	3	8	≤ 23
1	2	3	9	≤ 17

p_1	p_2	p_3	p_4	p_5
1	2	3	10	≤ 14
1	2	3	11	≤ 13
1	2	4	4	p_5
1	2	4	5	≤ 19
1	2	4	6	≤ 11
1	2	4	7	≤ 9
1	2	5	5	≤ 9
1	2	5	6	≤ 7

p_1	p_2	p_3	p_4	p_5
1	3	3	3	p_4
1	3	3	4	≤ 11
1	3	3	5	≤ 7
1	3	4	4	≤ 5
2	2	2	2	p_5
2	2	2	3	3
2	2	2	3	4
2	2	2	3	5

Tuples of (p_i)'s satisfying the asymptotic freedom bound.

Unitarity at the fixed point

- Besides checking asymptotic freedom, we should also make sure that the IR theory is a valid CFT.

Unitarity bound: $\Delta \geq 1 \leftrightarrow R \geq \frac{2}{3}$ for the chiral operators.

- For a SCFT, we need to deduce superconformal R-charges to check unitarity. It can be done using **a-maximization**:

[Intriligator-Wecht]

  - Consider a linear combination of the U(1) charges $R_{IR} = R_{UV} + \epsilon_i F_i$ and then maximize the trial a-function w.r.t to ϵ:

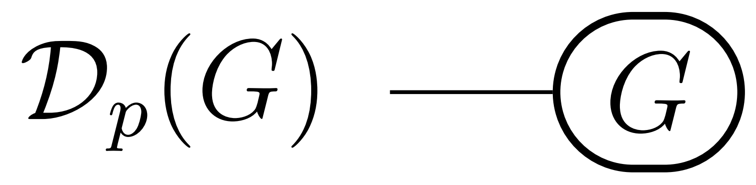
$$a = \frac{3}{32}(3\text{Tr}R^3 - \text{Tr}R), \qquad \frac{\partial a_{\text{trial}}}{\partial R} = 0 \ , \quad \frac{\partial^2 a_{\text{trial}}}{\partial R^2} < 0$$

- If all the (BPS) operators satisfy the bound, we are good to go. (*Not always sufficient!)

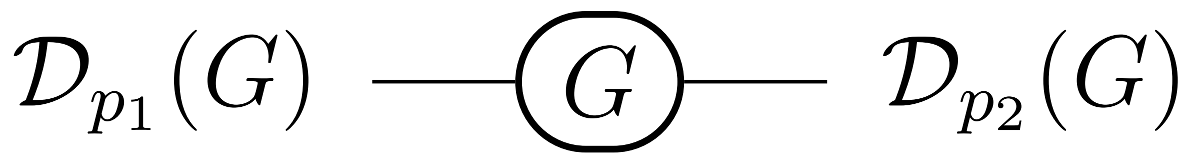
*[Maruyoshi-Nardoni-JS]

Results:

We need to check: $$-\frac{p_i+1}{3(p_i-1)} \leq \epsilon_i \leq \frac{1}{3(p_i+1)}\,, \quad \epsilon_i + \epsilon_{j\neq i} \geq -1$$

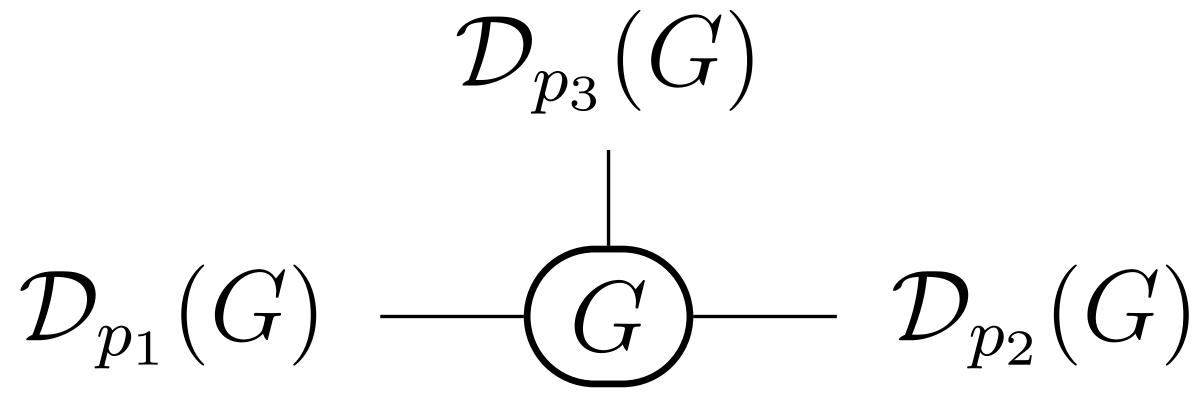


Gluing 1 $\mathscr{D}_p[G]$: no SCFT

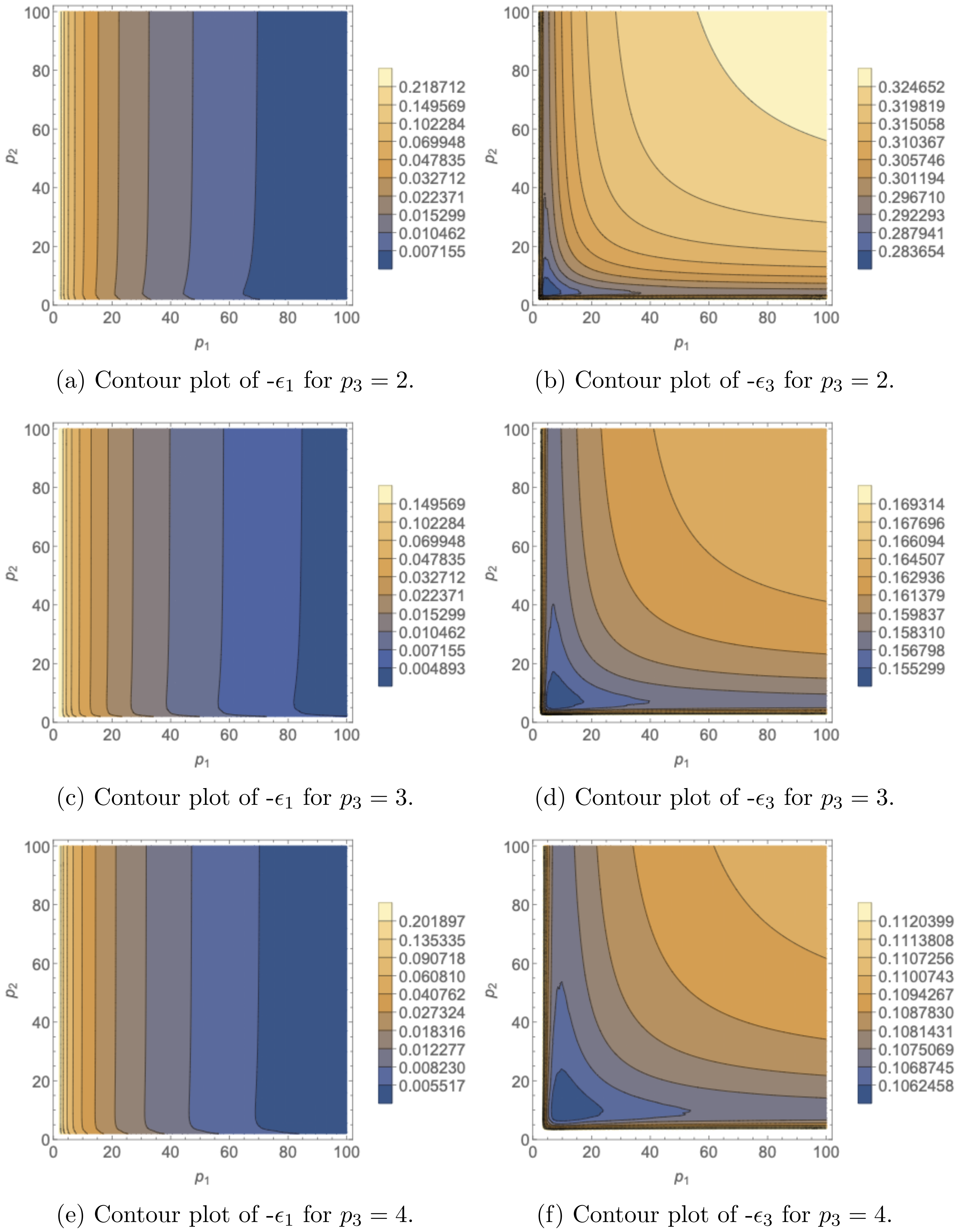


Gluing 2 $\mathscr{D}_p[G]$: $\quad p_1 \geq 3 \quad \text{and} \quad p_2 \geq 3$

$$a = c = \frac{9p(p-2)}{64(p-1)}\dim(G) \quad \text{when } p_1 = p_2 = p$$



Gluing 3 $\mathscr{D}_p[G]$: Need to check numerically.



(a) Contour plot of -ϵ_1 for $p_3 = 2$. (b) Contour plot of -ϵ_3 for $p_3 = 2$.

(c) Contour plot of -ϵ_1 for $p_3 = 3$. (d) Contour plot of -ϵ_3 for $p_3 = 3$.

(e) Contour plot of -ϵ_1 for $p_3 = 4$. (f) Contour plot of -ϵ_3 for $p_3 = 4$.

Figure 2.2: Contours plot of ϵ_1 and ϵ_3 in the (p_1, p_2) plane for $p_3 = 2, 3, 4$. They all satisfy the unitarity condition in equation (2.22).

Gluing 3 $\mathscr{D}_p[G]$:

no unitarity violations for generic p.

Gluing 4 $\mathscr{D}_p[G]$:

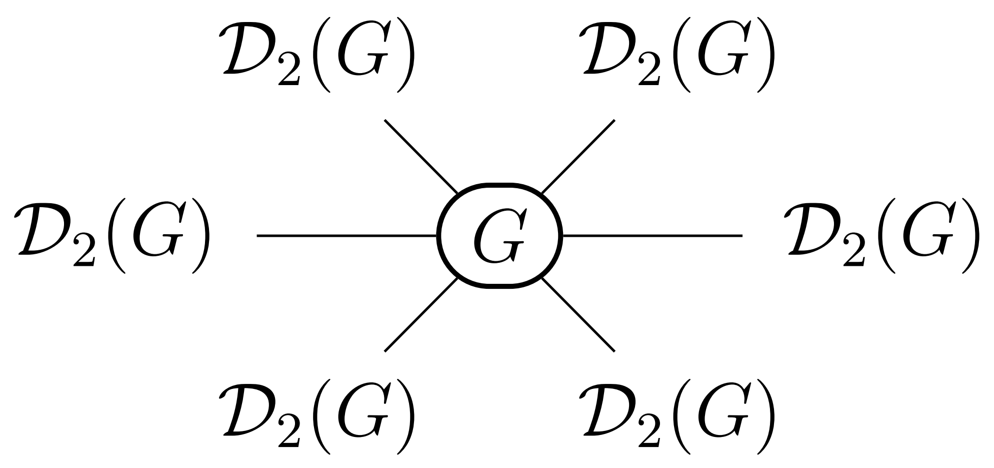
no unitary violations for generic p.

Gluing 5 $\mathscr{D}_p[G]$:

no unitary violations for generic p.

Gluing 6 $\mathscr{D}_p[G]$: “conformal gauging”

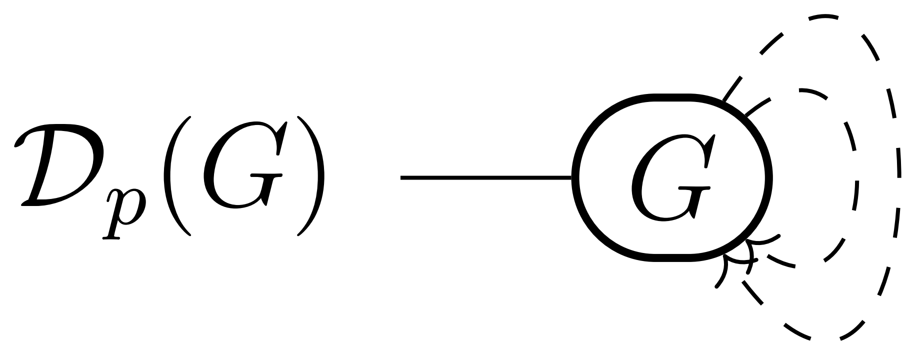
- vanishing beta function. It *does not flow*.
- unless there is an exactly marginal operator, no non-trivial SCFT.
- some of them are indeed non-trivial SCFT.



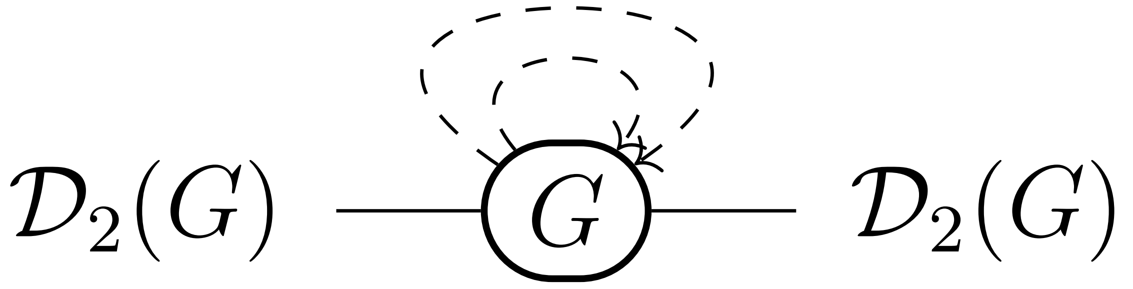
Landscape of $\mathcal{N}$=1 SCFTs with $a = c$

- In addition, one can add 1 or 2 **adjoint chiral** multiplets.
- 1 adjoint: can attach up to 4 $\mathscr{D}_p[G]$ theories.

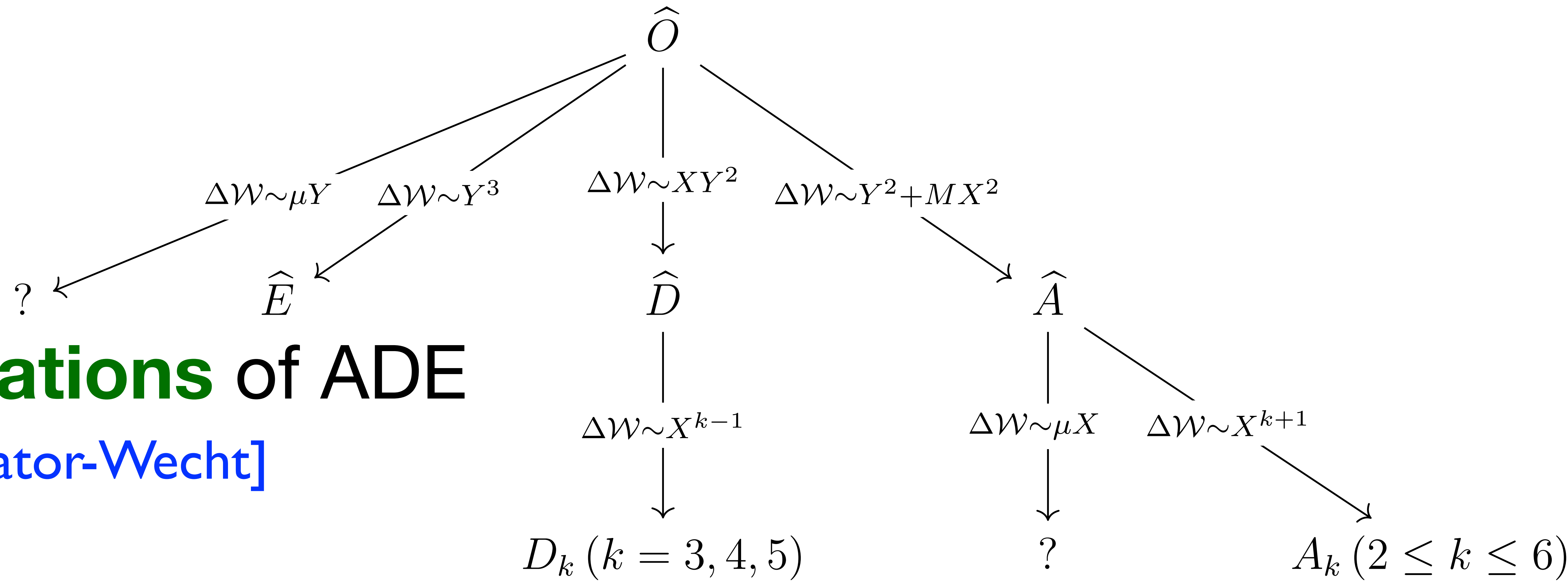
$$p_i = (p_1, p_2),\ (2,2,p_3),\ (2,3,\leq 6),\ (2,4,4),\ (3,3,3),\ (2,2,2,2)$$



- 2 adjoints: One can even have zero $\mathscr{D}_p[G]$ theories!



  - **The simplest *Lagrangian* model** with $a = c$: $\mathcal{N}$=1 gauge theory with 2 adjoints.
  - Can attach up to 2 $\mathscr{D}_p[G]$'s
- One can consider **superpotential deformations** of ADE type as in the case of adjoint SQCD. [Intriligator-Wecht]



Conclusion

Summary & future direction

- **Conformal anomalies** *a & c* of 4d CFTs capture many interesting aspects of underlying theory. (entropy-viscosity ratio, density of states, black hole entropy, entanglement entropy)
- The **scaling behavior** of *a & c* in the large *N* gauge theory is **not universal**: *c-a* can have either signs, $a \sim c \sim O(N^2)$ or $O(N^1)$
- We have constructed **genuinely** $\mathcal{N}$=1, 2 SCFTs with $a = c$, **exact in *N***. The 'landscape' of such theories is huge! What about $\mathcal{N}$=0?
- Such $\mathcal{N}$=2 SCFTs $\hat{\Gamma}(G)$ share many properties with $\mathcal{N}$=4 SYM. Especially, we find the Schur index to be almost identical upon rescaling:

  $$I_{\widehat{\Gamma}(G)}(q) = I_G^{\mathcal{N}=4}(q^{\alpha_\Gamma}, q^{\alpha_\Gamma/2-1})$$

  Why such a relation holds?
- What is the **holographic dual** of such $a = c$ theories? It should forbid particular type of corrections in SUGRA action without any symmetry constraints. How?

Thank you!