# *GINZBURG-LANDAU EFFECTIVE ACTION FOR A FLUCTUATING HOLOGRAPHIC SUPERCONDUCTOR*

Mitsutoshi Fujita (Sun Yat-Sen University)

Collaborators: Yanyan Bu and Shu Lin

Based on MF-Bu-Lin ***JHEP09(2021)168*** and work in progress

# Why is critical region?

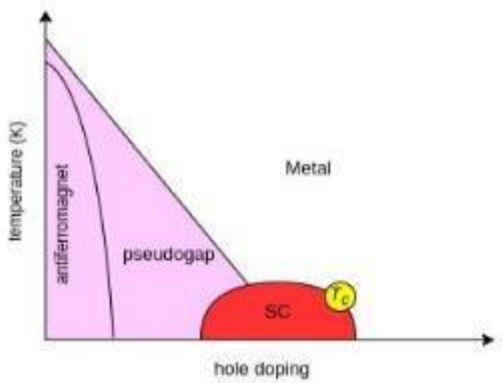



Figure: superconductivity
vs quantum criticality

Region near critical point (yellow dot)
Scaling behavior dominated by critical exponents (CE)

Nature of CE: symmetry, dimension, properties of order parameter, no details of interaction

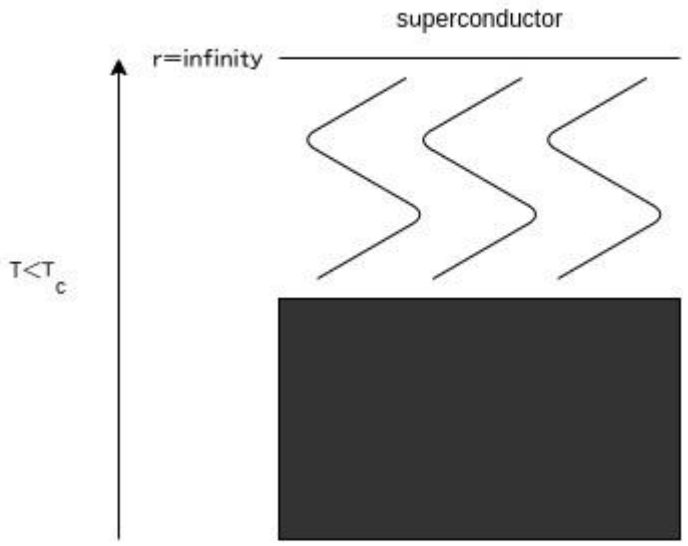
Holography to give discernment to dynamics in the critical region

# Holographic superconductor

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

- $A_a$ dual to $J_a$ current
- $\Psi$ dual to $\Delta$ : complex scalar hair

- Dynamics of AdS/CMT




- *Gubser, PRD 2008, Hartnoll-Herzog-Horowitz, PRL 2008*

# Analytic holographic superconductor

$$S_0 = \int d^5x\sqrt{-g}\left[-\frac{1}{4}F_{MN}F^{MN} - (D_M\Psi)(D^M\Psi)^* - m_0^2\Psi^*\Psi\right]$$

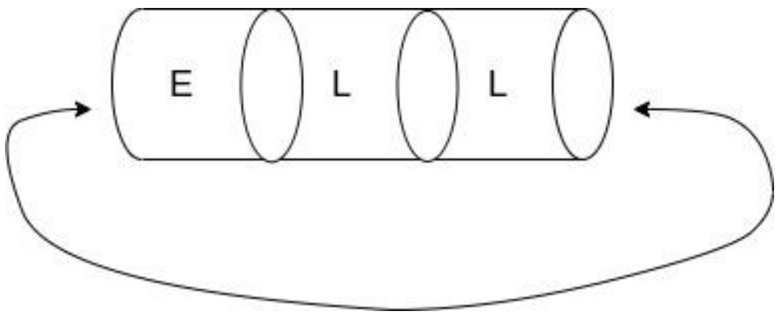
$$A_v = \mu_0\left(1-\frac{r_h^2}{r^2}\right),\ \ m_0^2 = -4$$

Phase transition at $\mu_0 = 2r_h$ *Herzog PRD 2010*

we fix $A = \Delta_s$ as vev of condensate

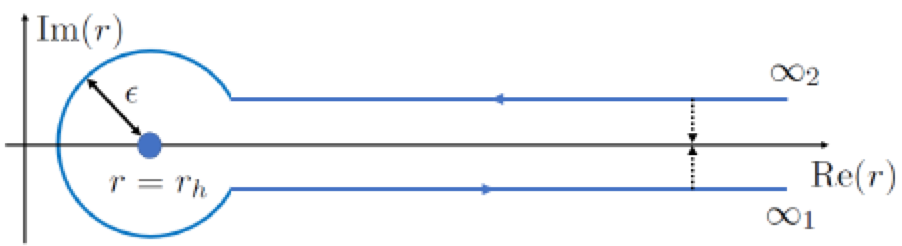
$$\Psi(r\to\infty_s) = \psi_{bs}\frac{\log r}{r^2} + \frac{\Delta_s}{r^2} + \cdots,$$
$$\Psi^*(r\to\infty_s) = \bar{\psi}_{bs}\frac{\log r}{r^2} + \frac{\bar{\Delta}_s}{r^2} + \cdots,$$
$$s = 1,2$$

# Complexified holography




Schwinger-Keldysh extended into the bulk

Two Lorentzian regions and one Euclidean black hole are glued




Radial coordinate complexified

*Glorioso, Liu 2018*

What we use

# Perturbation scheme

- $T$ slightly above $Tc$, off-equilibrium fluctuation of $\Delta$ small, justifies linear analysis keep $\Delta^2, \Delta^4$
- Slight deviation from critical point, $O(\delta\mu)$
- Low frequency expansion $O(\partial_v)$

Each field: $A_v^{(l)(m)(n)}, \Psi^{(l)(m)(n)}, \Psi^{*(l)(m)(n)}$

Sources: $j_{v,\Psi,\Psi^*}^{(l)(m)(n)}$

# Effective action in each order

$$\mathcal{L}_{eff}^{(0)(2)(0)} = \frac{i}{2\pi}\Delta_a^*\Delta_a$$

$$\begin{aligned}\mathcal{L}_{eff}^{(0)(2)(1)} &= \delta\mu\left[\frac{\log 2}{i\pi}(\Delta_2^* - \Delta_1^*)(\Delta_2 - \Delta_1) - (\Delta_2^*\Delta_2 - \Delta_1^*\Delta_1)\right] \\ &= \delta\mu\left[\frac{\log 2}{i\pi}\Delta_a^*\Delta_a + (\Delta_a\Delta_r^* + \Delta_a^*\Delta_r)\right].\end{aligned}$$

...

$$\mathcal{L}_{eff}^{(1)(2)(0)} = -\frac{1}{4}(1-3i)\Delta_a^*\partial_v\Delta_r + \frac{1}{4}(1+3i)\Delta_r^*\partial_v\Delta_a + \frac{\log 2}{4\pi}\Delta_a^*\partial_v\Delta_a,$$

$$\begin{aligned}\mathcal{L}_{eff}^{(0)(4)(0)} =& -0.000129006i(\Delta_a\Delta_a^*)^2 + 0.00466688\Delta_a\Delta_a^*(\Delta_a^*\Delta_r + \Delta_a\Delta_r^*) \\ & -0.000263406i\left[(\Delta_a^*\Delta_r)^2 + (\Delta_a\Delta_r^*)^2\right] - 0.00105363i\Delta_a\Delta_r\Delta_a^*\Delta_r^* \\ & +0.0208333(\Delta_a^*\Delta_r^*\Delta_r^2 + \Delta_a\Delta_r\Delta_r^{*2}),\end{aligned}$$

# Weak coupling results

- The Ginzburg-Landau action

$$S_{\rm GL} = 2\nu \int d^4x \left[ \Delta_{\mathcal{K}}^{\rm q*} (L^{-1})^R \Delta_{\mathcal{K}}^{\rm cl} + \Delta_{\mathcal{K}}^{\rm cl*} (L^{-1})^A \Delta_{\mathcal{K}}^{\rm q} + \Delta_{\mathcal{K}}^{\rm q*} (L^{-1})^K \Delta_{\mathcal{K}}^{\rm q} \right]$$

$$(L^{-1})^{R(A)} = \frac{\pi}{8T} \left[ \mp \partial_t + D(\nabla_r + 2ieA_{\mathcal{K}}^{\rm cl})^2 - \tau_{\rm GL}^{-1} - \frac{7\zeta(3)}{\pi^3 T_c} |\Delta_{\mathcal{K}}^{\rm cl}|^2 \right],$$

$$(L^{-1})^K = \coth \frac{\omega}{2T} \left[ (L^{-1})^R(\omega) - (L^{-1})^A(\omega) \right] \approx \frac{i\pi}{2},$$

$$\tau_{\rm GL} = \pi / [8(T - T_c)]$$

- v: the density of states and *D*: diffusion constant

- Theory at high temperature

# Comparison with weak coupling

- The Ginzburg-Landau action

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$\tau_{\rm GL} = \pi/[8(T - T_c)]$ dictated CE in the same dynamic universality class

$$\tau_{\rm GL} \sim \epsilon_T^{-z\nu} \sim \epsilon_T^{-1}$$

$$\mathcal{L}_{eff}^{(0)(2)(1)} = \delta\mu\left[\frac{\log 2}{i\pi}\Delta_a^*\Delta_a + (\Delta_a\Delta_r^* + \Delta_a^*\Delta_r)\right]$$

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# Summary

- Holography may provide a possible description of strongly coupled superconductor
- Time dependent GL effective action for holographic superconductor
- Results comparable to weak coupling counterpart in the same dynamic universality class
- Effective action for charge DOF and charge-condensate coupling
- How to incorporate Kibble-Zurek scaling
  - the influence of noises

# *Thank you!*