On Galilean Conformal Bootstrap

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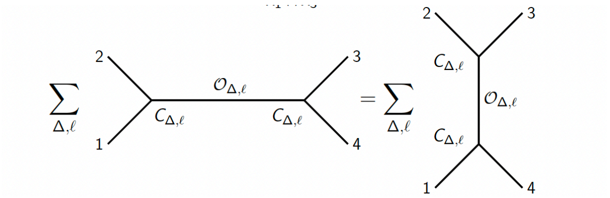
Based on the works
BC, Peng-xiang Hao, Reiko Liu and Zhe-fei Yu,
2011.11092, 2112.xxxxx, works in progress

Conformal bootstrap

A completely nonperturbative tool to study field theories!

Conformal bootstrap aims to constrain the CFT data by using the crossing symmetry and unitarity.



The crossing equation

$$v^{\Delta_{\mathcal{O}}} \sum_{\Delta,\ell} C_{12\Delta} C_{34\Delta} G_{\Delta,\ell}(u,v) = u^{\Delta_{\mathcal{O}}} \sum_{\Delta,\ell} C_{14\Delta} C_{23\Delta} G_{\Delta,\ell}(v,u)$$

where u, v are the conformal invariant cross-ratios, and $G_{\Delta,\ell}(u,v)$ is called the conformal block (CB).
CFT data: spectrum $\{\mathcal{O}_i\}$ with $\{\Delta_i, \ell_i\}$, and the OPE coefficients C_{ijk}.

It would be interesting to extend conformal bootstrap program to field theories with other conformal-like symmetries.

Schrödinger symmetry W. Goldberger et.al. 1412.8507
Carrollian conformal symmetry and Galilean conformal symmetry.
Warped conformal symmetry in 2D, Anisotropic Galilean conformal symmetry in 2D, ...

In this talk, I would like to report our study of 2D Galilean conformal field theories (GCFT) in the past few years.
(+ some recent studies on Carrollian CFT and GCFT in higher dimensions, if time permits)

Galilean conformal symmetry

Typical feature: in any dimensions, it is generated by an infinite dimensional algebra, being called Galilean conformal algebra (GCA) Bagchi and Gopakumar 0902.1385

Global part: could be obtained by a non-relativistic contraction of the conformal symmetry M. Negro et.al. (1997), J. Lukierski et.al. 0511259

- Translations, Isotropic scaling, Galilean transformations
- Analogues of special conformal transformations,

The full GCA could be obtained by taking the non-relativistic limit of conformal Killing equations, and is the maximal subset of non-relativistic conformal isometries C. Duval and P. Horvathy 0904.0531, D. Martelli and Y. Tachikawa, 0903.5184

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In particular, 2D GCA is isotropic to BMS_3

$\hookrightarrow$ Flat holography Bagchi 1006.3354, $\cdots$

2D Galilean conformal symmetry

Symmetry:

$$x \to f(x), \qquad y \to f'(x) y.$$
$$x \to x, \qquad y \to y + g(x).$$

The symmetry is generated by the Galilean conformal algebra Bagchi et.al. 0912.1090

$$[L_n, L_m] = (n-m)L_{n+m} + C_T n(n^2-1)\delta_{n+m,0},$$
$$[L_n, M_m] = (n-m)M_{n+m} + C_M n(n^2-1)\delta_{n+m,0},$$
$$[M_n, M_m] = 0.$$

Global subalgebra: $\{L_{\pm1}, L_0, M_{\pm1}, M_0\}$

Cartan subalgebra: $\{L_0, M_0\}$

Primary operators

The local operators in a GCFT_2 can be labelled by the eigenvalues (Δ, ξ) of the generators of the Cartan subalgebra (L_0, M_0)

$$[L_0, \mathcal{O}(0,0)] = \Delta\mathcal{O}(0,0), \quad [M_0, \mathcal{O}(0,0)] = \xi\mathcal{O}(0,0).$$

Δ: conformal weight ξ: boost charge

The highest weight representations require the primary operators satisfy

$$[L_n, \mathcal{O}(0,0)] = 0, \quad [M_n, \mathcal{O}(0,0)] = 0, \qquad \text{for } n > 0.$$

The tower of descendant operators can be got by acting L_{-n}, M_{-n} with $n > 0$ on the primary operators. A primary operator and its descendants form a module.

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The descendant states have negative norm states, reflecting the fact that the theory is not unitary. For example, for the level-1 states $L_{-1}|\Delta, \xi\rangle, M_{-1}|\Delta, \xi\rangle$, their inner products matrix has determinant $-\xi^2$.

Quasi-primary states

Hilbert space in 2D GCFT:

$$\mathcal{H} = \sum_{\text{primary module}} \mathcal{H}_{\Delta,\xi},$$

where each module is composed of a primary state and its descendants. However such a classification is not suited to bootstrap:

1. The conformal bootstrap is based on the global symmetry, rather than the local one;
2. The explicit form of the local GCA block is unknown.

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The Galilean conformal bootstrap is based on the global symmetry, generated by $L_{\pm1}, L_0, M_{\pm1}, M_0$. This means that we should start from "quasi-primary" operators. Actually this is feasible as the operators in GCFT_2 can be classified into different quasi-primary operators and their global descendants.

$$\mathcal{H} = \sum_{\text{quasiprimaries}} \mathcal{H}_{\Delta,\xi},$$

Subtlety

M_0 usually acts non-diagonally on these quasi-primary operators , even though L_0, M_0 act diagonally on the primary operators.

Consider the following level-2 descendant operators of a primary operators $\mathcal{O}$ with a weight Δ and a charge ξ

$$\mathcal{A} = L_{-2}\mathcal{O}, \quad \mathcal{B} = M_{-2}\mathcal{O}.$$

They are quasi-primary operators, on which M_0 acts as

$$M_0\mathcal{A} = \xi\mathcal{A} + 2\mathcal{B}, \quad M_0\mathcal{B} = \xi\mathcal{B}.$$

This phenomenon is typical in Galilean CFT, similar to Logarithmic CFT.

$\mathcal{A}$ and $\mathcal{B}$ share the same conformal dimension, and form a multiplet of rank 2.

A primary operator is referred to as a singlet, or a rank-1 multiplet.

The existence of multiplet structure is a typical feature in GCFT, no matter $\xi \neq 0$ or $\xi = 0$.

Multiplet

Simply speaking, the quasi-primary operators in a multiplet share the same scaling dimension. The action of boost M_0 gives a rank-r upper triangular Jordan block

$$
\begin{aligned}
&[L_0, \mathcal{O}^a] = \Delta \mathcal{O}^a, \qquad \forall a = 1, \cdots r, \\
&[M_0, \mathcal{O}^a] = \xi \mathcal{O}^a + \mathcal{O}^{a+1}, \\
&[L_1, \mathcal{O}^a] = 0, \qquad [M_1, \mathcal{O}^a] = 0.
\end{aligned}
$$

The quasi-primary operators in a multiplet together with their descendants form a (generalized) highest weight representation of the global group. This defines a rank-r multiplet : $\mathcal{V}_{\Delta,\xi,r}$.

For a rank-r multiplet $\mathcal{V}_{\Delta,\xi,r}$, the descendant states are

$$
|a, n, m\rangle_r = L_{-1}^n M_{-1}^m |\mathcal{O}^a_{\Delta,\xi,r}\rangle, \qquad n, m \in \mathbb{Z}^+,
$$

and $l = n + m$ is called the level since $L_0|a, n, m\rangle_r = (\Delta + l)|a, n, m\rangle_r$.

Hilbert space

Hilbert space:

$$\mathcal{H} = \sum_{multiplets} \mathcal{V}_{\Delta,\xi,r}$$

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Notice: there could be null states in $\xi = 0$ multiplet.

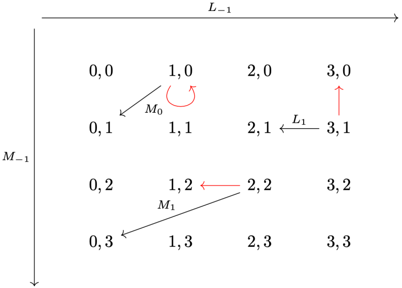
The null states are the vectors in the kernel space of the Gram matrix of inner product.
In 2D unitary relativistic CFT, the null states form sub-representations of highest weight repr.

$$\text{Physical Hilbert space} = \frac{\text{Hilbert space}}{\text{null states}}$$

The null states are orthogonal to the physical states, and thus lead to the differential equations on the correlation functions.

Null states in $\xi = 0$ singlets

If $\xi = 0$, the singlet representation $\mathcal{V}_{\Delta,0,1}$, $\Delta > 0$ is reducible and indecomposable, containing null states.



- (n, m) stands for descendant
- The states below the first row $|n, m\rangle$, $m \geq 1$ are null.
- The descendant state $|0, 1\rangle = M_{-1}|\mathcal{O}\rangle$ satisfies the quasiprimary conditions, hence they form a sub-representation $V_1 = \mathcal{V}_{\Delta+1,0,1}$ of $V_0 = \mathcal{V}_{\Delta,0,1}$.

Correlation functions with $\xi = 0$ singlet

With the null states, we may derive the differential equations on the correlators.

$$\frac{\partial}{\partial y}\langle \mathcal{O}_{\Delta,0}(x,y)\cdots\rangle = 0.$$

The three-point functions containing $\mathcal{O}_{\Delta,0}$ give the fusion rules of OPE.
For the singlet-singlet-$\mathcal{O}_{\Delta,0}$ case

$$\frac{\partial}{\partial y_3}\langle \mathcal{O}_{\Delta_1,\xi_1}\mathcal{O}_{\Delta_2,\xi_2}\mathcal{O}_{\Delta,0}(x_3,y_3)\rangle = 0,$$

implying that

$$c_{12,\xi=0}(\xi_1-\xi_2) = 0.$$

Either the boost charges satisfy $\xi_1 = \xi_2$, or the three point coefficient vanishes $c_{12,\xi=0} = 0$.
This is still true even the singlets $\mathcal{O}_1, \mathcal{O}_2$ are replaced with the multiplets.

Correlation functions of singlets

Two-point function:

$$\langle \mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2)\rangle = \delta_{\Delta_1,\Delta_2}\delta_{\xi_1,\xi_2}|x_{12}|^{-2\Delta}\exp(2\xi k_{12}),$$

where

$$x_{12} \equiv x_1 - x_2, \qquad k_{12} \equiv \frac{y_{12}}{x_{12}} = \frac{y_1 - y_2}{x_1 - x_2}.$$

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Three-point function:

$$\begin{aligned}\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle &= c_{123}|x_{12}|^{-\Delta_{12,3}}|x_{23}|^{-\Delta_{23,1}}|x_{31}|^{-\Delta_{31,2}} \\ &\quad \cdot \exp\{\xi_{12,3}k_{12} + \xi_{23,1}k_{23} + \xi_{31,2}k_{31}\},\end{aligned}$$

where c_{123} are the three-point coefficients and

$$\Delta_{ij,k} \equiv \Delta_i + \Delta_j - \Delta_k, \qquad \xi_{ij,k} \equiv \xi_i + \xi_j - \xi_k.$$

The 2-pt and 3-pt functions of multiplets can be determined by the Ward identities as well.

4-point functions of quasi-primary operators

$$G_4 = \langle \prod_{i=1}^{4} \mathcal{O}_i(x_i, y_i) \rangle = \prod_{i,j} x_{ij}^{\sum_{k=1}^{4} \frac{\Delta_{ij,k}}{3}} e^{-\frac{y_{ij}}{x_{ij}} \sum_{k=1}^{4} \frac{\xi_{ij,k}}{3}} \mathcal{G}(x, y)$$

where $\mathcal{G}(x, y)$ is called the stripped four-point function with x, y being the cross ratios,

$$x \equiv \frac{x_{12}x_{34}}{x_{13}x_{24}}, \quad \frac{y}{x} \equiv \frac{y_{12}}{x_{12}} + \frac{y_{34}}{x_{34}} - \frac{y_{13}}{x_{13}} - \frac{y_{24}}{x_{24}}.$$

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Crossing equation:

$$G_{34}^{21}(x, y) = G_{32}^{41}(1 - x, -y).$$

In the following discussions, we focus on the 4-pt functions of identical singlets.

1. The 4-pt function could be expanded by the conformal blocks , which are completely fixed by the conformal symmetry, depending on the external operators, the specific OPE channel, and the propagating operators.

2. It can also be expanded into an integral of the conformal partial waves over unphysical unitary principal series. Under suitable conditions, the block expansion is recovered from the inversion formula by a contour deformation.

Let's first look at the conformal block...

Global block (of the singlet) Bagchi 1612.01730,1705.05890

The contribution of the primary operator and its global descendant operators (which can be got by acting L_{-1} and M_{-1}) to the stripped four-point function $\mathcal{G}(x,y)$ could be written as

$$c_{12p}c_{34p}g_p(x,y)$$

where the indices $i=1,2,3,4$ label the operators $\mathcal{O}_i$ on the external legs, the index p labels the propagating primary operator $\mathcal{O}_p$. The function $g_p(x,y)$ is the global block (for identical $\mathcal{O}_i$), obeying the Casimir equations of the global algebra

$$\hat{C}_i g_p(x,y) = \lambda_i g_p(x,y), \quad i=1,2$$

where λ_i are the eigenvalues, and

$$\begin{aligned} \hat{C}_1 &= M_0^2 - M_1 M_{-1}, \\ \hat{C}_2 &= 4L_0 M_0 - L_{-1}M_1 - L_1 M_{-1} - M_1 L_{-1} - M_{-1}L_1. \end{aligned}$$

Solution:

$$g_p(x,y) = 2^{2\Delta_p-2}x^{\Delta_p-2\Delta}(1+\sqrt{1-x})^{2-2\Delta_p}e^{\frac{-\xi_p y}{x\sqrt{1-x}}+2\xi\frac{y}{x}}(1-x)^{-1/2}.$$

Global block of multiplets: $\xi \neq 0$ case

Different from the case of a singlet, the global block of a multiplet is not the eigenfunction of the Casimir operators. The stripped four-point functions can be expanded into

$$\mathcal{G}(x,y) = \sum_{\mathcal{O}_r} \frac{1}{d_r} f\left[\mathcal{O}_r\right]$$

where the propagating quasi-primary operator $\mathcal{O}_r$ is a rank-r multiplet with an overall normalization d_r, and $f\left[\mathcal{O}_r\right]$ satisfy the following Casimir equations

$$(\hat{C}_i - \lambda_i)^r f\left[\mathcal{O}_r\right] = 0, \quad \text{for } i = 1, 2.$$

The solution reads

$$f\left[\mathcal{O}_r\right] = \sum_{s=0}^{r-1} A_s g^{(s)}_{\Delta_r,\xi_r}.$$

Here $g^{(s)}_{\Delta_r,\xi_r}, s = 0, \cdots r-1$ make up the global block for the multiplet,

$$g^{(s)}_{\Delta_r,\xi_r} = \partial^s_{\xi_r} g^{(0)}_{\Delta_r,\xi_r}$$

where $g^{(0)}_{\Delta_r,\xi_r}$ is the global block for the singlet.

Global block expansion

It is more subtle to find the global block of multiplets for the $\xi = 0$ case, due to the existence of null states.

BC, Peng-xiang Hao, Reiko Liu and Zhe-fei Yu, in progress

The global block expansion of the stripped four-point function in GCFT is

$$\begin{aligned} \mathcal{G}(x,y) &= \sum_{\mathcal{O}_r|\xi_r\neq 0} \frac{1}{d_r} \sum_{s=0}^{r-1} \frac{1}{s!} \sum_{a,b|a+b+s+1=r} c_a c_b \partial_{\xi_r}^s g_{\Delta_r,\xi_r}^{(0)} \\ &\quad +(\xi = 0 \text{ sector}). \end{aligned}$$

where $q = y/x$. and c_a, c_b are 3-pt coefficients.

Conformal partial waves

One essential step in applying the inversion formula is to decompose the four-point function into a set of complete basis of conformal group in the Eulideanized space.

The complete basis consists of the normalizable eigenfunctions of the Hermitian Casimir operators. Dobrev et.al. (1977)

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Alternatively, we have developed the shadow formalism to read Galilean CPWs.

Shadow transform in GCFT_2

Since the 2d Galilean conformal group is isomorphic to the 3d Poincare group, the "unitary principal series" representations could be identified as unitary irreducible representations of the Poincare group.

For the unitary principal series $\mathcal{E}_{\Delta=1+is,\xi=ir}$, $s, r \in \mathbb{R}^{\neq 0}$, we define the associated shadow representation as $\mathcal{E}_{\widetilde{\Delta}=2-\Delta,\widetilde{\xi}=-\xi}$, and denote the virtual operator transforming in $\mathcal{E}_{\widetilde{\Delta},\widetilde{\xi}}$ as $\widetilde{\mathcal{O}}$.

For an operator $\mathcal{O}$ lying on the unitary principal series $\mathcal{E}_{1+is,ir}$, we construct the shadow transform $\mathcal{S}$ as

$$\begin{aligned}\mathcal{S}[\mathcal{O}](x,y) &= \int_{\mathbb{R}^2} dx_0 dy_0 \, \langle \widetilde{\mathcal{O}}(x,y)\widetilde{\mathcal{O}}(x_0,y_0)\rangle \mathcal{O}(x_0,y_0) \\ &= \int_{\mathbb{R}^2} dx_0 dy_0 \, |x-x_0|^{2\Delta-4} e^{-2\xi \frac{y-y_0}{x-x_0}} \mathcal{O}(x_0,y_0),\end{aligned}$$

which is an intertwining map between the two representations

$$\mathcal{S} : \mathcal{E}_{\Delta,\xi} \to \mathcal{E}_{\widetilde{\Delta},\widetilde{\xi}}.$$

If the representations $\mathcal{E}_{\Delta,\xi}$ and $\mathcal{E}_{\widetilde{\Delta},\widetilde{\xi}}$ are UIRs, then by the Schur lemma $\mathcal{S}$ is an isomorphism.

OPE block from shadow transform

The OPE relation can be written as

$$\mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2) = \sum_k c_{12}^k \, \mathcal{D}_{12k}(x_{12}, y_{12}, \partial_{x_2}, \partial_{y_2})\mathcal{O}_k(x_2, y_2)$$

The OPE block $\mathcal{D}$ encodes all the contributions of the derivative operators.

In the shadow formalism, the OPE block with respect to the two virtual operators should be

$$\mathcal{D}_{123}\mathcal{O}_3(x_2, y_2) = N_{123} \int_I dx_0 \, dy_0 \langle \mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2)\widetilde{\mathcal{O}_3}(x_0, y_0)\rangle \mathcal{O}_3(x_0, y_0)$$

$$\mathcal{D}_{123}(x, y, \partial_x, \partial_y) = x^{-\Delta_{12,3}} e^{\xi_{12,3} \frac{y}{x}}$$
$$\cdot \sum_{n,m} \frac{2^{-n-m} \xi_3^{-m}}{n!} (1+R)^n P_m^{(\Delta_{32,1}-1, \Delta_{31,2}+n-1)}(R)(x\partial_x + y\partial_y)^n (x\partial_y)^m,$$

where $R = \frac{\xi_1 - \xi_2}{\xi_3}$ and $P_n^{(a,b)}(z)$ is the Jacobi polynomial,

$$P_n^{(a,b)}(z) = \frac{(a+1)_n}{n!} {}_2F_1(-n, 1+a+b+n; a+1; \frac{1}{2}(1-z)).$$

Using the integral expression of the OPE blocks, we can construct the s-channel conformal blocks as

$$G^{(s)}_{\Delta_r, \xi_r}(x_i, y_i) = \mathcal{D}_{12r} \mathcal{D}_{43r} \langle \mathcal{O}_0(x_2, y_2) \mathcal{O}_0(x_3, y_3) \rangle$$

CPWs from shadow formalism

The s-channel unstripped conformal partial waves $\Psi_{\Delta_r,\xi_r}(x_i, y_i)$ with respect to four external virtual operators $\mathcal{O}_i \in \mathcal{E}_{\Delta_i,\xi_i}$, $\xi_i = \xi_r R_i$ and the propagating virtual operator $\mathcal{O} \in \mathcal{E}_{\Delta_r,\xi_r}$, $\xi_r \in i\mathbb{R}^{\neq 0}$, can be constructed as

$$\Psi_{\Delta_r,\xi_r}(x_i, y_i) = \int_{\mathbb{R}^2} dx_0 dy_0 \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}(x_0, y_0) \rangle \langle \widetilde{\mathcal{O}}(x_0, y_0) \mathcal{O}_3 \mathcal{O}_4 \rangle.$$

The stripped conformal partial waves $\Psi_{\Delta_r,\xi_r}(x, y)$ are defined by factoring out the kinematical factor $K^{(s)}$

$$\psi_{\Delta_r,\xi_r}(x_i, y_i) = K^{(s)}(x_i, y_i) \psi_{\Delta_r,\xi_r}(x, k).$$

They are combinations of two blocks,

$$\psi_{\Delta_r,\xi_r} = \mathcal{S}(\mathcal{O}_3, \mathcal{O}_4; \widetilde{\mathcal{O}}_{\Delta_r,\xi_r}) g_{\Delta_r,\xi_r}(x, k) + \mathcal{S}(\mathcal{O}_1, \mathcal{O}_2; \mathcal{O}_{\Delta_r,\xi_r}) g_{2-\Delta_r,-\xi_r}(x, k),$$

where the prefactors are simply the shadow coefficients.

GCPW expansion: $\xi \neq 0$

A 4-point function admits global block expansion in which the expansion coefficients contain the data of the theory.

It admits the GCPW expansion as well, where the expansion coefficients can be obtained by using the inversion formula.

The two expansions are related by the contour deformation.

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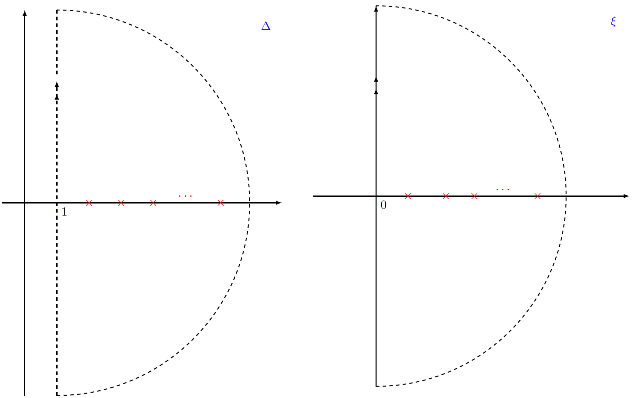
New features:

The multiplets appear as the multiple poles in the inversion function.

$$I(\Delta, \xi) = (\Psi_{\Delta,\xi}, \mathcal{G}) \sim - \sum_{\Delta_m, \xi_l, k} \Gamma(k+1) \frac{2^{2\Delta_m - 2}}{(\xi - \xi_l)^{k+1}} \frac{P_{\Delta_m, \xi_l, k+1}}{\Delta - \Delta_m}$$

where $\{\Delta_m, \xi_l\}$ are the physical poles.

GCA inversion function



(a) Δ-plane (b) ξ-plane

Figure: The contours in the Δ-plane and ξ-plane.

Generalized free theory

The generalized free field theory(GFT) or Mean Field Theory (MFT) plays an important role in analytic conformal bootstrap.

It provides the leading contribution to the correlators at large spin. The data in GFT is the starting point for many computations.

Holographically it is the dual of free field theories in AdS.

By definition, the correlators in GFT are simply sums of products of two-point functions.

The study of free field theory provides nontrivial check and guide to our formalism. We consider two free field theories: GGFT and BMS free scalar

Generalized Galilean free theory (GGFT)

We may start from the generalized Galilean free field theory (GGFT) which contains two fundamental scalar type operators $\mathcal{O}_1, \mathcal{O}_2$ with the conformal weights and the charges Δ_1, ξ_1 and Δ_2, ξ_2 respectively.

We would like to study the spectrum and 3-pt coefficients in such GGFT.

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Three different approaches

1. Operator construction: show "double trace" operator explicitly
2. Taylors expansion of 4-point function in terms of global block
3. Apply GCA inversion formula

They are consistent with each other.

Inversion function of GGFT

Consider the 4-pt function: $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$.

Inversion function:

$$I = (\Psi_{\Delta,\xi}, \mathcal{G}) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{1}{-\Delta + 2\Delta_{\mathcal{O}} + n} \frac{1}{(\xi - 2\xi_{\mathcal{O}})^{k+1}} P_{n,k}^{t,\text{inversion}}.$$

It shows explicitly

- the existence of double-twist operators $\Delta_n = 2\Delta_{\mathcal{O}} + n$.
- the spectrum of ξ is localized at $2\xi_{\mathcal{O}}$ in the propagating channel.
- the multipole structure, suggesting the appearance of multiplets.

BMS free scalar P.X.Hao et.al. 2111.04701, more in Wei Song's talk!

The discussions before could be applied to the field theory with BMS_3 symmetry. The BMS free scalar theory provides an example to see the block expansion in terms of $\xi = 0$ multiplet.

The action of a BMS-invariant free scalar on a cylinder parameterize by(σ, τ) with $\sigma \sim \sigma + 2\pi$ reads

$$S = \frac{1}{4\pi} \int d\sigma d\tau \, (\partial_\tau \phi)^2 \, .$$

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Two primary operators:

$$\mathcal{O}_0(x, y) \equiv i\partial_y \phi(x, y), \qquad \mathcal{O}_1(x, y) \equiv i\partial_x \phi(x, y)$$

They form a rank-2 multiplet: $\mathcal{O} = (\mathcal{O}_0, \mathcal{O}_1)$, with $\xi = 0$.

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Vertex operators

$$V_\alpha(x, y) \equiv: e^{\alpha\phi(x,y)} :, \qquad \alpha \in \mathbb{R} \text{ or } i\mathbb{R}.$$

are singlets with

$$\Delta = 0, \qquad \xi = -\frac{\alpha^2}{2}.$$

OPE of the vertex operators:

$$V_\alpha(x_1, y_1) V_\beta(x_2, y_2) = e^{-\alpha\beta \frac{y_2 - y_1}{x_2 - x_1}} \; V_{\alpha+\beta} + \cdots \, ,$$

Obviously

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We consider the following four-point function

$$\langle V_\alpha(x_1, y_1) V_{-\alpha}(x_2, y_2) V_\alpha(x_3, y_3) V_{-\alpha}(x_4, y_4) \rangle = e^{\alpha^2 \frac{y_{12}}{x_{12}}} e^{\alpha^2 \frac{y_{34}}{x_{34}}} + e^{\alpha^2 \frac{y_{14}}{x_{14}}} e^{\alpha^2 \frac{y_{23}}{x_{23}}}.$$

We use it to check the block expansion, and find consistent pictures.

BC, Peng-xiang Hao, Reiko Liu and Zhe-fei Yu, in progress

Conclusions

In this work, we tried to establish a framework to do Galilean conformal bootstrap.
Even though a Galilean conformal field theory is generically non-unitary, bootstrap may still be viable.

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1. We discussed the multiplets, and computed their conformal blocks.
2. We developed harmonic analysis of GCA, which paves the way for further analytic study.
3. We studied GGFT in three different ways, and found consistent picture.

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Even though a Galilean conformal field theory is generically non-unitary, bootstrap may still be viable.

1. We discussed the multiplets, and computed their conformal blocks.
2. We developed harmonic analysis of GCA, which paves the way for further analytic study.
3. We studied GGFT in three different ways, and found consistent picture.
4. We estimated the spectral density by using Hardy-Littlewood tauberian theorem.

On-going works

1. Developing shadow formalism

   BC, Peng-xiang Hao, Reiko Liu and Zhe-fei Yu, to appear

2. $\xi = 0$ sector: CB, CPW, BMS free scalar,...

   BC, Peng-xiang Hao, Reiko Liu and Zhe-fei Yu, in progress

3. Higher dimensional case

   BC, Reiko Liu and Yu-fan Zheng, to appear

Thanks for your attention!