Gravitational Positivity Bounds and the Standard Model

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based on arXiv:2104.09682 [PRL 127 091602]

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Gravitational Positivity Bounds and the Standard Model

Unitarity of scattering amplitudes is useful

- to explore how to UV complete low-energy scattering amplitudes

  ex. weak bosons, Higgs boson, string amplitudes

- to provide a necessary condition for an EFT to be UV completable

  → positivity bounds on low-energy scattering amplitudes

  ex. Higher derivative corrections to the Maxwell theory

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \cdots$$

※ positivity bounds imply $\alpha_{1,2} > 0$ [Adams et al '06]

Gravitational Positivity Bounds and the Standard Model

When applied to gravitational theories, they would provide

a necessary condition for a gravitational EFT to be UV completable

→ a criterion to distinguish Swampland from Landscape

※ In this talk, I will discuss

- how positivity bounds are generalized to gravitational theories
- their implications for the Standard Model of particle physics

<u>outline</u>

1. Gravitational Positivity Bounds [Tokuda-Aoki-Hirano '20]
2. Positivity in gravitational QED [Alberte-de Rham-Jaitly-Tolley '20]
3. Positivity in gravitational Standard Model [Aoki-Loc-TN-Tokuda '21]

1. Gravitational Positivity Bounds

Positivity bounds provide a necessary condition for a low-energy scattering amplitude to be UV completable.

- In this talk, we are interested in four-photon scattering.

- For s-u symmetric helicity amplitudes in the forward limit, let us write the IR amplitude as $\mathscr{M}(s, t=0) = \sum_{n=1}^{\infty} a_{2n} s^{2n}$.

  (Meantime, we ignore gravity and assume the above expansion)

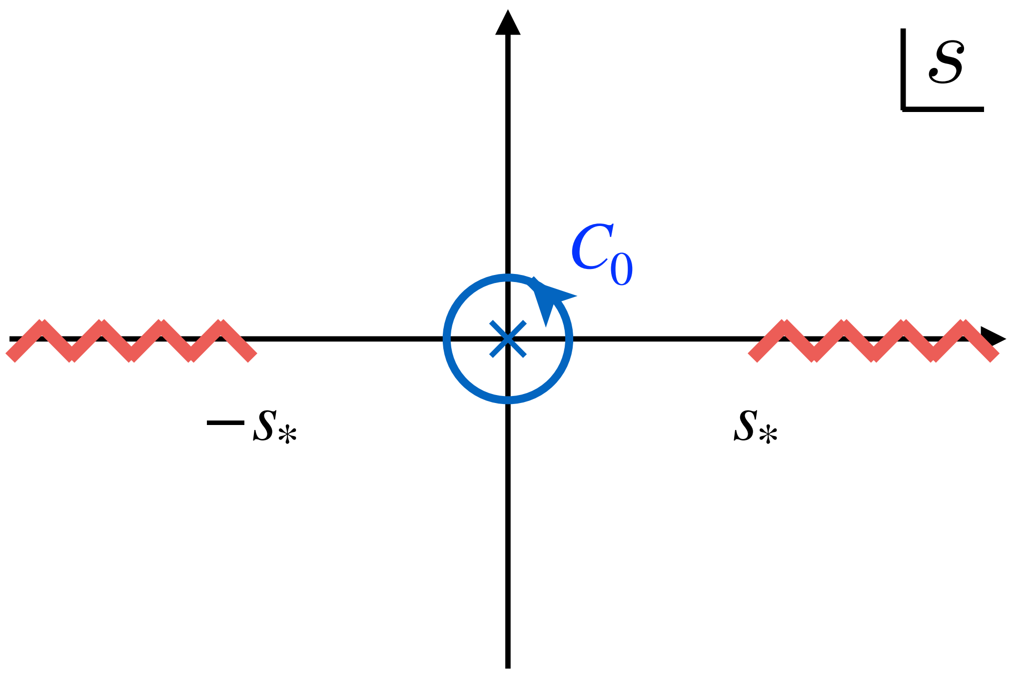
- Then, positivity implies $a_{2n} > 0$ as discussed in the next slides.

  ex. positivity of four-derivative couplings follows from $a_2 > 0$.

Positivity Bounds (w/o gravity) [Adams et al '06]

Consider an s-u crossing helicity sum of $\gamma\gamma \to \gamma\gamma$ scattering in the forward limit:

$$\mathcal{M} = \mathcal{M}_{++++} + \mathcal{M}_{----} + \mathcal{M}_{+-+-} + \mathcal{M}_{-++-+}$$



analytic structure of $\mathcal{M}(s, t = 0)$

IR behavior: $\mathcal{M}(s, t = 0) = a_2 s^2 + \mathcal{O}\left(s^4\right)$

$$a_2 = \oint_{C_0} \frac{ds}{2\pi i} \frac{\mathcal{M}(s, t = 0)}{s^3}$$

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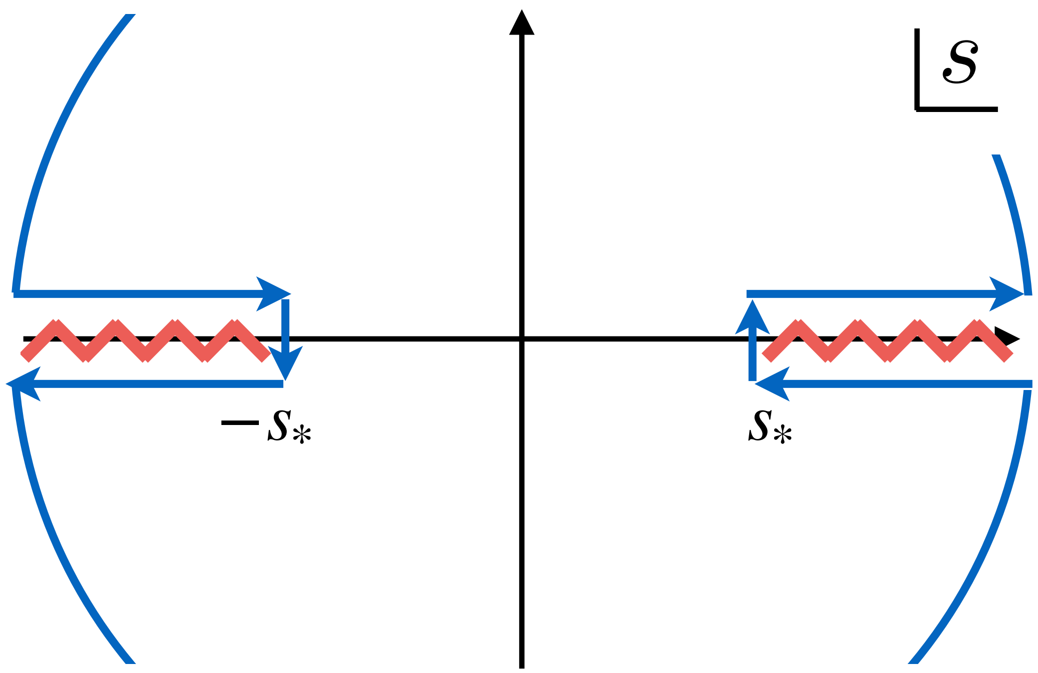

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$$a_2 = \oint_{C_0} \frac{ds}{2\pi i} \frac{\mathcal{M}(s, t=0)}{s^3}$$

Deform the integration contour to rewrite it in the UV language:

$$a_2 = \frac{2}{\pi} \int_{s_*}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3} + \oint_{C_\infty} \frac{ds}{2\pi i} \frac{\mathcal{M}(s, t=0)}{s^3}$$

※ used the s-u symmetry and Disc $\mathcal{M}(s, t=0) = 2i\, \mathrm{Im}\, \mathcal{M}(s, t=0)$

Positivity Bounds (w/o gravity) [Adams et al '06]

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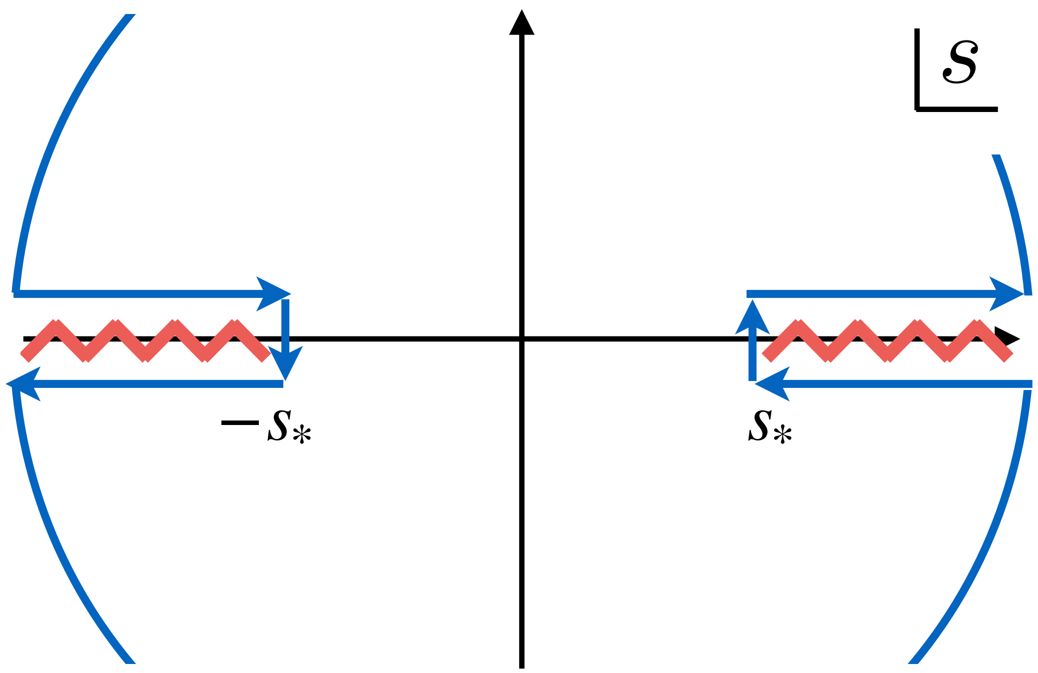


analytic s

IR behavior: $\mathcal{M}(s, t=0) = a_2 s^2 + \mathcal{O}\left(s^4\right)$

$$a_2 = \oint_{C_0} \frac{ds}{2\pi i} \frac{\mathcal{M}(s, t=0)}{s^3}$$

Positive because of unitarity!

Deform the integration contour to rewrite it in the UV language:

$$a_2 = \frac{2}{\pi} \int_{s_*}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3} + \oint_{C_\infty} \frac{ds}{2\pi i} \frac{\mathcal{M}(s, t=0)}{s^3} > 0$$

※ used the s-u symmetry and Disc $\mathcal{M}(s, t=0) = 2i\, \mathrm{Im}\, \mathcal{M}(s, t=0)$

※ assumed $|\mathcal{M}(s, t=0)| < |s|^2$ $(|s| \to \infty)$ (cf. Froissart bound)

Improved Positivity Bounds

To summarize, unitarity and analyticity imply the positivity bound:

$$a_2 = \frac{2}{\pi}\int_{s_*}^{\infty} ds \frac{\text{Im}\mathcal{M}(s, t=0)}{s^3} > 0, \text{ where } \mathcal{M}(s, t=0) = a_2 s^2 + \mathcal{O}\left(s^4\right)$$

It is convenient to rewrite it as [Bellazzini '16, de Rham-Melville-Tolley-Zhou '17, …]

$$B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\text{Im}\mathcal{M}(s, t=0)}{s^3} = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\text{Im}\mathcal{M}(s, t=0)}{s^3} > 0$$

- $B(\Lambda)$ is calculable within the EFT

- $B(\Lambda)$ monotonically decreases as Λ increases

Extension to gravitational theories

Gravitational Positivity Bounds [Tokuda-Aoki-Hirano '20]

In the forward limit, t-channel graviton exchange dominates over the a_2 term:

$$\mathcal{M}(s, t \to 0) \simeq -\frac{s^2}{M_{\rm Pl}^2 t} + \sum_{n=1}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$$

- Careful study of non-forward amplitudes is needed to derive a bound on a_2.

Assume the following Regge behavior of the imaginary part:

$$\mathrm{Im}\mathcal{M}(s,t) \simeq f(t) \left(\frac{s}{M_s^2}\right)^{2+\alpha' t + \alpha'' t^2 + \cdots} \quad (s > M_{\rm Regge} : \text{Reggeization scale}).$$

Then, the bound on a_2 reads [see Tokuda-Aoki-Hirano '20 for details]

$$B(\Lambda) = a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3} > -\frac{1}{M_{\rm Pl}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right).$$

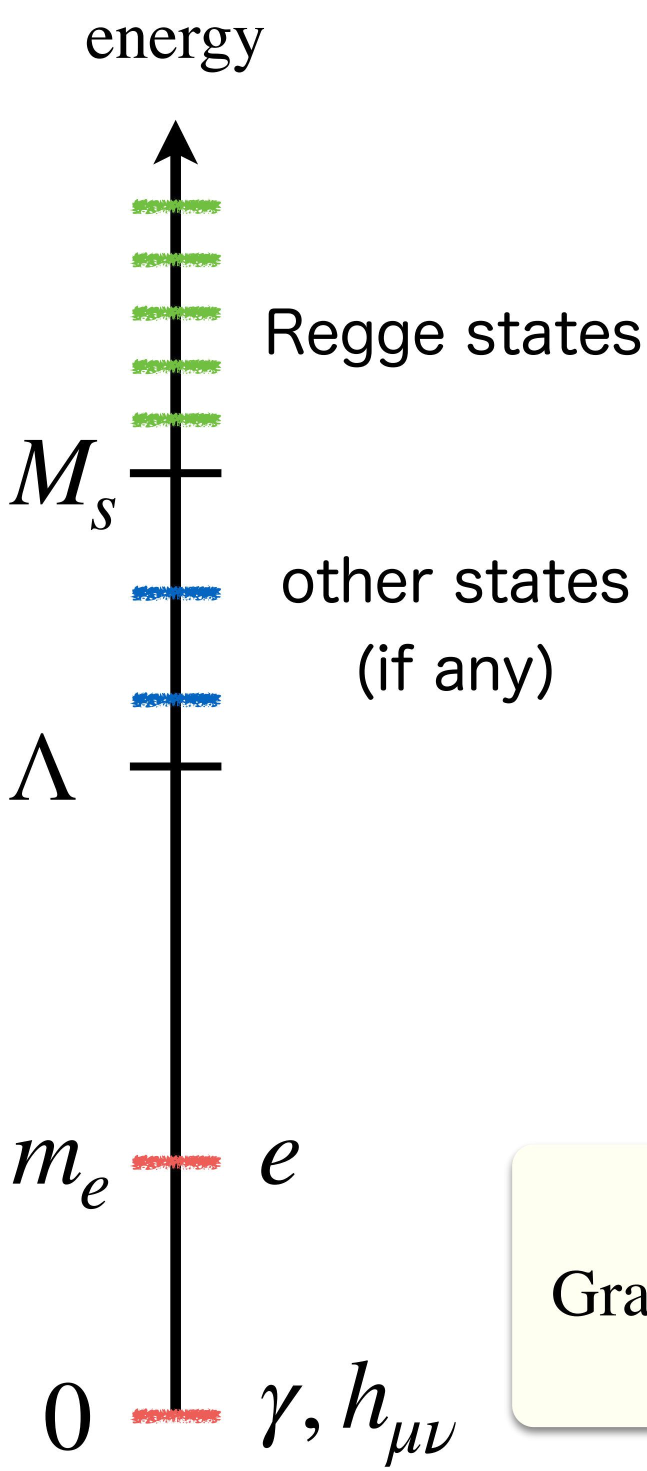
※ RHS depends on details of Regge amplitudes.

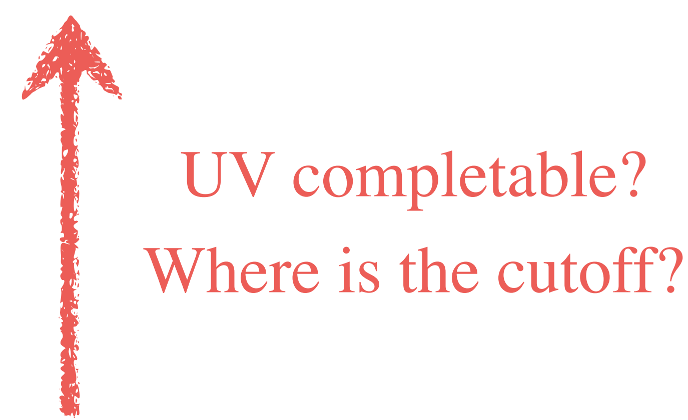
For related developments, see also Hamada-TN-Shiu '18, Herrero-Valea et al '20, Bellazzini et al'19, Alberte et al '20, Arkani-Hamed et al '20, Caron-Huot et al '21.

2. Positivity in Gravitational QED

[Alberte-de Rham-Jaitly-Tolley ’20, see also Aoki-Loc-TN-Tokuda ’21]

Gravitational QED as an EFT





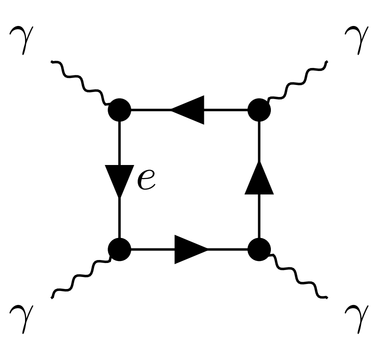
Gravitational QED: $\mathscr{L} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(\not{D} + m_e)\psi + \cdots$

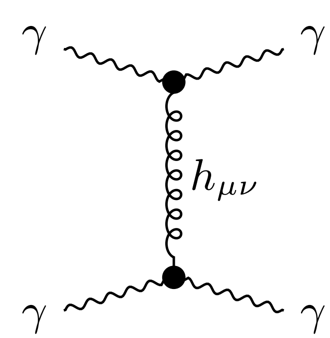
Decomposition of scattering amplitudes

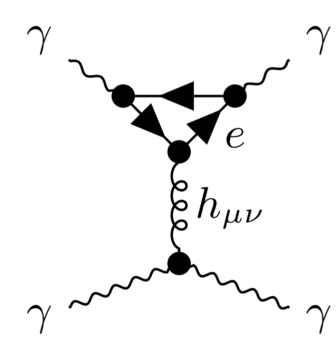
gravitational positivity bounds:

$$B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,0)}{s^3} > -\frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) =: \pm\frac{1}{M_{\mathrm{Pl}}^2 M^2}$$

- Decompose the $\gamma\gamma \to \gamma\gamma$ amplitude at IR as $\mathcal{M} = \mathcal{M}_{\mathrm{QED}} + \mathcal{M}_{\mathrm{GR}} + \mathcal{M}_{\mathrm{UV}}$

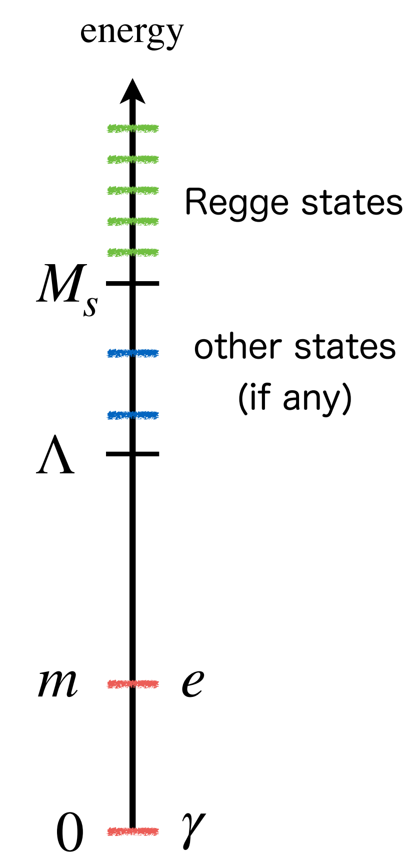
$\mathcal{M}_{\mathrm{QED}} =$ 

, $\mathcal{M}_{\mathrm{GR}} =$ 

$+$ 

$\mathcal{M}_{\mathrm{UV}}$: effects of UV dof $\lesssim \frac{1}{\Lambda^4}$

- We perform similar decompositions, e.g., as $B = B_{\mathrm{QED}} + B_{\mathrm{GR}} + B_{\mathrm{UV}}$



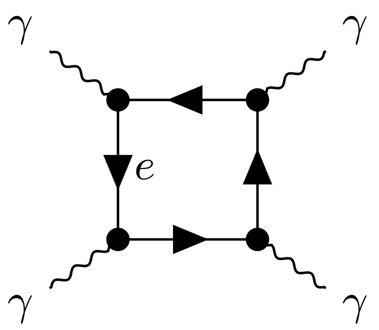
Evaluation of B's

gravitational positivity bounds:

$$B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,0)}{s^3} > -\frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) =: \pm\frac{1}{M_{\mathrm{Pl}}^2 M^2}$$

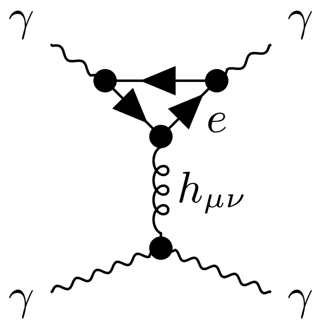
- Contribution of non-gravitational diagrams:

$$B_{\mathrm{QED}}(\Lambda) = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\mathscr{M}_{\mathrm{QED}}(s,0)}{s^3} = \frac{4e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right).$$



※ Notice in particular that $\lim_{\Lambda\to\infty} B_{\mathrm{QED}}(\Lambda) = 0$.

- A straightforward computation shows $B_{\mathrm{GR}}(\Lambda) = -\dfrac{11e^2}{90\pi^2 m_e^2 M_{\mathrm{Pl}}^2}$



This gives a negative contribution that survives even in the limit $\Lambda \to \infty$.

Cutoff scale of gravitational QED

gravitational positivity bounds:

$$B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,0)}{s^3} > -\frac{1}{M_{\rm Pl}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) =: \pm\frac{1}{M_{\rm Pl}^2 M^2}$$

Now the gravitational positivity bound reads

$$\frac{4e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right) - \frac{11e^2}{90\pi^2 m_e^2 M_{\rm Pl}^2} + \frac{\alpha_{\rm UV}}{\Lambda^4} > \pm\frac{1}{M_{\rm Pl}^2 M^2} \qquad (|\alpha_{\rm UV}| \lesssim 1).$$

If we assume a single scaling $M \sim M_{\rm Regge} \gg m_e$ of the Regge amplitude,

we find $\frac{64\alpha^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right) + \frac{\alpha_{\rm UV}}{\Lambda^4} > \frac{22\alpha}{45\pi m_e^2 M_{\rm Pl}^2}$, giving a bound on Λ:

$$\Lambda \lesssim \min\left[\sqrt{e m_e M_{\rm Pl}}, |\alpha_{\rm UV}|^{-1/4}\sqrt{m_e M_{\rm Pl}/e}\right] \sim 10^8\ \text{GeV}.$$

↑

for QED parameters in our real world

Cutoff scale of gravitational QED

gravitational positivity bounds:

$$B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,0)}{s^3} > -\frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) =: \pm\frac{1}{M_{\mathrm{Pl}}^2 M^2}$$

The bound is trivially satisfied if the RHS is negative and $M \sim m_e/e$

Now t

$$\frac{4e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right) - \frac{11e^2}{90\pi^2 m_e^2 M_{\mathrm{Pl}}^2} + \frac{\alpha_{\mathrm{UV}}}{\Lambda^4} > \pm\frac{1}{M_{\mathrm{Pl}}^2 M^2} \qquad (|\alpha_{\mathrm{UV}}| \lesssim 1).$$

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↑ for QED parameters in our real world

Summary so far

- gravitational positivity: $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{4m_e^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,0)}{s^3} > -\frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) =: \pm\frac{1}{M_{\mathrm{Pl}}^2 M^2}.$

- when applied to gravitational QED,

this implies either a cutoff $\Lambda \lesssim \sqrt{m_e M_{\mathrm{Pl}}/e} \sim 10^8$ GeV or a Regge amplitude w/$M \sim m_e/e$

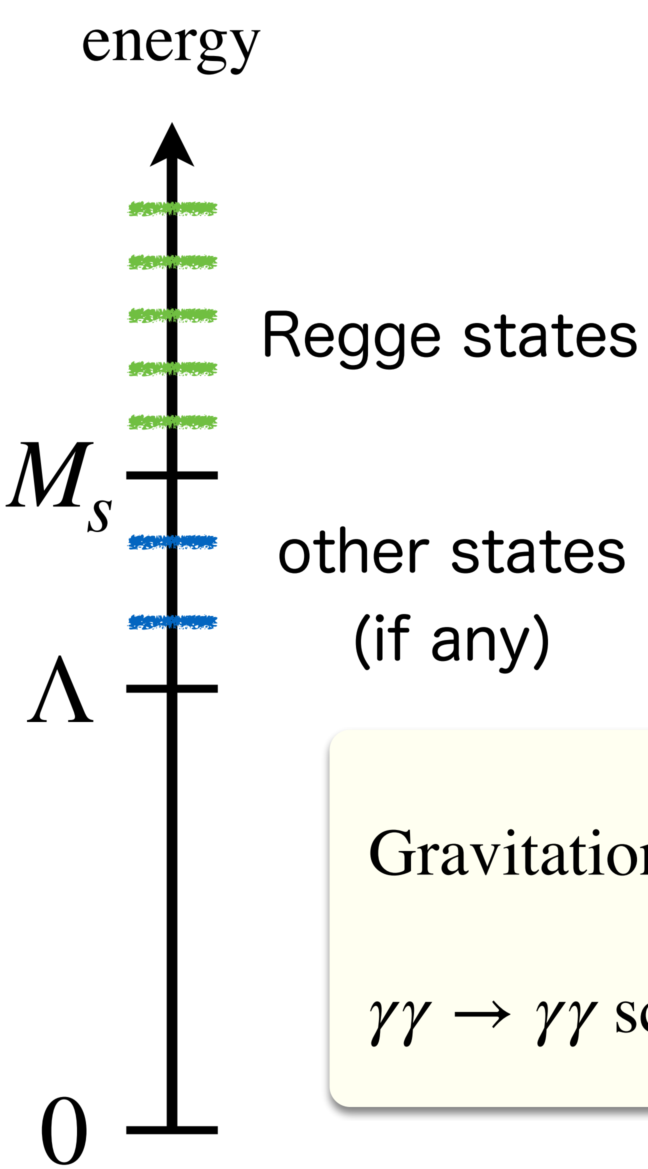
too small to believe the bound??? massless limit is not allowed???

→ we extended the analysis to the Standard Model

3. Positivity in Gravitational Standard Model

[Aoki-Loc-TN-Tokuda '21]

Gravitational Standard Model



Gravitational Standard Model: $\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{M_{\rm Pl}^2}{2} R + \cdots$

$\gamma\gamma \to \gamma\gamma$ scattering: $\mathscr{M} = \mathscr{M}_{\rm QED} + \mathscr{M}_{\rm weak} + \mathscr{M}_{\rm QCD} + \mathscr{M}_{\rm GR} + \mathscr{M}_{\rm UV}$

What to do is the same as the QED case except for

(A) there exist charged spin 1 particles (W bosons)

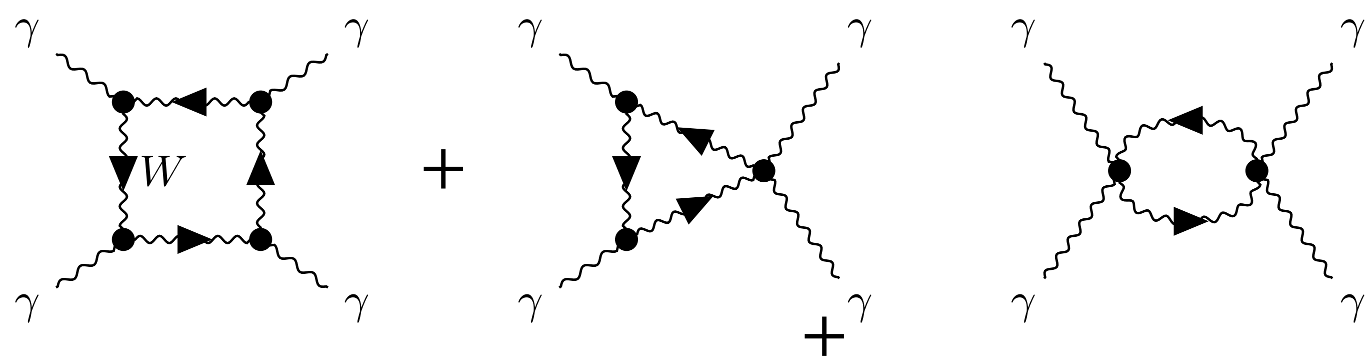
(B) hadrons may contribute if some of s, t, u is below the QCD scale

Weak sector analysis

gravitational positivity bounds: $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,0)}{s^3} > \pm \frac{1}{M_{\mathrm{Pl}}^2 M^2}$

- just like the QED case, we have $B_{\mathrm{weak}}(\Lambda) = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}_{\mathrm{weak}}(s,0)}{s^3}$.
- due to the spin 1 nature, W boson contributions grow faster than the QED case

$\mathcal{M}_{\mathrm{weak}} \simeq$



$$\simeq \frac{2e^4}{\pi^2 m_W^2} s \ln \frac{m_W^2}{-s} + (s \leftrightarrow -s) \qquad \text{cf. } \mathcal{M}_{\mathrm{QED}} \sim \ln^2 s$$

- we then find $B_{\mathrm{weak}}(\Lambda) = \frac{8e^4}{\pi^2 m_W^2 \Lambda^2} > B_{\mathrm{QED}}(\Lambda) = \frac{4e^4}{\pi^2 \Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right)$
- on the other hand, weak boson loops are sub-dominant in B_{GR}

QCD sector analysis

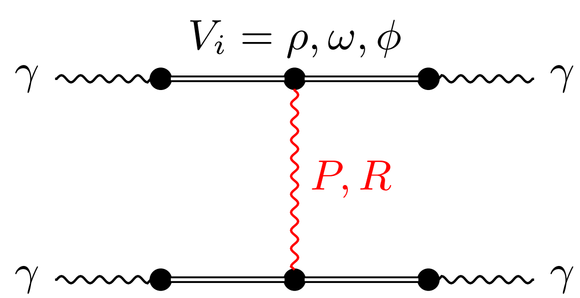
gravitational positivity bounds: $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,0)}{s^3} > \pm \frac{1}{M_{\mathrm{Pl}}^2 M^2}$

- again, we have $B_{\mathrm{QCD}}(\Lambda) = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}_{\mathrm{QCD}}(s,0)}{s^3}$.

- while the amplitude on the r.h.s. is high-energy, the momentum transfer is small

  → t-channel exchange of hadrons is relevant

$\mathrm{Im}\mathcal{M}_{\mathrm{QCD}} \simeq \mathrm{Im}$



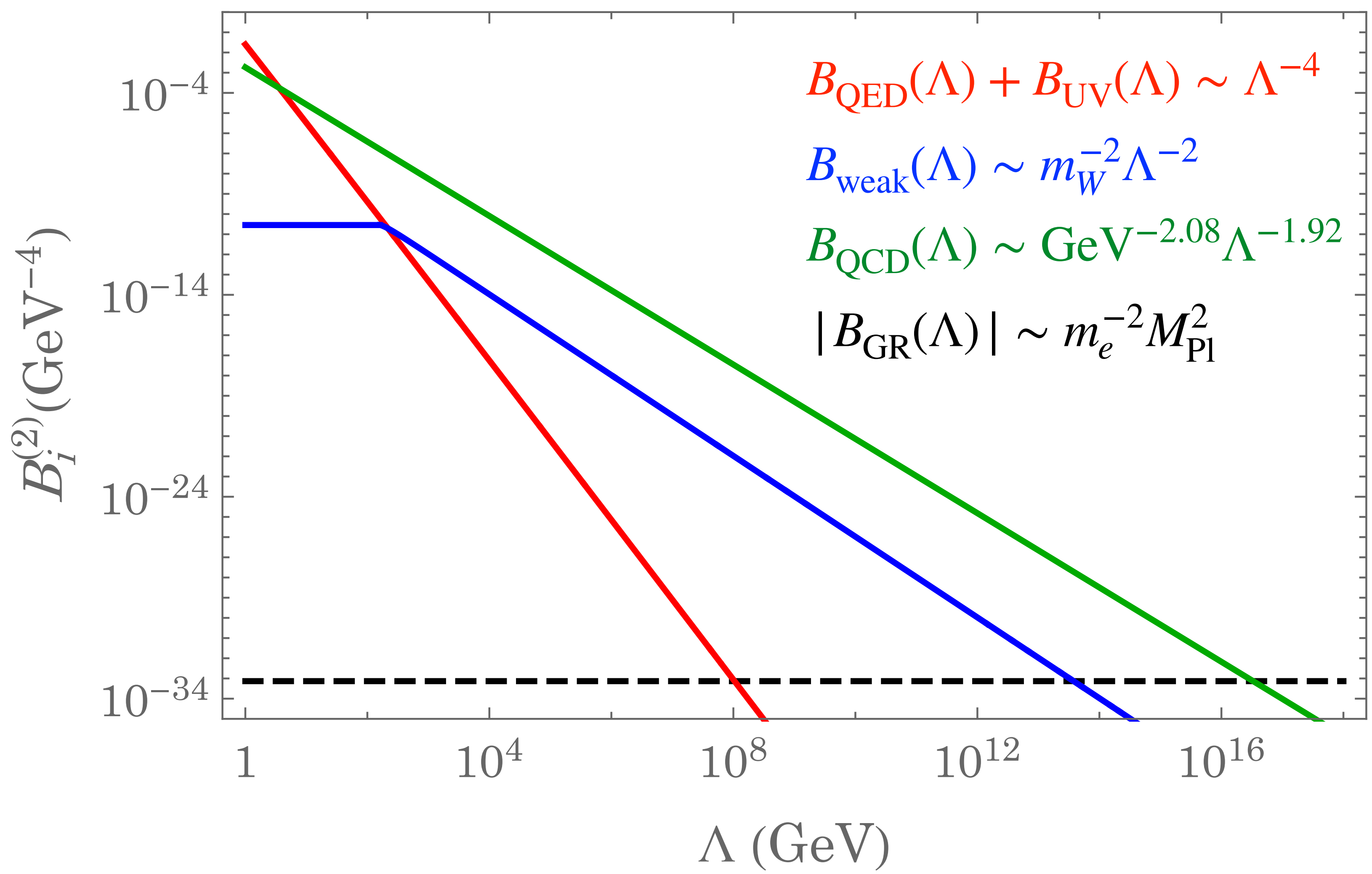
(P: Pomeroon, R: Reggeon)

- employing the Vector Meson Dominance (VDM) model,

$$\mathrm{Im}\mathcal{M}_{\mathrm{QCD}} \simeq \frac{25e^4}{16\pi^2}\left(\frac{s}{\mathrm{GeV}^2}\right)^{1.08}$$

(See our paper for model-(in)sensitivity)

Cutoff scale of gravitational SM

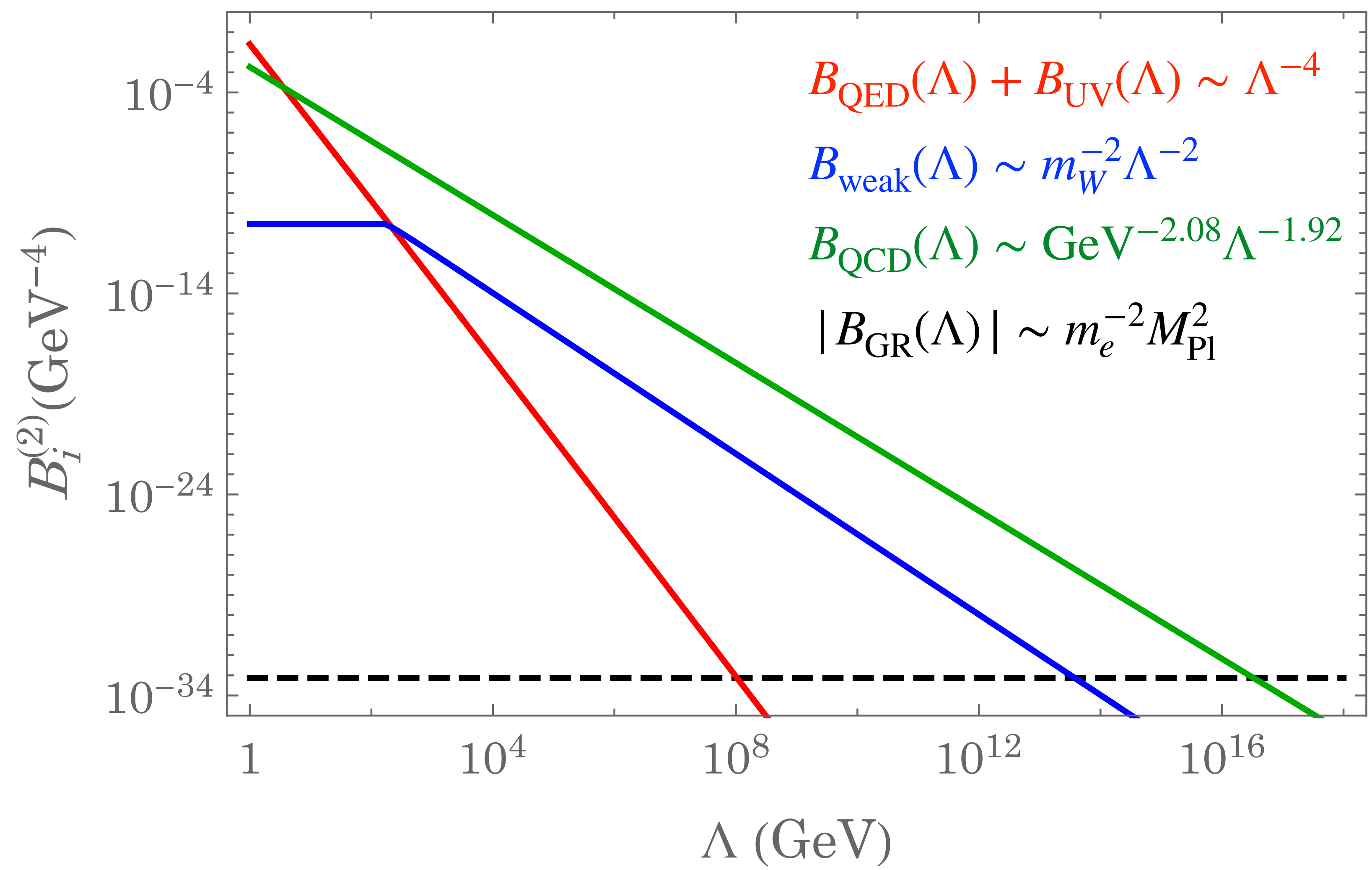


gravitational positivity with a single scaling $M \sim M_{\rm Regge} \gg m_e$:

$$B_{\rm QED}(\Lambda) + B_{\rm UV}(\Lambda) + B_{\rm weak}(\Lambda) + B_{\rm QCD}(\Lambda) > -B_{\rm GR}(\Lambda) \pm \frac{1}{M_{\rm Pl}^2 M^2}$$

→ this defines the cutoff of the gravitational SM $\Lambda \simeq 3 \times 10^{16}$ GeV.

A remark on EW theory w/o QCD



the same bound in the absence of the QCD sector reads

$$B_{\rm weak}(\Lambda) > -B_{\rm GR}(\Lambda) \;\rightleftarrows\; \frac{m_W}{M_{\rm Pl}} < \sqrt{\frac{720}{11}}\,e\,\frac{m_e}{\Lambda} \;\rightleftarrows\; \Lambda < \sqrt{\frac{1440}{11}}\,y_e \sin\theta_W M_{\rm Pl}$$

- Possible explanation for the hierarchy between the EW scale and the Planck scale??

- Massless limit $m_e \to 0$ is allowed if we take the limit $m_W \to 0$ simultaneously

Summary and prospects

Summary

1. Positivity bounds on low-energy scattering amplitudes provide

a criterion for a low-energy EFT to be UV completable in the standard manner

- provides a Swampland condition when applied to gravitational EFTs

2. Puzzles on positivity in gravitational QED [Alberte-de Rham-Jaitly-Tolley '20]

- implies a cutoff scale $\Lambda \sim 10^8$ GeV (too low to believe???)

- implies that massless QED $m_e \to 0$ is in the Swampland (sounds strange???)

under the single scaling assumption $M \sim M_{\rm Regge} \gg m_e$.

3. Positivity in gravitational Standard Model [Aoki-Loc-TN-Tokuda '21]

- the cutoff scale is improved up to $\Lambda \sim 10^{16}$ GeV

- massless limit $m_e \to 0$ is allowed if we take $m_W \to 0$ simultaneously

Future directions

- How generic the single scaling assumption is? $\rightarrow$ detailed study of string amplitudes

  cf. [Alberte-de Rham-Jaitly-Tolley '21] of the last week on graviton-photon scattering

- connections to other principles such as energy conditions, entropy bounds?

- phenomenological applications

  e.g., bounds on scalar potentials [TN-Tokuda '21], dark matters, neutrinos, …

- possible implications for Higgs mechanism in string theory (brane recombination)?

Thank you!