Entanglement entropy in Schwarzschild spacetime

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Based on [arXiv:2110.13898]

East Asia Joint Symposium on Fields and Strings 2021 @ Osaka City U.

Additivity conjecture in quantum information theory

$\mathcal{A}_1$, $\mathcal{A}_2$: two sets of states (density matrices)

$\mathcal{N}_i$: quantum channel (CPTP)

$$\mathcal{N}_i\colon \mathcal{A}_i \to \mathcal{B}_i$$

Minimum output entropy

$$S_{\min}(\mathcal{N}_i) = \min_{\rho\in\mathcal{A}_i} S\big(\mathcal{N}_i(\rho)\big)$$

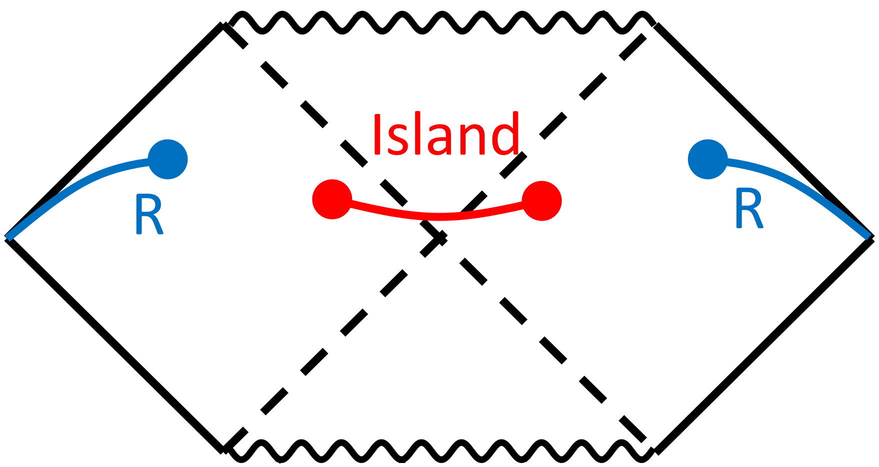
satisfies additivity condition

$$S_{\min}(\mathcal{N}_1 \otimes \mathcal{N}_2) = S_{\min}(\mathcal{N}_1) + S_{\min}(\mathcal{N}_2)$$

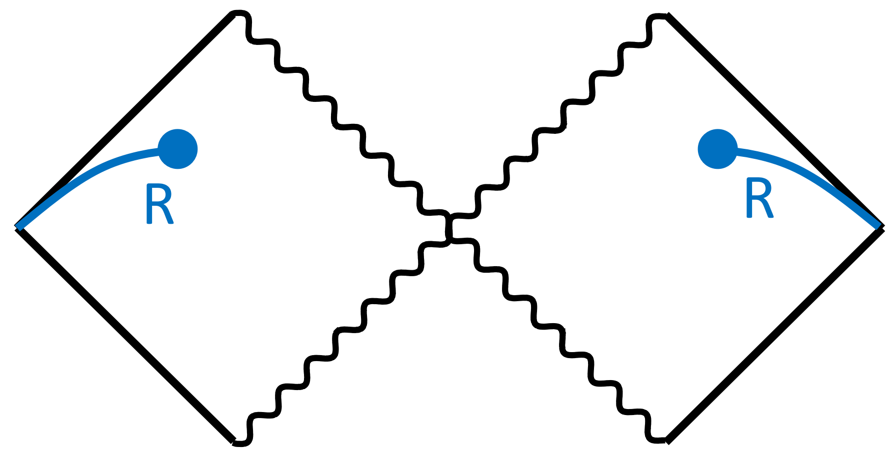
Geometries with disconnected exteriors should be considered

Entanglement entropy of region R in Schwarzschild spacetime

Hartle-Hawking vacuum



Other vacuum states



State with minimal entropy should be considered

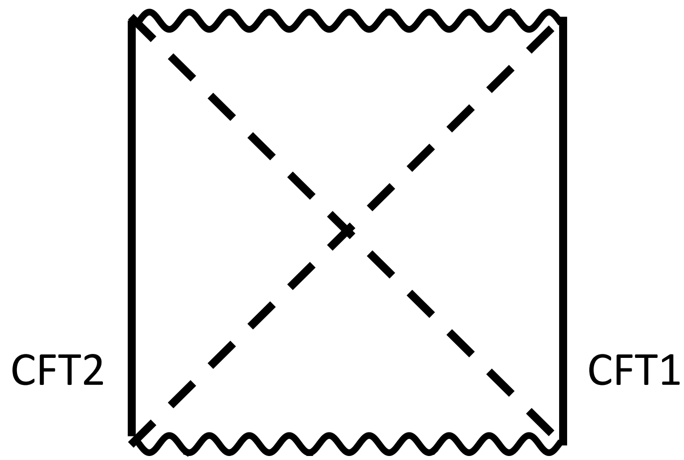
General vacuum states give disconnected geometries

⇨ gives consistent entropy to additivity conjecture

Additivity conjecture in AdS/CFT correspondence

[Hayden-Penington,'20]

Eternal BH spacetime ⟺ Thermofield double state



$$|\mathrm{TFD}\rangle = \sum_n e^{-\frac{\beta}{2}E_n}\,|E_n\rangle_1 |E_n\rangle_2$$

$\mathcal{A}_1$, $\mathcal{A}_2$: States in CFT1 and CFT2

$\mathcal{N}_i$: partial trace in CFTi

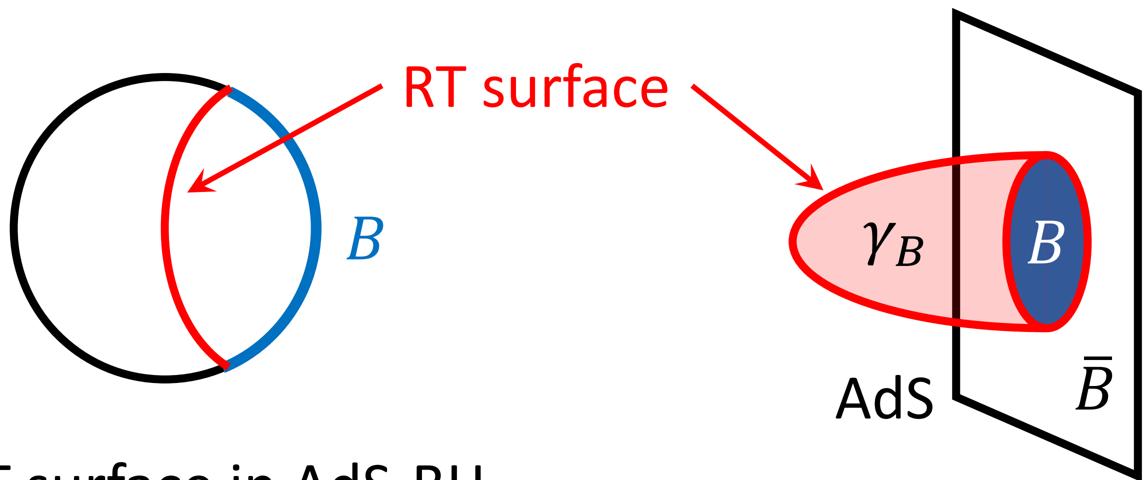
$$\mathcal{N}(\rho) = \mathrm{tr}_{\bar{B}}\rho = \rho_B$$

Is additivity condition satisfied in AdS/CFT setup?

$$S_{\min}(\mathcal{N}_1 \otimes \mathcal{N}_2) \stackrel{?}{=} S_{\min}(\mathcal{N}_1) + S_{\min}(\mathcal{N}_2)$$

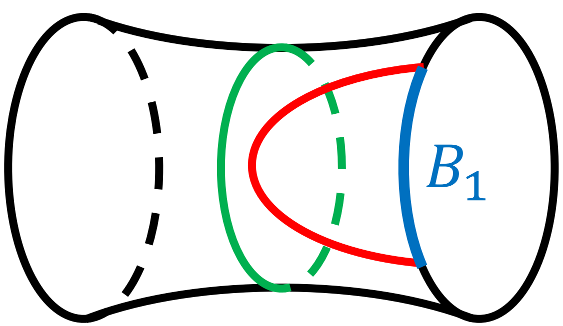
Entanglement entropy in AdS/CFT

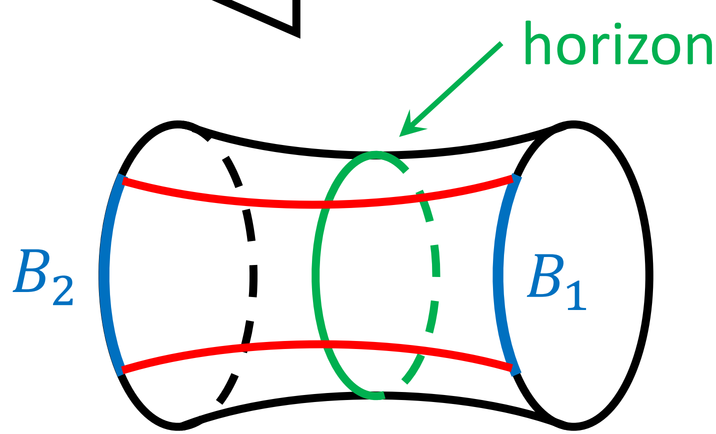
Entanglement entropy is given by Ryu-Takayanagi formula



$$S_{\mathrm{B}} = \frac{\mathrm{Area}(\gamma_B)}{4G_N}$$

RT surface in AdS-BH

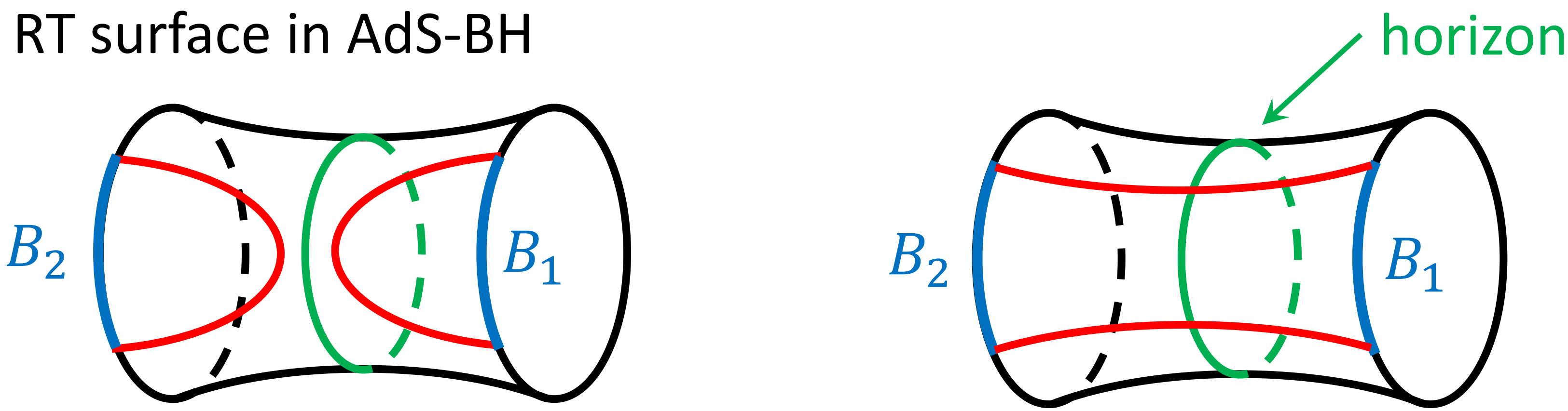




Additivity conjecture in AdS/CFT correspondence

[Hayden-Penington,'20]

RT surface in AdS-BH



$$S(B_1) + S(B_2) > S(B_1 \cup B_2)$$

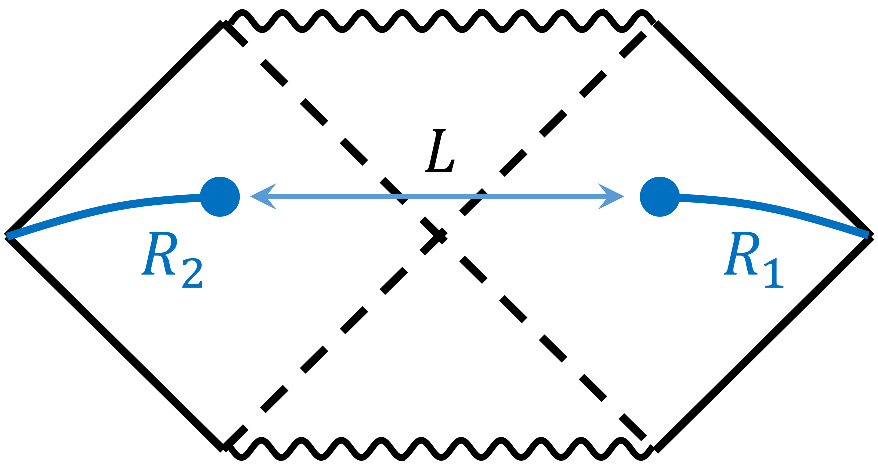
Additivity conjecture is not satisfied by $S(B_i)$

Two possibilities

- Two exteriors are disconnected $\quad S(B_i) \neq S_{\min}(\mathcal{N}_i)$
- Large violation of additivity conjecture $\quad S(B_i) = S_{\min}(\mathcal{N}_i)$

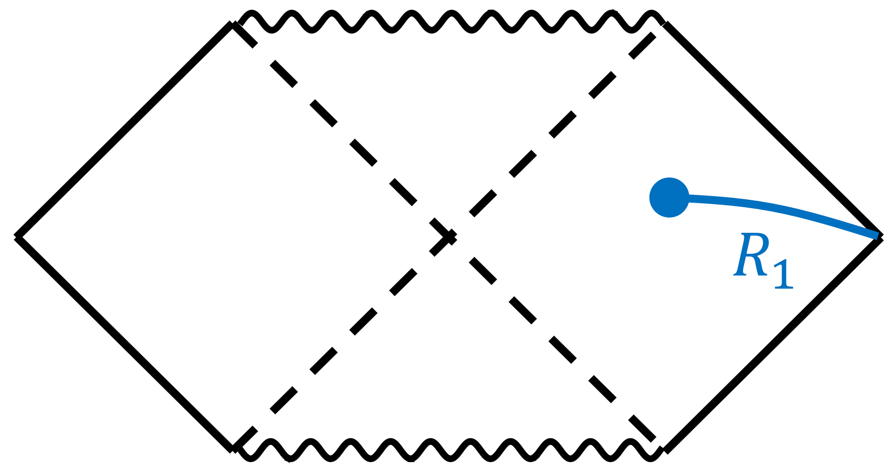
Entanglement entropy in Schwarzschild spacetime

Entanglement entropy of regions in both exteriors



$$S = \frac{c}{3} \log L$$

Entanglement entropy of region in one exterior



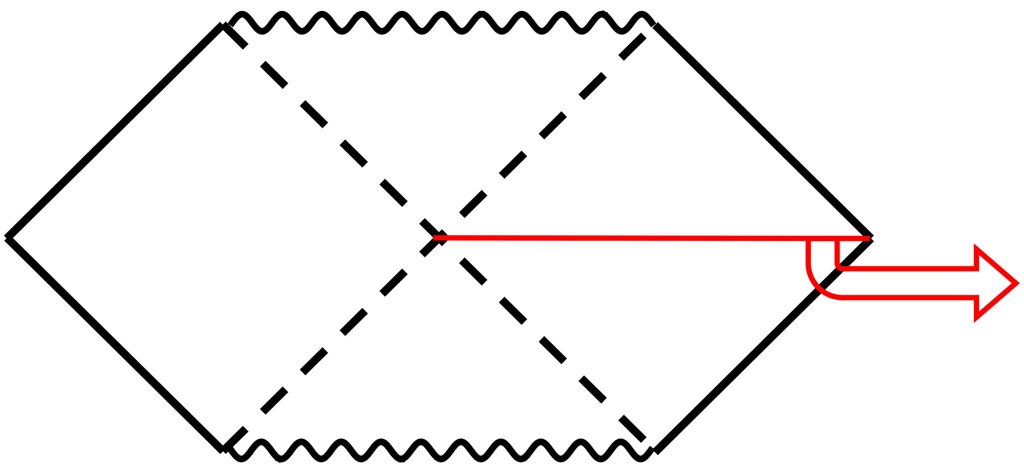
$$S = \frac{c}{6} \log \Lambda \to \infty$$

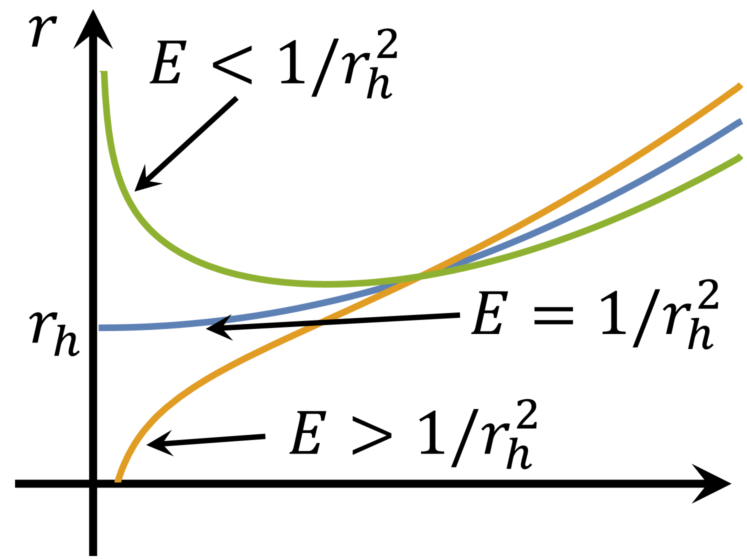
$$S(R_1 \cup R_2) < S(R_1) + S(R_2)$$

Additivity conjecture is not satisfied

Semi-classical Schwarzschild

Classical Schwarzschild solution is
good approximation only in the Hartle-Hawking vacuum



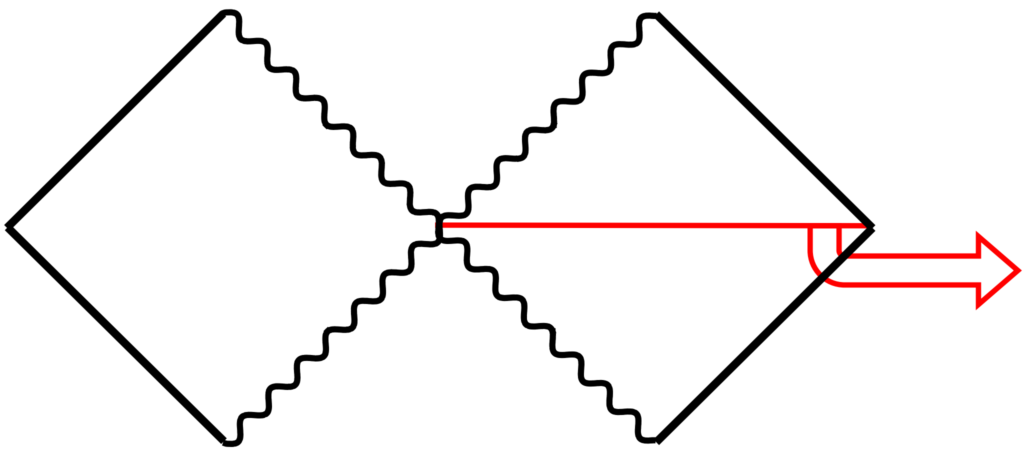


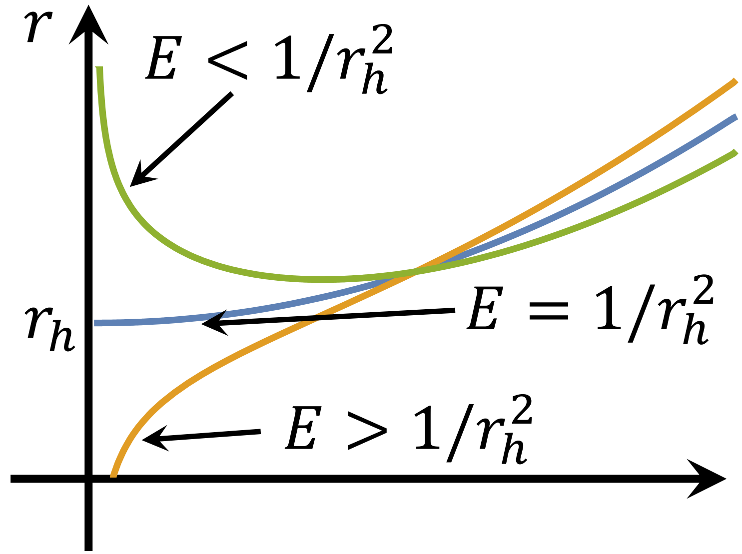
If radiation of vacuum state is $E \neq 1/r_h^2$ (different from that in HH), semi-classical geometry ends around the horizon.

⇨ Two exteriors are disconnected

Semi-classical Schwarzschild

Classical Schwarzschild solution is
good approximation only in the Hartle-Hawking vacuum





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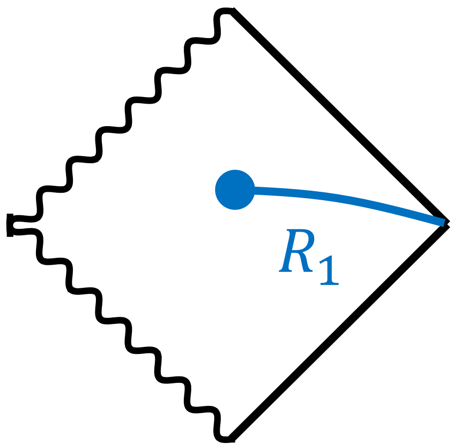
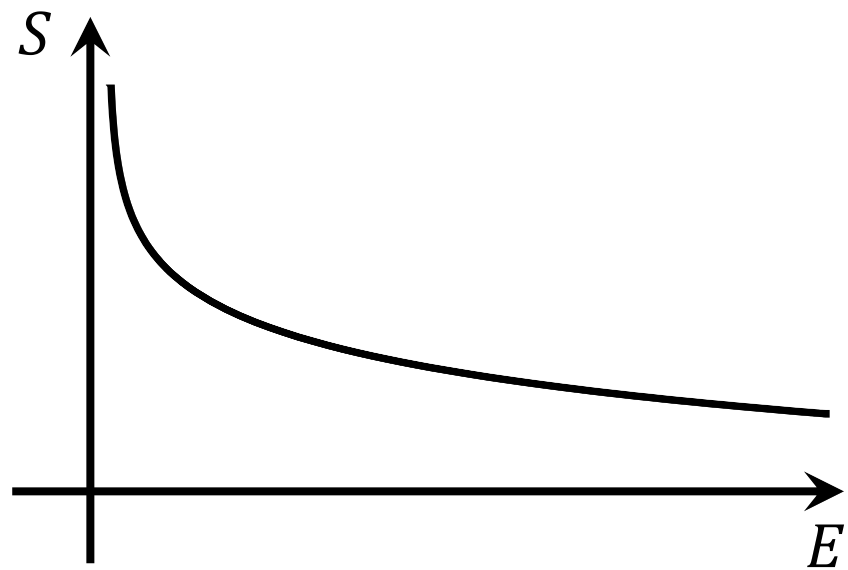
Entanglement entropy in semi-classical Schwarzschild



$$S \sim -\log E$$



E : Energy of radiation in the vacuum state

In disconnected cases, region R_1 and R_2 are disentangled.

$$S(R_1 \cup R_2) = S(R_1) + S(R_2)$$

Additivity conjecture is satisfied

Geometries with disconnected exteriors should be considered

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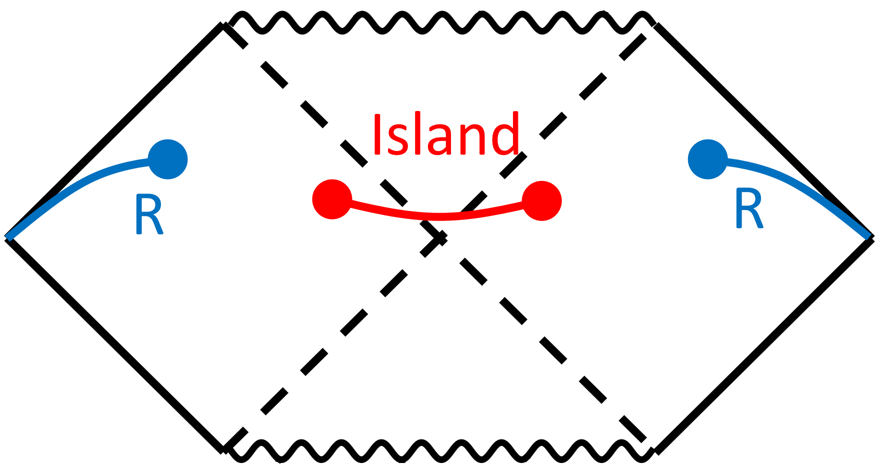

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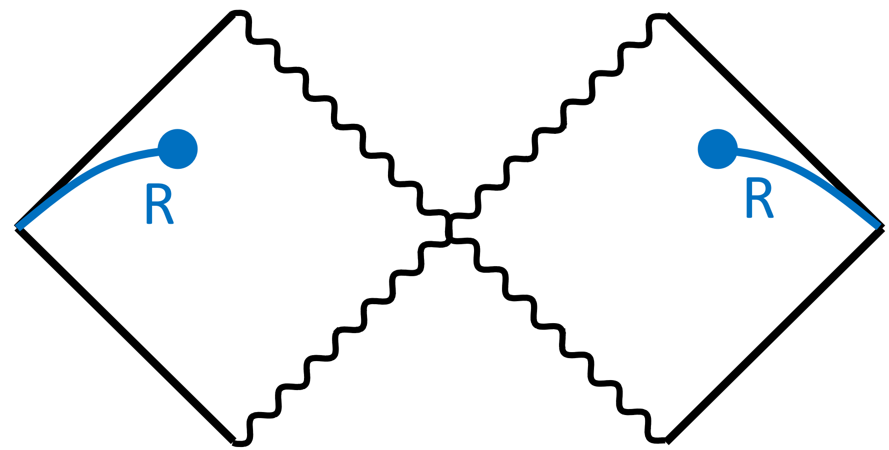

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Thank you