Regge conformal blocks from the Rindler-AdS black hole and the pole-skipping phenomena

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[arXiv:2105.07778]

with Keun-Young Kim, Kyung-Sun Lee

Regge limit is constrained by consistencies of theories

Regge limit in scattering amplitude

Regge limit of bulk S-matrix

$$M(s,t) \quad (s \to \infty, \text{fixed } t)$$

Regge limit in CFTs

Regge limit of Mellin amplitude

$$\mathcal{M}(s,t) \ (s \to \infty, t \text{ fixed})$$

Regge limit of CFT correlators

$$\mathcal{A}(z,\bar{z}) \ \ (1-z) \to e^{-2\pi i}(1-z), \bar{z} \text{ fixed}$$

$$z, \bar{z} \to 0, \ \frac{\bar{z}}{z} \ \text{ fixed}$$

[L. Cornalba, M. S. Costa, J. Penedones, R. Schiappa, 2006]

Regge limit in CFTs is related to out-of-time-order correlator (OTOC)

OTOC is a measure of quantum chaos.

$$\frac{\langle W(t,\mathbf{d})V(0,0)W(t,\mathbf{d})V(0,0)\rangle}{\langle W(t,\mathbf{d})W(t,\mathbf{d})\rangle\langle V(0,0)V(0,0)\rangle} \sim 1 - \varepsilon e^{\lambda_L(t-\mathbf{d}/v_B)} + \cdots$$

[A. Larkin, Y. N. Ovchinnikov, 1969], [A. Kitaev, 2014]

Regge limit corresponds to an analytic continuation from Euclidean correlator to OTOC

[D. A. Roberts, D. Stanford, 2014] [E. Perlmutter, 2016]

Lyapunov exponent λ_L is bounded: $\lambda_L \leq 2\pi/\beta$, which is related to Regge growth bound of S-matrix

[J. Maldacena, S. H. Shenker, D. Stanford, 2015], [D. Chandorkar, S. D. Chowdhury, S. Kundu, S. Minwalla, 2021]

Regge conformal block with $T_{\mu\nu}$ exchange captures λ_L and v_B of holographic CFTs

Conformal block expansion in d-dim CFTs

$$\langle W(x_1)W(x_2)V(x_3)V(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_W} x_{34}^{2\Delta_V}} \sum_{\mathcal{O}} C_{WW\mathcal{O}} C_{VV\mathcal{O}} G_{\Delta,\ell}(z,\bar{z})$$

Regge conformal blocks in OTOCs

$$G^{\text{Regge}}_{\Delta,\ell}(z,\bar{z}) \propto \underline{e^{(\ell-1)t-(\Delta-1)\mathbf{d}}}\ {}_2F_1\left(\Delta-1, \frac{d}{2}-1, \Delta+1-\frac{d}{2}; e^{-2\mathbf{d}}\right)$$

$e^{t-(d-1)\mathbf{d}}$ with $T_{\mu\nu}$ exchange $(\ell=2, \Delta=d)$

captures $\lambda_L = \dfrac{2\pi}{\beta}, v_B = \dfrac{1}{d-1}$ in the Rindler spacetime

$(\beta = 2\pi)$

[D. A. Roberts, D. Stanford, 2014] [E. Perlmutter, 2016]

Proposal of pole-skipping phenomena

λ_L and v_B in theories with Einstein gravity duals are related to a pole-skipping point (ω_*, k_*) in $G_{T_{00}T_{00}}(\omega, k)$.

$$e^{-i\omega_* t + ik_* \mathbf{d}} = e^{\lambda_L (t - \mathbf{d}/v_B)}$$

[S. Grozdanov, K. Schalm, V. Scopelliti, 2017], [M. Blake, H. Lee, H. Liu, 2018],

Pole-skipping points (ω_*, k_*) of $G(\omega, k)$

Intersections between lines of poles and lines of zeros in momentum space of Green's function

e.g. $G(\omega, k) = \dfrac{\omega + k}{\omega - k}$

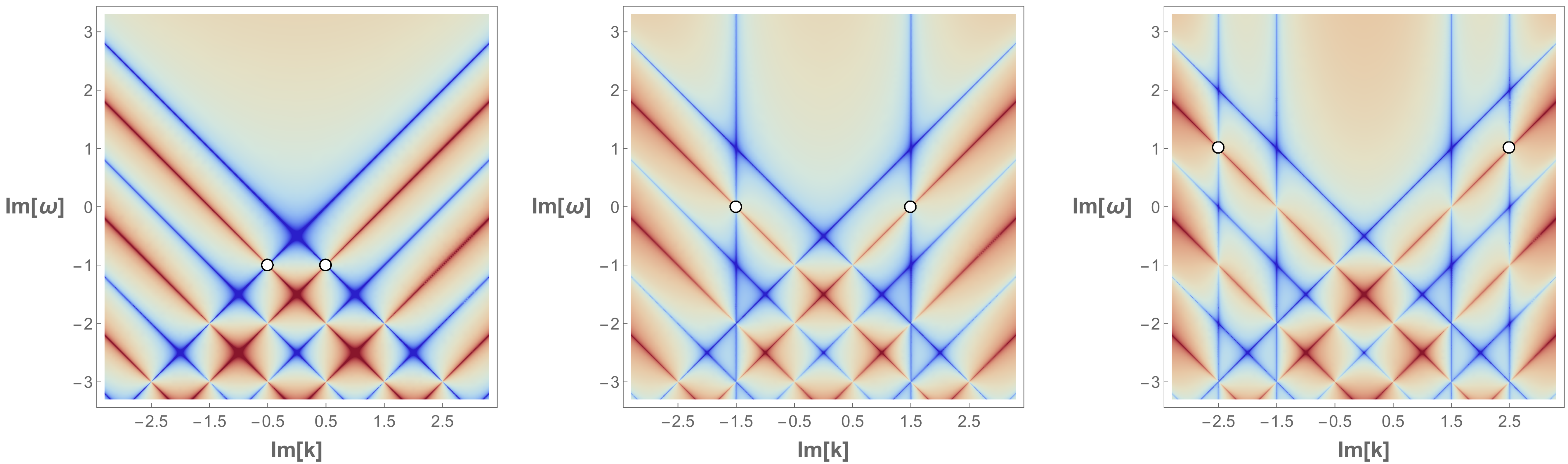
Pole-skipping point

$\omega_* = k_* = 0$

Leading pole-skipping points are related to Regge conformal blocks

$$G^{\text{Regge}}_{\Delta,\ell} \propto e^{(\ell-1)t-(\Delta-1)\mathbf{d}}$$

$\ell = 0$ $\qquad$ $\ell = 1$ $\qquad$ $\ell = 2$



[Y. Ahn, V. Jahnke, H.-S. Jeong, K.-Y. Kim, K.-S. Lee, MN, 2020]

[F. M. Haehl, W. Reeves, M. Rozali, 2019]

red lines: poles $\quad$ blue lines: zeros

intersections: pole-skipping points

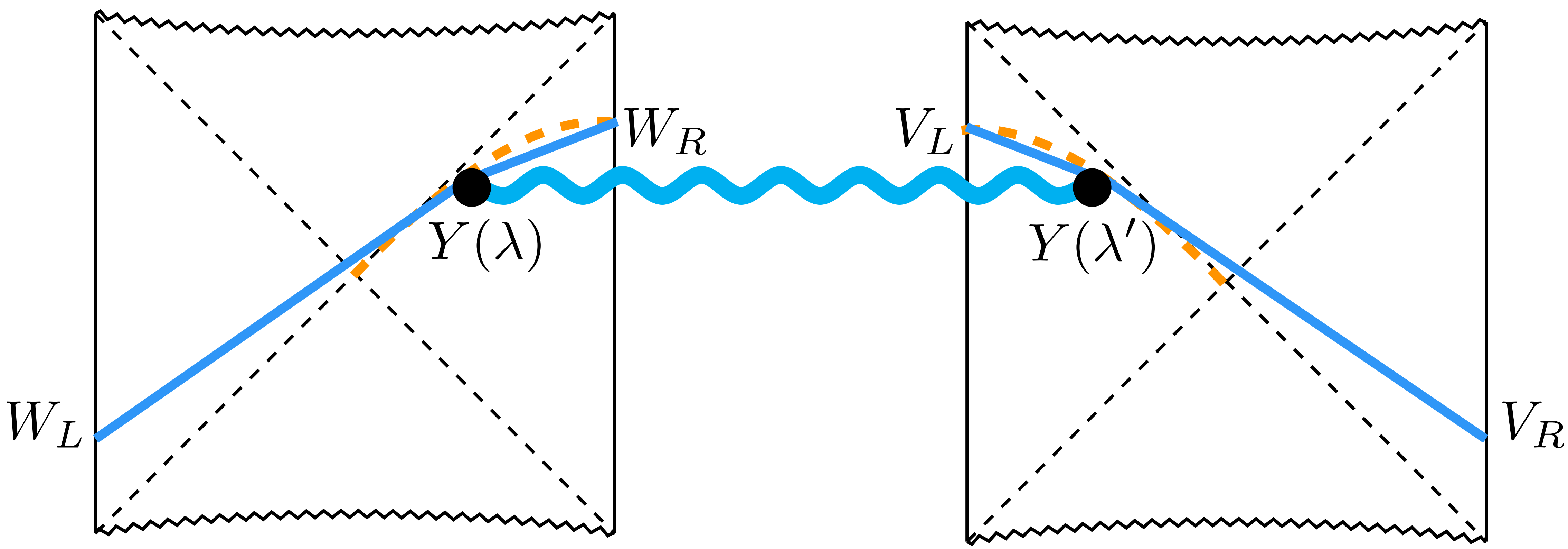
Our motivation

Holographic understanding why pole-skipping points capture Regge conformal blocks

- Near-horizon analysis is a holographic method to detect pole-skipping points [M. Blake, R. A. Davison, S. Grozdanov, H. Liu, 2018]
- Gravity duals of Regge conformal or OPE blocks were studied [L. Cornalba, M. S. Costa, J. Penedones, R. Schiappa, 2006] [N. Afkhami-Jeddi, T. Hartman, S. Kundu, A. Tajdini, 2017] [N. Kobayashi, T. Nishioka, Y. Okuyama, 2020]
- We want to connect Regge conformal blocks and near-horizon analysis in Rindler-AdS BH

Our result 1

We construct diagrams in the Rindler-AdS BH
that have the Regge behaviors of conformal blocks



Half-geodesic Witten diagrams

$$\mathcal{W}^{\mathcal{R}}_{\Delta,\ell} := \int_{\gamma^R_W} d\lambda \int_{\gamma^L_V} d\lambda' G_{b\partial}\left(Y(\lambda), W_L; \Delta_W\right) G_{b\partial}\left(Y(\lambda), W_R; \Delta_W\right) G_{bb}\left(Y(\lambda), Y(\lambda'); \frac{dY(\lambda)}{d\lambda}, \frac{dY(\lambda')}{d\lambda'}; \Delta, \ell\right)$$
$$\times G_{b\partial}\left(Y(\lambda'), V_L; \Delta_V\right) G_{b\partial}\left(Y(\lambda'), V_R; \Delta_V\right)$$
$$\simeq \frac{\left(\mathcal{C}_{\Delta_W,0}\mathcal{C}_{\Delta_V,0}\right)^2 \mathcal{C}_{\Delta,0}}{2^{2(\Delta_W+\Delta_V)+\ell}(\Delta-1)} \log\left(\frac{1}{\epsilon}\right) e^{(\ell-1)t_R-(\Delta-1)\mathbf{d}}\, {}_2F_1\left(\Delta-1, \frac{d}{2}-1, \Delta+1-\frac{d}{2}; e^{-2\mathbf{d}}\right)$$

Our result 2

Near-horizon analysis in the Rindler-AdS BH captures equations of the Regge behaviors with any integer spin

$$(\nabla_\mu \nabla^\mu - \Delta(\Delta - d) + \ell) h_{v\dots v}(v, r, \mathbf{x}) = 0$$

↓ Near-horizon analysis

$$e^{(\ell-1)t} \qquad \left[\Box_{\mathbb{H}} - (\Delta-1)(\Delta-d+1)\right] h_{v\dots v}\Big|_{r\to 1} = 0$$

↓ ↓

$$G^{\text{Regge}}_{\Delta,\ell}(z,\bar{z}) \propto e^{(\ell-1)t_R - (\Delta-1)\mathbf{d}} \, {}_2F_1\left(\Delta - 1, \frac{d}{2} - 1, \Delta + 1 - \frac{d}{2}; e^{-2\mathbf{d}}\right)$$

Integrals of the bulk-to-bulk propagators are related to the near-horizon analysis

Summary

- Pole-skipping points in momentum Green's functions are related to Regge conformal blocks in OTOCs
- We construct half-geodesic Witten diagrams on Rindler-AdS BH as gravity duals of Regge conformal blocks
- We derive equations for Regge conformal blocks with integer spin from near-horizon analysis on Rindler-AdS BH

Future directions

- Fermion exchange
- Gravity duals with retarded or advanced propagators
- CFT’s picture of the relation between Regge conform blocks and pole-skipping points
- Sub-leading analysis of the pole-skipping points and the half-geodesic Witten diagrams