

A BMS-invariant free scalar model

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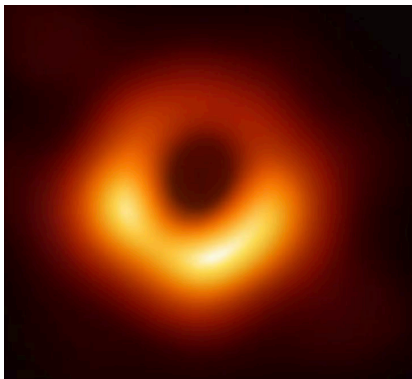
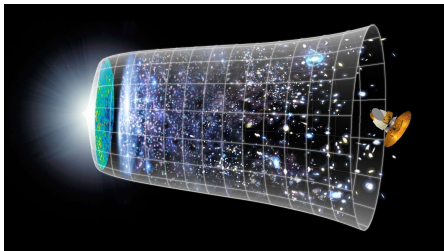
Pengxiang Hao, WS, Xianjin Xie and Yuan Zhong, 2111.04701

& earlier work with Luis Apolo, Hongliang Jiang, Qiang Wen, and Yuan Zhong

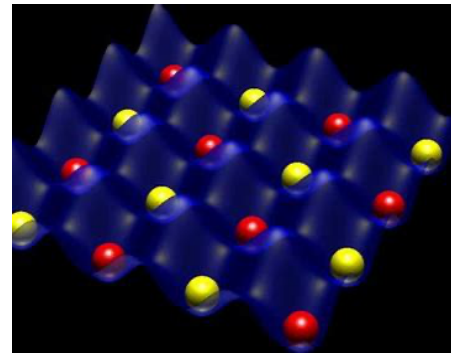
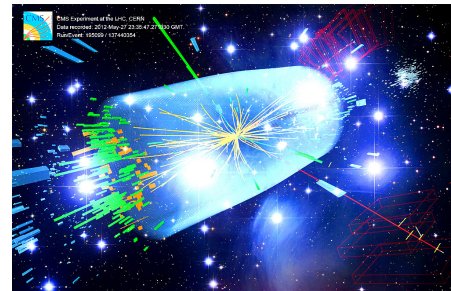
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The study of the AdS/CFT correspondence has been very fruitful. But more steps are necessary to understand our real world.

Non-AdS geometries



Non-CFT quantum systems



Question:

Will holography still work for asymptotically flat spacetime, and if so, how does it work?

Top down approach:

-BFSS matrix model

-TsT / $T\bar{T}$ deformation

- ...

Bottom up approach:

-glue AdS to flat space

-flat/celestial CFT in 4d

-flat/BMSFT in 3d

- ...

➤ The setup: flat holography in 3d

- Holographic entanglement entropy
- A free scalar model of BMSFT

Warmup: AdS_3 / CFT_2 from bottom up approach

<i>Einstein gravity w/ negative c.c & the Brown-Henneaux boundary conditions</i>	<i>CFT_2</i>
<i>Global AdS_3</i>	vacuum on the cylinder
<i>Poincare AdS_3</i>	vacuum on the plane
<i>Isometry group $SL(2,R) \times SL(2,R)$</i>	<i>symmetry for the vacuum $SL(2,R) \times SL(2,R)$</i>
Asymptotic symmetry Virasoro x Virasoro	2d conformal symmetry Virasoro x Virasoro
Entropy for BTZ black holes	Cardy formula
HHRT formula	Entanglement entropy

[Bondi, van der Burg, Metzner, Sachs 62']

[Barnich-Compere 06', Bagchi-Gopakumar 09', Barnich-Troessaert, Bagchi 10']

Flat holography for 3d Einstein gravity From AdS_3 : $l = \frac{1}{\epsilon} \rightarrow \infty$

Asymptotic flat solutions of pure Einstein gravity in Bondi gauge

$$ds^2 = P(\phi)du^2 - 2dudr + [J[\phi] + u\partial_\phi P(\phi)]dud\phi + r^2d\phi^2,$$

← Bananos solutions

$$\phi \sim \phi + 2\pi$$

Zero mode solutions: $ds^2 = Mdu^2 - 2dudr + Jdud\phi + r^2d\phi^2$

- $M = -1, J = 0$ global Minkowski spacetime, with isometry

generators: $L_1, L_0, L_{-1}, M_1, M_0, M_{-1}$

global AdS

- $M > 0$, Flat space cosmological solutions(FSC) with a Cauchy horizon

BTZ with the inner horizon

- $M < 0$, conical defects

Alternative perspective: flat limit of AdS_3 gravity

The asymptotic symmetry generators includes superrotations L_m and supertranslations M_n .

BMS₃ algebra with central extension,

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12}(n^3 - n)\delta_{n+m,0}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12}(n^3 - n)$$

$$[M_n, M_m] = 0$$

For Einstein gravity, the gravitational anomaly

$$c_L = 0, \quad \text{but} \quad c_M = \frac{3}{G}$$

For topologically massive gravity $c_L \neq 0$

$$c^+ - c^- \rightarrow c_L, \quad c^+ + c^- \rightarrow \ell c_M$$

Asymptotic symmetry analysis suggests a holographic relation between the following theories:

- Three dimensional Einstein gravity with asymptotic flat boundary conditions imposed at null infinity
- Two dimensional quantum field theories with BMS invariance (BMSFT)

BMSFT: field theory invariant under BMS transformations

$$\tilde{\sigma} = f(\sigma), \quad \tilde{\tau} = f'(\sigma)\tau + g(\sigma), \quad \sigma \sim \sigma + 2\pi$$

From cylinder to plane: $x = e^{i\sigma}$, $y = i e^{in\sigma} \tau$

Generators in the plane: $l_n = -x^{n+1} \partial_x - (n+1)yx^n \partial_y$
 $m_m = -x^{n+1} \partial_y$

States fall into representations of the centrally extended BMS algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} (n^3 - n)\delta_{n+m,0}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12} (n^3 - n)$$

$$[M_n, M_m] = 0$$

BMSFT as the UR (ultra relativistic) limit of CFT_2

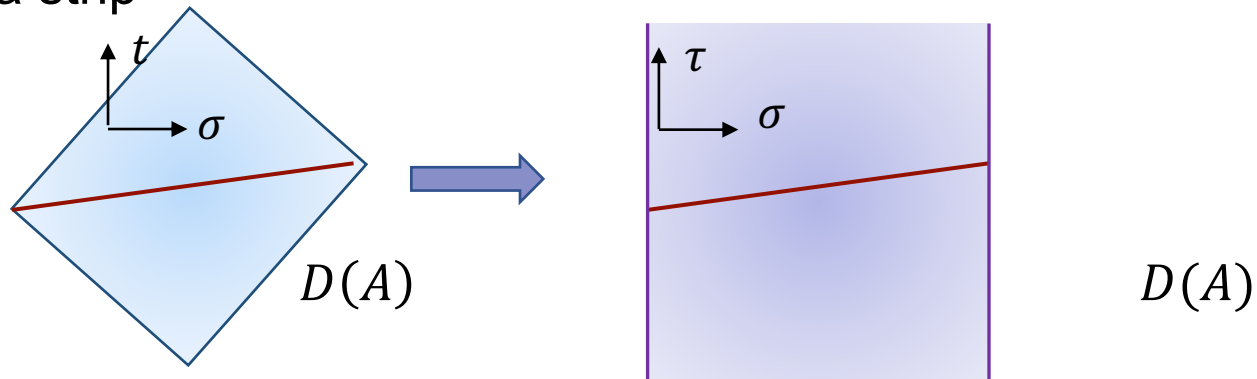
consistent with the bulk $\epsilon \propto \frac{1}{l}$

From CFT_2 : ultra relativistic limit on the cylinder $t \rightarrow \epsilon \tau, \sigma \rightarrow \sigma$

$$L_n^+ - L_{-n}^- \rightarrow L_n, \quad L_n^+ + L_{-n}^- \rightarrow \frac{M_n}{\epsilon},$$

$$c^+ - c^- \rightarrow c_L, \quad c^+ + c^- \rightarrow \frac{c_M}{\epsilon}$$

Effectively the speed of light is going to zero, and the causal diamond becomes a strip



Galilean conformal algebra (**GCA**): **non-relativistic** limit $t \rightarrow t, \sigma \rightarrow \epsilon \sigma$

[Duval-Horvathy 09', Martelli-Tachikawa, 09']

BMS algebra is isomorphic to GCA, but lots of properties are different.

- Torus partition function for BMSFT is modular invariant.

$$(\tau, \sigma) \sim (\tau + i\bar{a}, \sigma - ia) \sim (\tau + i\bar{b}, \sigma - ib)$$

- `Cardy' formula for BMSFT

Barnich

Bagchi-Detournay-Fareghbal-Simón

$$S_{\bar{b}|b}(\bar{a}|a) = -\frac{\pi^2}{3} \left(c_L \frac{b}{a} + c_M \frac{(\bar{a}b - a\bar{b})}{a^2} \right). \quad \text{reproduces the entropy of FSC}$$

- Torus 1pf vs asymptotic structure constant

Kraus-Moloney

WS-Xu


Bagchi-Nandi-Saha-Zodinmawia

- Conformal bootstrap

Bagchi-Gary, 16'

Chen-Hao-Liu-Yu, 20'

flat/BMSFT from bottom up approach

<i>3d Einstein gravity in asy. flat spacetime</i>	<i>BMSFT</i>
<i>Global Minkovski</i>	vacuum on the cylinder
<i>Poincare</i>	vacuum on the plane
<i>Isometry group</i> <i>Poincare</i> $L_n, M_n, n = 0, \pm 1$	<i>symmetry for the vacuum</i> $L_n, M_n, n = 0, \pm 1$
Asymptotic symmetry Generated by L_n, M_n	BMS symmetry Generated by L_n, M_n
Entropy for FSC black holes	Cardy formula
 Holographic entanglement entropy	Entanglement entropy

Holographic entanglement entropy in AdS/CFT

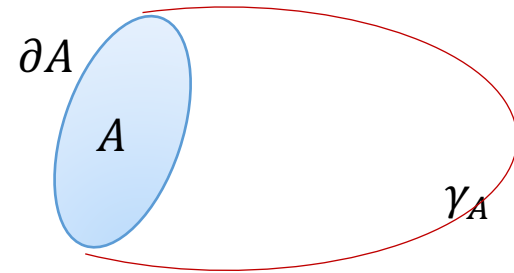
*Ryu-Takayanagi,
Hubeny-Rangamani-Takayanagi*

$$S_A = \min \frac{\text{Area}(\gamma_A)}{4G}$$

γ_A : co-dimension 2 extremal
surface homologous to ∂A

Well established in AdS/CFT, for Einstein Gravity

- Rindler method: *Casini-Huerta-Myers (CHM)*
- Generalized gravitational entropy
Lewkowycz-Maldacena, Dong-Lewkowycz-Rangamani



Question: what is the holographic entanglement entropy in holographic dualities for non-AdS spacetimes?

Our approach:

use the “proof” to “derive” the bulk picture

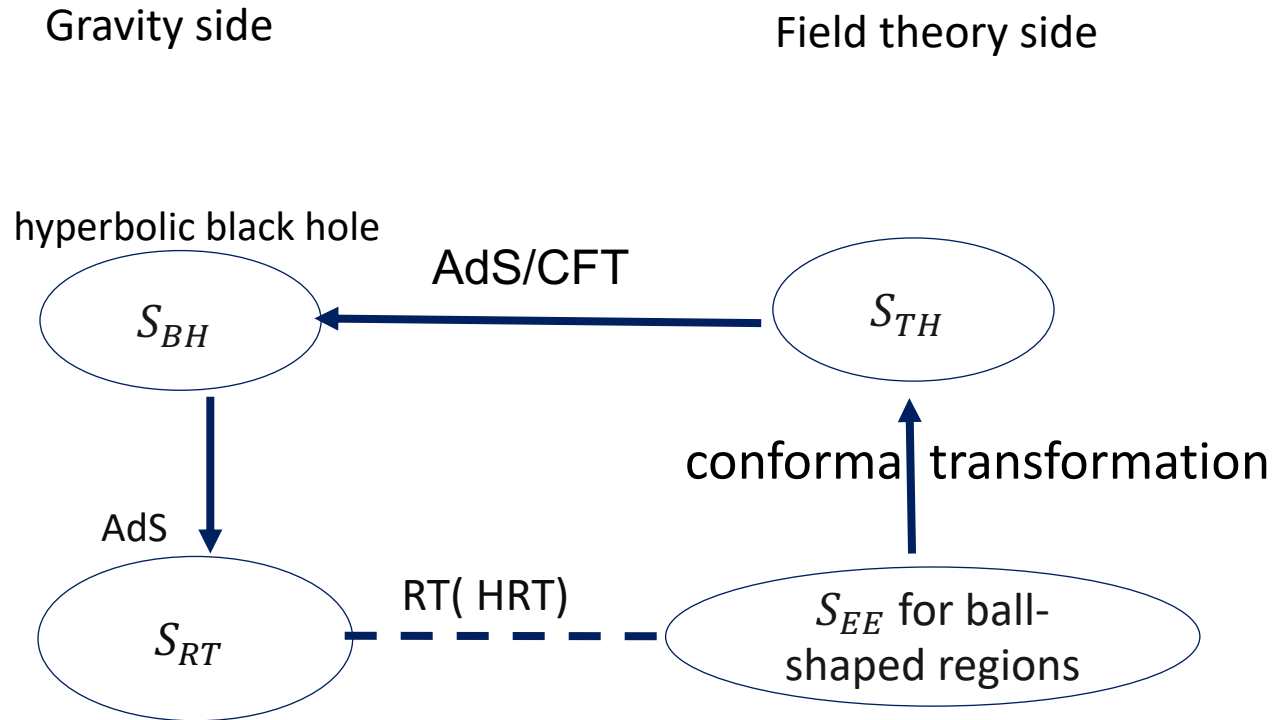
- Assumptions: flat holography exists.

Asymptotic symmetry in gravity = symmetry of the dual field theory=BMS

- A new geometric picture: the swing surface

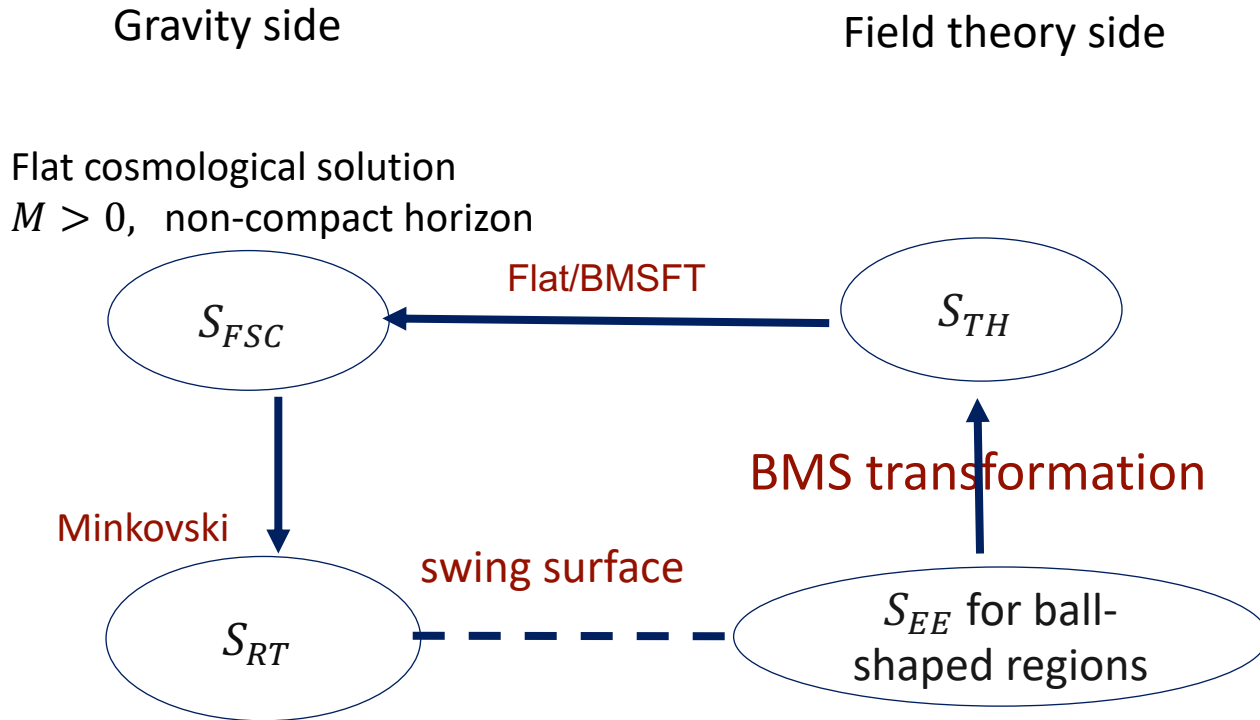
The Rindler method in AdS/CFT :
derivation of RT for ball-shaped subregions

Casini-Huerta-Myers



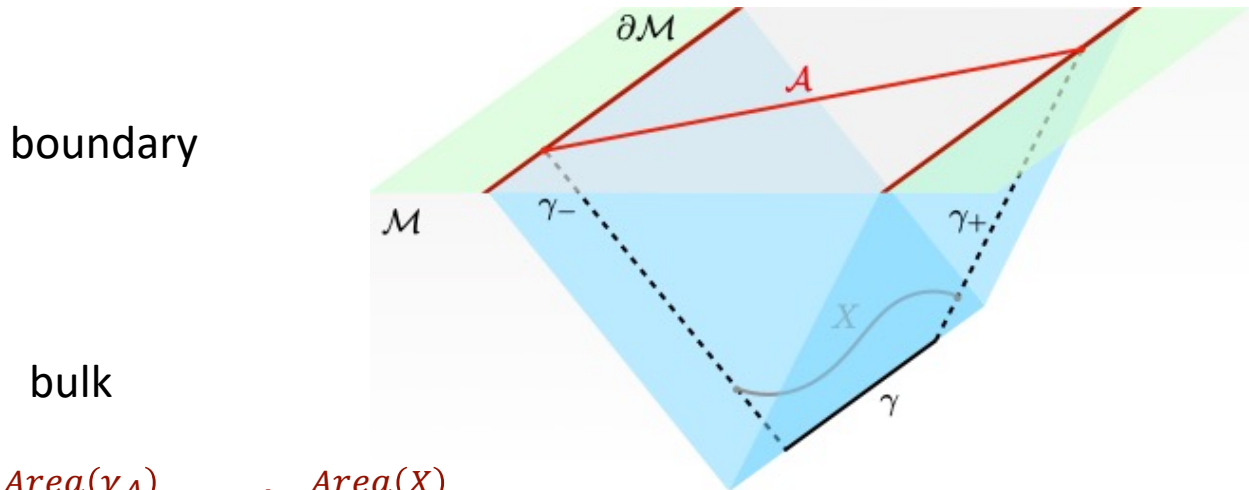
The generalized Rindler method in non-AdS holography :
derivation for ball-shaped subregions

Castro-Hofman-Iqbal
WS-Wen-Xu
Jiang-WS-Wen



The new geometric picture:

holographic entanglement entropy is given by an extremal **swing surface**



$$S_A = \frac{\text{Area}(\gamma_A)}{4G} = \min_{X \sim A} \frac{\text{Area}(X)}{4G}$$

swing surface: $\gamma_A = \gamma_+ \cup \gamma_- \cup \gamma$

ropes γ_{\pm} : null geodesics emanating from points in ∂A , along the modular flow direction

bench γ : extremal surface between γ_+ and γ_-

Consistency checks and related developments

Pure field theory calculation:

- Replica trick: 2pf of twist operators *Bagchi-Basu-Grumiller-Riegler*
- Generalized Rindler method: $S_{EE} \rightarrow S_{TH} \rightarrow S_{Generalized\ Cardy}$ *Jiang-WS-Wen*

Holographic calculation:

- Wilson lines in Chern-Simons formalism *Basu-Riegler*
- Rindler method *Jiang-WS-Wen*
- Generalized gravitational entropy argument *Apolo-Jiang-WS-Zhong 20'*

Flat limit from AdS_3 /Wigner contraction from CFT

Geodesic Witten diagrams *Hijano-Rabideau, Hijano*

Quantum energy conditions *Grumiller-Praekh-Riegler*

Entanglement 1st law vs the Einstein equation *Godet-Marteau, Fareghbal-Shalamzari*

Modular Hamiltonian *wen 18', Apolo-Jiang-WS-Zhong 20'*

- ✓ The setup: flat holography in 3d
- ✓ Holographic entanglement entropy
- A free scalar model of BMSFT

BMS free scalar theory

The action on the plane is $S = \frac{1}{4\pi} \int dydx (\partial_y \phi)^2$.

The equation of motion $\partial_y^2 \phi = 0$.

The classical solution to the equation of motion is given by

$$\phi(\sigma, \tau) = \sum_{n=-\infty}^{\infty} x^{-n} (A_n + iy/x B_n)$$

The reality condition then implies the adjoint relation

$$A_n^\dagger = A_{-n}, \quad B_n^\dagger = -B_{-n}$$

This model is a building block of the string worldsheet theory at the tensionless limit
See e. g. Bagchi 13', Bagchi- Banerjee-Chakraborty 20'

Canonical quantization

After canonical quantization, we have the commutation relations

$$[A_n, B_m] = \delta_{n+m,0}, \quad [A_n, A_m] = [B_n, B_m] = 0$$

Stress tensors are given by

$$T(x, y) = - : \partial_x \phi \partial_y \phi :, \quad M(x) = -\frac{1}{2} : \partial_y \phi \partial_y \phi :,$$

The conserved charges on the plane

$$L_n = \frac{1}{2\pi i} \oint ((n+1)x^n y M + x^{n+1} T) dx, \quad M_n = \frac{1}{2\pi i} \oint dx x^{n+1} M.$$

generate the BMS transformations

$$l_n = -x^{n+1} \partial_x - (n+1)yx^n \partial_y, \quad m_n = -x^{n+1} \partial_y.$$

The vacuum

For CFT_2 , the vacuum is invariant under $SL(2, R) \times SL(2, R)$

For BMSFT, the vacuum is invariant under the Poincaré subgroup

$$L_{\pm 1, 0}|0\rangle = M_{\pm 1, 0}|0\rangle = 0$$

Option I: the highest weight vacuum $A_n|0\rangle = 0, n > 0, B_m|0\rangle = 0, m \geq 0$

$$M_n|0\rangle = 0, \quad L_n|0\rangle = 0, n \geq -1.$$

BMS algebra with $c_L = 2, c_M = 0$.

Option II: the induced vacuum $B_n|0_I\rangle = 0, \forall n \in \mathbb{Z}$.

$$L_0|0_I\rangle = M_n|0_I\rangle = 0, \quad \forall n \in \mathbb{Z}.$$

Highest weight representation

Singlet: L_0 and M_0 can be diagonalized

Highest weight operator at the origin of the plane

$$[L_0, O] = \Delta O, \quad [M_0, O] = \xi O. \quad \Delta: \text{conformal weight}, \quad \xi: \text{boost charge}$$

$$[L_n, O] = 0, \quad [M_n, O] = 0, \quad n > 0.$$

Local operators obtained from $O(x, y) = UO(0, 0)U^{-1}$.

Subtleties:

1. non-unitarity
2. the action of L_0 and M_0 on the descendant operators are not simultaneously diagonal

The appearance of multiplets in descendants is a generic feature of BMS algebra.

Highest weight representation: multiplet Chen-Hao-Liu-Yu, 20'

The action of M_0 is block diagonal

$$\begin{aligned} [L_0, O_a] &= \Delta O_a, & [M_0, O_a] &= (\boldsymbol{\xi} O)_a, & a &= 0, \dots, r-1 \\ [L_n, O_a] &= 0, & [M_n, O_a] &= 0, & n &> 0, \end{aligned}$$

where O_a denotes the a -th component of the multiplet \mathbf{O} ,

$\boldsymbol{\xi}$ is a Jordan cell with rank r ,

$$\boldsymbol{\xi} = \begin{pmatrix} \xi & & & & \\ 1 & \xi & & & \\ & \ddots & \ddots & & \\ & & & 1 & \xi \end{pmatrix}_{r \times r} .$$

r is called the rank of the multiplet.
When $r = 1$ we get back the singlet.

The free scalar model

Hao-WS-Xie-Zhong, 21'

Primary operators should satisfy the following OPE

$$T(\tilde{x}, \tilde{y})\mathbf{O}(x, y) \sim \frac{\Delta\mathbf{O}}{(\tilde{x} - x)^2} + \frac{2(\tilde{y} - y)\xi\mathbf{O}}{(\tilde{x} - x)^3} - \frac{\partial_x\mathbf{O}}{\tilde{x} - x} - \frac{(\tilde{y} - y)\partial_y\mathbf{O}}{(\tilde{x} - x)^2},$$
$$M(\tilde{x}, \tilde{y})\mathbf{O}(x, y) \sim \frac{\xi\mathbf{O}}{(\tilde{x} - x)^2} + \frac{\partial_y\mathbf{O}}{\tilde{x} - x}.$$

- Primary singlets:

The vacuum $|0\rangle \sim 1$, $\Delta = \xi = 0$

vertex operator $V_\alpha =: e^{\alpha\phi}:$, $\Delta = 0$, $\xi = -\frac{\alpha^2}{2}$

- Primary multiplet (rank 2):

$$O_0 = i\partial_x\phi, \quad O_1 = i\partial_y\phi, \quad \Delta = 1, \quad \xi = 0.$$

Quasi-pramary multiplets

$$T(x', y')\mathbf{O}(x, y) \sim \dots + \frac{\Delta\mathbf{O}}{(x'-x)^2} - \frac{2(y'-y)\xi\mathbf{O}}{(x'-x)^3} + \frac{\partial_x\mathbf{O}}{x'-x} - \frac{(y'-y)\partial_y\mathbf{O}}{(x'-x)^2},$$

$$M(x', y')\mathbf{O}(x, y) \sim \dots + \frac{\xi\mathbf{O}}{(x'-x)^2} + \frac{\partial_y\mathbf{O}}{x'-x}.$$

From general analysis one may expect that T and M form a rank 2 multiplet with $\Delta=2$, $\xi=0$.

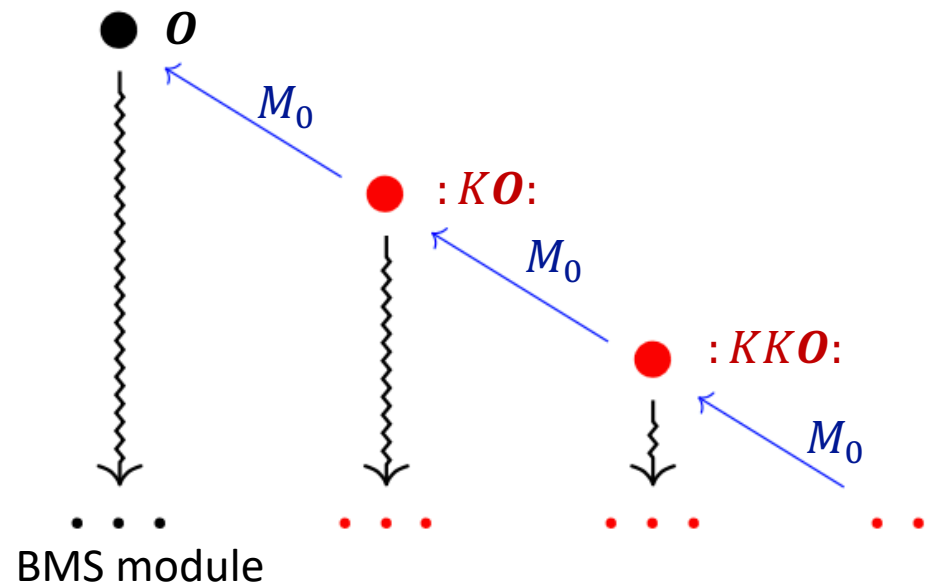
In the free scalar model, however, there is an extra level 2 quasi-primary

$$K \equiv -\frac{1}{2} : \partial_x \phi \partial_x \phi :$$

so that $\mathbf{T} = \{2M, T, K\}$ is a rank 3 multiplet, with $\Delta = 2$, $\xi = 0$.

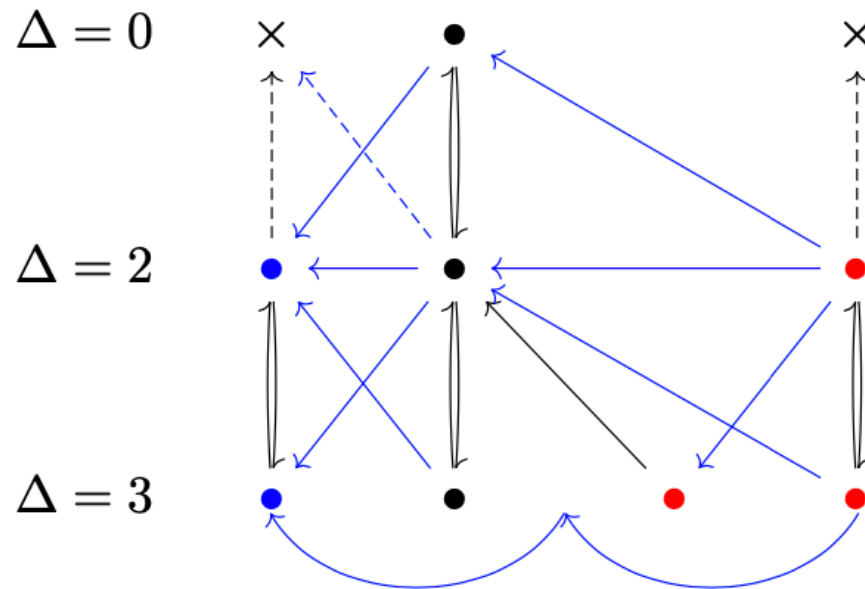
The staggered module

Due to the existence of the extra quasi-primary operator at level 2, states are now organized by an enlarged BMS module, named staggered module.



Similar structure also appears in logarithmic CFTs [Gaberdiel-Kausch, Rohsiepe, 96']

The vacuum module

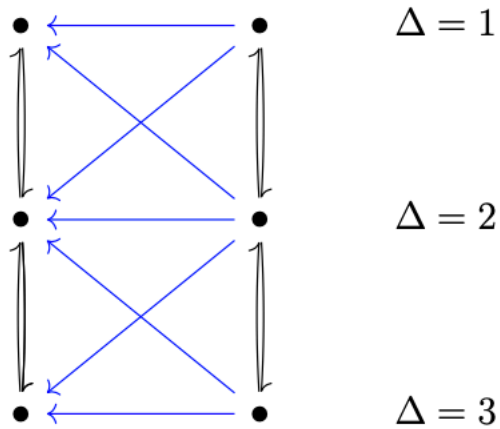


states with $\Delta = 2$ form a quasi-primary multiplet, from left to right: $|M\rangle$, $|T\rangle$, $|K\rangle$;

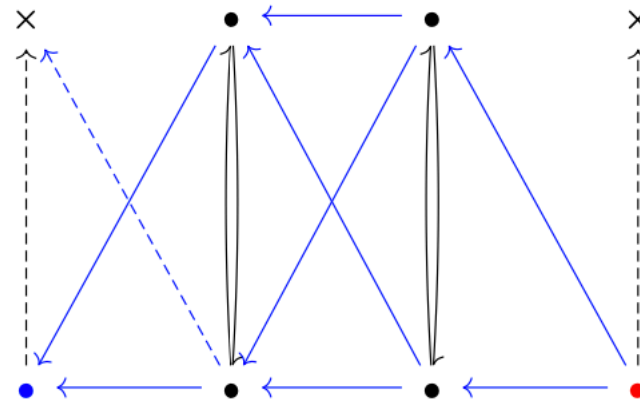
states with $\Delta = 3$ from left to right: $L_{-1}|M\rangle$, $L_{-1}|T\rangle$, $M_{-1}|K\rangle$, $L_{-1}|K\rangle$.

The four states at level 3 split into two multiplets: a singlet $L_{-1}|T\rangle - 3M_{-1}|K\rangle$, and a triplet consisting of $L_{-1}|M\rangle$, $L_{-1}|T\rangle + M_{-1}|K\rangle$ and $L_{-1}|K\rangle$.

The O module



states with $\Delta = 1$: $|O_0\rangle, |O_1\rangle$;
 states with $\Delta = 2$: $L_{-1}|O_0\rangle, L_{-1}|O_1\rangle$;
 states with $\Delta = 3$: $L_{-1}^2|O_0\rangle, L_{-1}^2|O_1\rangle$.



states with $\Delta = 1$: $|O_0\rangle, |O_1\rangle$;
 states with $\Delta = 3$:
 $M_{-2}|O_0\rangle, L_{-2}|O_0\rangle, L_{-2}|O_1\rangle, |KO_1\rangle$.

Correlation functions

$$\text{Singlets: } \langle \prod_{k=1}^n V_{\alpha_k}(x_k, y_k) \rangle = \exp\left\{ \sum_{i < j}^n (-\alpha_i \alpha_j) \frac{y_i - y_j}{x_i - x_j} \right\}, \quad \sum_k \alpha_k = 0$$

Multiplets:

$$\langle O_1 O_1 \rangle = -\frac{1}{x^{2\Delta}} \frac{2y}{x}$$

$$\langle O_0 O_1 \rangle = \frac{1}{x^{2\Delta}}$$

$$\langle O_0 O_0 \rangle = 0$$

Consistent with general discussions.

The **Hilbert space** is spanned by $|\vec{i}, \vec{j}; \alpha\rangle = A_{-1}^{i_1} A_{-2}^{i_2} \cdots B_{-1}^{j_1} B_{-2}^{j_2} \cdots |\alpha\rangle$

The **Torus partition function** on the torus defined by

$$\textit{spatial circle} : (\tau, \sigma) \sim (\tau, \sigma + 2\pi)$$

$$\textit{thermal circle} : (\tau, \sigma) \sim (\tau - 2\pi ib, \sigma - 2\pi ia)$$

is given by

$$Z(a, b) \equiv \text{Tr} e^{-2\pi a(L_0 - \frac{cL}{24}) - 2\pi b(M_0 - \frac{cM}{24})} = \sqrt{\frac{2}{|b|}} \frac{1}{\eta^2(ia)}$$

which is manifestly modular invariant $Z(a, b) = Z(\frac{1}{a}, -\frac{b}{a^2})$

More to be understood

- Further understanding of primary multiplets and staggered module
- The model has central charges $c_L = 2$, $c_M = 0$, whereas the dual of Einstein gravity has $c_L = 0$.
model of BMSFT with $c_L = 0$?
- Supersymmetric version
- Interpretation of the primary multiplets and staggered module in the holographic dual
- Implications in tensionless strings

Summary and discussion

- Finding holographic duality for non-AdS spacetime is very challenging
- Some progress can be made in the bottom up approach based on asymptotic symmetries
- Geometric picture of holographic entanglement entropy becomes a swing surface in the examples of $flat_3$ /BMSFT and (W)AdS/WCFT dualities.
- A free scalar model of BMSFT can be studied explicitly. Novel features including highest weight multiplets and staggered module appear appear in the model.
- Further study is needed to understand the putative duality, or to kill it.

Thank you!