FZZT branes in JT gravity and topological gravity

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based on work with Kazuhiro Sakai (Meiji Gakuin U.)

arXiv: 2108.03876, 2111.09551 [KO-Sakai]

JT gravity

- JT gravity is an example of 2d dilaton gravity

$$S = -\frac{1}{2}\int d^2x\sqrt{g}\phi(R+2)$$

- JT gravity is a useful toy model for studying quantum gravity

Various problems in quantum gravity

- Using JT gravity, one can study various important problems in quantum gravity
  - Black hole and quantum chaos
    [Cotler et al, Kitaev, Maldacena-Stanford-Shenker]
  - Entropy of Hawking radiation and Page curve
    [Almheiri et al, Penington et al]
  - Sum over topologies in gravitational path integral
    [Saad-Shenker-Stanford, Stanford-Witten]

JT gravity as a random matrix model

- In 2019 [Saad-Shenker-Stanford] showed that JT gravity is equivalent to a random matrix model

$$Z_{\mathrm{JT}} = \Big\langle Z(\beta) \Big\rangle = \Big\langle \operatorname{Tr} e^{-\beta H} \Big\rangle_{\text{random matrix } H}$$

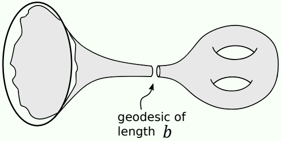
- This is an example of holography involving ensemble average

Sum over topologies in JT gravity

- In general, we do not know how to sum over topologies in gravitational path integral
- In JT gravity, sum over topology is realized by as the genus expansion of matrix model

- Amplitude is obtained by gluing trumpet and Weil-Petersson (WP) volume [Saad-Shenker-Stanford]

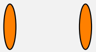
$$\langle Z(\beta) \rangle = \sum_{g=0}^{\infty} g_s^{2g-1} \int_0^\infty b\,db\, \underbrace{\frac{e^{-\frac{b^2}{2\beta}}}{\sqrt{2\pi\beta}}}_{\text{trumpet}} \times \underbrace{V_{g,1}(b)}_{\text{WP volume}}$$



Two-boundary correlator

- One-point function $\langle Z(\beta)\rangle$ corresponds to spacetime with one boundary
- Two-point function $\langle Z(\beta_1)Z(\beta_2)\rangle$ receives a contribution of wormhole corresponding to the connected correlator

$$\left\langle Z(\beta_1)Z(\beta_2)\right\rangle =$$



disconnected

$+$

wormhole

Factorization puzzle

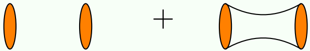
- This is in conflict with the usual rules of AdS/CFT correspondence and called the factorization puzzle

- Partition function of two separate CFTs should factorize

$$Z(\mathrm{CFT}_1 \otimes \mathrm{CFT}_2) = Z(\mathrm{CFT}_1) \cdot Z(\mathrm{CFT}_2)$$

- But the ensemble average of two-boundary correlator does not factorize due to wormhole contribution

$$\langle Z(\beta_1) Z(\beta_2) \rangle = \langle Z(\beta_1) \rangle \langle Z(\beta_2) \rangle + \langle Z(\beta_1) Z(\beta_2) \rangle_{\mathrm{conn}}$$

$=$ 

FZZT branes

- One can fix some of the eigenvalues of matrix by introducing FZZT branes (eigenbranes) [Blommaert-Mertens-Verschelde]
- This might resolve the issue of factorization puzzle
- This eigenbrane picture comes from the fact that FZZT brane in matrix model corresponds to the determinant operator

$$\psi(E) = \left\langle \det(E - H) \right\rangle_{\text{random } H}$$

FZZT-branes in JT gravity

- We found a general prescription to introduce FZZT-branes in JT gravity [KO-Sakai]

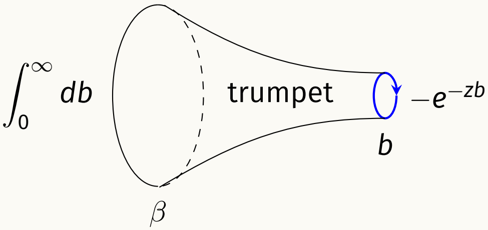
- Our result is closely related to the matrix model of end-of-the-world brane by [Gao-Jafferis-Kolchmeyer]

- Amplitude in the presence of FZZT brane $\det(E - H)$ is obtained by gluing three building blocks:

trumpet	WP volume	$-e^{-zb}, \quad (z = \sqrt{-E})$

Trumpet can end on a FZZT brane

- We attach the factor $-e^{-zb}$ along the geodesic boundary with length b and integrate over b
- In particular, this implies that trumpet can end on a FZZT brane

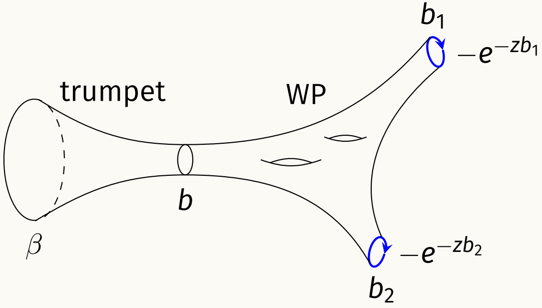


Higher genus correction

- We can compute higher genus corrections by gluing trumpet, WP volume, and the factor $-e^{-zb}$

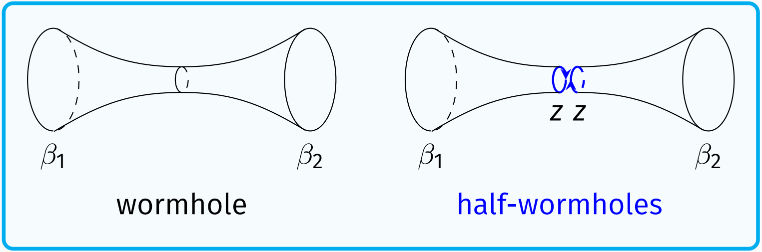
Higher genus correction

$$\int_0^\infty db\,db_1\,db_2$$



Half-wormhole

- In the presence of FZZT brane, two-point function $\langle Z(\beta_1)Z(\beta_2)\rangle$ receives contribution from "half-wormholes" introduced by [Saad-Shenker-Stanford-Yao]



- This might be a clue for solving the factorization puzzle
  see e.g. [Blommaert-Iliesiu-Kruthoff] for an attempt in this direction

Application to the Page curve

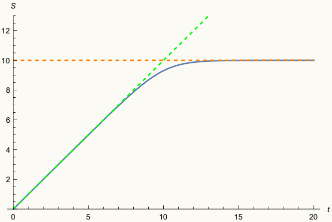
- In our recent paper, we considered Page curve of Hawking radiation as an application of our formalism [KO-Sakai]
- We consider a toy model of Hawking radiation introduced by [Penington-Shenker-Stanford-Yang] (PSSY)

PSSY model

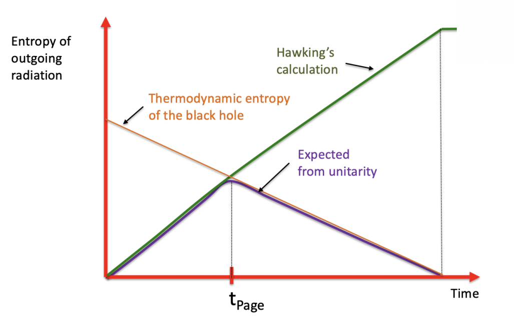
- JT gravity + branes
- Number of branes $\Leftrightarrow$ Number of Hawking quanta
- $\log$(Number of branes) $\Leftrightarrow$ time

Page curve and replica wormhole

- In Hawking's calculation, entropy of radiation grows indefinitely, which leads to a violation of unitarity
- In PSSY model, this problem is cured by the contribution of replica wormhole
  - In their model, entropy approaches a constant at late time



Page curve of evaporating black hole



- For an evaporating black hole, entropy decreases at late-time
- This decreasing behavior of entropy is not captured by the PSSY model

Back-reaction of branes

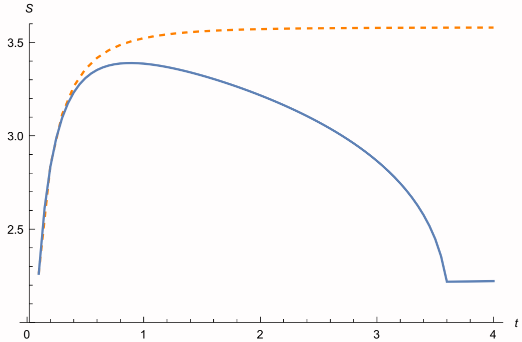
- This difference comes from the fact that PSSY used the probe brane approximation

- We take account of the back-reaction of branes
  - In matrix model description, eigenvalue density is deformed due to back-reaction of branes
  - For a technical reason, we consider anti-FZZT branes corresponding to the inverse determinant $\det(E-H)^{-1}$

It turns out that JT gravity with anti-FZZT branes is a good toy model of evaporating black hole!

Result of von Neumann entropy

Plot of von Neumann entropy



Orange curve: Result of PSSY (probe brane approximation)
Blue curve: Our result with back-reaction of branes

We recover the decreasing behavior of entropy at late-time!

Summary

- We find a prescription to introduce FZZT branes in JT gravity
- In the presence of branes, two-boundary correlator receives contributions from "half-wormholes"
- Taking account of the back-reaction of branes, we find the Page curve with decreasing behavior of entropy at late-time

Lot more to be learned from JT gravity!