3D rank 0 N=4 SCFTs and Non-unitary TQFTs

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Introduction

- **Superconformal field theories with 8 Qs**
  - $4D\ \mathcal{N}=2$ **theories** : **Seiberg-Witten, Class S theories, 2D chiral algebra...**
  - $3D\ \mathcal{N}=4$ **theories** : **Mirror symmetry, Rozansky-Witten, 1D TQM,...**
  - $5D, 6D$: **Predicted by String/M-theory**
- **Classification by rank**
  - **Rich Structures in vacuum moduli space** X
  - **rank** $:=$ **dim**X_C , X_C : **Coulom branch vacuum moduli space**
  - **No rank-0 non-trivial SCFTs in** $D \geq 4$
  - **In Today's talk : Classification of rank-0 3D** $\mathcal{N}=4$ **SCFTs**

Most previous approaches are not applicable

3D $\mathcal{N}=4$ SCFTs

- $SO(4) \simeq SU(2)_L \times SU(2)_R$ **R-symmetry**

Mirror symmetry $\left\{\begin{array}{l} \textbf{Coulomb branch :} \\ \textbf{Higgs branch :} \end{array}\right.$

$\Rightarrow$ **We define** $r(\text{rank}) :=$ **max**$\{\dim_{\mathbb{H}} X^C, \dim_{\mathbb{H}} X^H\}$

- $\exists$ **Intereacting 3D** $\mathcal{N}=4$ **SCFTs of rank 0**!!

Ex) **The minimal** $\mathcal{N}=4$ **SCFT** [Gang, Yamazaki:2018]

The minimal $\mathcal{N}=4$ SCFT

[Gang, Yamazaki:2018]

- **UV description : only 3D $\mathcal{N}=2$ SUSY**

  $U(1)$ $\mathcal{N}=2$ **vector multiplet coupled to a chiral multiplet Φ of charge +1 with CS level $k=-3/2$**

  Symmetry : $(\mathcal{N}=2$ **SUSY**$) + (U(1)_{\text{top}}$ **flavor symmetry**$)$

$$J^{\mu}_{\text{top}} = \epsilon^{\mu\nu\rho}F_{\nu\rho}$$

- **At IR : 3D $\mathcal{N}=4$ SCFT of rank 0**

  SUSY enhancement : $\mathcal{N}=2 \to \mathcal{N}=4 \quad (U(1)_R \times U(1)_{\text{top}} \longrightarrow SO(4)_R)$

  No vacuum moduli : rank 0

  Very small C_T and F

$$C_T = \frac{8}{26}\left(8 - \frac{5\sqrt{5+2\sqrt{5}}}{10}\right) \simeq 0.992549, \qquad F = -\log\left(\sqrt{\frac{5-\sqrt{5}}{10}}\right) = 0.652965$$

$$cf)\ C_T = 1, \quad F = \frac{1}{2}\log 2 \simeq 0.346572 \qquad (\textbf{free chiral})$$

3D $\mathcal{N} = 4$ SCFTs

- $SO(4) \simeq SU(2)_L \times SU(2)_R$ **R-symmetry**

Coulomb branch :

Mirror symmetry

Higgs branch :

$\Rightarrow$ **We define** $r(\text{rank}) :=$ **max**$\{\dim_{\mathbb{H}} X^C, \dim_{\mathbb{H}} X^H\}$

- $\exists$ **Intereacting 3D** $\mathcal{N} = 4$ **SCFTs of rank 0**!!

Ex) **The minimal** $\mathcal{N} = 4$ **SCFT**

$\Rightarrow$ **Classification program should start with** **rank 0** **Difficult to bootstrap**

(**only stress-energy tensor multiplets**)

3D $\mathcal{N} = 4$ SCFTs

- $SO(4) \simeq SU(2)_L \times SU(2)_R$ **R-symmetry**

Mirror symmetry

Coulomb branch :

Higgs branch :

$\Rightarrow$ **We define** $r(\text{rank}) := $ **max**$\{\dim_{\mathbb{H}} X^C, \dim_{\mathbb{H}} X^H\}$

- $\exists$ **Intereacting 3D** $\mathcal{N} = 4$ **SCFTs of rank 0**!!

Ex) **The minimal** $\mathcal{N} = 4$ **SCFT**

$\Rightarrow$ **Classification program should start with rank 0**

- **We attack the classification of rank-0 by establishing**

(3D rank-0 theory) $\longrightarrow$ **(a pair of 3D non-unitary TQFTs)**

Correlation functions of local/loop operators

Correlation functions of ~~local/~~loop operators

Degenerate limits

(Coulomb/Higgs branch limits)

Rank-0/(Non-unitary TQFTs) correspondence

(3D rank-0 SCFT $\mathscr{T}_{\text{rank 0}}$) $\longrightarrow$ **(a pair of 3D non-unitary TQFTs $\text{TFT}_{\pm}[\mathscr{T}_{\text{rank 0}}]$)**

$$\mathbf{Z}^{\mathbb{B}}_{\mathscr{T}_{\text{rank 0}}}(b^2, m, \nu; s) \xrightarrow{m=0,\ \nu=\pm 1} \mathbf{Z}^{M}_{\text{TFT}_{\pm 1}}(s)$$

$\mathbb{B}$: **Rigid supersymmetric background**

M : **Topological structure of $\mathbb{B}$** $\quad M = \mathscr{M}_{g,p}$

$$\begin{tikzcd} S^1 \arrow[r, "p"] & \mathscr{M}_{g,p} \arrow[d] \\ & \Sigma_g \end{tikzcd}$$

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b^2(**or** q) : **Squashing (or Omega-deformation) parameter** (**only for** $g = 0$)

s : **Spin-structure choice along fiber** S^1, $s = 1(s = \pm 1)$ **for odd** p (**for even** p)

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m(**or** $\eta := e^m$) : **real mass parameter (or fugacity) for** $U(1)_A$

ν : **Mixing between superconformal** $U(1)$ **R-symmetry and** $U(1)_A$

$R, R' \in \mathbb{Z}/2$: **Cartans of** $SU(2)_L \times SU(2)_R$

$R_\nu = R_{\nu=0} + \nu A$

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- $\mathbb{B}$ = **Superconformal index** $(M = S^2 \times S^1 = \mathscr{M}_{g=0,p=0})$ [S. Kim:2009] [Imamura,Yokoyama:2011]

$$\mathbf{Z}^{\mathbb{B}}(b^2, m, \nu, ; s) \to I^{\text{sci}}(q, \eta, \nu; s) := \begin{cases} \mathbf{Tr}_{\mathscr{H}_{\text{rad}}}(-1)^{R_\nu} q^{\frac{R_\nu}{2}+j_3}\eta^{A}, & s = -1 \\ \mathbf{Tr}_{\mathscr{H}_{\text{rad}}}(-1)^{2j_3} q^{\frac{R_\nu}{2}+j_3}\eta^{A}, & s = 1 \end{cases}$$

$R, R' \in \mathbb{Z}/2$: **Cartans of** $SU(2)_L \times SU(2)_R$

$R_{\nu=0} = R + R',\ A := R - R' \qquad R_\nu = R_{\nu=0} + \nu A$

$\mathscr{H}_{\text{rad}}$: Radially quantized Hilbert space (space of local oprators)

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From superconformal multiplet analysis, [Cordova, Dumitrescu, Intrilligator;2016]

one can prove that, for rank 0 theories $(\eta = e^m)$

$$I^{\text{sci}}(q, \eta = 1, \nu = 1; s) := \begin{cases} \mathbf{Tr}_{\mathcal{H}_{\text{rad}}}(-1)^{R} q^{R}, & s = -1 \\ \mathbf{Tr}_{\mathcal{H}_{\text{rad}}}(-1)^{2j_3} q^{R}, & s = 1 \end{cases} = 1\ (q\text{-independent})$$

since only Coulomb branch operator $(\Delta = R', j = 0, R = 0)$ **contributes to the index**

compatible with the fact that $Z^{M=S^2\times S^1} = 1$ **for all TQFTs** (S^2 **is homotopically trivial**)

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- $\mathbb{B}$ = **Squashed 3-sphere ptn** $\mathbf{Z}^{S^3_b}(b^2, m, \nu)$ $\quad (M = S^3 = \mathscr{M}_{g=0,p=1})$

$$S^3_b = \left\{ z, w \in \mathbb{C} : \ b^2 |z|^2 + \frac{1}{b^2} |w|^2 = 1 \right\}$$

[Hama,Hosomichi,Lee:2011]

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The ptn becomes b-indepenent in the degenerate limits (for rank 0 theories)

$$|\mathbf{Z}^{S^3_b}(b^2, m = 0, \nu = \pm 1)| = \mathbf{Z}^{M=S^3}_{\text{TFT}_{\pm}}$$

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- $\mathbb{B} =$ **Twisted indices** $(M = \Sigma_g \times S^1 = \mathcal{M}_{g,p=0})$

[Benini, Zaffaroni : 2015,2016]
[Closset, Kim, Willet : 2016]

$$I^{\Sigma_g}(\eta, \nu; s) := \begin{cases} \mathrm{Tr}_{\mathcal{H}_{\text{top}}(\nu)} (-1)^{R_\nu} \eta^A, & s = -1 \\ \mathrm{Tr}_{\mathcal{H}_{\text{top}}(\nu)} (-1)^{2j_3} \eta^A, & s = 1 \end{cases}$$

$$\text{top'l twisting} \; : \; \frac{1}{2\pi} \int_{\Sigma_g} F_{R_\nu} = (1 - g)$$

The index becomes GSD (ground state dengeneracy) on Σ_g

$$I^{\Sigma_g}(\eta = 1, \nu = \pm 1; s) := \mathbf{GSD}_g(\mathbf{TFT}_\pm; s)$$

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$$|\mathbf{Z}^{S^3_b}(b^2, m = 0, \nu = \pm 1)| = \mathbf{Z}^{M=S^3}_{\text{TFT}_{\pm}} = S_{00}[\textbf{TFT}_{\pm}]$$

- $\mathbb{B} =$ **Twisted indices** $(M$

$S_{\alpha\beta}$: **Modular S-matrix of TFT**

$$I^{\Sigma_g}(\eta, \nu; s) := \begin{cases} \textbf{Tr}_{\mathcal{H}_{\text{top}}(\nu)} \cdots \\ \textbf{Tr}_{\mathcal{H}_{\text{top}}(\nu)} (-1)^{2j_3} \eta^A, \quad s = 1 \end{cases}$$

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$$I^{\Sigma_g}(\eta = 1, \nu = \pm 1; s) := \textbf{GSD}_g(\textbf{TFT}_{\pm}; s) = \sum_{\alpha=0}^{N-1} (S_{0\alpha})^{2-2g}$$

Ex) The minimal $\mathcal{N} = 4$ SCFT

- **UV description : only 3D $\mathcal{N} = 2$ SUSY**

  $U(1)$ $\mathcal{N} = 2$ **vector multiplet coupled to a chiral multiplet Φ of charge +1 with CS level $k = -3/2$**

- **Squashed 3-sphere ptn**

$$\mathcal{Z}^{S_b^3}_{\mathcal{T}_{\min}}(b, m, \nu) = \int \frac{dZ}{\sqrt{2\pi\hbar}}\, e^{-\frac{Z^2+2Z\left(m+(i\pi+\frac{\hbar}{2})\nu\right)}{2\hbar}} \psi_\hbar(Z)$$

degenerate limits

$$\left|\mathcal{Z}^{S_b^3}_{\mathcal{T}_{\min}}(b, m=0, \nu \to \pm 1)\right| = \sqrt{\frac{1}{10}\left(\sqrt{5}+5\right)}$$

- **Twisted indices**

$$I^{\Sigma_g}(\eta = 1, \nu = \pm 1; s) := \left(\frac{5+\sqrt{5}}{10}\right)^{1-g} + \left(\frac{5-\sqrt{5}}{10}\right)^{1-g}$$

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- **Twisted indices**

$$I^{\Sigma_g}(\eta = 1, \nu = \pm 1; s) := \left(\frac{5 + \sqrt{5}}{10}\right)^{1-g} + \left(\frac{5 - \sqrt{5}}{10}\right)^{1-g}$$

$$S_{00}(\mathrm{Lee-Yang}) = \sqrt{\frac{5 + \sqrt{5}}{10}}$$

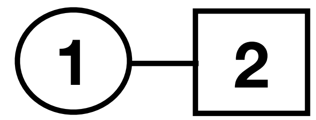
$$S_{01}(\mathrm{Lee-Yang}) = -\sqrt{\frac{5 - \sqrt{5}}{10}}$$

- **This partition function coincide with the ptn of (Lee-Yang TFT)**

$$Z^{\mathrm{Lee-Yang}}[S^3] = \sqrt{\frac{1}{10}(5 + \sqrt{5})} \qquad Z^{\mathrm{Lee-Yang}}[\Sigma_g \times S^1] = \left(\frac{5 + \sqrt{5}}{10}\right)^{1-g} + \left(\frac{5 - \sqrt{5}}{10}\right)^{1-g}$$

Other Examples

$$\frac{T[SU(2)]}{SU(2)^{\text{diag}}_k} := \text{Gauging } SU(2)^{\text{diag}} \text{ of } \underline{T[SU(2)]} \text{ theory with CS level } k$$



[Gaiotto,Witten:2007]

$\mathcal{T}_{\text{rank }0}$	$\text{TFT}_\pm[\mathcal{T}_{\text{rank }0}]$	Set of $\{\lvert S^\pm_{0\alpha}\rvert\}$
$\frac{T[SU(2)]}{SU(2)^{\text{diag}}_{\lvert k\rvert=3}}$	$(\text{Lee-Yang})^{\otimes 2}\otimes U(1)_2$	$\left\{\frac{1}{\sqrt{10}}^{\otimes 4}, \frac{5+\sqrt{5}}{10\sqrt{2}}^{\otimes 2}, \frac{5-\sqrt{5}}{10\sqrt{2}}^{\otimes 2}\right\}$
$\frac{T[SU(2)]}{SU(2)^{\text{diag}}_{\lvert k\rvert=4}}$	$\frac{\text{Gal}_{\varsigma^7_{10}}(SU(2)_{10})\times SU(2)_2}{\mathbb{Z}_2^{\text{diag}}}$	$\left\{\frac{1}{2}, \frac{1}{2\sqrt{3}}^{\otimes 5}, \frac{3+\sqrt{3}}{12}^{\otimes 2}, \frac{3-\sqrt{3}}{12}^{\otimes 2}\right\}$
$\frac{T[SU(2)]}{SU(2)^{\text{diag}}_{\lvert k\rvert=5}}$	$\text{Gal}_d\big((G_2)_3\big)\otimes U(1)_{-2}$ $\left(d=\sqrt{\frac{5}{84}+\frac{1}{4\sqrt{21}}}\right)$	$\left\{\frac{1}{\sqrt{6}}^{\otimes 2}, \frac{1}{\sqrt{14}}^{\otimes 6}, \sqrt{\frac{5}{84}\pm\frac{1}{4\sqrt{21}}}^{\otimes 2}\right\}$
$\frac{T[SU(2)]}{SU(2)^{\text{diag}}_{\lvert k\rvert\geq 6}}$	?	$\left\{\frac{1}{\sqrt{2\lvert k\rvert-4}}^{\otimes(\lvert k\rvert-3)}, \frac{1}{\sqrt{2\lvert k\rvert+4}}^{\otimes(\lvert k\rvert+1)}, \left(\frac{1}{\sqrt{8\lvert k\rvert-16}}+\frac{1}{\sqrt{8\lvert k\rvert+16}}\right)^{\otimes 2}, \left(\frac{1}{\sqrt{8\lvert k\rvert-16}}-\frac{1}{\sqrt{8\lvert k\rvert+16}}\right)^{\otimes 2}\right\}$

New non-unitary TQFTs?

Dictionary for F

So far, we relate the ptn of rank 0 SCFT at $\nu = \pm 1$ with modular date of $\text{TFT}_\pm$

$$\mathbf{Z}^{\mathbb{B}}_{\mathcal{T}_{\text{rank } 0}}(b^2, m, \nu; s) \xrightarrow{m=0,\ \nu=\pm 1} \mathbf{Z}^{M}_{\text{TFT}_{\pm 1}}(s)$$

Supringly, we found that

$$\boxed{\begin{aligned} F &= -\log|\mathbf{Z}^{S^3_b}(b=1, m=0, \nu=0)| && \textbf{quantity at superconformal point } (\nu = 0) \\ &= -\log(\min_\alpha |S_{0\alpha}| \textbf{ of TFT}_\pm) && \textbf{at non-superconformal point } (\nu = \pm 1) \end{aligned}}$$

proper measure of degrees of freedom , $F_{UV} > F_{IR}$ $\quad cf)\ C_T$

ex) Minimal Theory

$$\mathcal{Z}^{S^3_b}_{\mathcal{T}_{\min}}(b, m, \nu) = \int \frac{dZ}{\sqrt{2\pi\hbar}}\, e^{-\frac{Z^2 + 2Z\left(m + (i\pi + \frac{\hbar}{2})\nu\right)}{2\hbar}} \psi_\hbar(Z)$$

$$\left|\mathcal{Z}^{S^3_b}_{\mathcal{T}_{\min}}(b, m=0, \nu \to \pm 1)\right| = \sqrt{\frac{1}{10}\left(\sqrt{5}+5\right)}$$

$$\left|\mathcal{Z}^{S^3_b}_{\mathcal{T}_{\min}}(\overset{=1}{b}, m=0, \nu \to 0)\right| = \sqrt{\frac{1}{10}(-\sqrt{5}+5)}$$

$$S_{00}(\text{Lee}-\text{Yang}) = \sqrt{\frac{5+\sqrt{5}}{10}}$$

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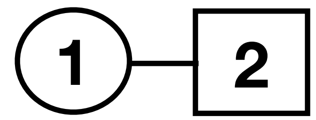

[Gaiotto,Witten:2007]

$\mathcal{T}_{\rm rank\ 0}$	$\mathrm{TFT}_\pm[\mathcal{T}_{\rm rank\ 0}]$	Set of $\{\lvert S^\pm_{0\alpha}\rvert\}$	$\exp(-F)$
$\frac{T[SU(2)]}{SU(2)^{\rm diag}_{\lvert k\rvert=3}}$	$(\text{Lee-Yang})^{\otimes 2}\otimes U(1)_2$	$\left\{\frac{1}{\sqrt{10}}^{\otimes 4}, \frac{5+\sqrt{5}}{10\sqrt{2}}^{\otimes 2}, \frac{5-\sqrt{5}}{10\sqrt{2}}^{\otimes 2}\right\}$	$\frac{5-\sqrt{5}}{10\sqrt{2}}$
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New non-unitary TQFTs?

$$F = -\log\lvert \mathbf{Z}^{S^3_b}(b=1, m=0, \nu=0)\rvert$$ **quantity at superconformal point** $(\nu=0)$

$$= -\log(\min_\alpha \lvert S_{0\alpha}\rvert \text{ of } \mathrm{TFT}_\pm)$$ **at non-superconformal point** $(\nu=\pm 1)$

Lower bounds on F

$F = -\log|\mathbf{Z}^{S^3_b}(b=1, m=0, \nu=0)|$ **quantity at superconformal point** $(\nu=0)$

$= -\log(\min_\alpha |S_{0\alpha}|$ **of TFT**$_\pm)$ **at non-superconformal point** $(\nu=\pm 1)$

The S-matrix should satisfy

$$\textbf{GSD}_g = \sum_{\alpha=0}^{N-1}(S_{0\alpha})^{2-2g} = \begin{cases} 1 & g=0 \\ \mathbb{Z}_{>0} & g>0 \end{cases}$$

For $N=2$, **let** $x = S_{00}^2, y = S_{01}^2$ $(x>y)$

$$x+y=1, \frac{1}{x}+\frac{1}{y}=k\in\mathbb{Z}_{>0} \quad \Rightarrow \quad x=\frac{1}{2}+\frac{1}{2}\sqrt{\frac{k-4}{k}}, y=\frac{1}{2}-\frac{1}{2}\sqrt{\frac{k-4}{k}}$$

$\Rightarrow k\geq 5$ **and** $y \leq \frac{1}{2}(1-\frac{1}{\sqrt{5}})$ $\Rightarrow F \geq -\log\left(\sqrt{\frac{5-\sqrt{5}}{10}}\right) = 0.652965$

Similarly for $N=3$, $\quad F\geq \log 2 = 0.693147$

$$\boxed{\Rightarrow F \geq -\log\left(\sqrt{\frac{5-\sqrt{5}}{10}}\right) = 0.652965}$$

Saturated by the minimal SCFT

Summary

- **We initiate the classification of 3D $\mathcal{N}=4$ rank-0 SCFTsby establishing a correspondence between**

(3D rank-0 SCFT $\mathcal{T}_{\text{rank }0}$) $\longrightarrow$ **(a pair of 3D non-unitary TQFTs $\text{TFT}_{\pm}$)**

It give a physical bulk realization of non-unitary TQFT

- **As an non-trivial dictionary**

$(F$ **of rank-0 SCFT**$) = -\log(\min_\alpha |S_{0\alpha}|$ **of TFT**$_{\pm})$

- **Using the dictioary, we derive**

$$F \geq \underline{-\log\left(\sqrt{\frac{5-\sqrt{5}}{10}}\right)}\ , \quad \text{for any rank-0 } \mathcal{N}=4 \text{ SCFTs}$$

Saturated by the minimal SCFT

- **Future direction**

(3D rank-0 SCFT $\mathcal{T}_{\text{rank }0}$) $\longleftarrow$ **(a pair of 3D non-unitary TQFTs $\text{TFT}_{\pm}$)**

Thank you for your attention!!