Nonvanishing Finite Scalar Mass in Flux Compactification

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1. Introduction

By the discovery of Higgs,
the standard model is established !!

But !!

There are some problems…

One of these problems is

Hierarchy Problem

1. Introduction

As one of approaches to solution of hierarchy problem,

Extra-dimension + magnetic flux

This theory（four dimension + two dimension）is called

torus

Flux Compactification

$$A_5 = -\frac{1}{2} f x_6, \quad A_6 = \frac{1}{2} f x_5$$

The configuration of higher dimensional gauge field

f : background magnetic flux

x_5, x_6 : coordinates in extra space

1. Introduction

Quantum corrections in flux compactification

W. Buchmuller, M. Dierigl and E. Dudas (2017, 2018)

➡6D scalar QED, QED @1-loop

N. Maru and T. H. (2019, 2021)

➡6D SU(2) Yang-Mills theory, + higher dimensional operators@1-loop

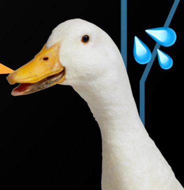
M. Honda and T. Shibasaki (2019)

➡6D QED @2-loop

<u>Quantum corrections to higher dimensional gauge field</u>

<u>(WL scalar) mass are canceled</u>.

Although WL scalar want to be regarded as
Higgs field,
we cannot solve the hierarchy problem
as long as quantum correction becomes zero.



1. Introduction

We want an interaction term
such that quantum corrections are finite…

We generalize loop integrals in the quantum correction to WL scalar mass.

From the finiteness of the loop integral, we guess the interaction terms.

Finite quantum correction to mass is obtained

only when we add certain interaction terms.

2. Loop integral and interaction terms

Kaluza-Klein masses in flux compactification are determined.
(Analogy to the quantum mechanics in magnetic field)

- Scalar field

$$m^2_{scalar} = \alpha\left(n + \frac{1}{2}\right) \qquad (\alpha = 2gf)$$

- Fermion field

$$m^2_{fermion} = \alpha(n+1)$$

- SU(2) gauge field

$$m^2_{YM} = \alpha\begin{pmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3+1 \end{pmatrix}$$

2. Loop integral and interaction terms

The general form of loop integral in the quantum correction

$$I(x;a,b) = \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{k^{2a}}{(k^2+\alpha(n+x))^b}$$

$$= \frac{1}{\alpha^{b-a}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} \boxed{\frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)} \zeta[\epsilon+b-a-2, x]}$$

x : parameter specifying KK mass spectrum

Calculate this part…

$$\Gamma(\epsilon-1)\zeta[\epsilon-p,x] =$$ finite (if p = even)

This condition
+
a(the number of derivatives)
+
b(the number of propagator)

We can guess the interaction terms generating finite quantum correction to WL scalar mass !!

2. Loop integral and interaction terms

Four-point interaction（b=1）

- Scalar field : $\bar{\varphi}\varphi\partial_{\mu_1}\cdots\partial_{\mu_a}\overline{\Phi}\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi$
- Fermion field : $\bar{\varphi}\varphi\bar{\psi}(\partial\!\!\!/)^{2a-1}\Psi$
- SU(2) gauge field : $\bar{\varphi}\varphi\partial_{\mu_1}\cdots\partial_{\mu_a}A^a_\nu\partial^{\mu_1}\cdots\partial^{\mu_a}A^{a\nu}$

Three-point interaction (b=2)

The simplest one
because of no derivatives !!

- Scalar field : $\bar{\varphi}\overline{\Phi}\Phi + \varphi\overline{\Phi}\Phi$

  $\bar{\varphi}\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi + \varphi\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}\bar{\Phi}\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}\Phi$
- Fermion field : $\bar{\varphi}\bar{\psi}(\partial\!\!\!/)^{a-1}\psi + \varphi\bar{\psi}(\partial\!\!\!/)^{a-1}\psi$
- SU(2) gauge field : $\bar{\varphi}\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^a_\nu\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu} + \varphi\partial_{\mu_1}\cdots\partial_{\mu_{a/2}}A^a_\nu\partial^{\mu_1}\cdots\partial^{\mu_{a/2}}A^{a\nu}$

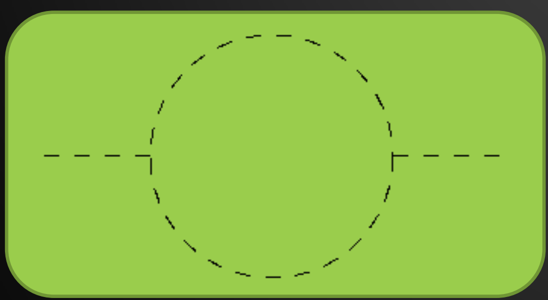
3. Finite WL scalar mass

Our set up（6D scalar QED + a new interaction term）

$$\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN} - D_M\bar{\Phi}D^M\Phi + \kappa(\bar{\phi}\bar{\Phi}\Phi + \phi\bar{\Phi}\Phi)$$

↓WL scalar

$(\phi = \langle\phi\rangle + \varphi)$



$$\mathcal{I} = -i\frac{\kappa^2|N|\ln 2}{32\pi^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon} + \mathcal{O}(\epsilon)$$

$$\delta m^2 = i\mathcal{I} = \frac{|N|\ln 2}{32\pi^2}\frac{\kappa^2}{L^2}$$

3.　Finite WL scalar mass

Our set up（6D scalar QED + a new interaction term）

$$\delta m^2 = i\mathcal{I} = \frac{|N| \ln 2}{32\pi^2} \frac{\kappa^2}{L^2}$$

- κ=0：Cancellation of quantum correction is reproduced
- κ≠0：Finite quantum correction to WL scalar mass is generated！

Even if the compactification scale is near Planck scale,

Higgs mass could be realized if

$$\kappa \sim \mathcal{O}(m_{Higgs}/m_{Planck})$$

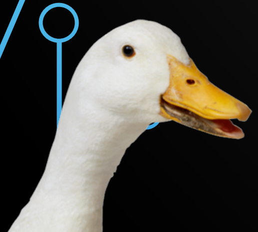
3. Finite WL scalar mass

Q. Why are the finite quantum correction generated？

A. New interaction is NOT invariant under the translation in extra space

- $\kappa=0$：Lagrangian is invariant under the translation
- $\kappa\neq0$： $\varphi\Phi\Phi$ is NOT invariant under the translation

➡ φ is pseudo NG boson, the mass is generated！



the same as π meson

Can we apply this result to another physics？

Extranatural Flux Inflation

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4. Application to inflation

What is Inflation？

➡ a theory of exponential expansion of space

in the early universe

Can we regard WL scalar as inflaton？

There are some inflation theories with extra-dimension .

N. Arkani-Hamed, H. C. Cheng, P. Creminelli and L. Randall (2003)

➡ 5D theory, the fifth gauge field = inflaton

T. Inami, Y. Koyama, C.-M. Lin and S. Minakami (2011)

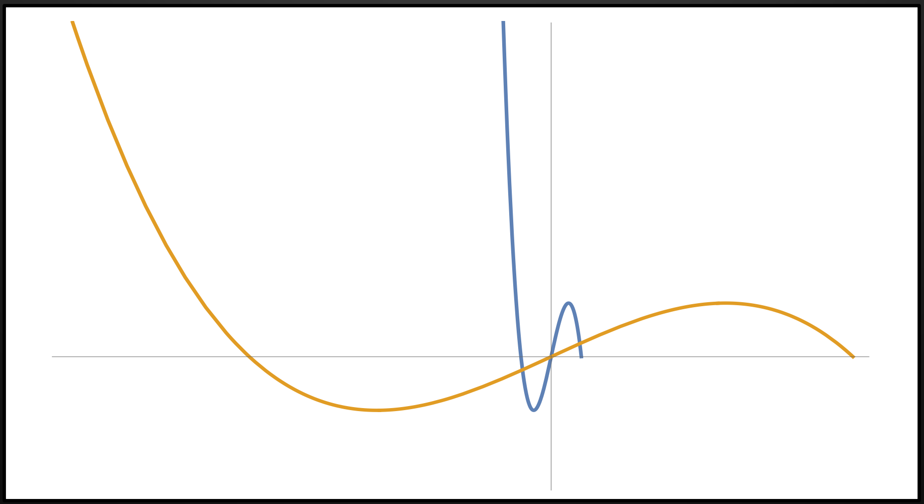
➡ 6D theory, the fifth gauge field = inflaton

4. Application to inflation

The effective potential of inflaton

$$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2,\frac{1}{2}-2xy\right]$$

$$z=\frac{\varphi}{M_P},\quad y=M_P\frac{\kappa}{\alpha},\quad \mathrm{Re}\ z=x$$



Schematic picture of the effective potential

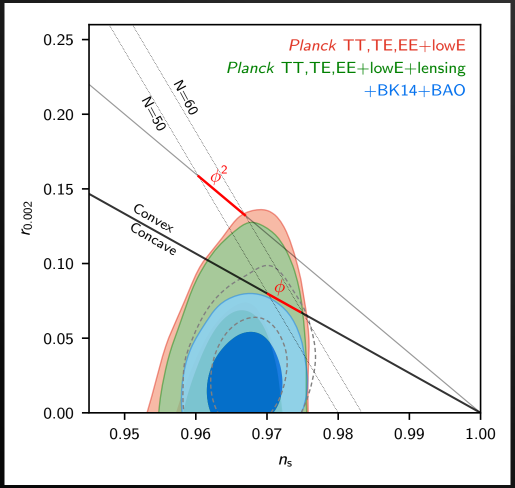
blue : y=1

yellow : y=0.1

4. Application to inflation

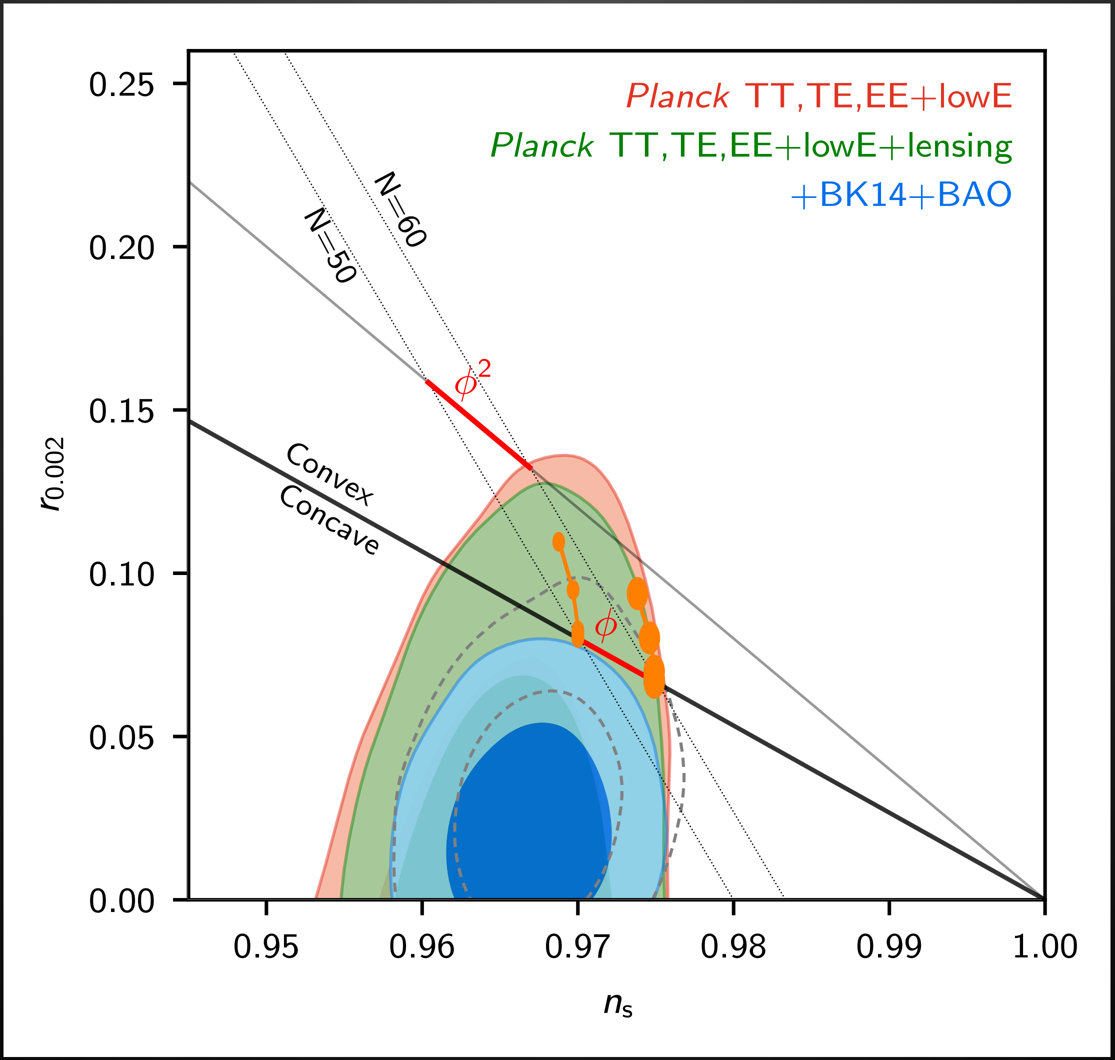
- Slow-roll parameters ε, η
- e-folding N_*
- Spectral index n_s、tensor-scalar ratio r

We calculate these parameters in our model, compare them to Planck 2018 data



Compare to this result…

4. Application to inflation



: Our results

5. Summary

- Generalization of loop integral in flux compactification
- guessed the interaction terms generating finite WL scalar mass

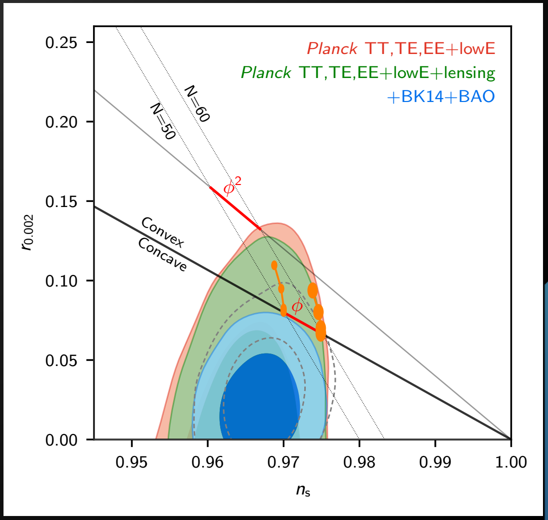
→ $\bar{\varphi}\bar{\Phi}\Phi + \varphi\bar{\Phi}\Phi$ is the most interesting interaction term！

- By the above interaction term, finite WL scalar mass is generated！

$$\delta m^2 = i\mathcal{I} = \frac{|N|\ln 2}{32\pi^2}\frac{\kappa^2}{L^2}$$

- A possibility of the connection of finite WL scalar mass to Higgs mass
- Proposed inflation theory in flux compactification
- Calculated n_s and r, and

  compare to Planck 2018 data



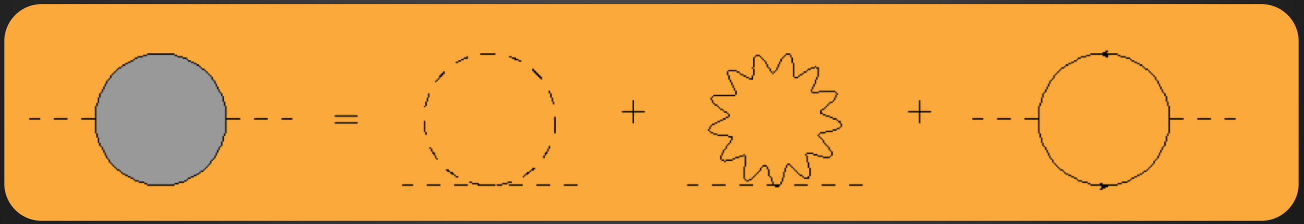
5. Summary

There are still some issues to be explored…

- What is the dynamics that gives $\kappa \sim \mathcal{O}(m_{Higgs}/m_{Planck})$?
- What is a new bulk scalar field Φ ?
- What is the origin term of $\varphi\Phi\Phi$?
- The extension to gauge-Higgs unification

• Appendix

What's hierarchy problem？



- This problem is the large difference between weak scale and new physics scale.
- Concretely, the large difference between Higgs mass 125 GeV and its mass correction

$$\delta m_H^2 \propto \Lambda^2$$

- If we admit the difference, we need unnatural fine tuning (Naturalness)

- # Appendix

What are merits of
flux compactification ?

1. To obtain chiral theory in 4D D. Cremades, L. E. Ibanez and F. Marchesano (2004)
2. To compute Yukawa coupling
3. To break SUSY spontaneously C. Bachas (1995)
4. To explain the number of generation

Appendix

We can generalize the loop integral…

$$I'(x;a,b) = \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{k^{2a} f(n)}{\left(k^2+\alpha(n+x)\right)^b}$$

$$= \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(a+\frac{d}{2}\right)\Gamma\left(b-a-\frac{d}{2}\right)}{\Gamma(b)\Gamma\left(\frac{d}{2}\right)} \sum_{n=0}^{\infty} \frac{f(n)}{\left(\alpha(n+x)\right)^{b-a-\frac{d}{2}}},$$

a coefficient depending on KK mode ↓ $f(n)$

Four-point interaction (b=1)

- Scalar field : $\bar{\varphi}\varphi\overline{\Phi}\left(a^\dagger a + \frac{1}{2}\right)\Phi$

  $\bar{\varphi}\varphi\partial_{\mu_1}\cdots\partial_{\mu_a}\overline{\Phi}\left(a^\dagger a + \frac{1}{2}\right)\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi$
- Fermion field : $\bar{\varphi}\varphi\bar{\psi}(\partial\!\!\!/)^{2a-1}(a^\dagger a + 1)\psi$
- SU(2) gauge field : $\bar{\varphi}\varphi\partial_{\mu_1}\cdots\partial_{\mu_a}A^a_\nu(a^\dagger a)\partial^{\mu_1}\cdots\partial^{\mu_a}A^a_\mu$

• Appendix

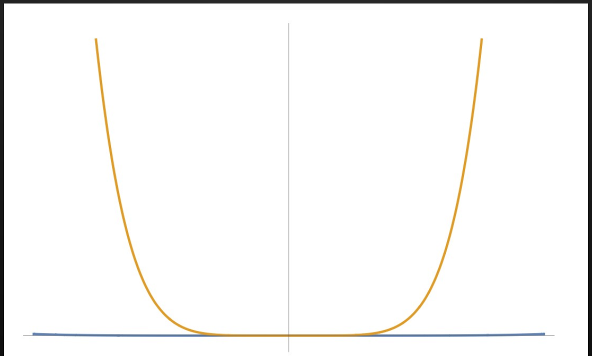
The effective potential of inflaton

Sec. 4→ $$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2,\frac{1}{2}-2xy\right] \qquad (g \ll \kappa L)$$

other case→ $$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2,\frac{1}{2}-2xy+4Gx^2\right] \qquad (g \gg \kappa L)$$

$$z = \frac{\varphi}{M_P}, \quad y = M_P\frac{\kappa}{\alpha}, \quad x = \mathrm{Re}\ z, \quad G = \frac{g^2 M_P^2}{\alpha}$$

In the case g≫κ L, ns and r is inconsistent with the result of Planck 2018.



blue :$G=10^3$
yellow:$G=10^2$

Appendix

y : large

y : small

	ϵ	η	n_s	r
A_{50}	0.00683107	0.00494671	0.968907	0.109297
A_{60}	0.00582958	0.00444092	0.973904	0.0932733
B_{50}	0.00594149	0.00271376	0.969779	0.0950639
B_{60}	0.00502904	0.00245613	0.974738	0.0804646
C_{50}	0.00517205	0.000586594	0.970141	0.0827528
C_{60}	0.00432933	0.000534704	0.975093	0.0692693
D_{50}	0.00507359	0.000296245	0.970151	0.0811775
D_{60}	0.00423959	0.000270297	0.975103	0.0678334
E_{50}	0.00499408	0.0000597248	0.970155	0.0799053
E_{60}	0.00416705	0.0000545352	0.975107	0.0666728

Table 2: Inflation parameters ϵ, η, n_s, r obtained from our model.

For Planck 2018 data,

$$n_s = 0.9649 \pm 0.0042, \quad r < 0.10$$