An alternative bulk construction by the flow equation

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work in progress with
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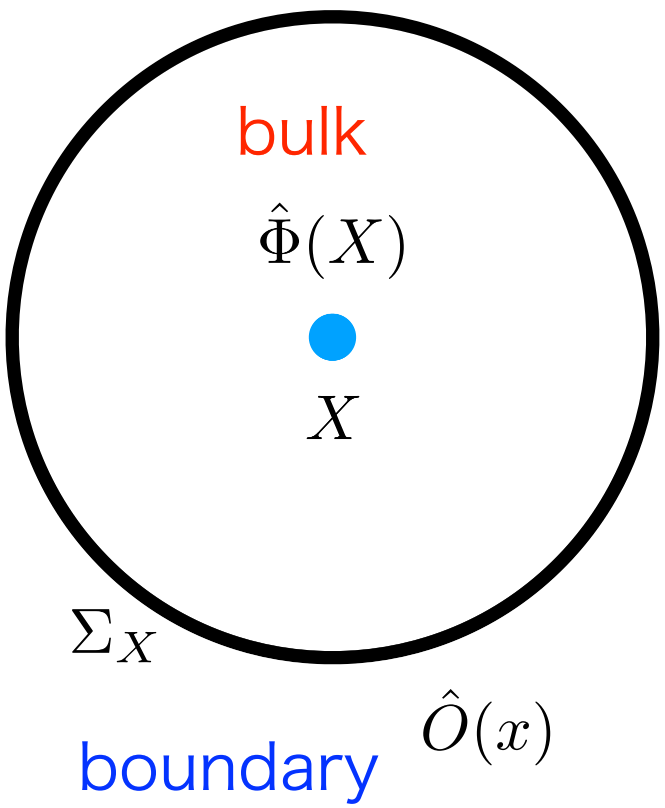
Motivation

to understand AdS/CFT form field theories without using string theory

HKLL bulk reconstruction

Hamilton, Kabat, Lifschytz, Lowe 2006



free scalar field operator on AdS $\hat{\Phi}(X)$

↓

Banks, Douglas, Horowitz, Martinec, 1998

BDHM relation

$$\lim_{z\to 0} \hat{\Phi}(z,x) = \hat{O}(x)$$

CFT field operator at the boundary $\hat{O}(x)$

↓ express $\Phi(x)$ in terms of $O(x)$

$$\hat{\Phi}(X) = \int_{\Sigma_X} d^d y\, K(X,y)\hat{O}(y)$$

c.f. S. Terashima, 2021: CFT<->AdS without BDHM relation in the large N.

Aoki, Balog, Onogi, Yokoyama (in preparation)

Our proposal: bulk construction

Starting point CFT in d-dimensions

$$\langle \varphi^a(x)\varphi^b(y)\rangle = \delta^{ab}\frac{1}{|x-y|^{2\Delta}}$$ non-singlet primary field

Smeared field $\sigma^a(X) = \int d^d y\, h(z, x-y)\varphi^a(y)$ field in d+1 dimensions $X := (x, z)$

$$S(X) := \phi^a(X)\phi^a(X)$$ (gauge) singlet composite field

Correlation functions $\langle S(X_1)S(X_2)\cdots S(X_n)\rangle$ → structure of bulk

Ex. metric $\langle g_{AB}(X)\rangle$ → **AdS ?**

Euclidean path integral ↔ HKLL: Lorentzian, canonical operator

Determination of smearing kernel $h(z,x)$

$$\sigma^a(X) = \int d^d y\, h(z, x-y)\varphi^a(y)$$

conditions

1. normalization $\langle \sigma^a(X)\sigma^a(X)\rangle = 1$

2. symmetry

conformal $\mathrm{SO}(d+1,1)$ ⟶ bulk symmetry $\mathrm{SO}(d+1,1)$

$$U\varphi^a(y)U^\dagger := \tilde{\varphi}^a(y) = J(y)^\Delta \varphi^a(\tilde{y})$$

$$U\sigma^a(X)U^\dagger := \tilde{\sigma}^a(X) = \sigma^a(\tilde{X})$$

translation/rotation/dilatation/inversion

↓

$$h(z,x) = \Sigma_0 \left(\frac{z}{x^2+z^2}\right)^{d-\Delta}, \qquad \Delta < \frac{d}{2}$$

$h(z, x-y)$ agree with $K(X,y)$ of HKLL but with $\Delta > d-1$. HKLL, PRD74(2006)066009.

Symmetry

$$U\varphi^a(y)U^\dagger := \tilde{\varphi}^a(y) = J(y)^\Delta \varphi^a(\tilde{y}) \quad \longrightarrow \quad U\sigma^a(X)U^\dagger := \tilde{\sigma}^a(X) = \sigma^a(\tilde{X})$$

conformal $\mathrm{SO}(d+1,1)$ **bulk** $\mathrm{SO}(d+1,1)$

1. translation $\tilde{y}^\mu = y^\mu + a^\mu,\ J(y) = 1$ $\tilde{x}^\mu = x^\mu + a^\mu,\ \tilde{z} = z$

2. rotation $\tilde{y} = \Omega^\mu{}_\nu y^\nu,\ J(y) = 1$ $\tilde{x}^\mu = \Omega^\mu{}_\nu x^\mu,\ \tilde{z} = z$

3. dilatation $\tilde{y}^\mu = \lambda y^\mu,\ J(y) = \lambda$ $\tilde{X}^A = \lambda X^A$

4. inversion $\tilde{y}^\mu = \dfrac{y^\mu}{y^2},\ J(y) = \dfrac{1}{y^2}$ $\tilde{X}^A = \dfrac{X^A}{X^2}$

Some properties

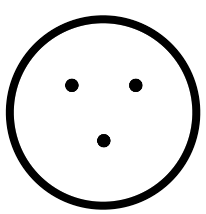
$$\sigma^a(X) = \int d^d y\, h(z, x-y)\varphi^a(y), \qquad h(X) = \Sigma_0 \left(\frac{z}{x^2+z^2}\right)^{d-\Delta}$$

Differential equation $\quad (\Box_{\rm AdS} - m^2)\sigma^a(X) = 0. \qquad \Box_{\rm AdS} := z^2(\partial_z^2 + \Box) - (d-1)z\partial_z$

$\sigma^a(X)$ satisfies EOM of a free scalar field on AdS with $m^2 = (\Delta - d)\Delta$.

flow equation = equation for $z^{-\Delta}\sigma^a(X)$

BDHM relation $\quad \lim_{z\to 0} z^{-\Delta}\sigma(X) \simeq \varphi^a(x)$



$$\lim_{z\to 0} h(z,x) = \frac{\Sigma_0}{\Lambda} z^\Delta \delta^{(d)}(x).$$

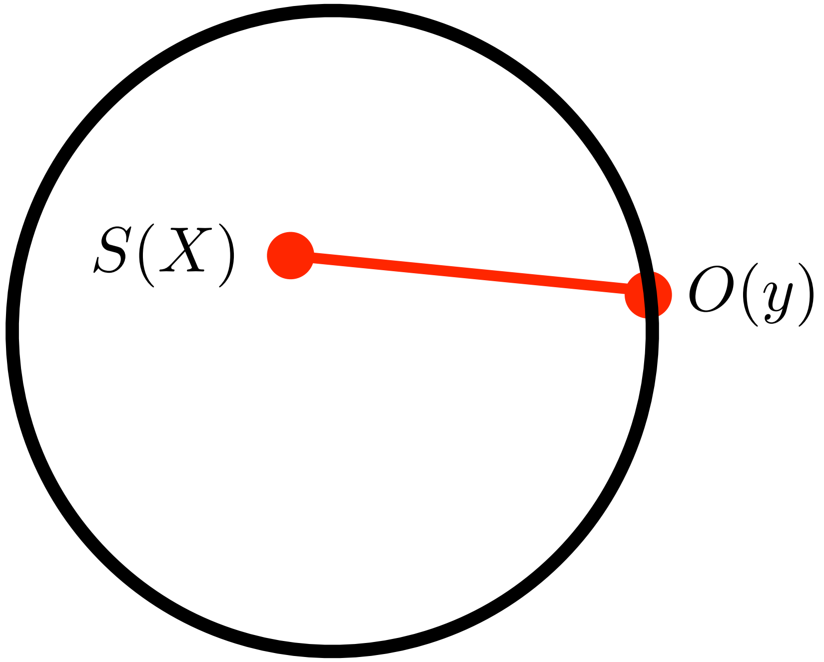
Boundary to bulk correlation function

$\mathcal{F}(X,y) := \langle S(X) O(y) \rangle$ a singlet scalar primary field of weight Δ_O: $O(y)$

a singlet bulk scalar field: $S(X) := \sigma^a(X)\sigma^a(X)$ composite

Symmetries(bulk & boundary) ⟶ $F(X,y) = C_O \left(\frac{z}{(x-y)^2 + z^2} \right)^{\Delta_O}$ exact result

The bulk to boundary propagator in the AdS/CFT correspondence is reproduced.



$$(\Box^X_{\rm AdS} - m_O^2) F(X,y) = 0$$

$$m_O^2 = \Delta_O(\Delta_O - d)$$

Metric field

$$g_{AB}(X) := \ell^2 \sum_{a=1}^{N} \partial_A \sigma^a(X) \partial_B \sigma^a(X)$$

ℓ: some length scale

1. simplest among singlet symmetric 2nd rank tensors

2. finite without UV divergence

3. metric becomes classical in the large N limit

$$\langle g_{AB}(X_1) g_{CD}(X_2) \rangle = \langle g_{AB}(X_1) \rangle \langle g_{CD}(X_2) \rangle + O(1/N)$$

large N factorization

$$\longrightarrow \langle G_{AB}(g_{CD}) \rangle = G_{AB}(\langle g_{CD} \rangle) + O(1/N)$$

classical geometry after quantum averages

4. VEV of metric operator = information metric

Bures Information metric

define a "distance" between density matrices ρ and $\rho + d\rho$ ρ can be a mixed state.

$$d^2(\rho, \rho + d\rho) := \frac{1}{2}\mathrm{tr}(d\rho G) \longleftarrow \rho G + G\rho = d\rho$$

Our case $\rho(X) := \sum_{a=1}^{N} |\sigma^a(X)\rangle\langle\sigma^a(X)|$ mixed state (N entangled pairs)

$$\longrightarrow \boxed{\ell^2 d^2(\rho(X), \rho(X + dX)) = \langle g_{AB}(X)\rangle dX^A dX^B}$$

VEV of $g_{AB}(X)$ defines a distance in the bulk as $d^2(\rho(X), \rho(X + dX))$.

VEV of the metric field

Symmetry $\longrightarrow$ $\langle 0|g_{AB}(X)|0\rangle = a_0 \dfrac{\delta_{AB}}{z^2}$

AdS metric in the Poincare patch

explicit calculation $\longrightarrow$ $a_0 = \ell^2 \dfrac{\Delta(d-\Delta)}{d+1} > 0$

Conformal symmetry $\longrightarrow$ Bulk symmetry $\longrightarrow$ AdS isometry on $\langle 0|g_{AB}(X)|0\rangle$

$\uparrow$ CFT $\qquad\qquad\qquad\qquad$ $\downarrow$ AdS

AdS/CFT correspondence is realized.

Excited state contribution

Mixed matrix element

$$e^{JO(0)}|0\rangle \simeq |0\rangle + J|S\rangle +$$

$|S\rangle$: primary scalar state

$$\bar{g}_{AB}(X) := \langle 0|g_{AB}(X)|0\rangle + J\langle 0|g_{AB}(X)|S\rangle$$

$$\langle 0|g_{AB}(X)|S\rangle = \lim_{y^2\to 0} G_{AB}(X,y) \qquad G_{AB}(X,y) := \langle 0|g_{AB}(X)O(y)|0\rangle$$

scalar operator

Symmetry →

$$G_{AB}(X,y) = T^{\Delta_O}(X,y)\left[a_1 \frac{\delta_{AB}}{z^2} + a_2 \frac{T_A(X,y)T_B(X,y)}{T^2(X,y)}\right]$$

$$T(X,y) := \frac{z}{(x-y)^2+z^2} \qquad T_A(X,y) := \partial_{X^A} T(X,y)$$

→

$$\bar{g}_{AB}(X) = \frac{\delta_{AB}}{z^2}\left[a_0 + a_1 J\left(\frac{z}{x^2+z^2}\right)^{\Delta_O}\right] + a_2\, J\, T_A T_B \left(\frac{z}{x^2+z^2}\right)^{\Delta_O - 2}$$

3 known parameters, a_0, a_1, a_2

$z \to 0$ asymptotically AdS

Summary

CFT $\varphi^a(y)$ $\Delta < \frac{d}{2}$ non-singlet, elementary field

conformal symmetry

↓ $\sigma^a(X) = \int d^d y\, h(z, x-y)\varphi^a(y)$ ↓

bulk non-singlet, elementary ("quarks")

bulk "quantum" symmetry

bulk to boundary $\langle \sigma^a(X)\sigma^a(X)O(y)\rangle$ singlet, composite ("hadrons")

geometrical interpretation

Metric field $g_{AB}(X) := \ell^2 \sum_{a=1}^{N} \partial_A \sigma^a(X) \partial_A \sigma^a(X)$

classical in the large N limit ⟶ "geometry"

vacuum ⟶ AdS bulk symmetry = isometry

excited state ⟶ asymptotically AdS

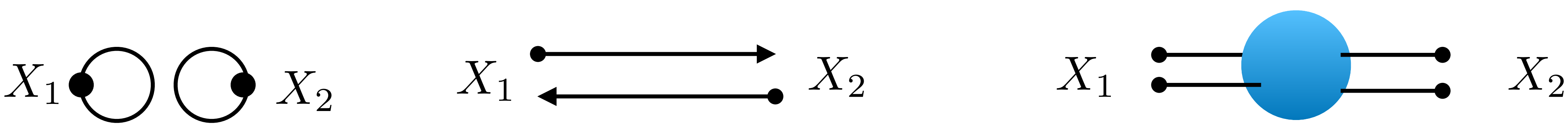
bulk "quantum" symmetry controls all properties in the bulk.

Discussions

Bulk to bulk correlation function for scalars

$$S(X) = \sigma^a(X)\sigma^a(X)$$

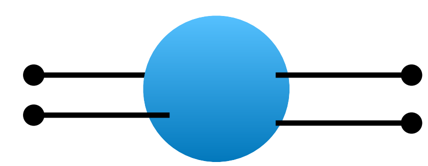
$$\langle S(X_1)S(X_2)\rangle = 1 + \frac{2}{N}G^2(X_1, X_2) + \langle S(X_1)S(X_2)\rangle_c$$



$G(X_1, X_2)$ non-singular at $X_1 = X_2$

free CFT → bulk theory is non-local (stringy)

connected contribution



$$\langle S(X_1)S(X_2)\rangle_c = \prod_{i=1}^{4}\int d^d y_i \, h(X_1, y_1)h(X_1, y_2)h(X_2, y_3)h(X_2, y_4)\langle\varphi^a(y_1)\varphi^a(y_2)\varphi^b(y_3)\varphi^b(y_4)\rangle_c$$

CFT interaction is required to recover bulk locality.

strong coupling CFT → bulk theory becomes local ?

Bulk matter content for asymptotically AdS

What is a matter action, which gives asymptotically AdS metric as a solution to the Einstein equation ?

$$R_{AB} - \frac{1}{2} g_{AB} R + \Lambda g_{AB} = 2\kappa T_{AB}$$

bulk scalar ?

$$S_\Phi = -\frac{1}{2} \int d^{d+1}X \sqrt{g} \left[g^{AB} \partial_A \Phi(X) \partial_B \Phi(X) + m^2 \Phi^2(X) \right]$$

g_{AB} : Euclidean AdS metric

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