反強磁性絶縁体アクシオン

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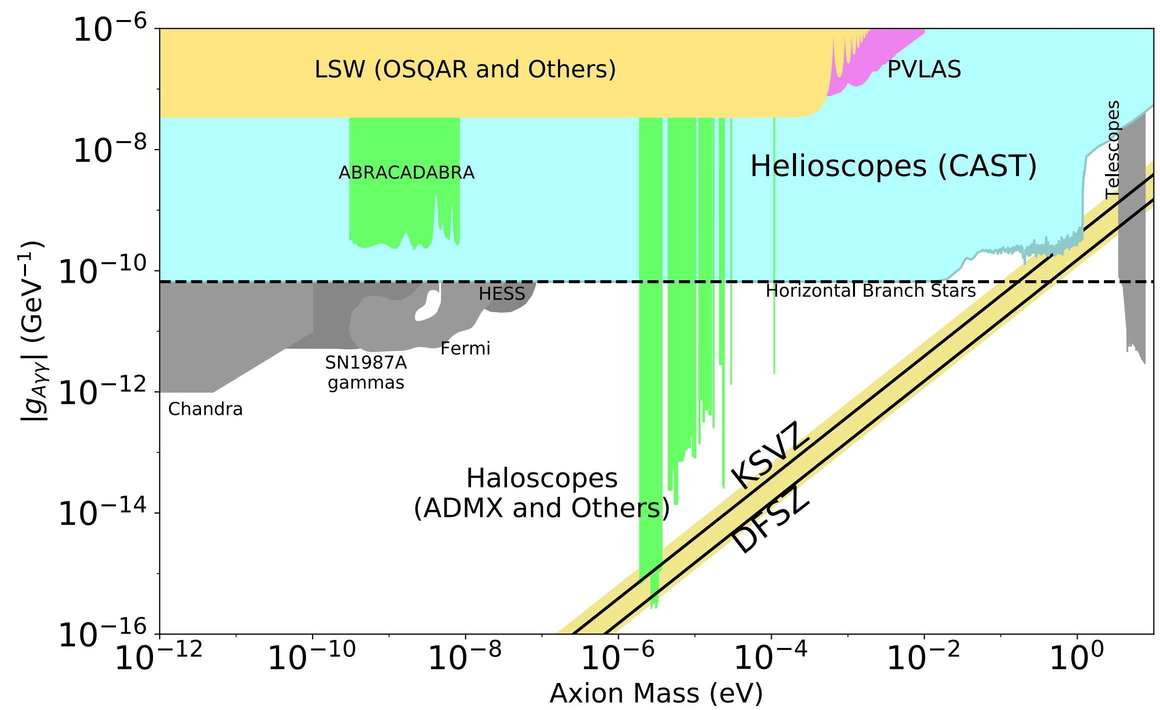
1. Introduction

Axion and axion-like particles (ALPs)

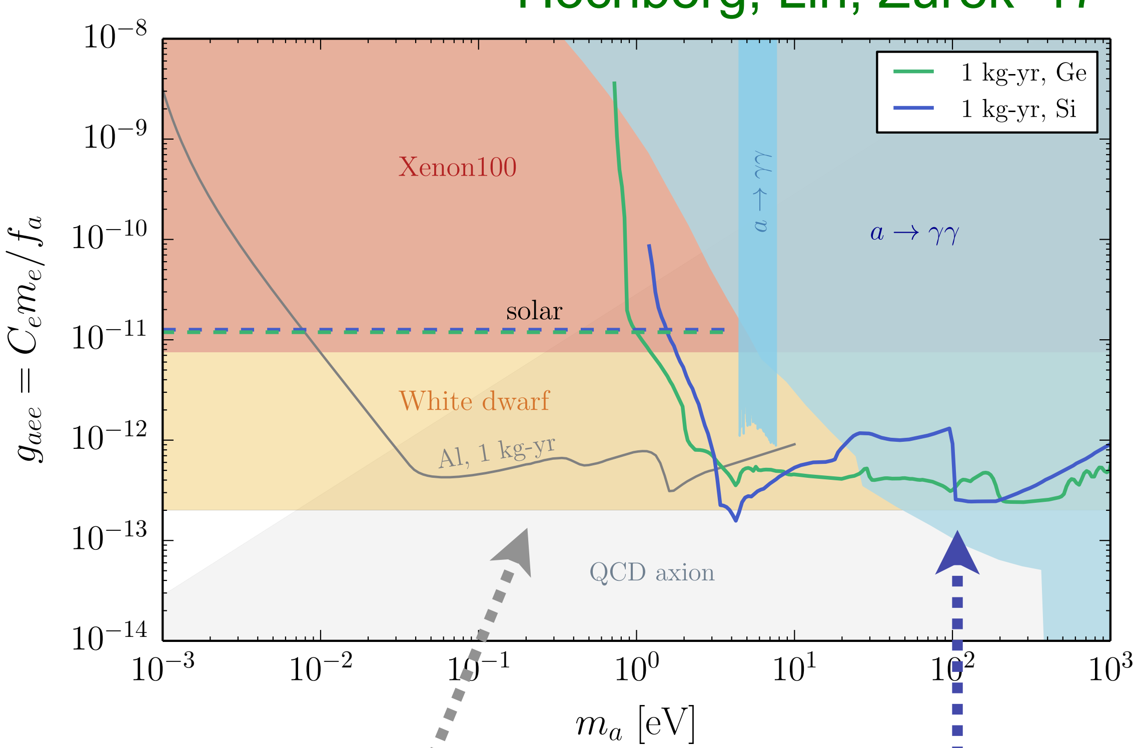
- A solution to the strong CP problem (for axion)
- DM candidates
- Inspired by superstring theory
- Impacts on cosmology (axion strings, domain walls, mini-clusters, etc.)

Axion and ALPs search

PDG ’20



Hochberg, Lin, Zurek ’17

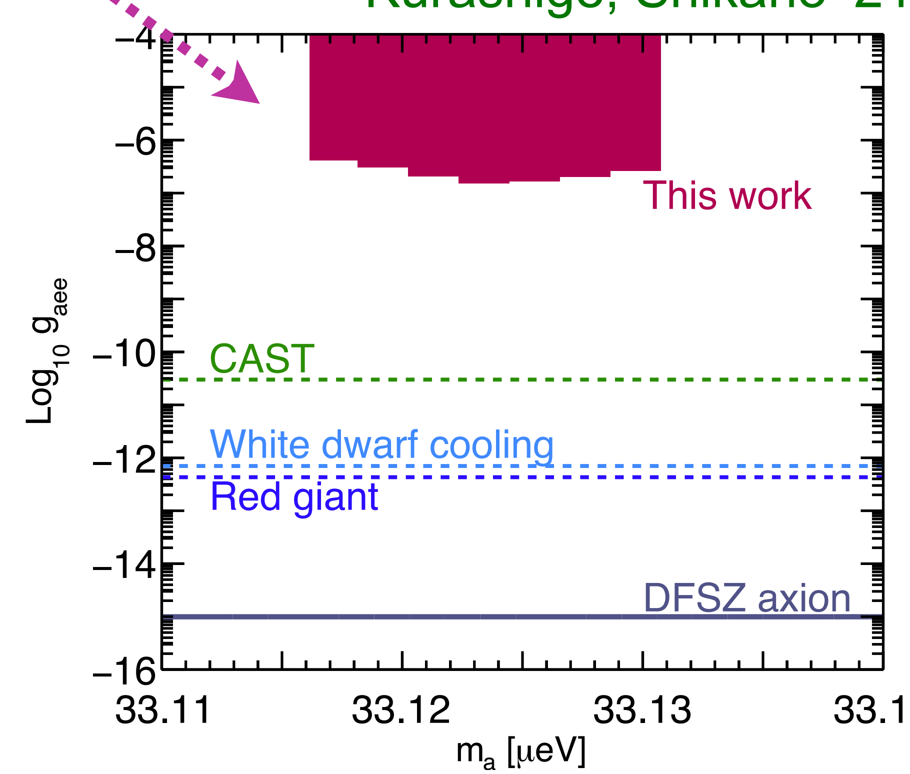


superconductors

semiconductors

magnon

Ikeda, Ito, Miuchi, Soda, Kurashige, Shikano ’21



Axion and axion-like particles (ALPs)

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- A solution to the strong CP problem (for axion)
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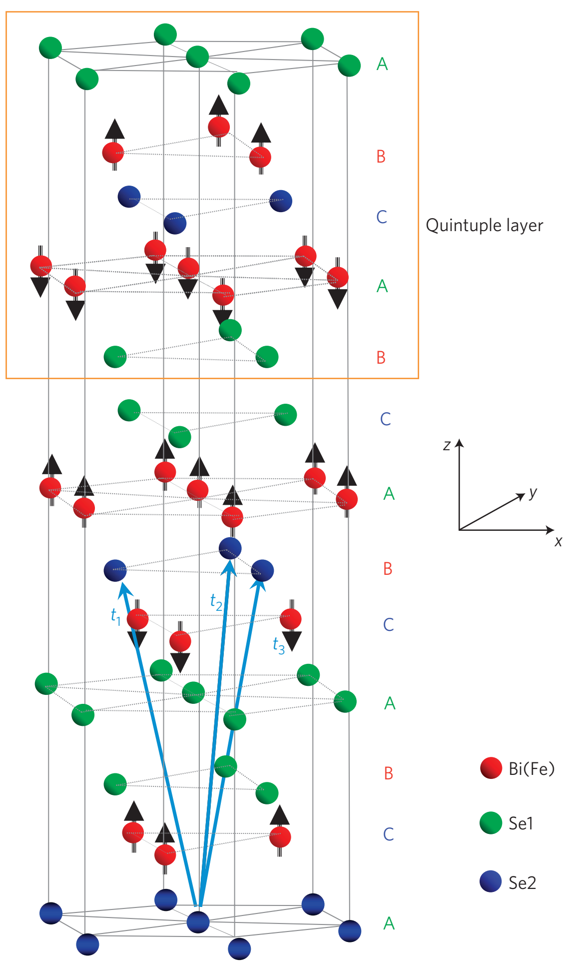
- Lots of searching using various techniques are ongoing
- ‘Axion’ is predicted in topological insulators
- ‘Axion’ in insulators can be used for axion detection

Axion is predicted in topological magnetic insulators

ARTICLES
PUBLISHED ONLINE: 7 MARCH 2010 | DOI: 10.1038/NPHYS1534
nature physics

Dynamical axion field in topological magnetic insulators

Rundong Li[1], Jing Wang[1,2], Xiao-Liang Qi[1] and Shou-Cheng Zhang[1]*



Bi_2Se_3

(Topological insulator)

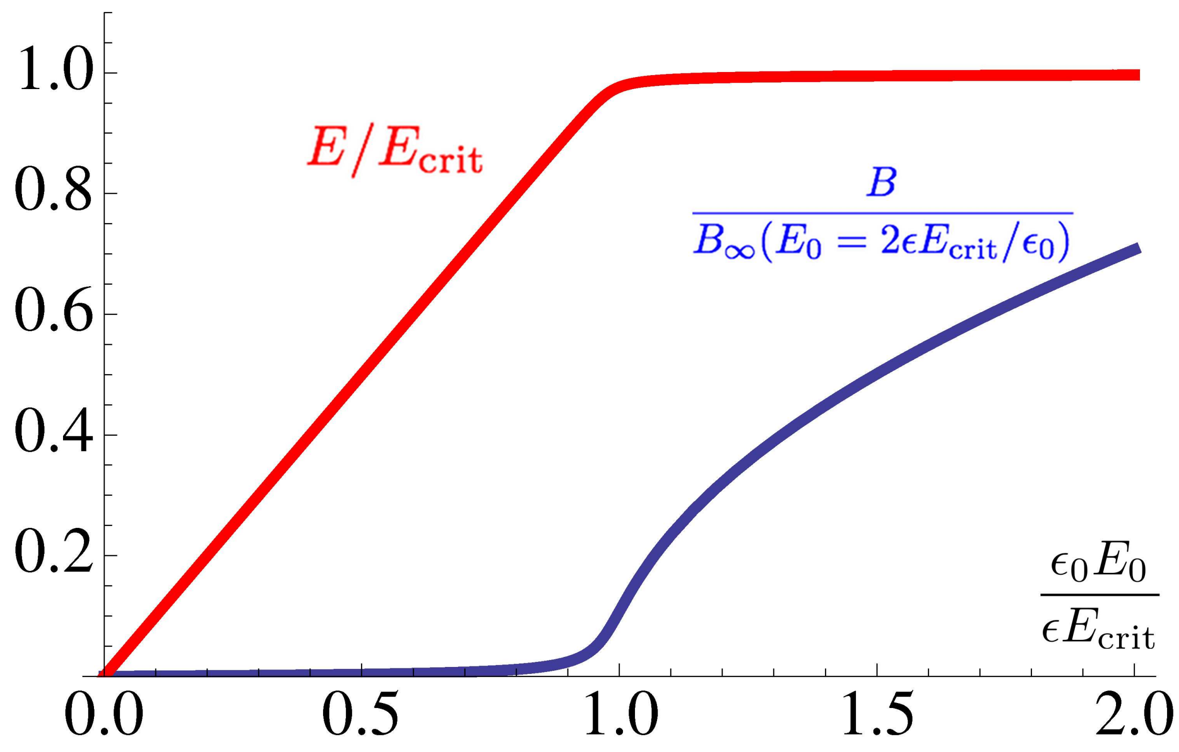
$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\text{tot}} &= \mathcal{S}_{\text{Maxwell}} + \mathcal{S}_{\text{topo}} + \mathcal{S}_{\text{axion}} \\ &= \frac{1}{8\pi}\int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\alpha}{4\pi^2}\int d^3x dt (\theta_0 + \delta\theta)\mathbf{E}\cdot\mathbf{B} \\ &\quad + g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\text{meV})$

Axion induces instability in insulators

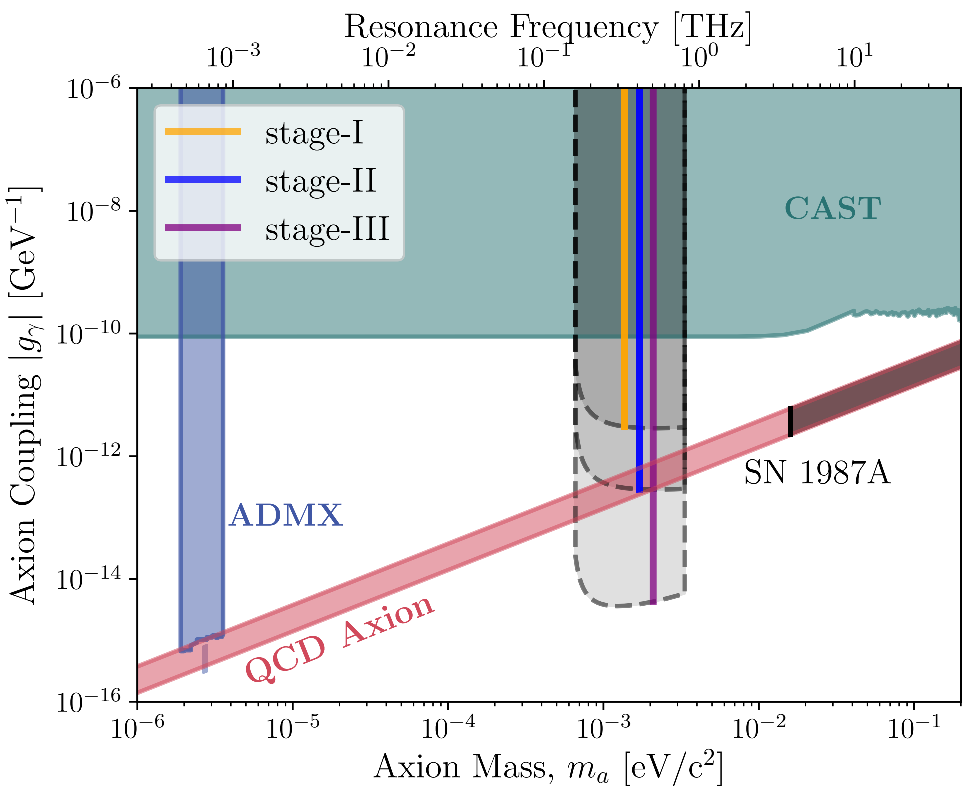
Ooguri, Oshikawa '12



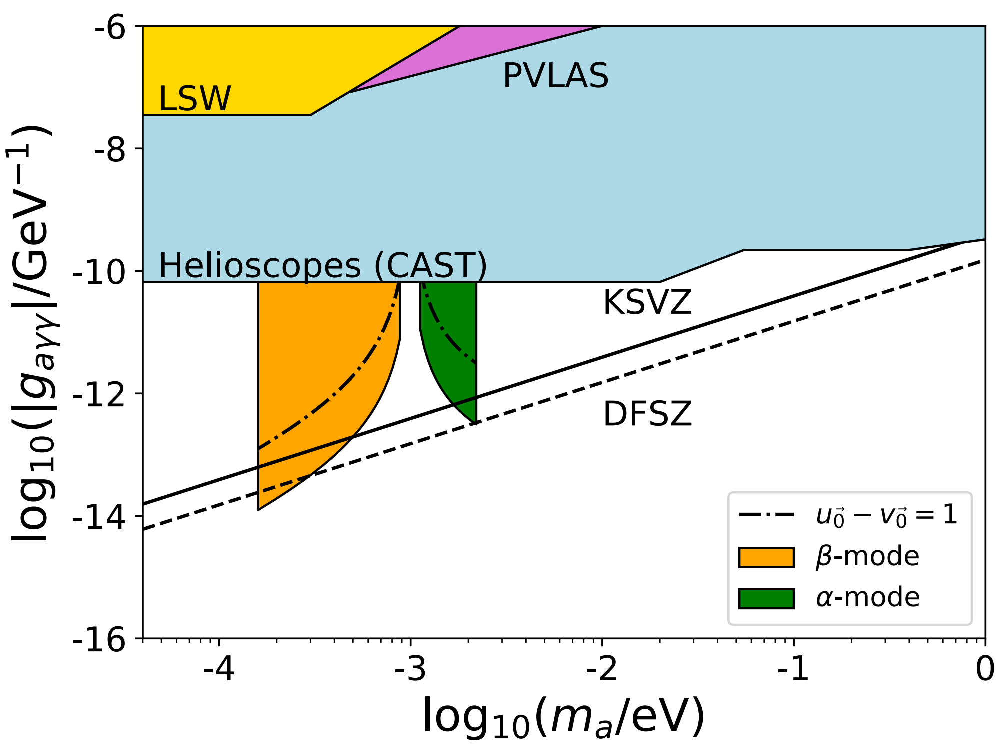
Axion-photon coupling → Instability of photon

→ Magnetic field is induced

Proposals for axion/ALPs search using 'axion' in insulators



Marsh, Fong, Lentz, Smejkal, Ali '19



Chigusa, Moroi, Nakayama '21

Keywords: topological insulator, magnetism

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Today, I would like to address

- What is topological insulator?
- How does magnetism play a role?
- How is ‘axion’ in insulators described?

Plan to talk

1. Introduction

2. Brief review of condensed matter physics (related to axion)

3. Axion in antiferromagnetic topological insulators

4. Conclusions and discussion

2. Brief review of condensed matter physics (related to axion)

Topics related to axion in condensed matter physics

a). Insulators

b). Anomalous quantum Hall effect

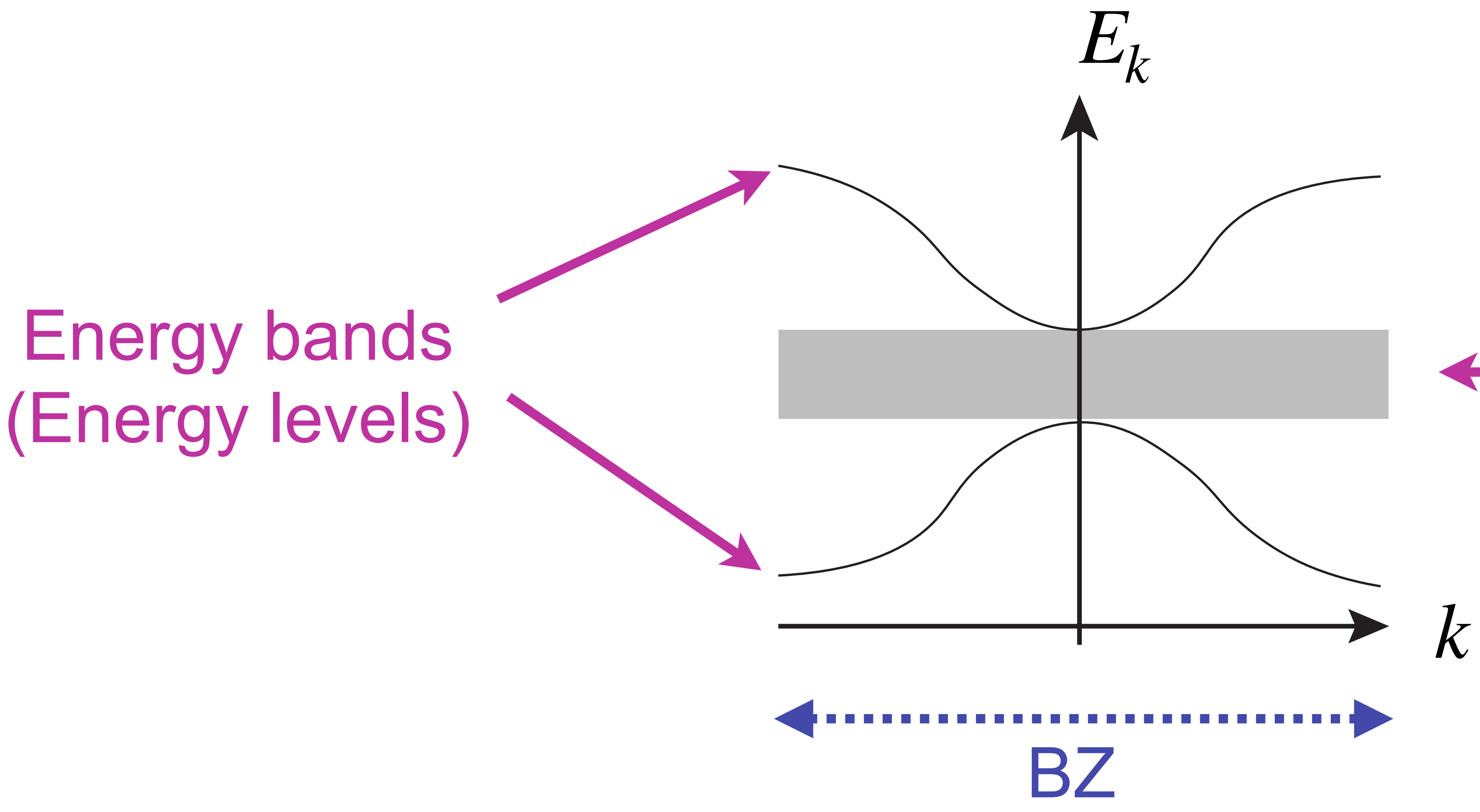
c). Topological insulators

d). Magnetoelectric effect

Minimum basics

E_k : energy of electron

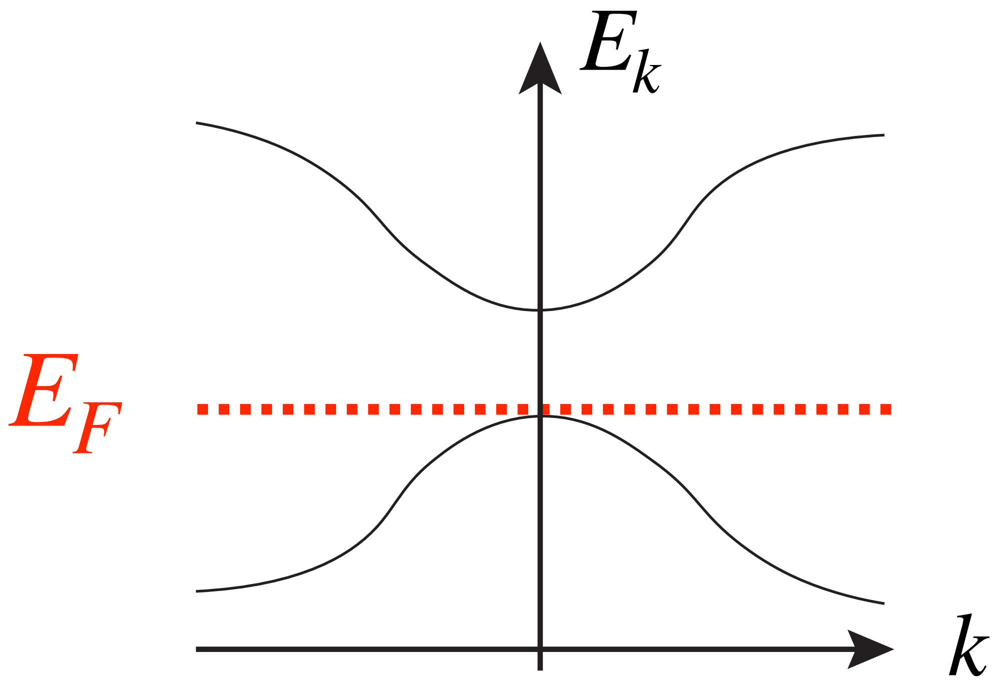
k : wavenumber of electron



BZ (Brillouin zone)

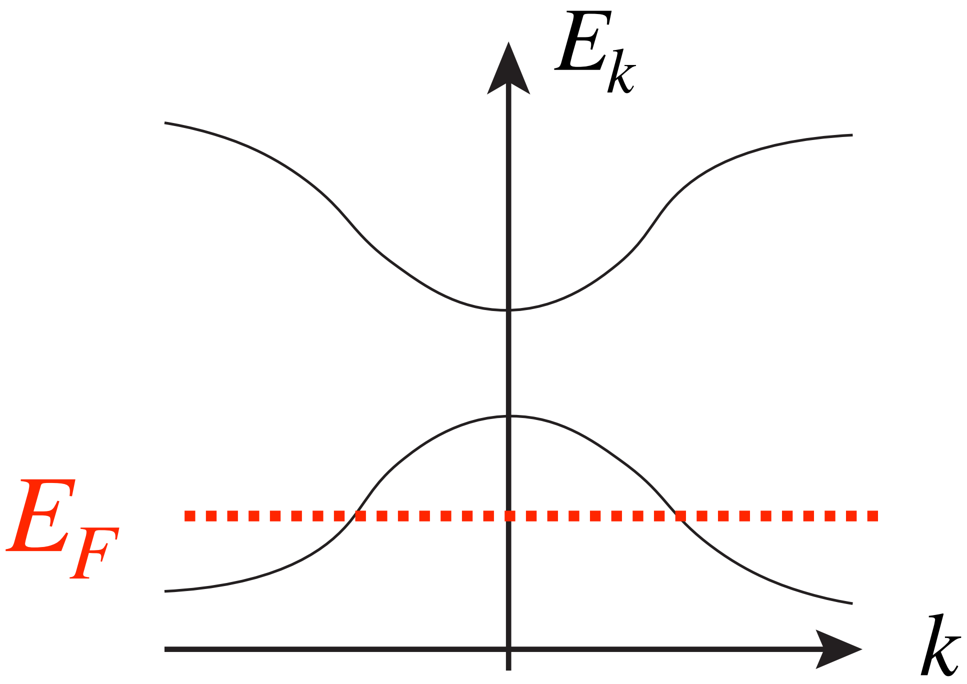
The region where there is no energy level is called band gap
(important for insulators)

Insulator



Fermi energy is in the band gap

Metal

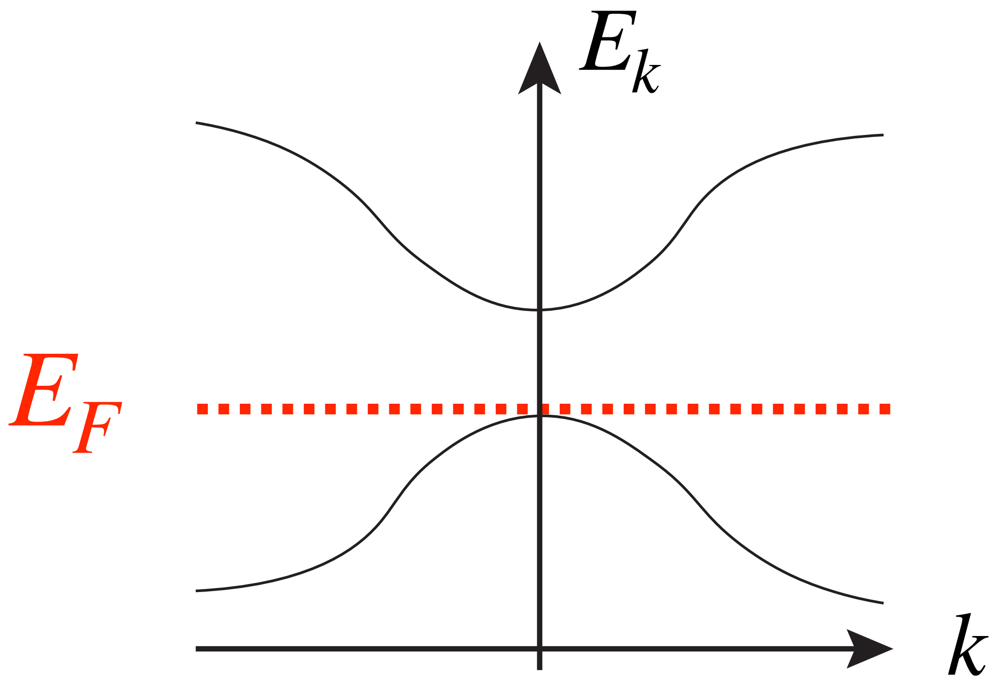


Fermi energy is in the band

E_F : Fermi energy

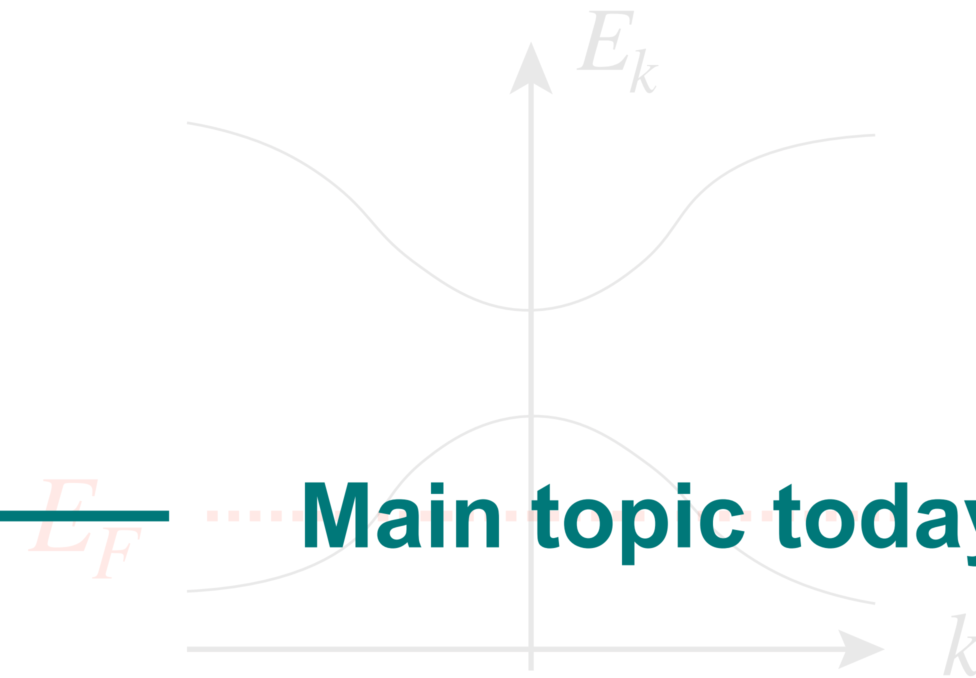
Fermi energy are important to distinguish insulators and metals

Insulator



Fermi energy is in the band gap

Metal



Main topic today

Fermi energy is in the band

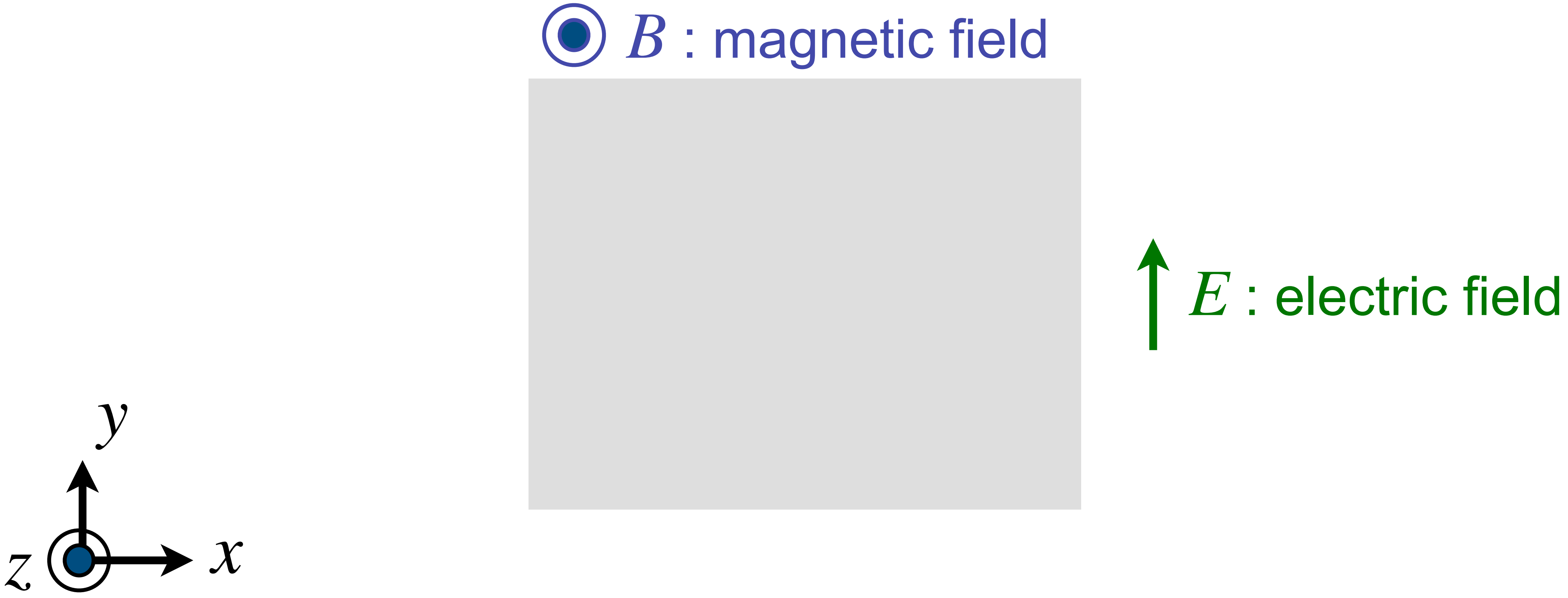
E_F : Fermi energy

Fermi energy are important to distinguish insulators and metals

Quantum Hall (QH) effect

e.g., 2D insulator

QH effect appears in semiconductor too



Quantum Hall (QH) effect

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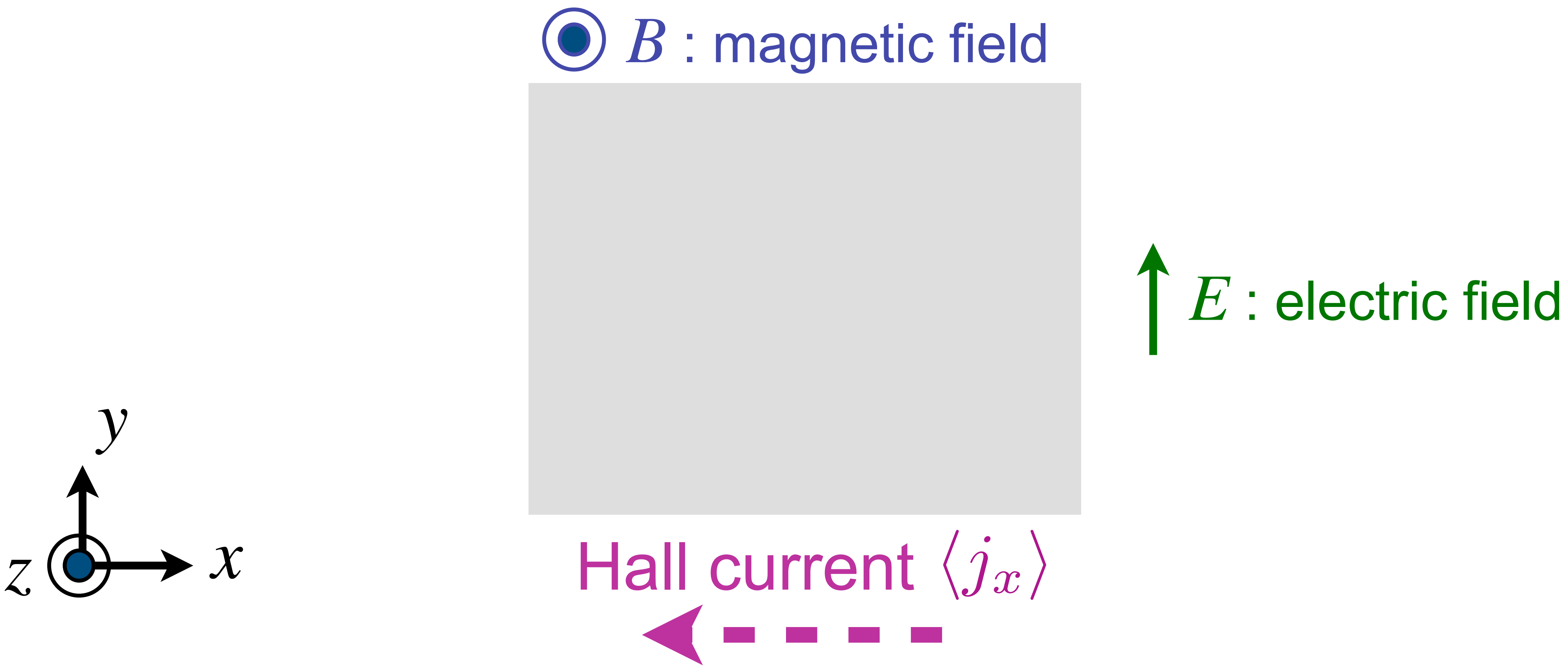


Quantized electric current is induced in x direction

Thouless, Kohmoto, Nightingale, den Nijs '82

It seems weird but there was theoretical prediction:

Hall conductivity:

$$\sigma_{xy} \equiv \langle j_x \rangle / E_y$$
$$= \nu \frac{e^2}{h}$$

TKNN formula

$$\nu \equiv \sum_n \int_{\mathrm{BZ}} \frac{d^2k}{2\pi} [\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{a}_n(\boldsymbol{k})]_z$$
$$\boldsymbol{a}_n(\boldsymbol{k}) \equiv -i \langle u_{n\boldsymbol{k}} | \frac{\partial}{\partial \boldsymbol{k}} | u_{n\boldsymbol{k}} \rangle$$

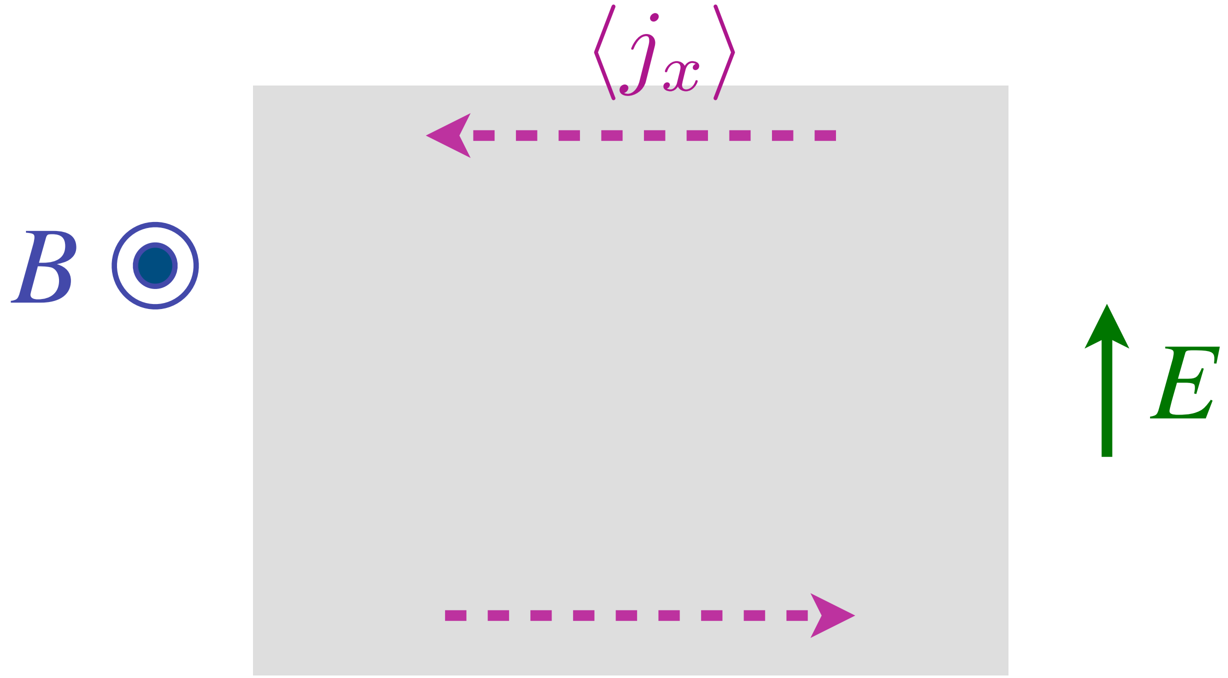
$|u_{n\boldsymbol{k}}\rangle$: Bloch state

n : label of band

→ ν is given by (half-) integer

"(Integer) QH effect"

The current flows at the edge



e.g., a toy model in 2D

$$H = \begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix} \qquad = \boldsymbol{d} \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{d} = (m, k_x, k_y)$$

$\longrightarrow$

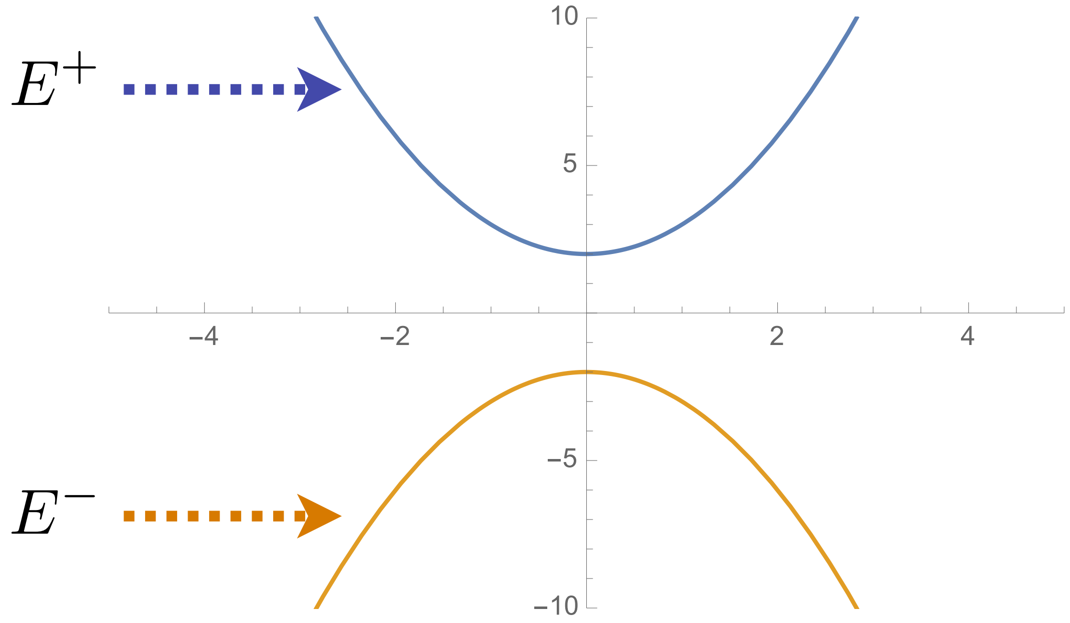
$$E^+ \simeq m + \frac{|\boldsymbol{k}|^2}{2|m|}$$

$$E^- \simeq -m - \frac{|\boldsymbol{k}|^2}{2|m|}$$

around $\boldsymbol{k} = 0$

The band structure

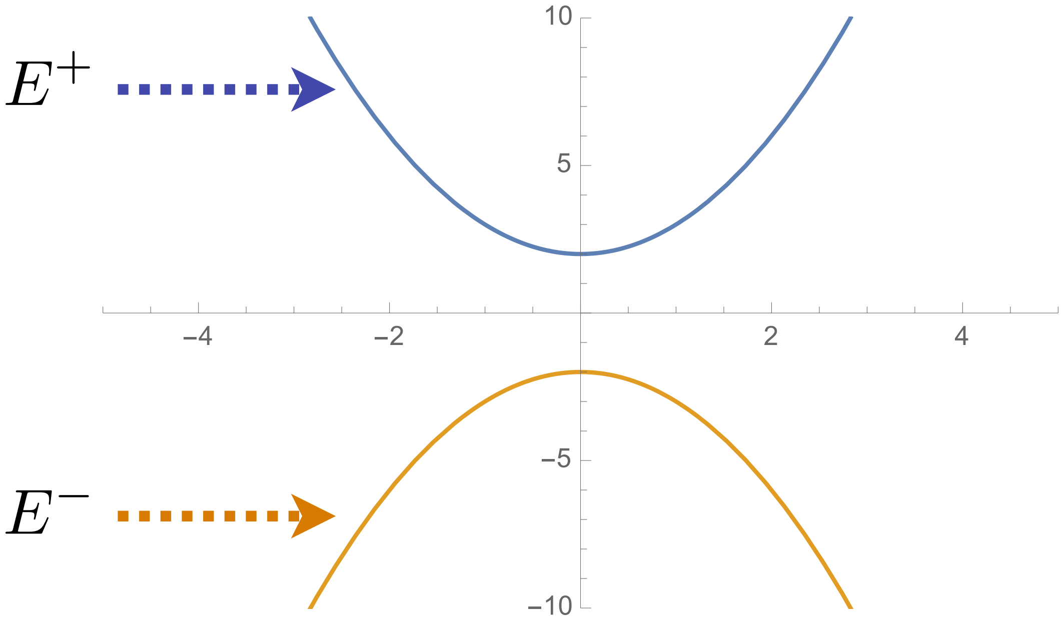
$m > 0$



Normal insulator

The band structure

$m > 0$



Normal insulator

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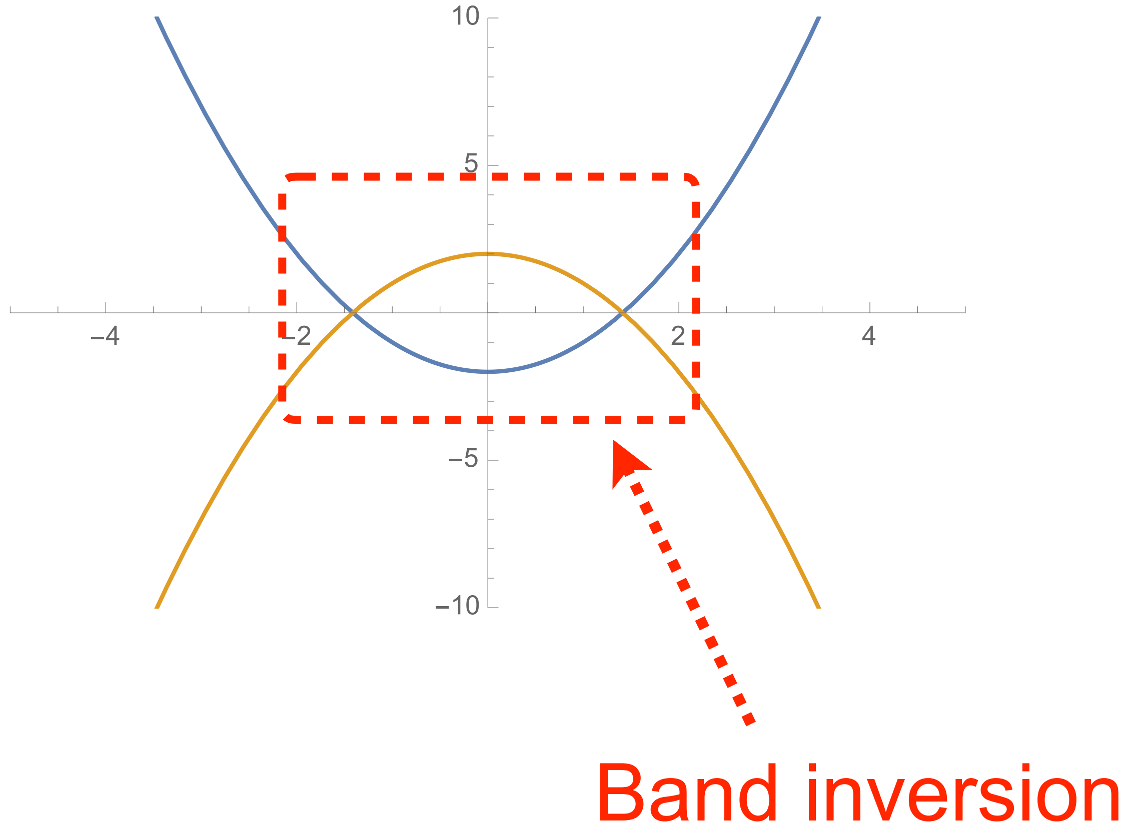
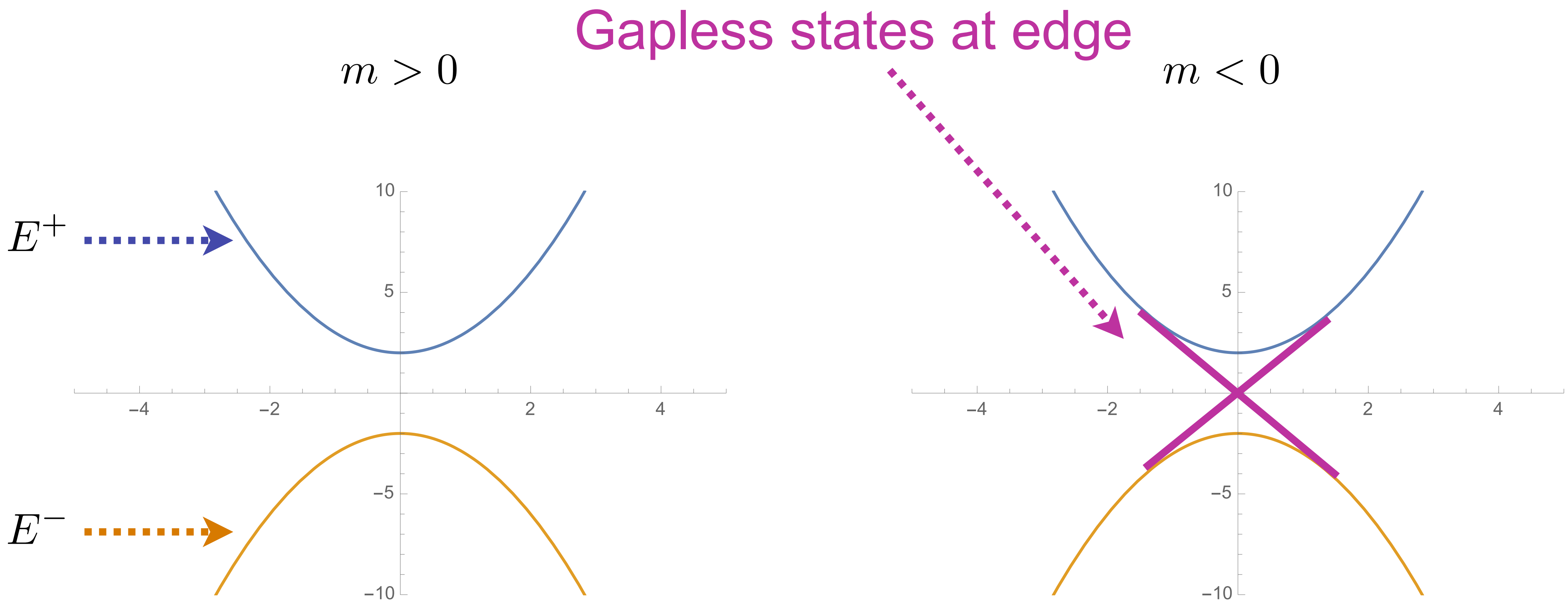


QH insulator

The band structure

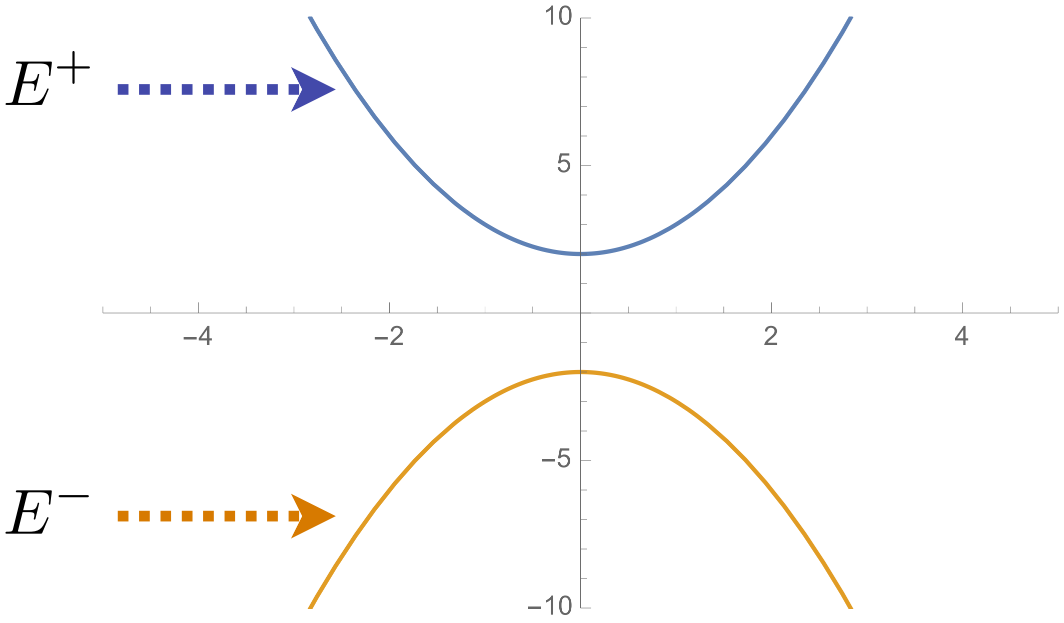


Normal insulator

QH insulator

The band structure

$m > 0$



Normal insulator

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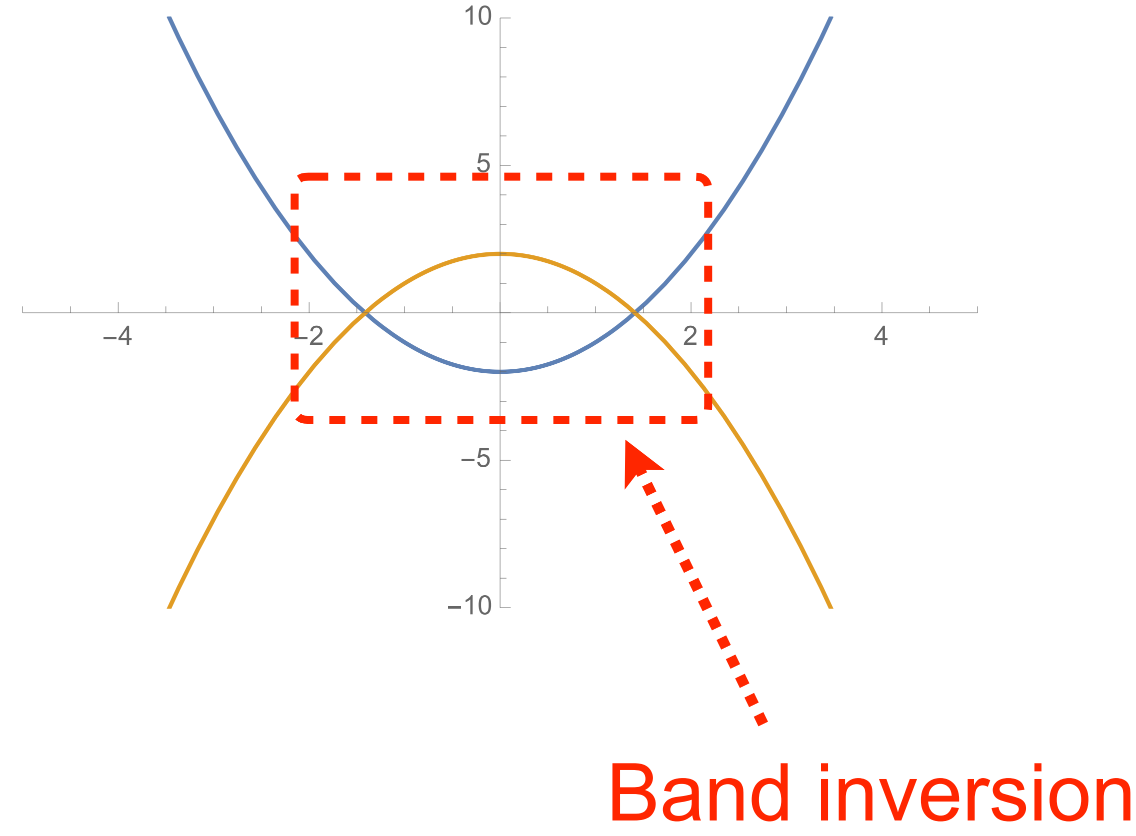

QH insulator

Band inversion is important

Anomalous quantum Hall (AQH) effect

The same effect by magnetization M, not magnetic field

$\longrightarrow$ *Anomalous* QH effect

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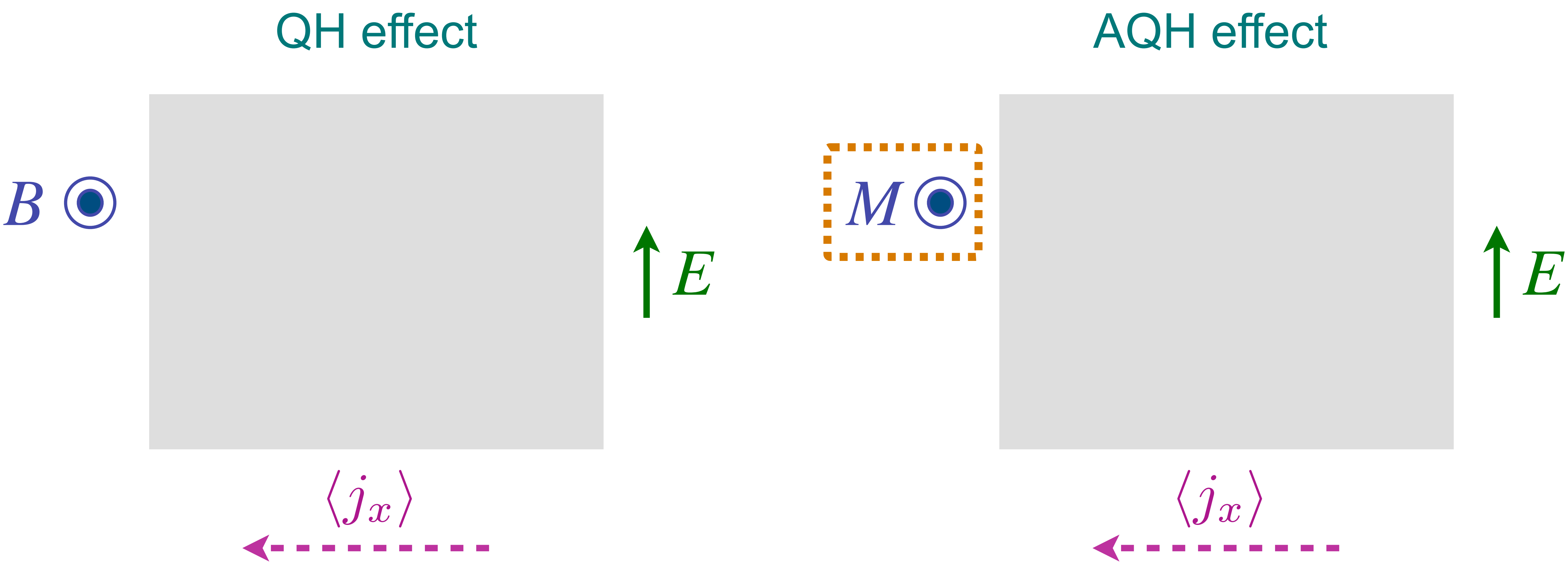
The same effect by magnetization M, not magnetic field

⟶ *Anomalous* QH effect



"AQH insulators"

"Chern insulators"

Topics related to axion in condensed matter physics

a). Insulators

b). Anomalous quantum Hall effect

c). Topological insulators

d). Magnetoelectric effect

Topological insulators (TIs)

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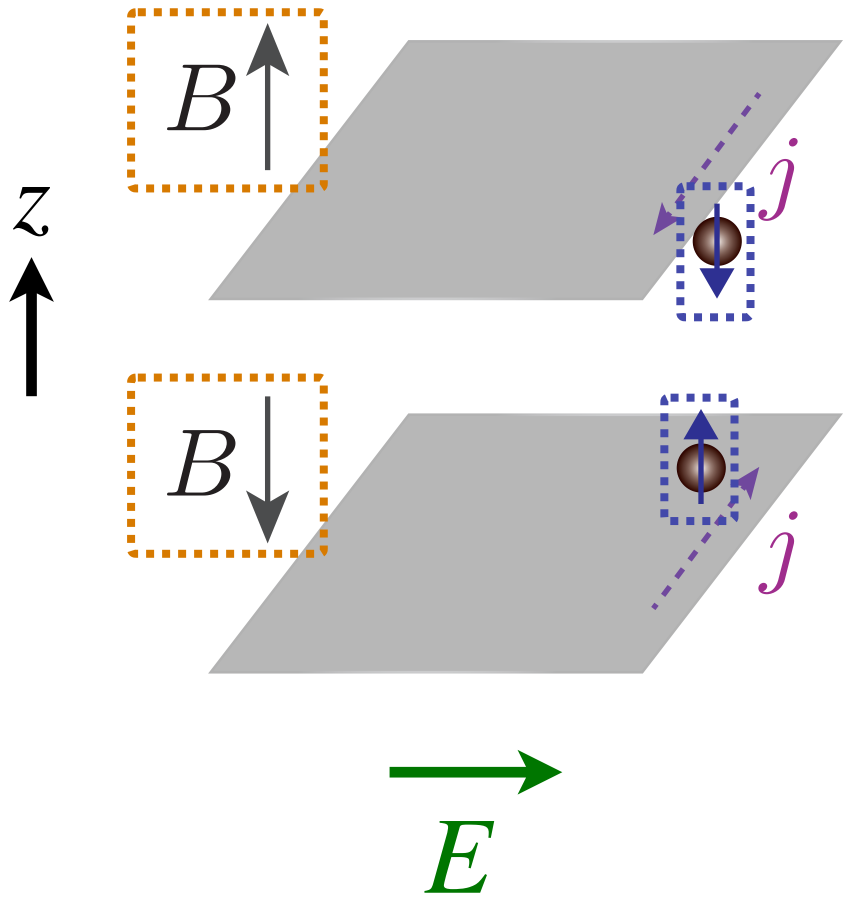
Kane, Mele '05

Idea: combination of two QH insulators



Topological insulators (TIs)

Kane, Mele '05

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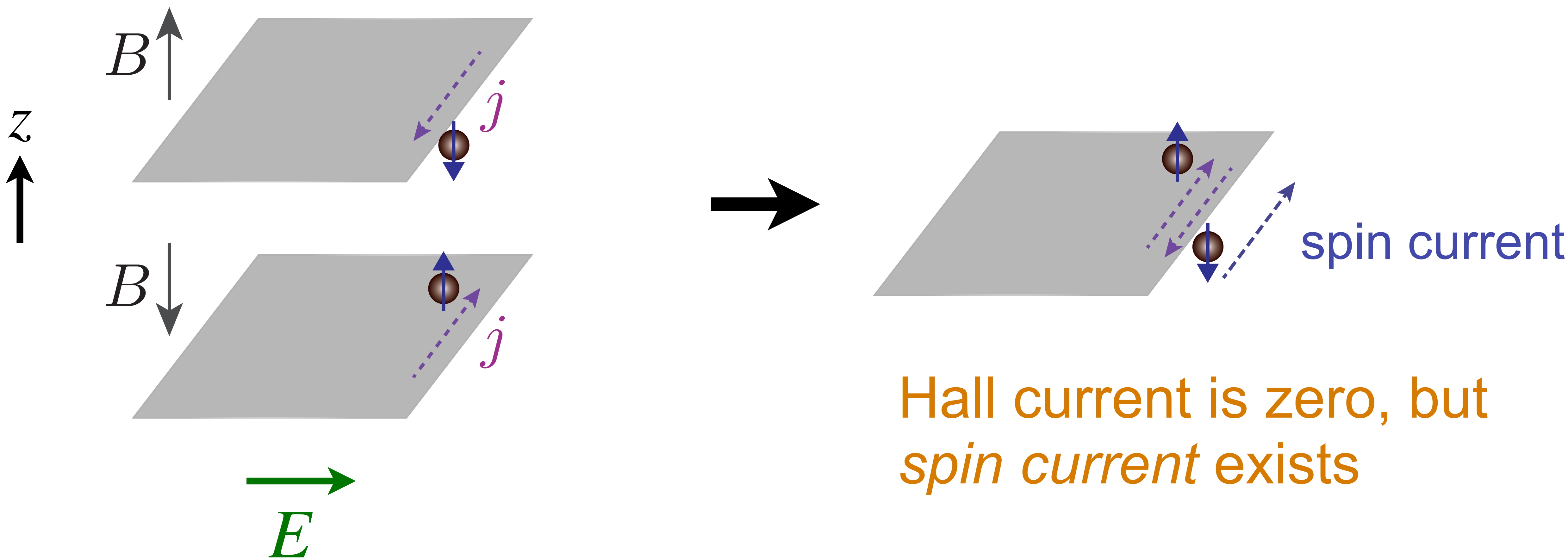

Hall current is zero, but
spin current exists

Topological insulators (TIs)

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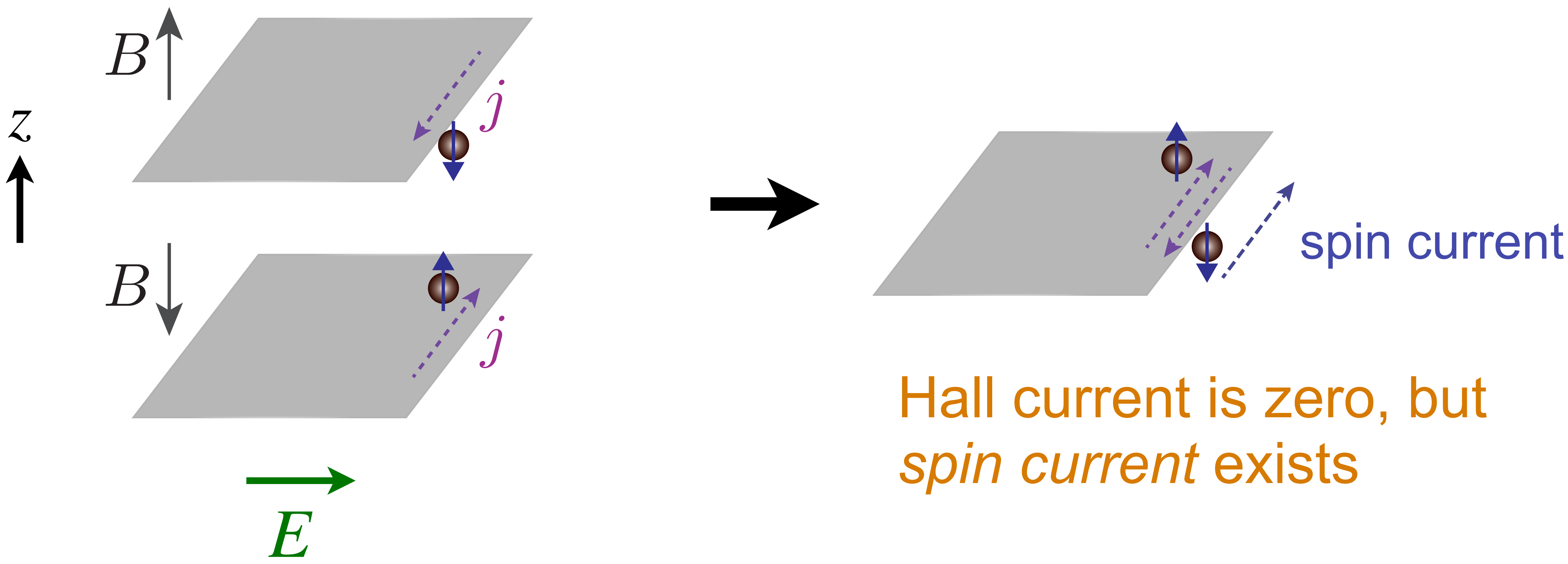


Hall current is zero, but
spin current exists

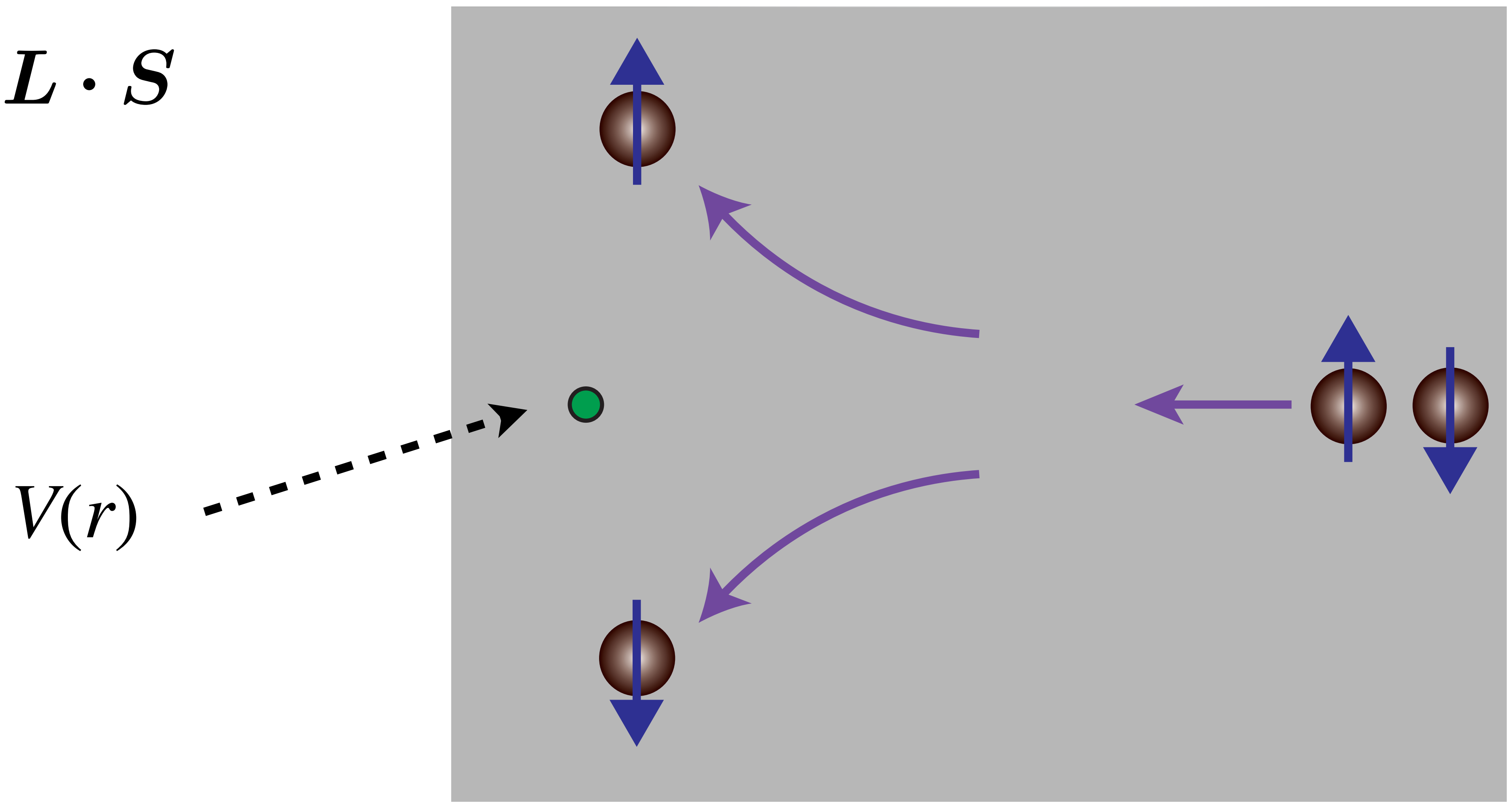
Such a system can be realized due to SOC
(without magnetic field)

SOC: spin orbit coupling

Spin-orbit coupling (SOC)

e.g.,

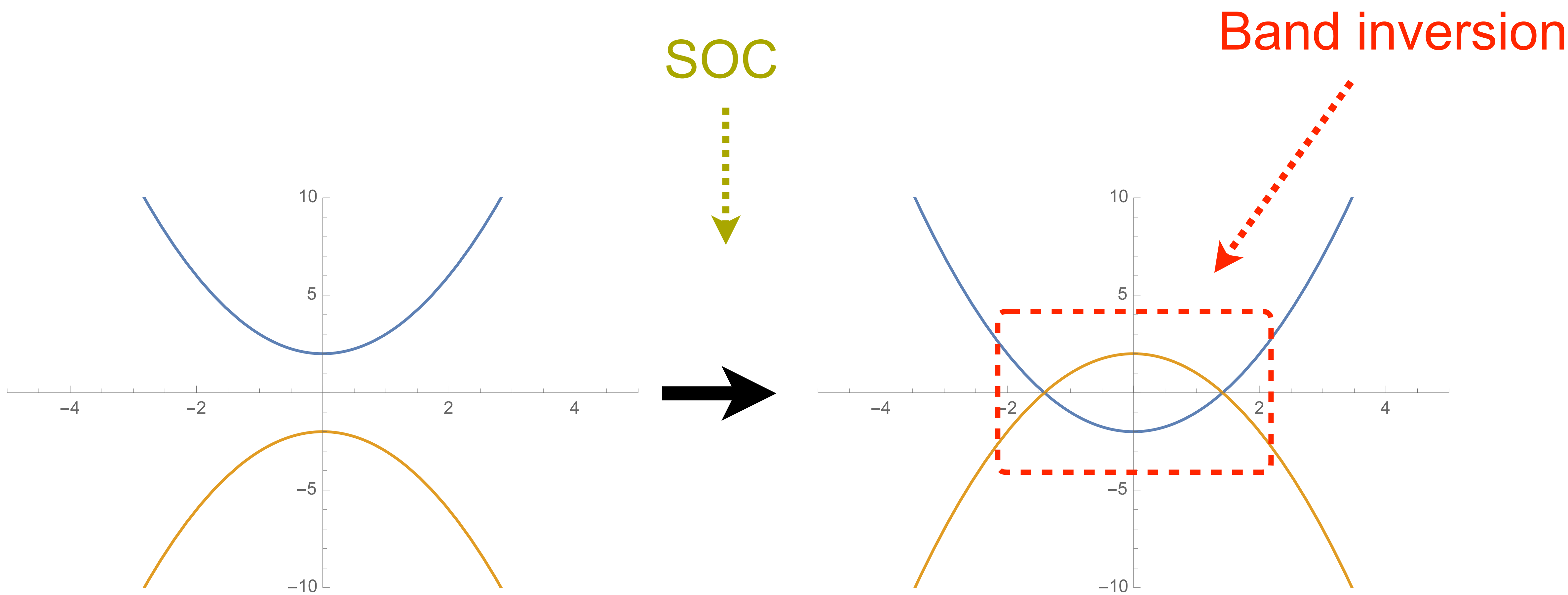
$$H_{\mathrm{SO}} = V(r)\boldsymbol{L}\cdot\boldsymbol{S}$$



Electrons with spin up or down are scattered off
to the opposite directions

The band structure

→ The same as (A)QH insulators



Normal insulator
(NI)

Topological insulator
(TI)

Keywords for topological insulators

- Time reversal invariance ($\mathcal{T}$)
- Strong spin-orbit coupling (SOC)

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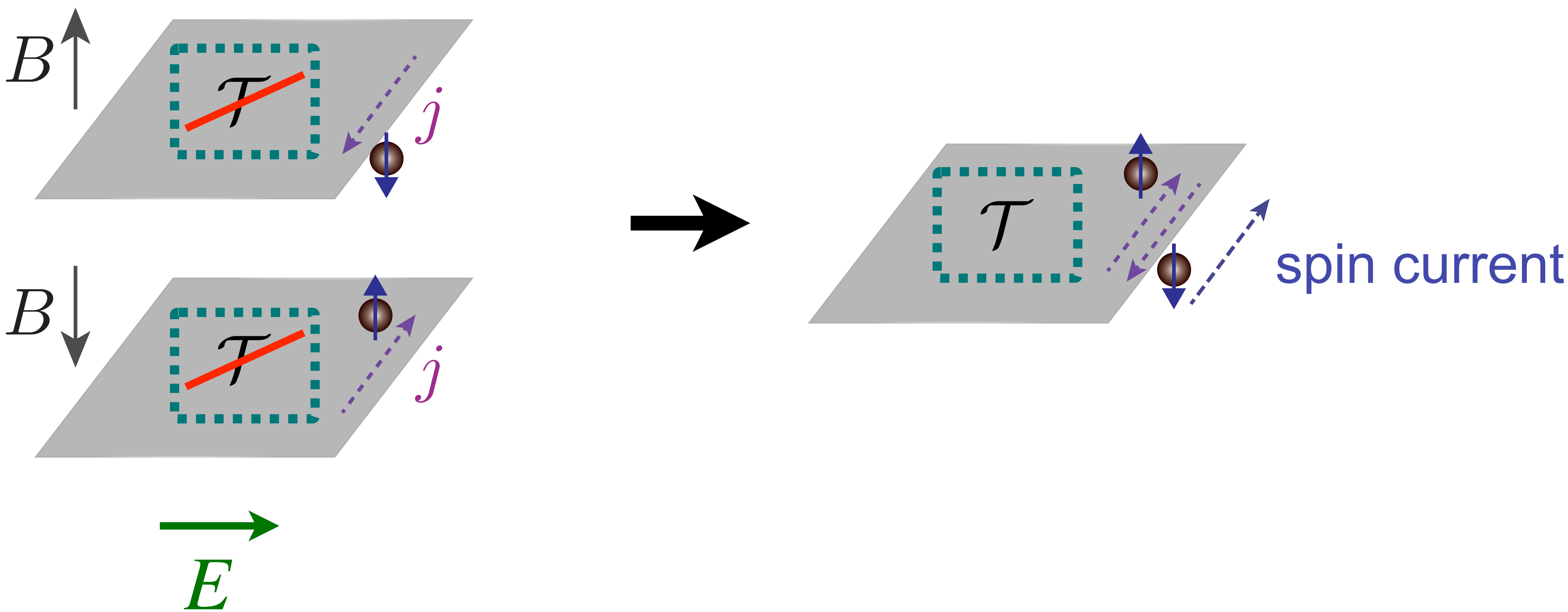
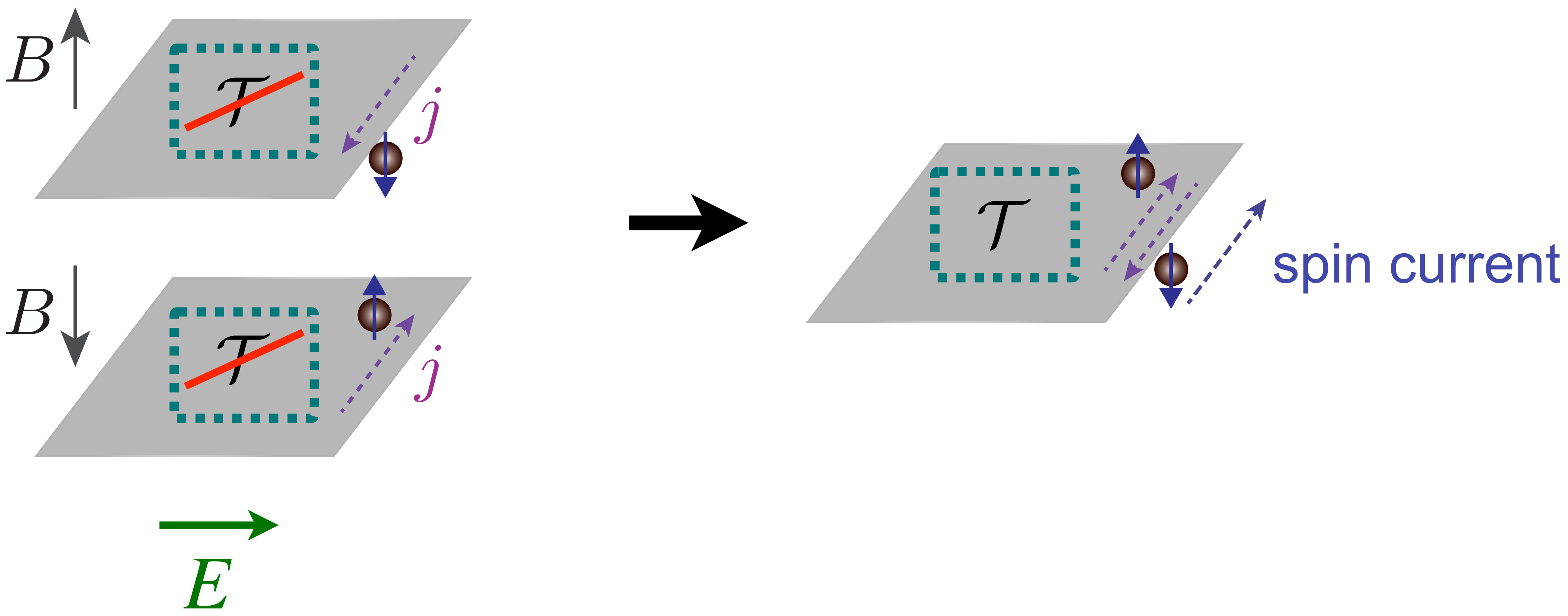


B breaks $\mathcal{T}$ but the combination of B and $-B$ keeps $\mathcal{T}$

Keywords for topological insulators

- Time reversal invariance ($\mathcal{T}$)
- Strong spin-orbit coupling (SOC)



Strong SOC is crucial for the realization

Magnetoelectric (ME) effect

predicted by Landau&Lifshitz

discovered by Dzyaloshinskii '60

- Electric field ($\boldsymbol{E}$) induces magnetization $\boldsymbol{M}$
- Magnetic field ($\boldsymbol{B}$) induces electric polarization $\boldsymbol{P}$

$$F = -\frac{1}{\mu_0 c} \int d^3x \; \alpha_{ij} E_i B_j$$

$$M_j = \alpha_{ij} E_i$$
$$P_i = \alpha_{ij} B_j$$

←

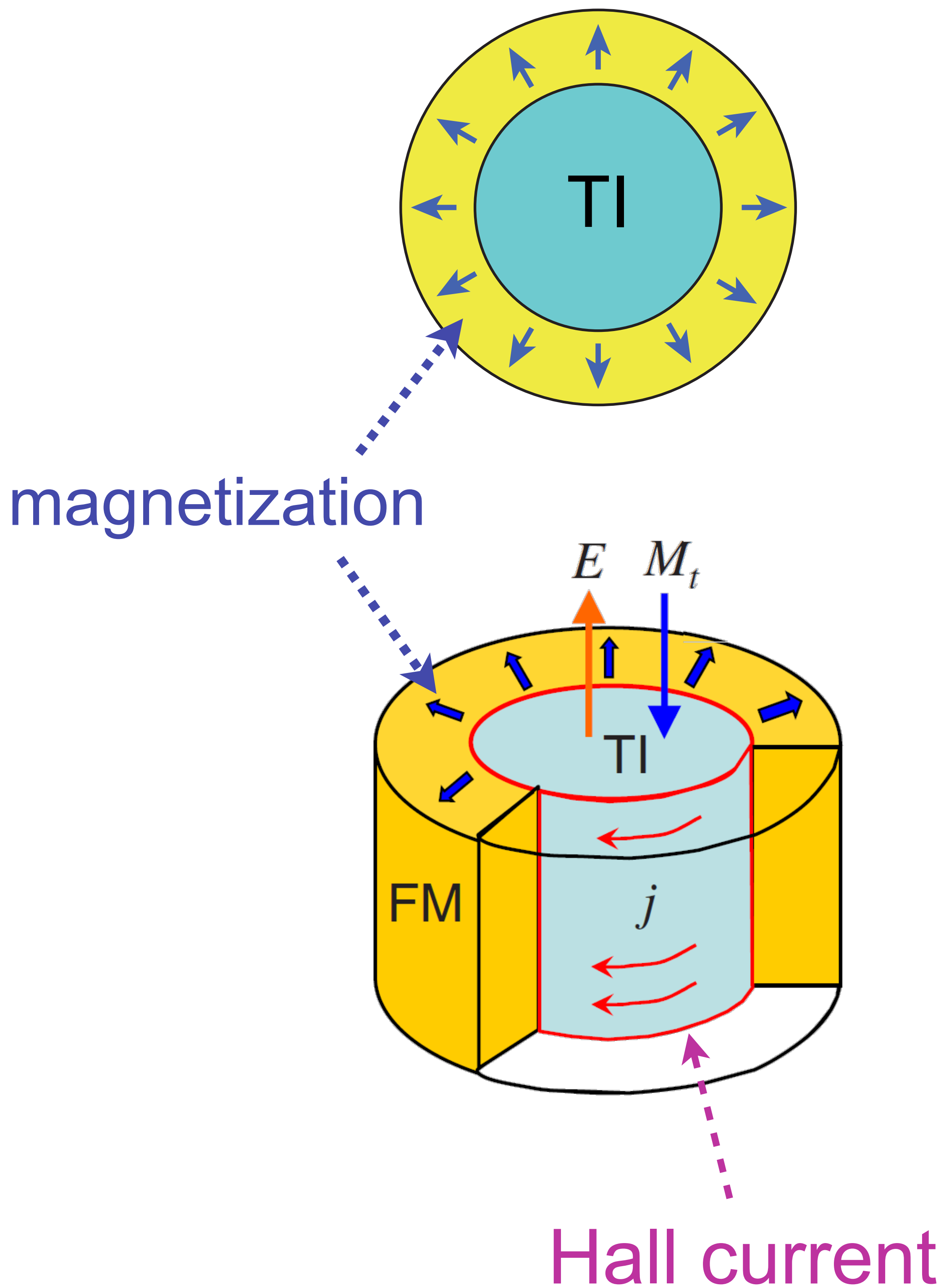
$$M_i = -\frac{1}{V} \left. \frac{\partial F}{\partial E_i} \right|_{B=0}$$

$$P_i = -\frac{1}{V} \left. \frac{\partial F}{\partial B_i} \right|_{E=0}$$

α_{ij} : constant

3D TI cylinder coated with magnetization directing outside

Qi, Hughes, Zhang '08



Half-integer AQH current

↓

The current induces magnetization in z direction

$$\boldsymbol{M} = \pm \frac{\alpha}{\mu_0 c} \boldsymbol{E}$$

α : fine-structure constant

3D TI cylinder coated with magnetization directing outside

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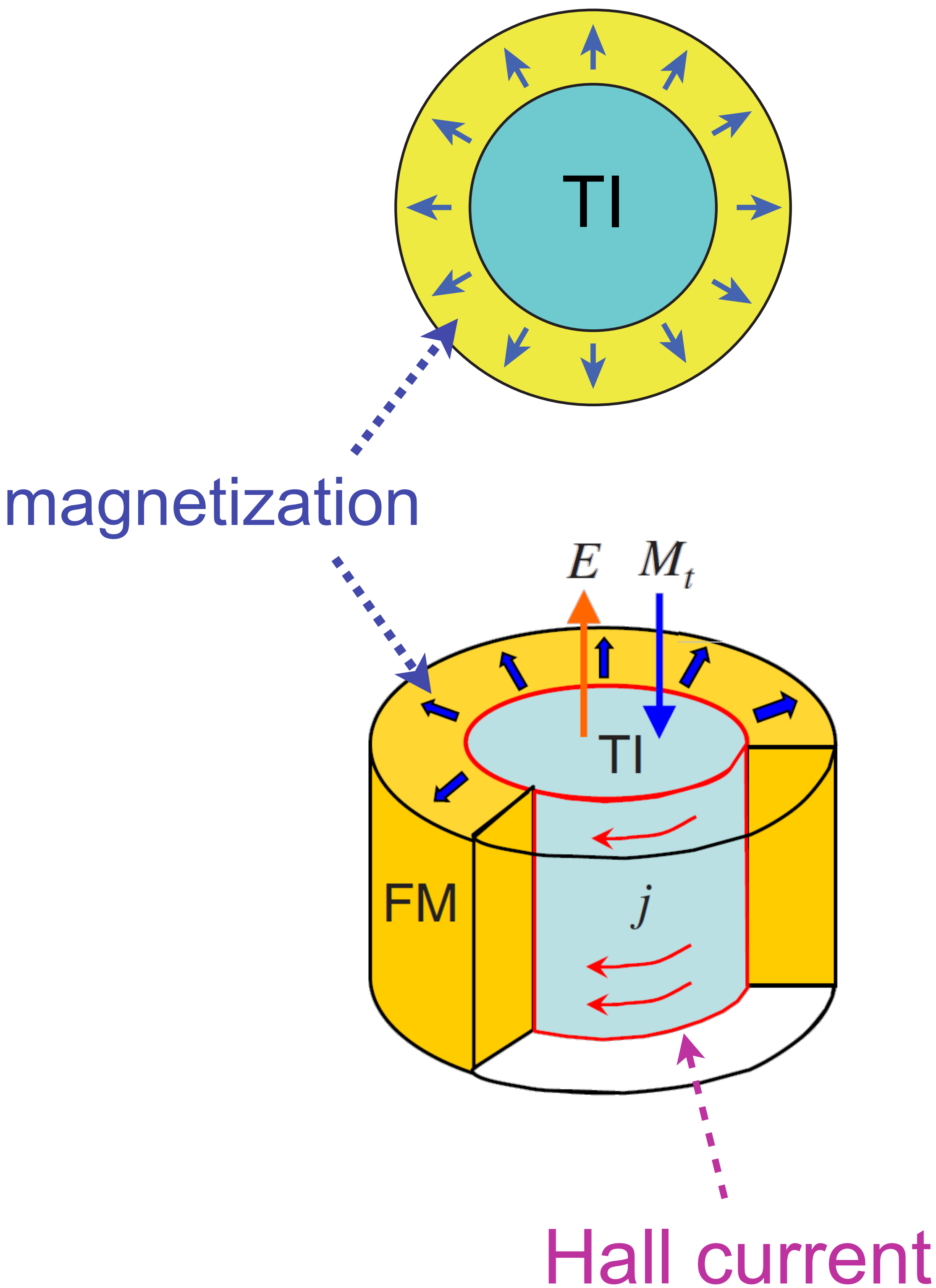

Half-integer AQH current

↓

The current induces magnetization in z direction

$$\boldsymbol{M} = \pm \frac{\alpha}{\mu_0 c} \boldsymbol{E}$$

The coefficient is given by fine-structure constant

α : fine-structure constant

This ME effect can be understood from the following free energy:

$$F_\theta = -\frac{1}{\mu_0}\int d^3x \ \frac{\alpha}{c\pi}\theta \ \boldsymbol{E}\cdot\boldsymbol{B} \qquad \text{with} \quad \theta = \pm\pi$$

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$$\longrightarrow \quad -\frac{\alpha}{4\pi}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$\theta = \pm\,\pi$ is called static axion

($\theta = 0$ in NI)

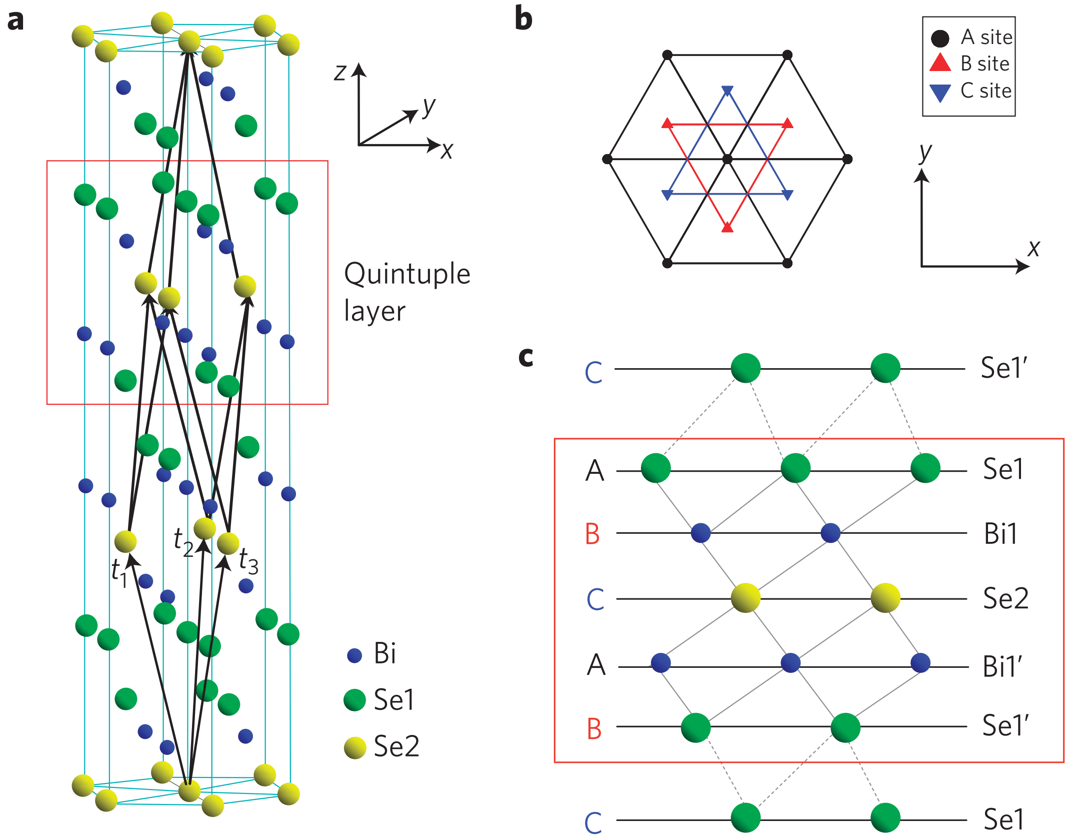
Quick summary

- Fermi energy is in the band gap for insulators
- SOC and $\mathcal{T}$ are crucial for TI
- ME effect in TI is described by “static axion”

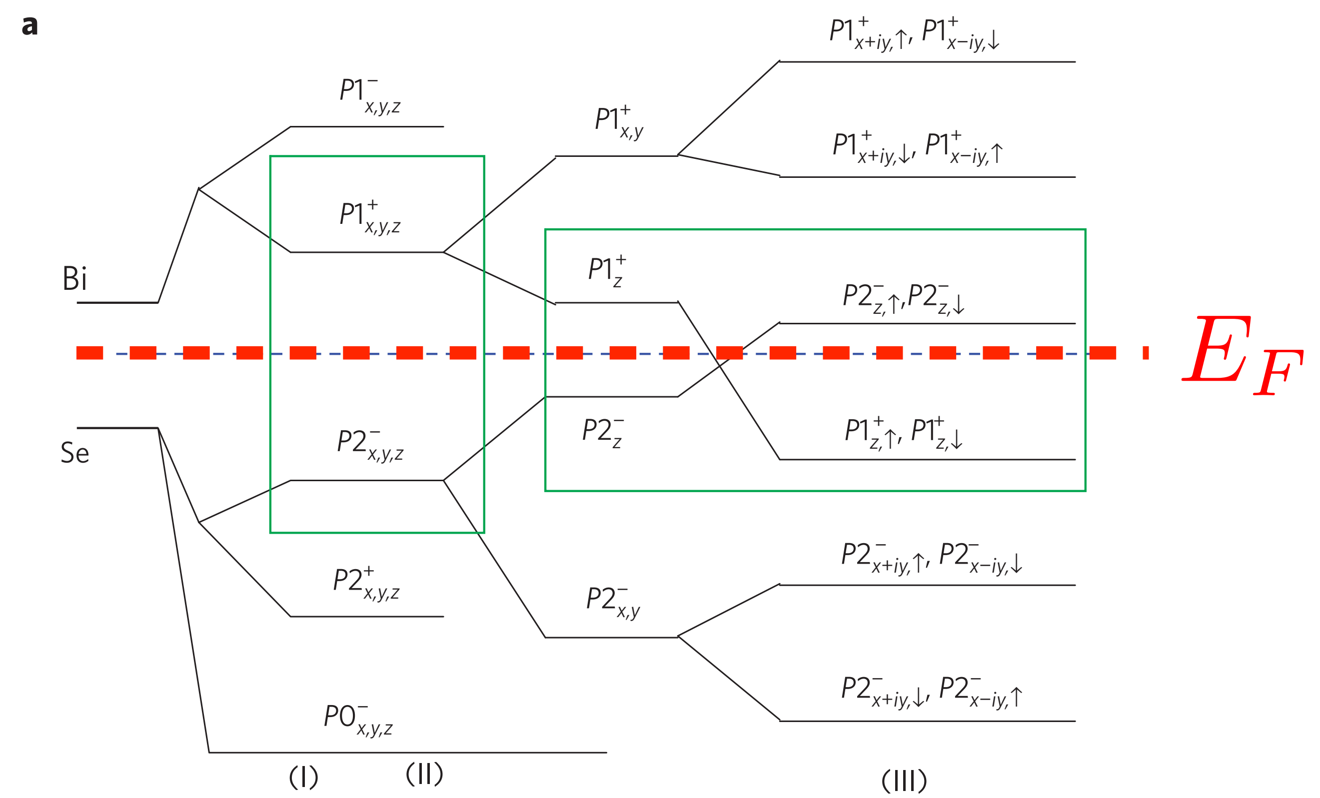
3. Axion in antiferromagnetic topological insulators

Let's consider 3D TI, Bi_2Se_3

H. Zhang et al. '09



Cristal structure



Energy levels

Let's consider 3D TI, Bi_2Se_3

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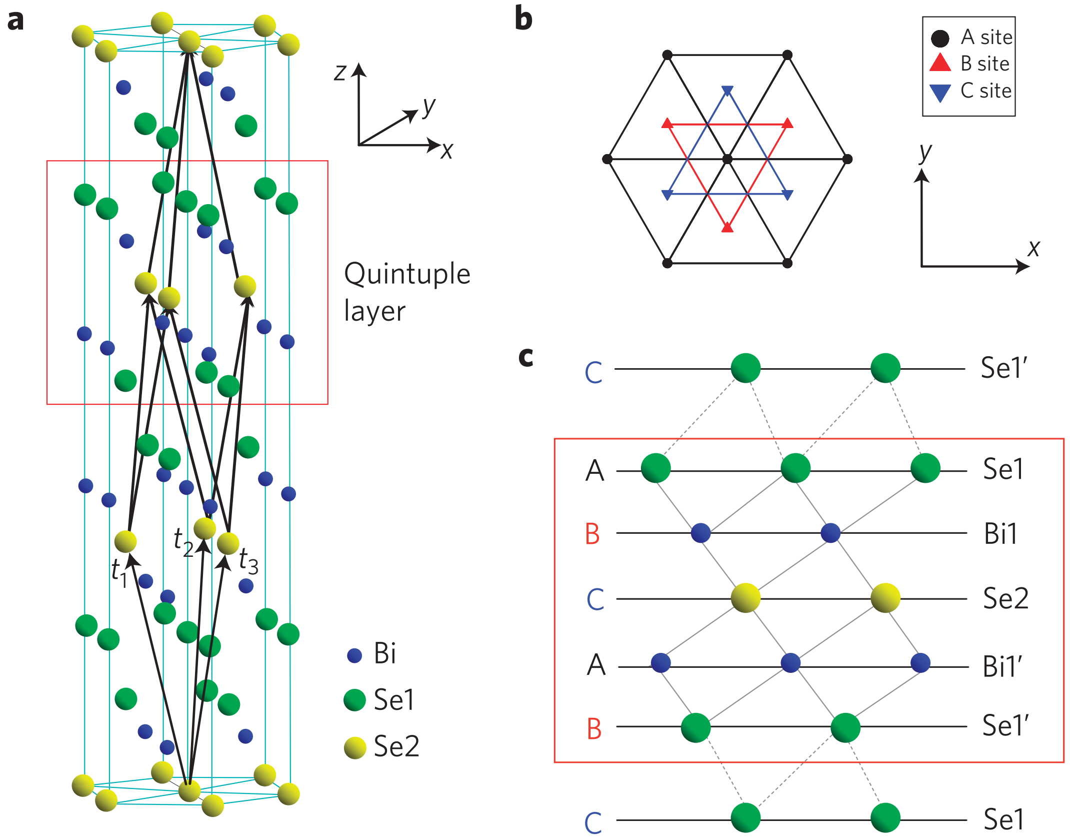

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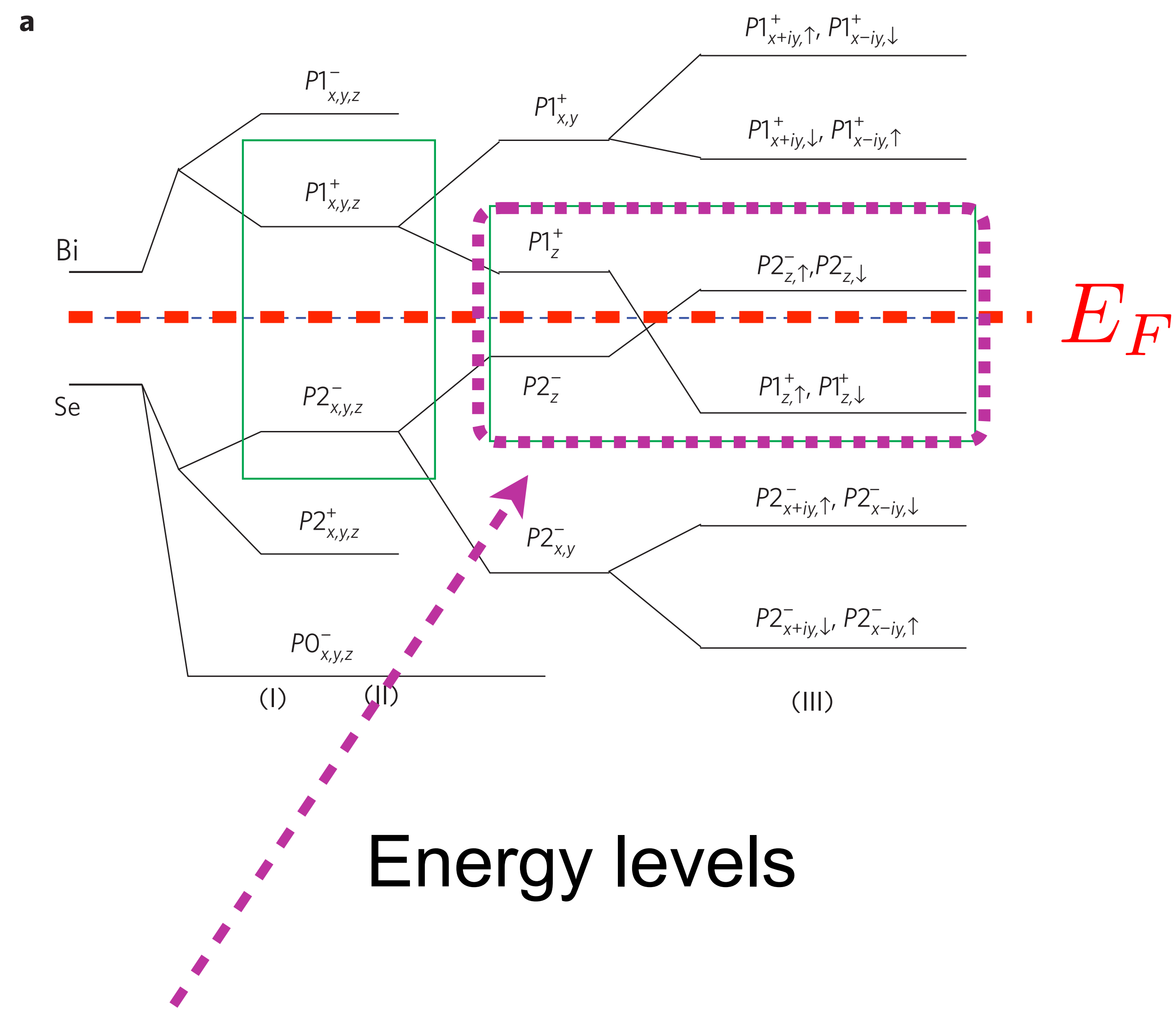
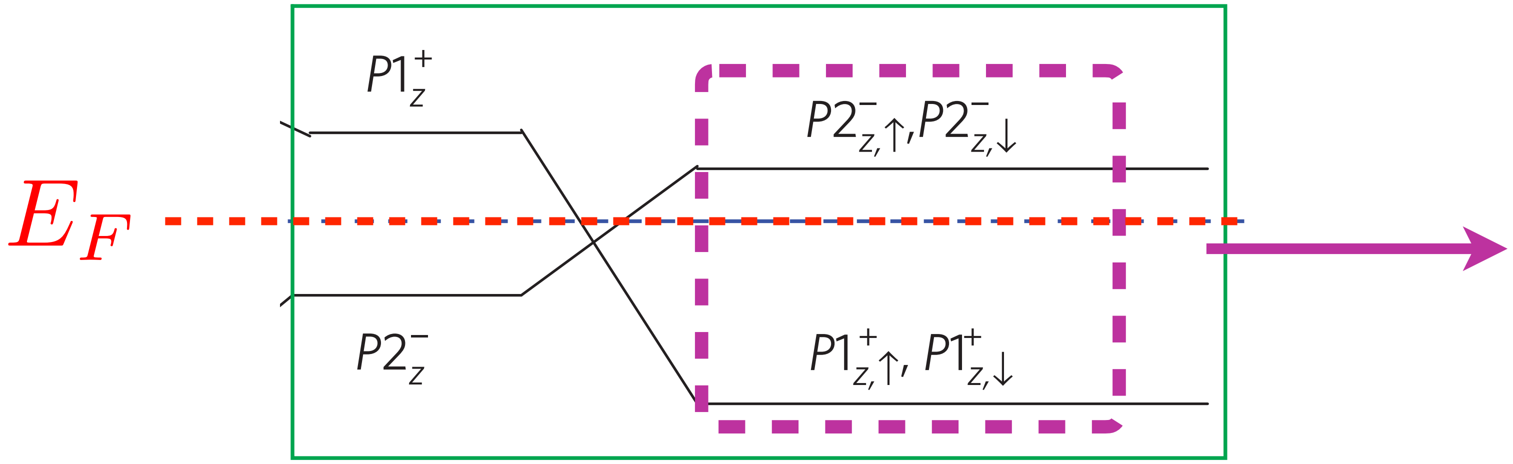


Energy levels

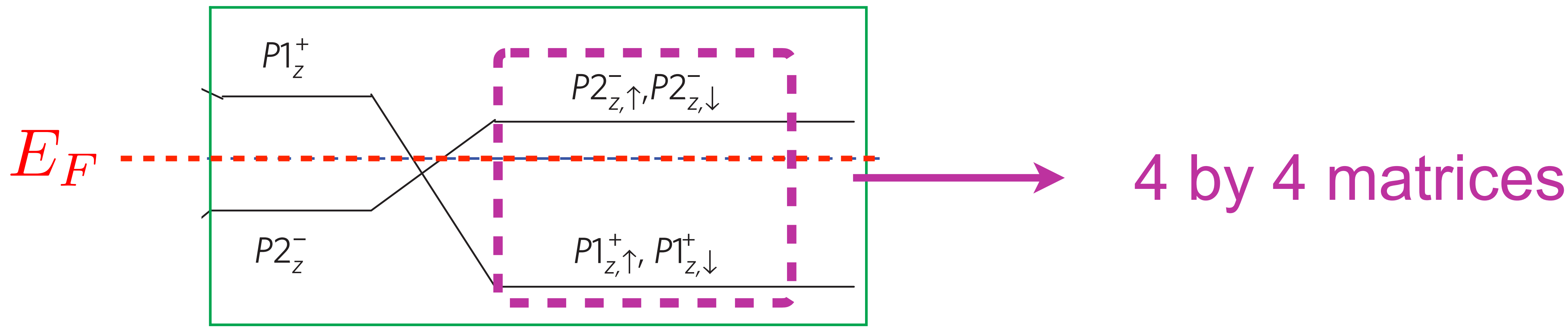
Band inversion due to strong SOC

Two bands near the Fermi energy are important



4 by 4 matrices

Two bands near the Fermi energy are important



$$H_0(\boldsymbol{k})$$

$$= \begin{pmatrix} \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & 0 & -iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) \\ 0 & \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & A_2(\sin k_x + i \sin k_y) & -iA_1 \sin k_z \\ iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) & 0 \\ A_2(\sin k_x + i \sin k_y) & iA_1 \sin k_z & 0 & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) \end{pmatrix}$$

basis: $\left(\left|P1_z^+, \uparrow\right\rangle, \left|P1_z^+, \downarrow\right\rangle, \left|P2_z^-, \uparrow\right\rangle, \left|P2_z^-, \downarrow\right\rangle\right)$

"Effective Hamiltonian for 3D TI"

The Hamiltonian can be written in the Gamma matrices:

$$\begin{pmatrix} \epsilon_0(\boldsymbol{k})+\mathcal{M}(\boldsymbol{k}) & 0 & -iA_1\sin k_z & A_2(\sin k_x - i\sin k_y) \\ 0 & \epsilon_0(\boldsymbol{k})+\mathcal{M}(\boldsymbol{k}) & A_2(\sin k_x + i\sin k_y) & -iA_1\sin k_z \\ iA_1\sin k_z & A_2(\sin k_x - i\sin k_y) & \epsilon_0(\boldsymbol{k})-\mathcal{M}(\boldsymbol{k}) & 0 \\ A_2(\sin k_x + i\sin k_y) & iA_1\sin k_z & 0 & \epsilon_0(\boldsymbol{k})-\mathcal{M}(\boldsymbol{k}) \end{pmatrix}$$

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$$= \epsilon_0 \mathbf{1}_{4\times 4} + \sum_{a=1}^{5} d^a \Gamma^a$$

cf. 2D toy model

$$H = \boldsymbol{d} \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{d} = (m, k_x, k_y)$$

$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x,\ A_2 \sin k_y,\ A_1 \sin k_z,\ \mathcal{M}(\boldsymbol{k}), 0)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \quad \Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \quad \Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

The Hamiltonian can be written in the Gamma matrices:

$$\begin{pmatrix} \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & 0 & -iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) \\ 0 & \epsilon_0(\boldsymbol{k}) + \mathcal{M}(\boldsymbol{k}) & A_2(\sin k_x + i \sin k_y) & -iA_1 \sin k_z \\ iA_1 \sin k_z & A_2(\sin k_x - i \sin k_y) & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) & 0 \\ A_2(\sin k_x + i \sin k_y) & iA_1 \sin k_z & 0 & \epsilon_0(\boldsymbol{k}) - \mathcal{M}(\boldsymbol{k}) \end{pmatrix}$$

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$$H = \boldsymbol{d} \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{d} = (m, k_x, k_y)$$

$$(d^1, d^2, d^3, d^4, d^5) = (A_2 \sin k_x, A_2 \sin k_y, A_1 \sin k_z, \mathcal{M}(\boldsymbol{k}), 0)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \quad \Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \quad \Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

$d_5 = 0$

(see later discussion)

The Hamiltonian can be written in the Gamma matrices:

$$\begin{pmatrix} \epsilon_0(\boldsymbol{k})+\mathcal{M}(\boldsymbol{k}) & 0 & -iA_1\sin k_z & A_2(\sin k_x - i\sin k_y) \\ 0 & \epsilon_0(\boldsymbol{k})+\mathcal{M}(\boldsymbol{k}) & A_2(\sin k_x + i\sin k_y) & -iA_1\sin k_z \\ iA_1\sin k_z & A_2(\sin k_x - i\sin k_y) & \epsilon_0(\boldsymbol{k})-\mathcal{M}(\boldsymbol{k}) & 0 \\ A_2(\sin k_x + i\sin k_y) & iA_1\sin k_z & 0 & \epsilon_0(\boldsymbol{k})-\mathcal{M}(\boldsymbol{k}) \end{pmatrix}$$

$$= \epsilon_0 \mathbf{1}_{4\times 4} + \sum_{a=1}^{5} d^a \Gamma^a$$

important for topological state

cf. 2D toy model

$$H = \boldsymbol{d}\cdot\boldsymbol{\sigma}$$

$$\boldsymbol{d} = (m, k_x, k_y)$$

$$(d^1,\, d^2,\, d^3,\, d^4,\, d^5) = (A_2\sin k_x,\; A_2\sin k_y,\; A_1\sin k_z,\; \mathcal{M}(\boldsymbol{k}), 0)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1\cos k_z + 2B_2(\cos k_x + \cos k_y)$$

$$\Gamma^1 = \begin{pmatrix} 0 & \sigma^x \\ \sigma^x & 0 \end{pmatrix} \qquad \Gamma^2 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix} \qquad \Gamma^3 = \begin{pmatrix} 0 & -i\mathbf{1} \\ -i\mathbf{1} & 0 \end{pmatrix}$$

$$\Gamma^4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \qquad \Gamma^5 = \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}$$

Partition function (given by path integral)

$$S_0 = \int d^4x \; \psi^\dagger(x) \left[i\partial_t - H_0 \right] \psi(x)$$

$$\longrightarrow \quad Z_0 = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger e^{iS_0}$$

$\psi(x)$: wavefunction of electron

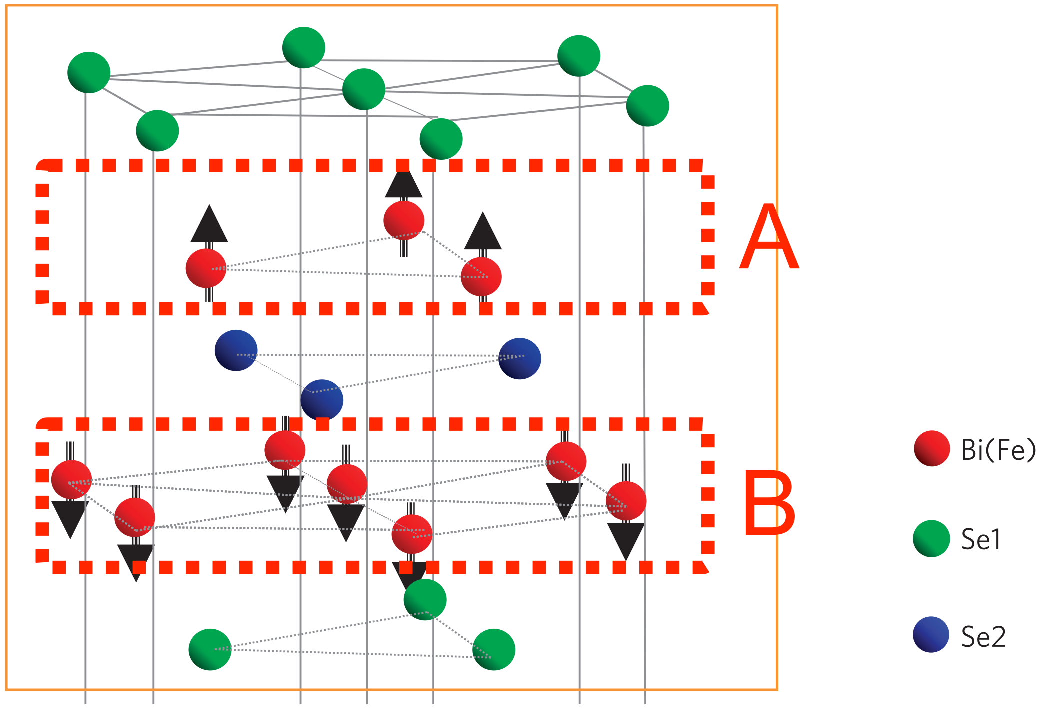
$$\sim \left(\left| P1_z^+, \uparrow \right\rangle, \left| P1_z^+, \downarrow \right\rangle, \left| P2_z^-, \uparrow \right\rangle, \left| P2_z^-, \downarrow \right\rangle \right)^T$$

Now we introduce *antiferromagnetism (AFM)* by hand

($\mathcal{T}$ violation)

R. Li et al. '10

Suppose electrons at Bi are AFM order

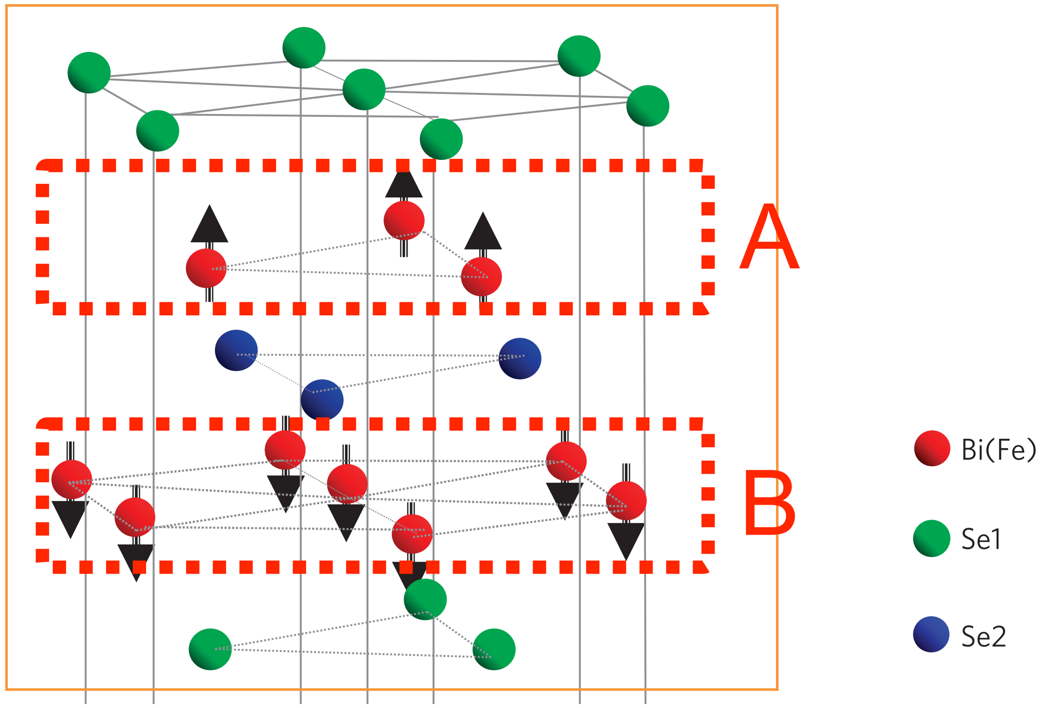


Now we introduce *antiferromagnetism (AFM)* by hand

($\mathcal{T}$ violation)

R. Li et al. ’10

$$\mathcal{H}_{\rm int} = \frac{UV}{N}\int d^3x \; (n_{A\uparrow}n_{A\downarrow} + n_{B\uparrow}n_{B\downarrow})$$



“Hubbard term”

U : parameter to give AFM

V : volume

N : number of site

$$n_{A\sigma} = \psi^\dagger_{A\sigma}\psi_{A\sigma}$$

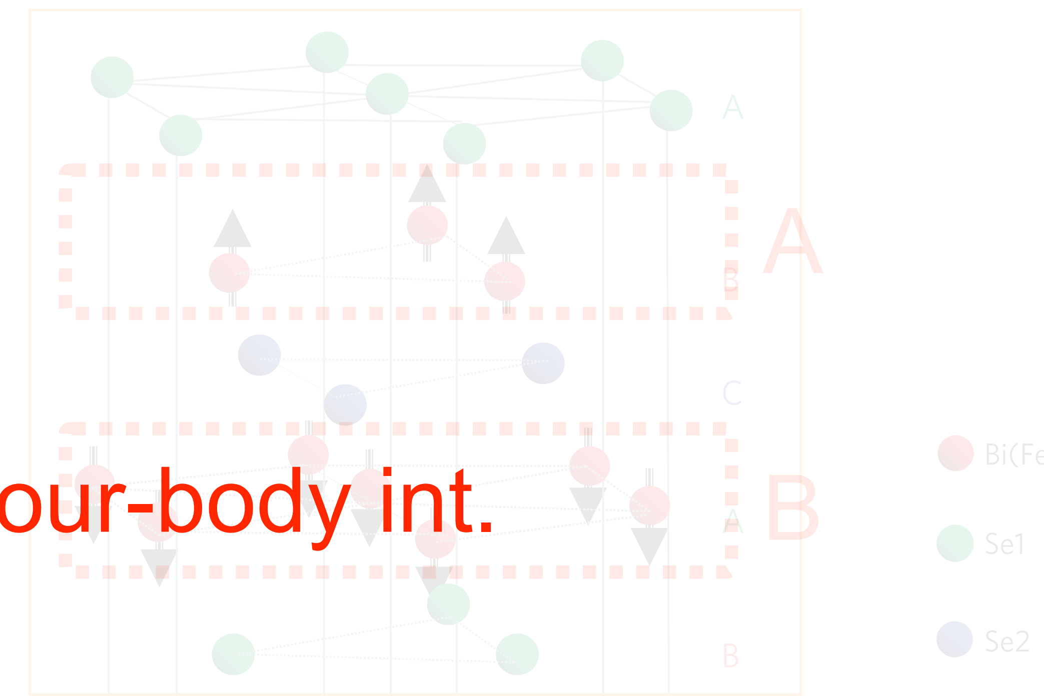
$$n_{B\sigma} = \psi^\dagger_{B\sigma}\psi_{B\sigma}$$

Now we introduce *antiferromagnetism (AFM)* by hand

($\mathcal{T}$ violation)

R. Li et al. '10

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \; (n_{\rm A\uparrow} n_{\rm A\downarrow} + n_{\rm B\uparrow} n_{\rm B\downarrow})$$



"Hubbard term"

U : parameter to give AFM

Difficult to handle since they're four-body int.

V : volume

N : number of site

$$n_{\rm A\sigma} = \psi^\dagger_{\rm A\sigma} \psi_{\rm A\sigma}$$

$$n_{\rm B\sigma} = \psi^\dagger_{\rm B\sigma} \psi_{\rm B\sigma}$$

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \, \left(n_{{\rm A}\uparrow} n_{{\rm A}\downarrow} + n_{{\rm B}\uparrow} n_{{\rm B}\downarrow} \right)$$

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\text{int}} = \frac{UV}{N} \int d^3x \, (n_{\text{A}\uparrow} n_{\text{A}\downarrow} + n_{\text{B}\uparrow} n_{\text{B}\downarrow})$$

↓ HS transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ ($d_5 = \phi$)
- Mass term of ϕ

This can be understood from

HS transformation

$\sim$ Inverse of integrating out a scalar

This can be understood from

HS transformation

$\sim$ Inverse of integrating out a scalar

$n_{\mathrm{A}\uparrow} n_{\mathrm{A}\downarrow},\ n_{\mathrm{B}\uparrow} n_{\mathrm{B}\downarrow}$

$(\psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow^\dagger \psi_\downarrow)$

Four Fermi int.

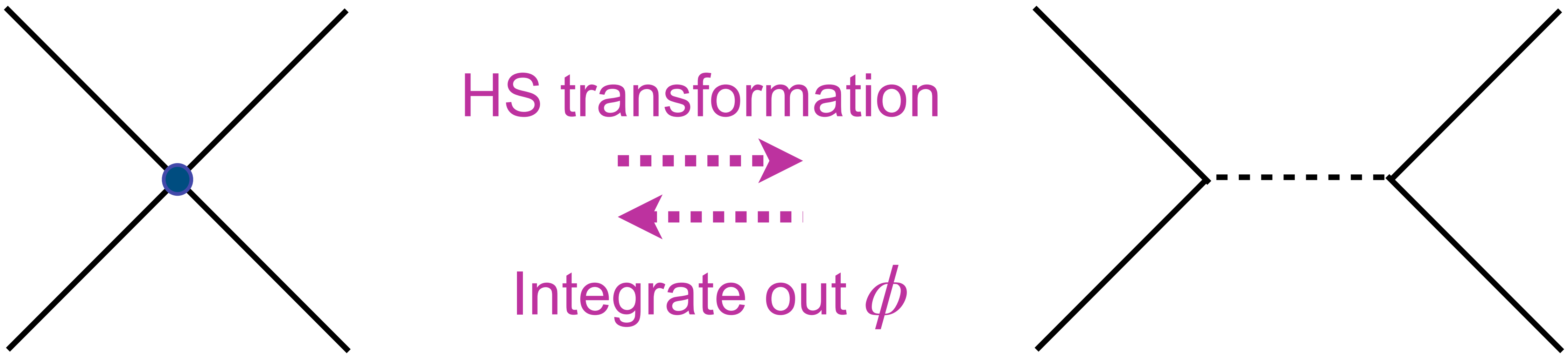
$\phi \psi_\uparrow^\dagger \psi_\uparrow\,,\ \phi \psi_\downarrow^\dagger \psi_\downarrow$

Yukawa int.

This can be understood from

HS transformation

$\sim$ Inverse of integrating out a scalar



$n_{A\uparrow}n_{A\downarrow},\ n_{B\uparrow}n_{B\downarrow}$
$(\psi^\dagger_\uparrow\psi_\uparrow\psi^\dagger_\downarrow\psi_\downarrow)$

Four Fermi int.

$\phi\psi^\dagger_\uparrow\psi_\uparrow\ ,\ \phi\psi^\dagger_\downarrow\psi_\downarrow$

Yukawa int.

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\mathrm{int}} = \frac{UV}{N} \int d^3x \, (n_{\mathrm{A}\uparrow} n_{\mathrm{A}\downarrow} + n_{\mathrm{B}\uparrow} n_{\mathrm{B}\downarrow})$$

↓ HS transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ $(d_5 = \phi)$
- Mass term of ϕ ← missed in Sekine, Nomura '16, Sekine, Nomura '20, Schütte-Engel '21

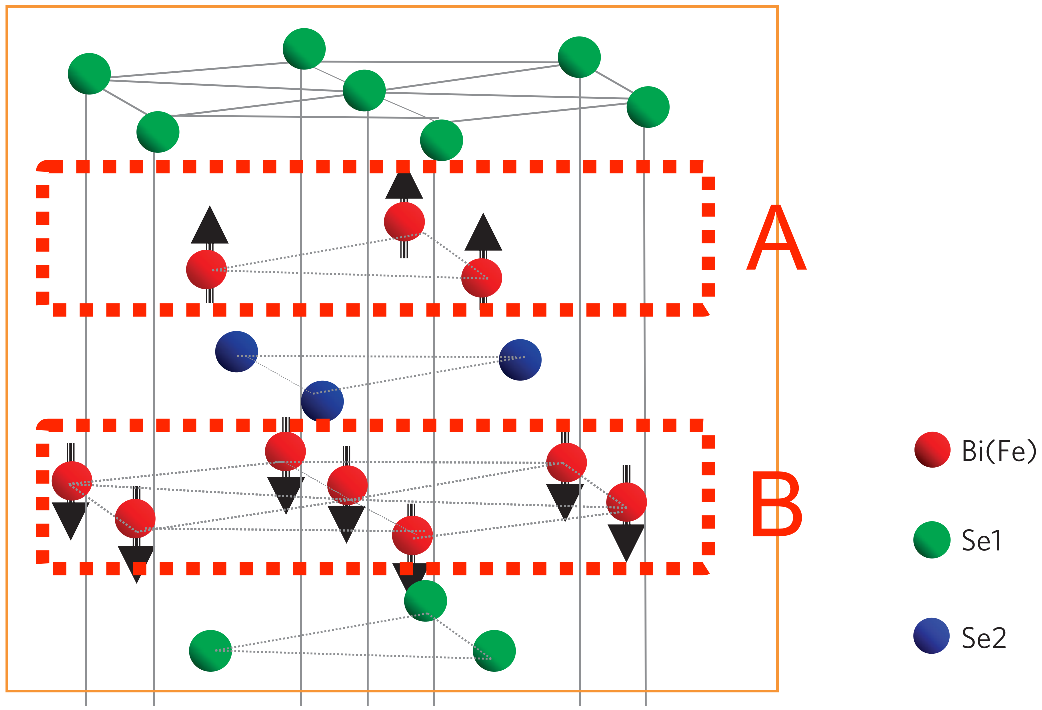
(confirmed by private communication with Sekine-san)

Hubbard-Stratonovich (HS) transformation

$$\mathcal{H}_{\rm int} = \frac{UV}{N} \int d^3x \, \left(n_{\rm A\uparrow}n_{\rm A\downarrow} + n_{\rm B\uparrow}n_{\rm B\downarrow}\right)$$

$\Big\downarrow$ HS transformation

- A dynamical scalar ϕ that gives $\Gamma^5 d_5$ ($d_5 = \phi$)
- Mass term of ϕ
- VEV of ϕ is the order parameter of the AFM



$$\langle \phi \rangle \sim \langle S_{\mathrm{A}} \rangle = -\langle S_{\mathrm{B}} \rangle$$

- $\langle \phi \rangle = 0$ → PM order
- $\langle \phi \rangle \neq 0$ → AFM order

PM (paramagnetic)

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi \; e^{iS + iS_\phi^{\text{mass}}}$$

$$S = \int d^4x \; \psi^\dagger(x) \left[i\partial_t - H \right] \psi(x)$$

$$H = H_0 + \delta H$$

$$S_\phi^{\text{mass}} = -\int d^4x \; M_\phi^2 \phi^2$$

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi \, e^{iS + iS_\phi^{\text{mass}}}$$

$$S = \int d^4x \; \psi^\dagger(x) \left[i\partial_t - H \right] \psi(x)$$

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$$\Gamma^5 \phi$$

$$H = H_0 + \delta H$$

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Partition function (TI + AFM)

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\phi \, e^{iS + iS_\phi^{\mathrm{mass}}}$$

$\Gamma^5 \phi$

$$S = \int d^4x \; \psi^\dagger(x) \left[i\partial_t - H \right] \psi(x)$$

$$H = H_0 + \delta H$$

$$S_\phi^{\mathrm{mass}} = - \int d^4x \; M_\phi^2 \phi^2$$

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Integrate out ψ , $\psi^\dagger$

Effective action for ϕ

Effective potential for ϕ

KI '21

$$V_\phi = -2 \int \frac{d^3k}{(2\pi)^3} \left(\sqrt{|d_0|^2 + \phi^2} - |d_0| \right) + M_\phi^2 \phi^2$$

$$|d_0|^2 = \sum_{a=1}^{4} |d^a|^2$$

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3} \frac{2}{U}$$

Effective potential for ϕ

KI '21

$$V_\phi = -2\int \frac{d^3k}{(2\pi)^3}\left(\sqrt{|d_0|^2+\phi^2}-|d_0|\right) + M_\phi^2\phi^2$$

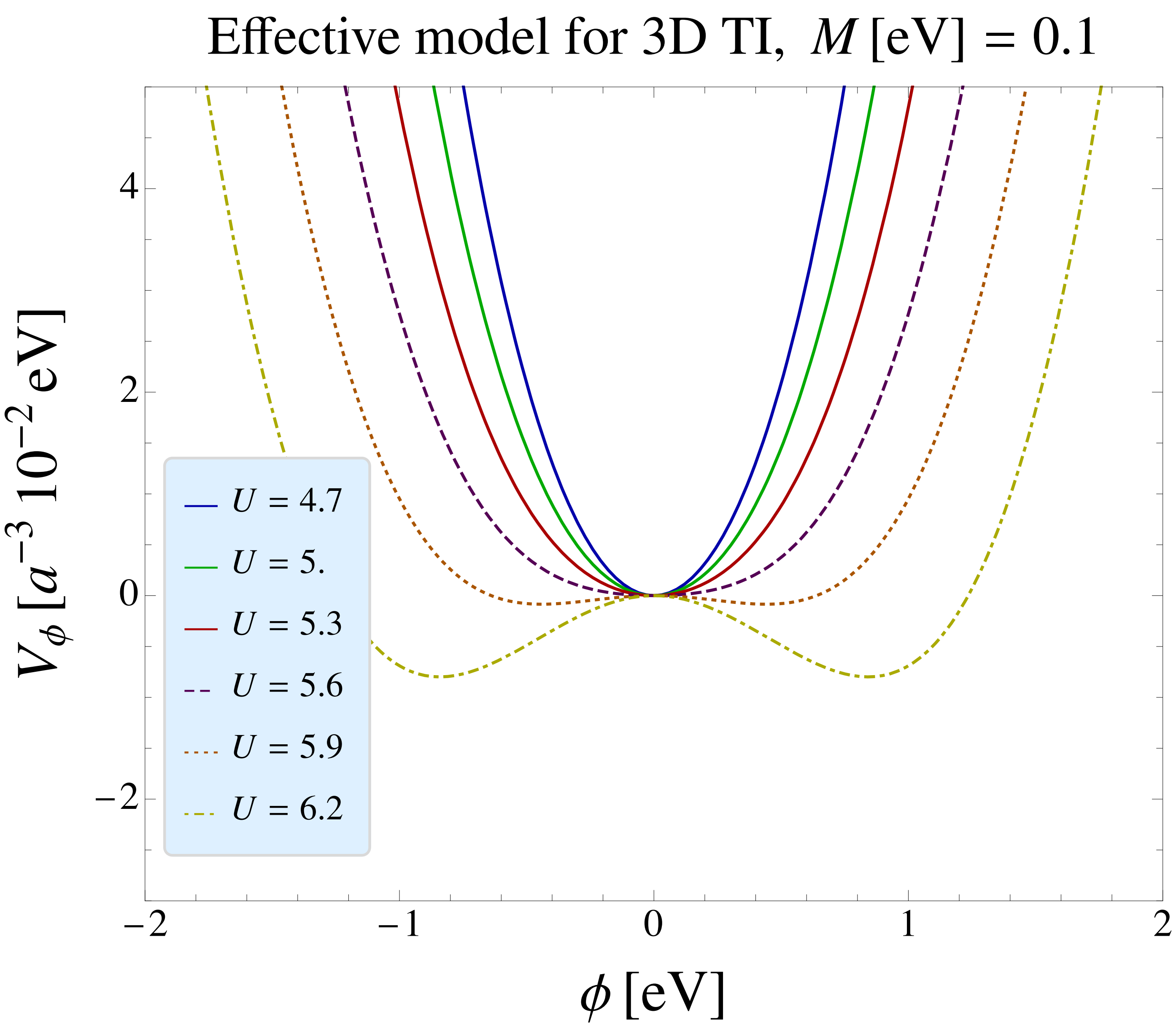
Negative potential

The mass term stabilizes the potential

$$|d_0|^2 = \sum_{a=1}^{4} |d^a|^2$$

$$M_\phi^2 = \int \frac{d^3k}{(2\pi)^3}\frac{2}{U}$$

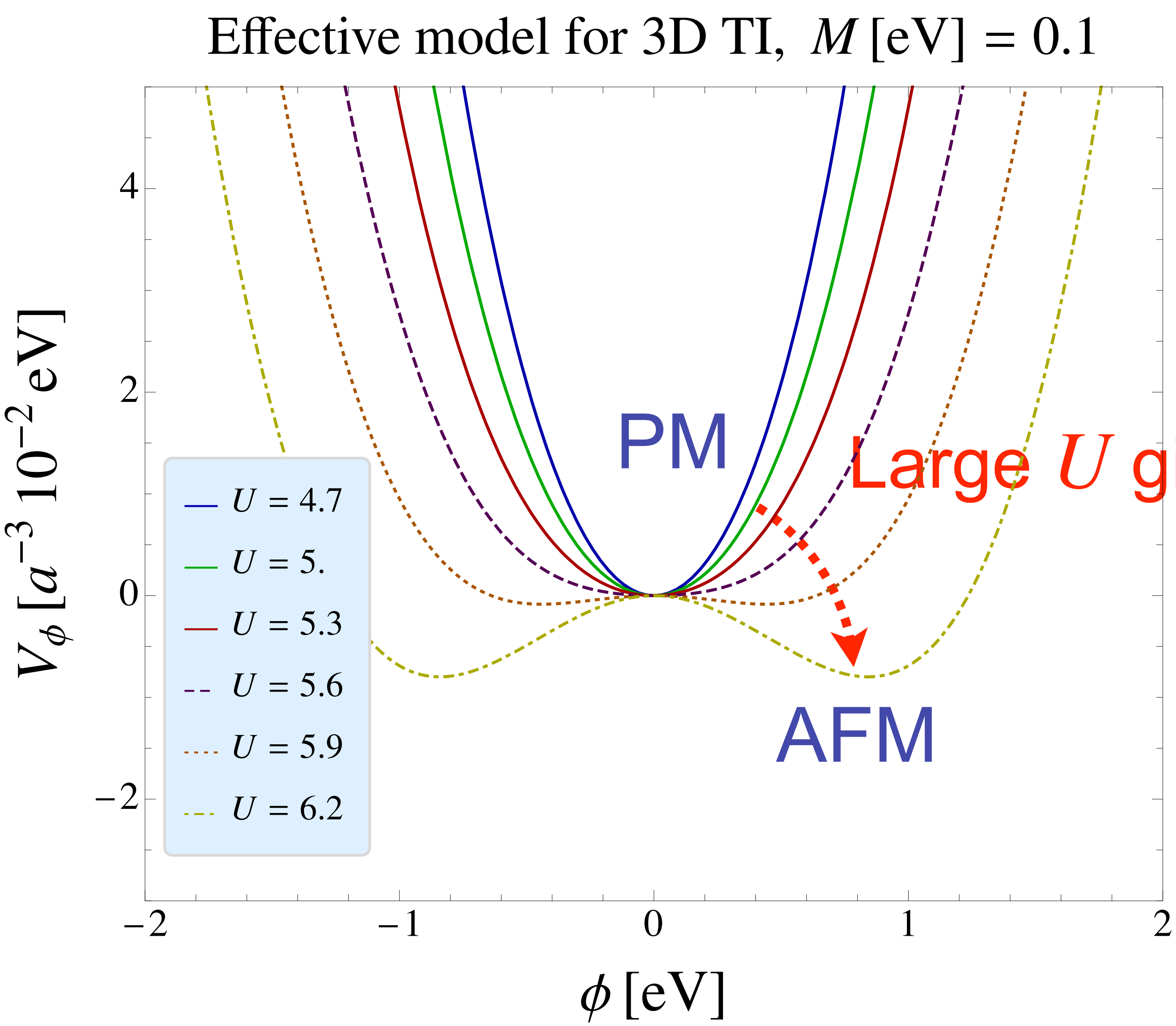
KI '21



$A_1 = A_2 = 1$

$B_1 = B_2 = -0.5$

KI ’21



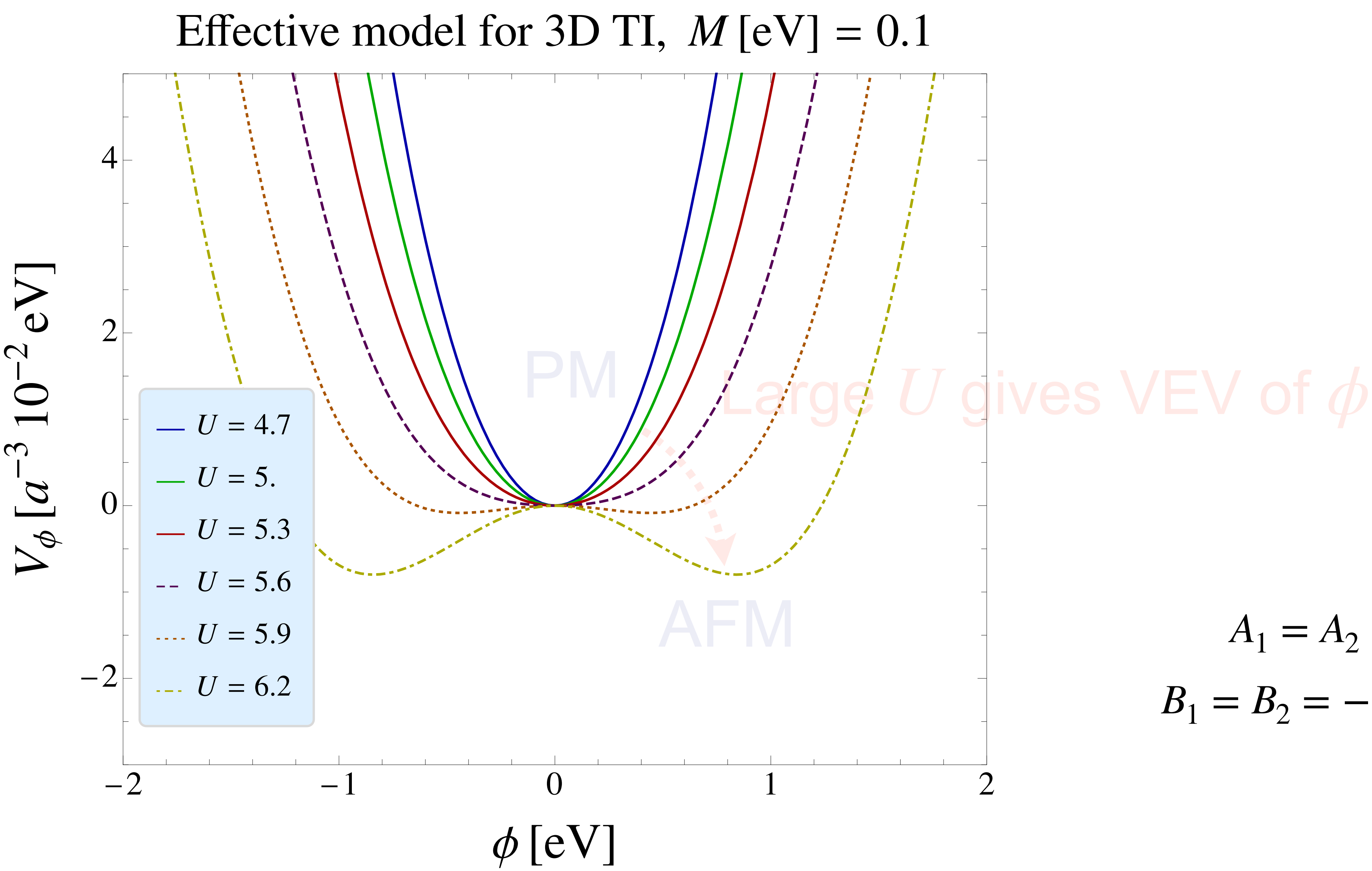
$A_1 = A_2 = 1$

$B_1 = B_2 = -0.5$

Phase transition from PM to AFM

$$\because \quad M_\phi^2 \propto 1/U$$

KI ’21



$A_1 = A_2 = 1$

$B_1 = B_2 = -0.5$

Where is axion?

θ can be computed from Hamiltonian:

- $$\theta = \frac{1}{4\pi}\int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2|d|^3}\epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

R. Li et al. '10

- Approximately given by chiral anomaly (Fujikawa method)

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- $$\theta = \frac{1}{4\pi}\int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2|d|^3}\epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

R. Li et al. '10

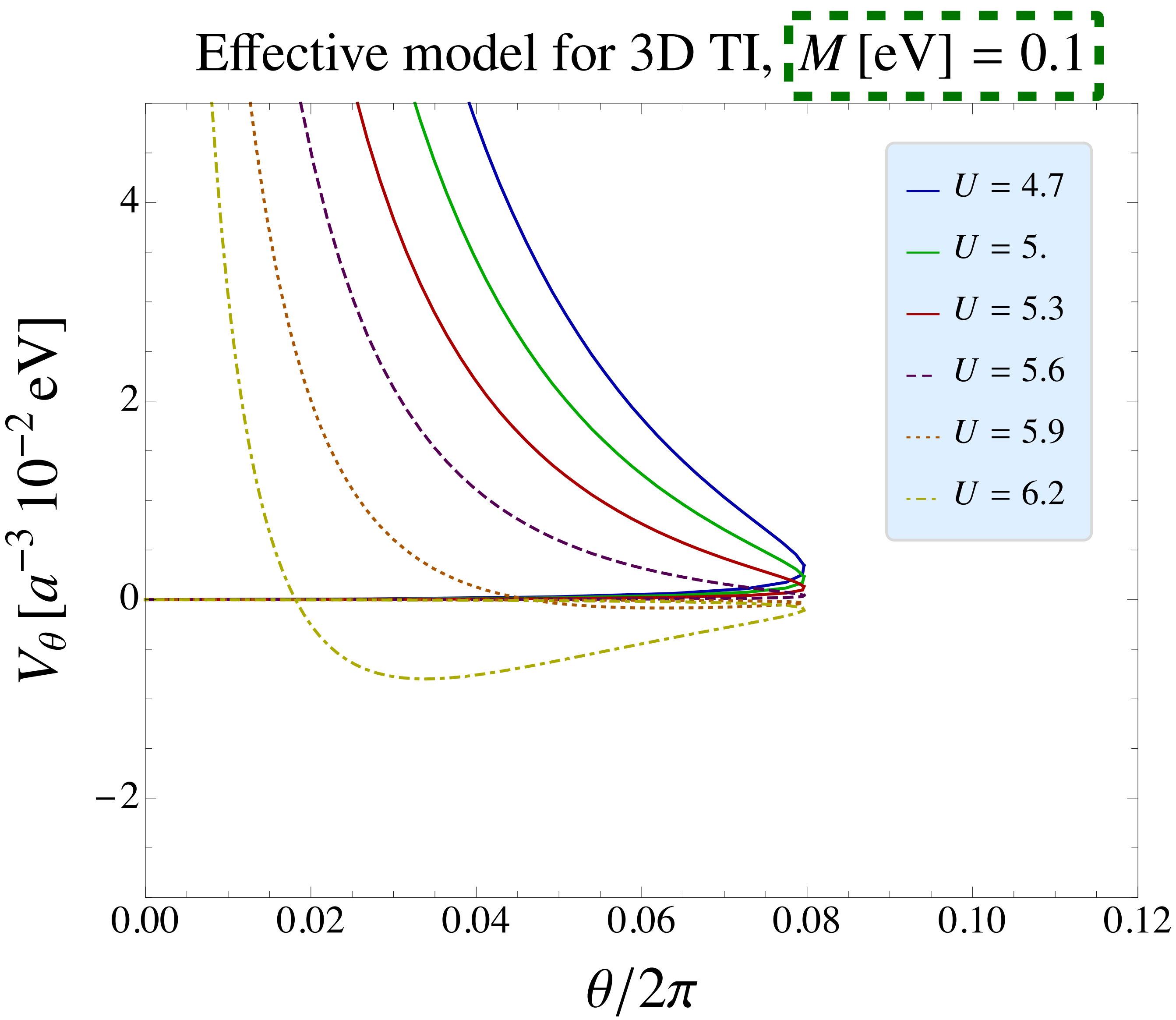
- Approximately given by chiral anomaly (Fujikawa method)

$$\phi = \phi(\theta)$$

$$V_\phi(\phi) \longrightarrow V_\theta(\theta)$$

Effective potential in terms of θ

NI phase

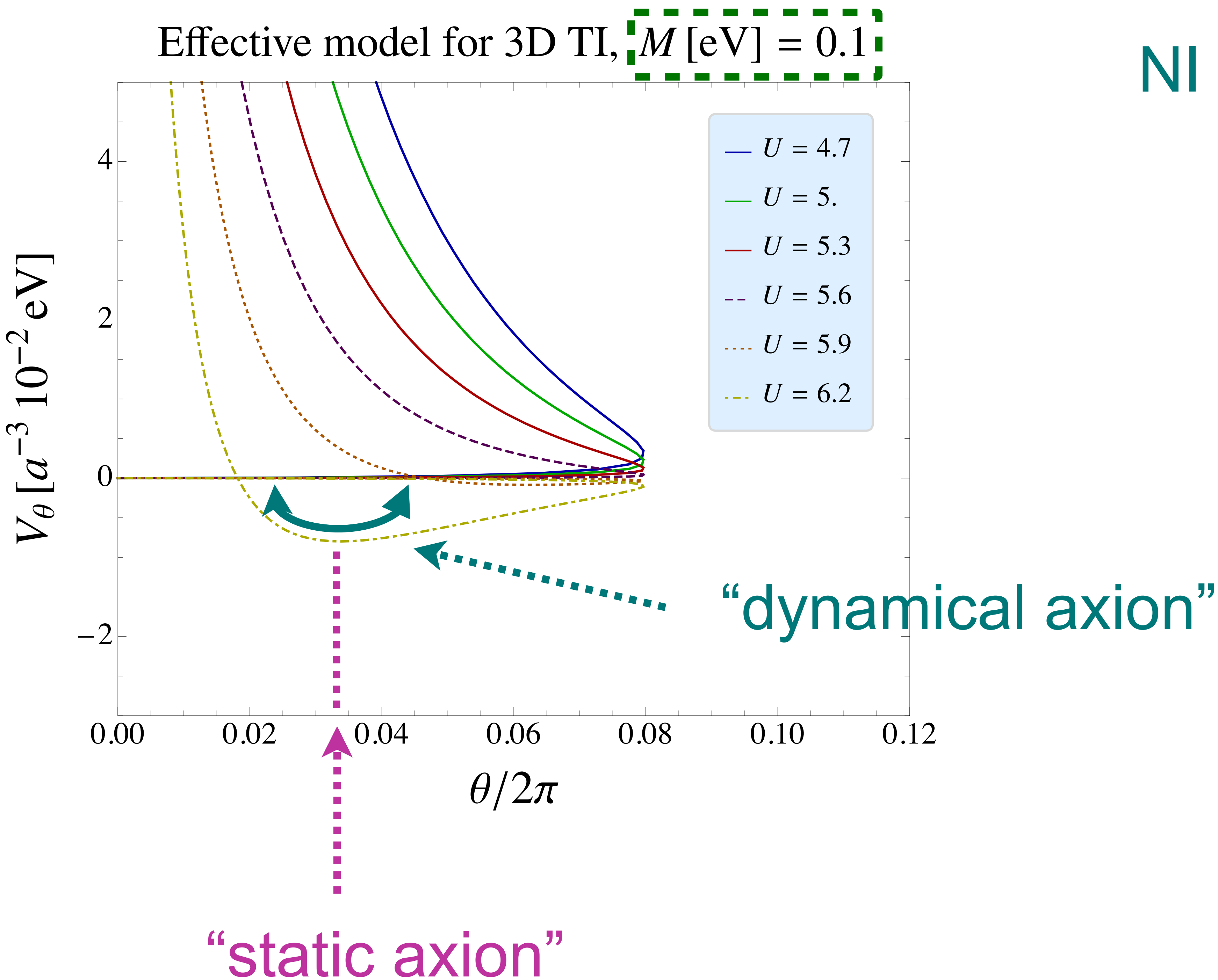


Potential minimum:

- $\theta = 0$ (small U, i.e., PM)
- $\theta \neq 0$ (large U, i.e., AFM)

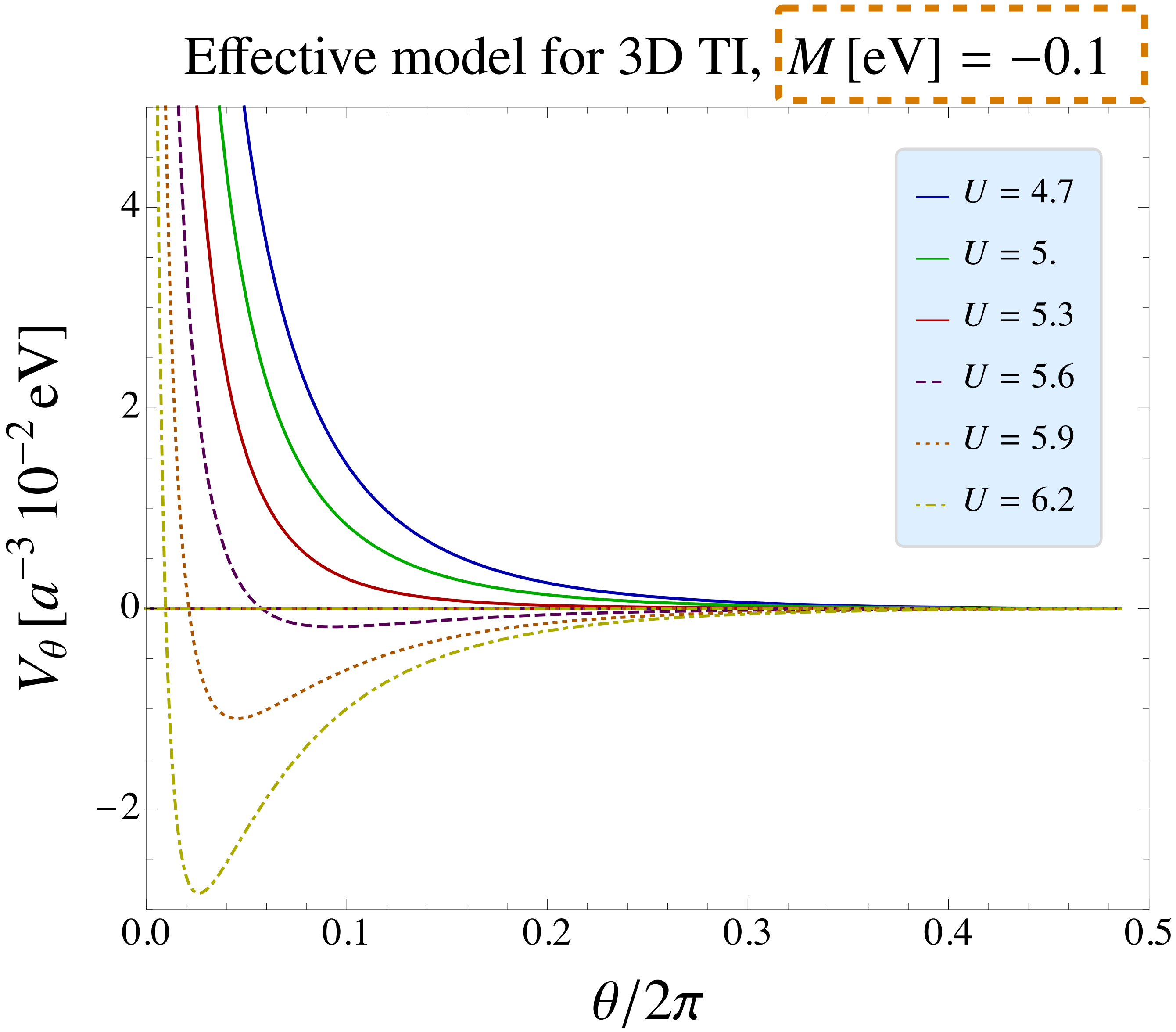
Effective potential in terms of θ

NI phase



Effective potential in terms of θ

TI phase

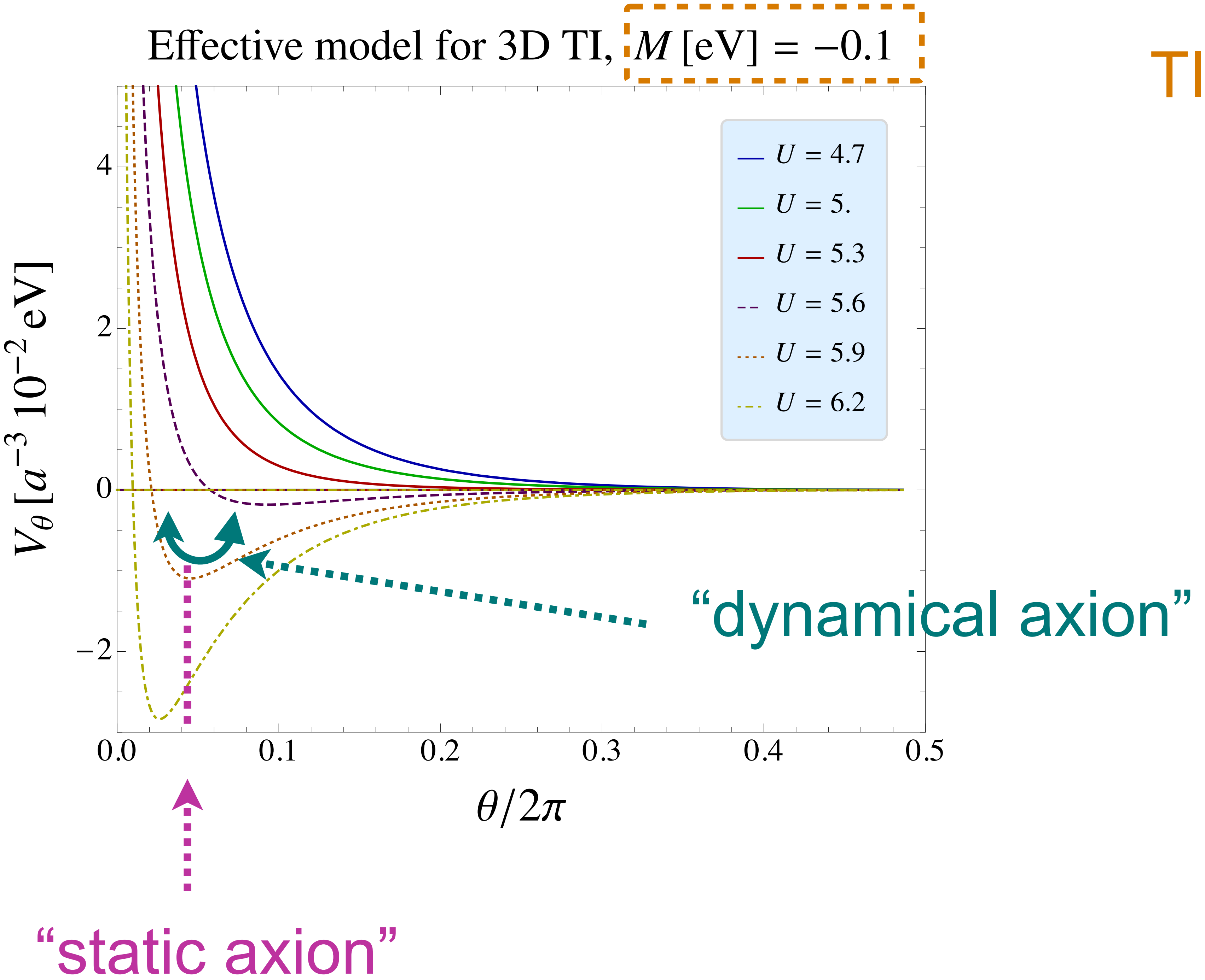


Potential minimum:

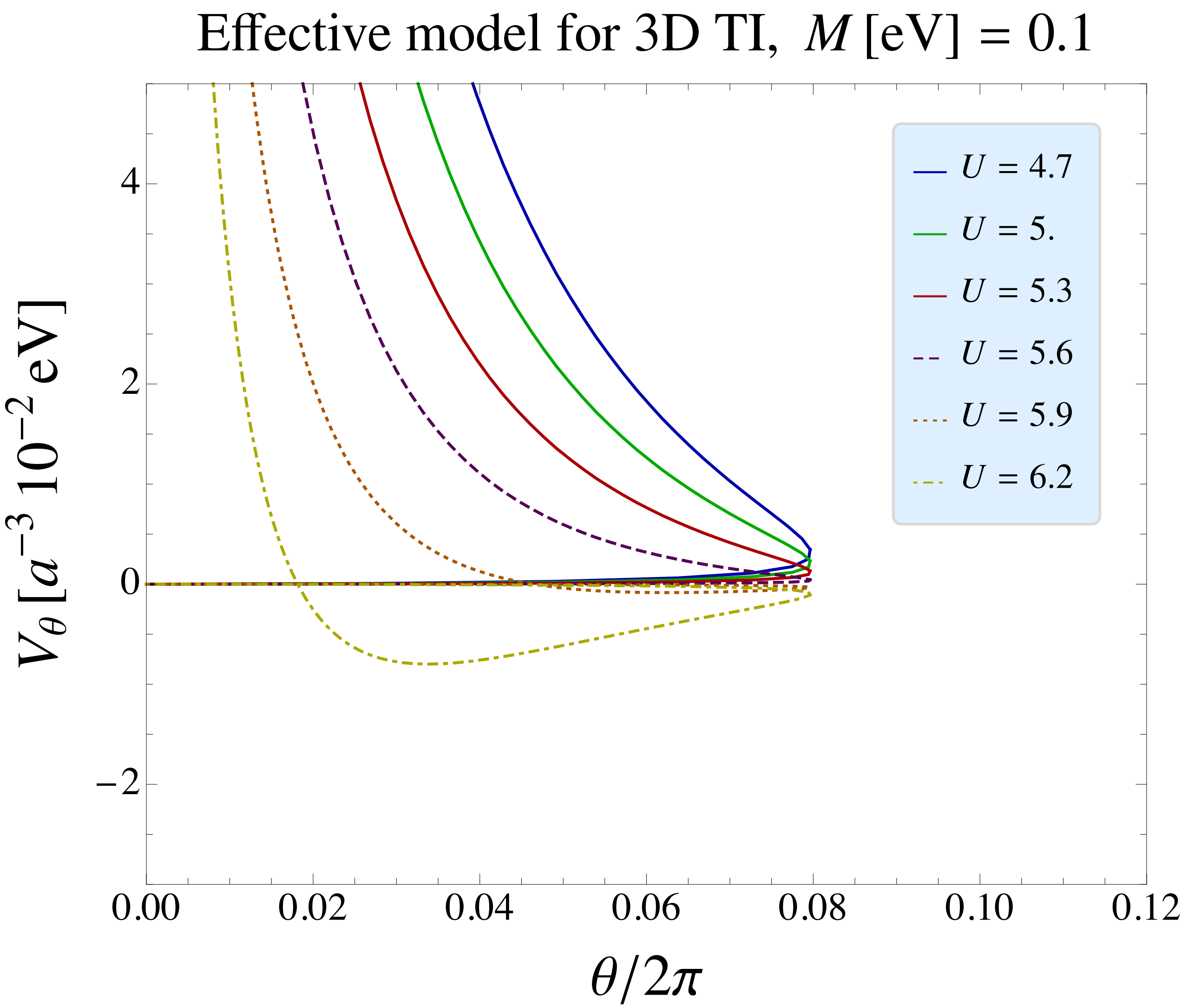
- $\theta = \pi$ (small U, i.e., PM)
- $\theta \neq 0$ (large U, i.e., AFM)

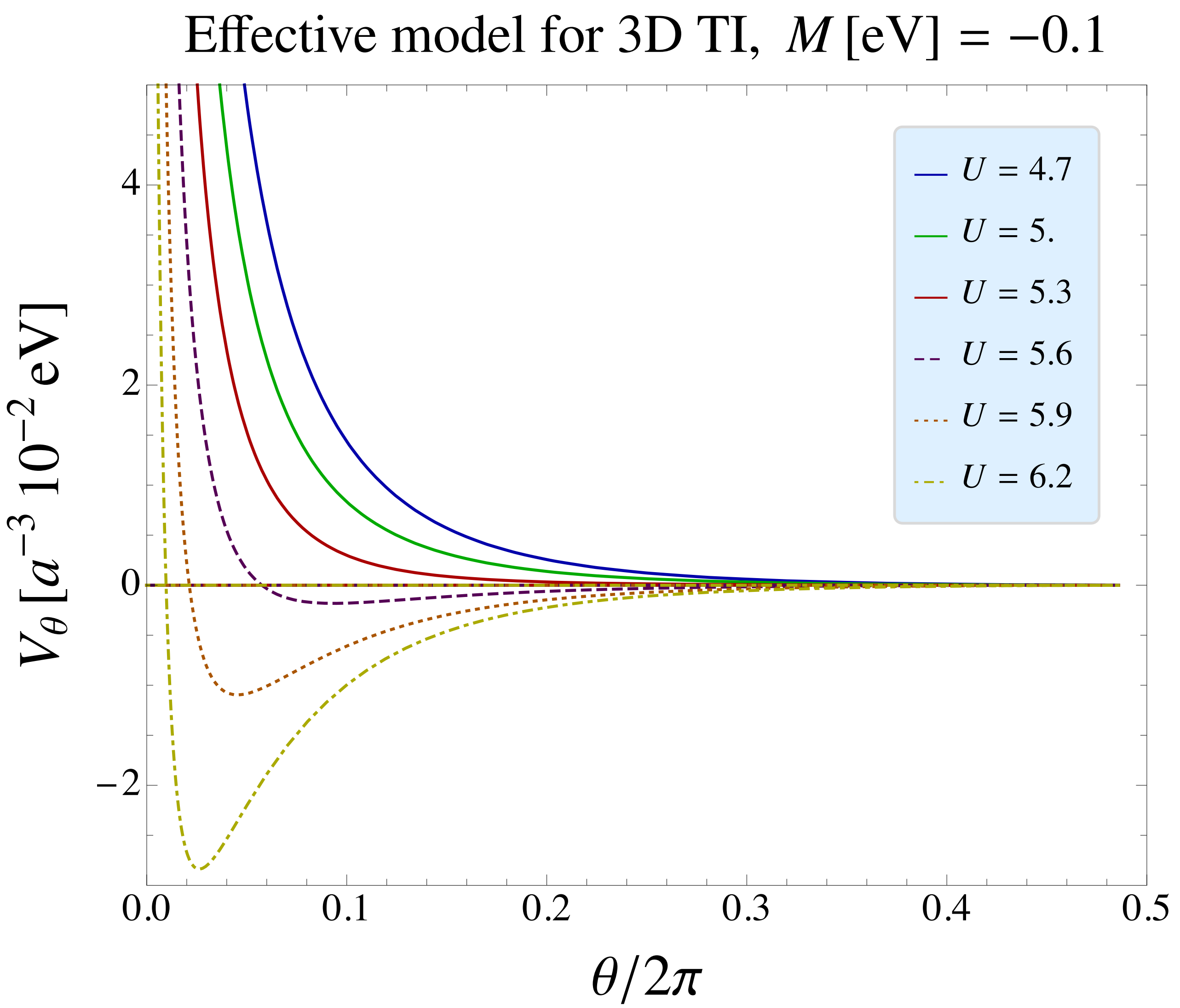
Effective potential in terms of θ

TI phase



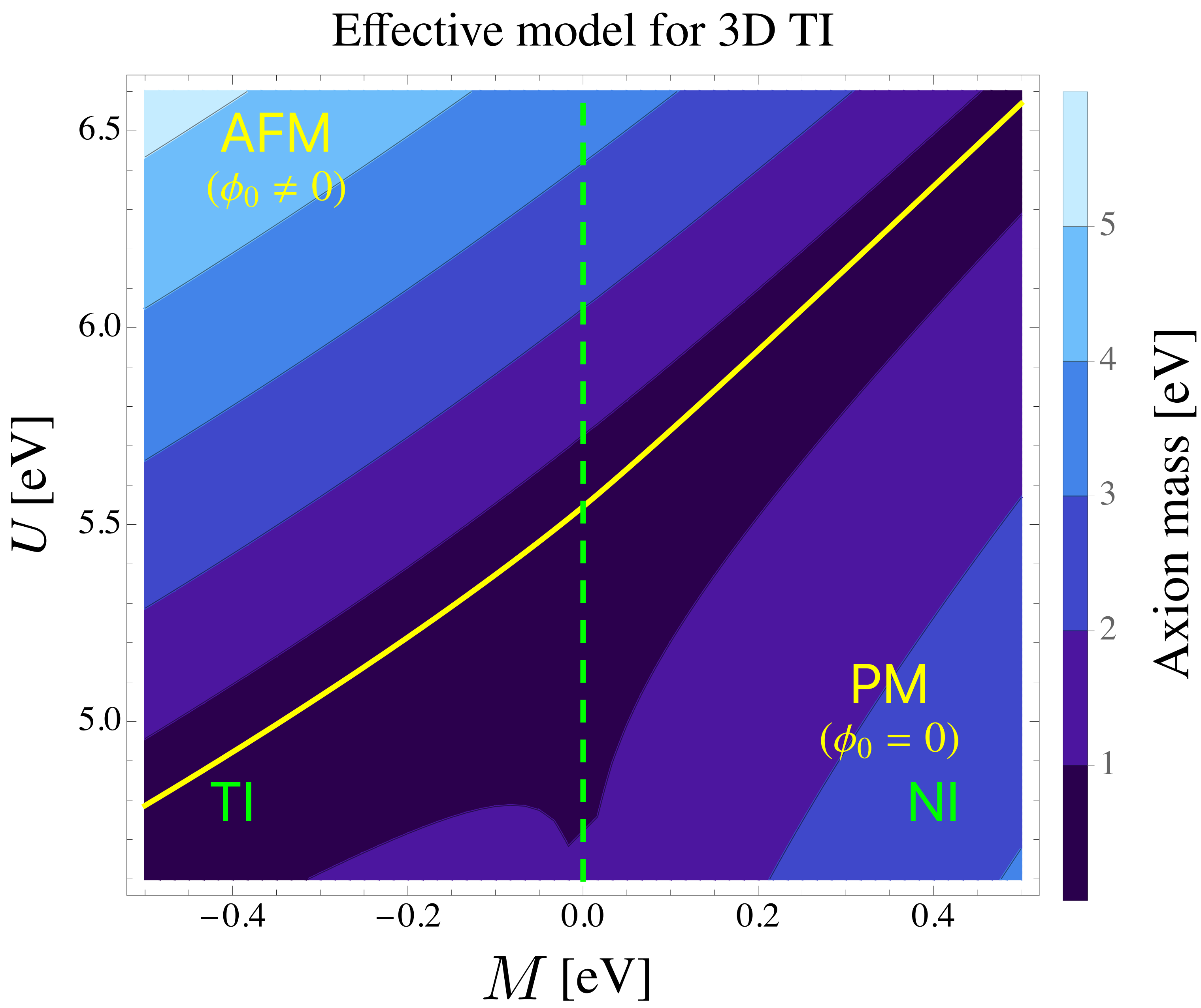
Axion in antiferromagnetic insulators





Dynamical axion exits in both TI and NI phases

Axion mass



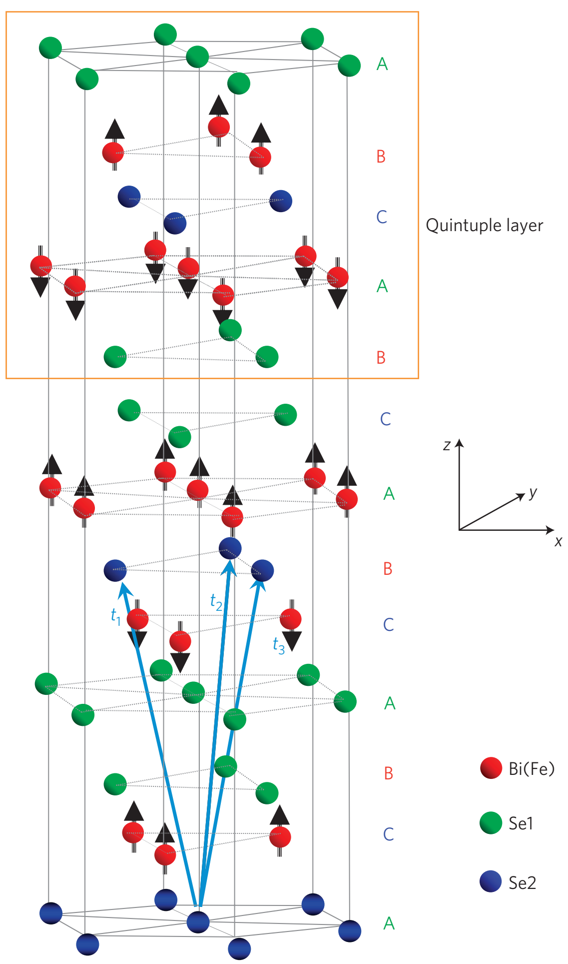
Axion mass is $\mathcal{O}(\mathrm{eV})$

Axion is predicted in topological magnetic insulators

ARTICLES
PUBLISHED ONLINE: 7 MARCH 2010 | DOI:10.1038/NPHYS1534
nature physics

Dynamical axion field in topological magnetic insulators

Rundong Li[1], Jing Wang[1,2], Xiao-Liang Qi[1] and Shou-Cheng Zhang[1]*



Bi_2Se_3

$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\text{tot}} &= \mathcal{S}_{\text{Maxwell}} + \mathcal{S}_{\text{topo}} + \mathcal{S}_{\text{axion}} \\ &= \frac{1}{8\pi}\int d^3x dt \left(\epsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\alpha}{4\pi^2}\int d^3x dt (\theta_0 + \delta\theta)\mathbf{E}\cdot\mathbf{B} \\ &\quad + g^2 J \int d^3x dt [(\partial_t \delta\theta)^2 - (v_i \partial_i \delta\theta)^2 - m^2 \delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\text{meV})$

Axion is predicted in topological magnetic insulators

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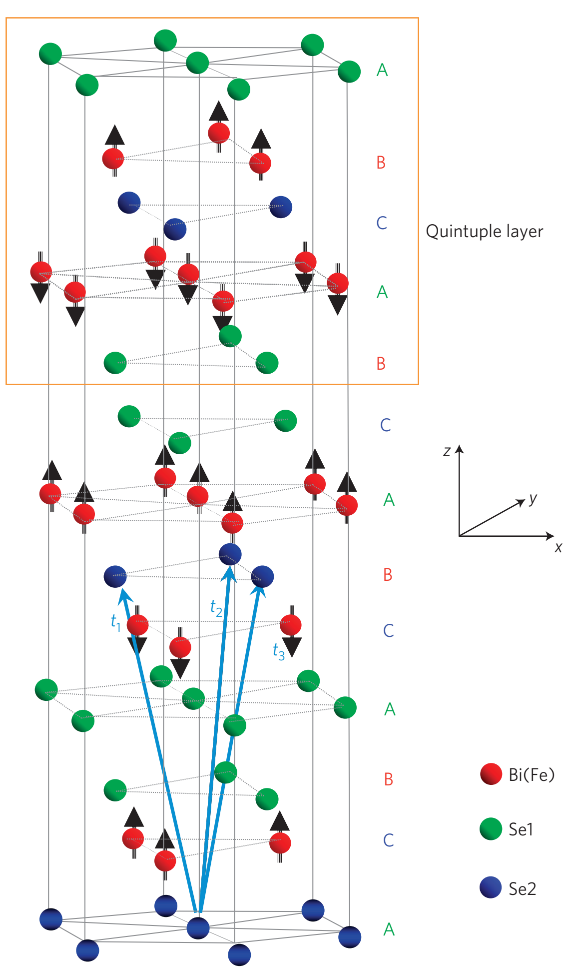


$\mathrm{Bi_2Se_3}$

$$\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\begin{aligned}\mathcal{S}_{\mathrm{tot}} &= \mathcal{S}_{\mathrm{Maxwell}} + \mathcal{S}_{\mathrm{topo}} + \mathcal{S}_{\mathrm{axion}} \\ &= \frac{1}{8\pi}\int \mathrm{d}^3x\mathrm{d}t\left(\epsilon\mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\alpha}{4\pi^2}\int \mathrm{d}^3x\mathrm{d}t(\theta_0+\delta\theta)\mathbf{E}\cdot\mathbf{B} \\ &\quad + g^2J\int \mathrm{d}^3x\mathrm{d}t[(\partial_t\delta\theta)^2 - (v_i\partial_i\delta\theta)^2 - m^2\delta\theta^2] \qquad (4)\end{aligned}$$

Axion mass ~ $\mathcal{O}(\mathrm{meV})$

?

The thing would be …

R. Li et al. ’10

- $\langle \phi \rangle \ (= m_5) = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle \phi \rangle$ is considered to be a free parameter)

- AFM order is *assumed*

The thing would be …

R. Li et al. '10

- $\langle\phi\rangle\ (= m_5)\ = 1\ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because\ m_a^2 \propto m_5^2)$

- AFM order is *assumed*

The thing would be …

R. Li et al. '10

- $\langle\phi\rangle\ (= m_5) = 1\ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

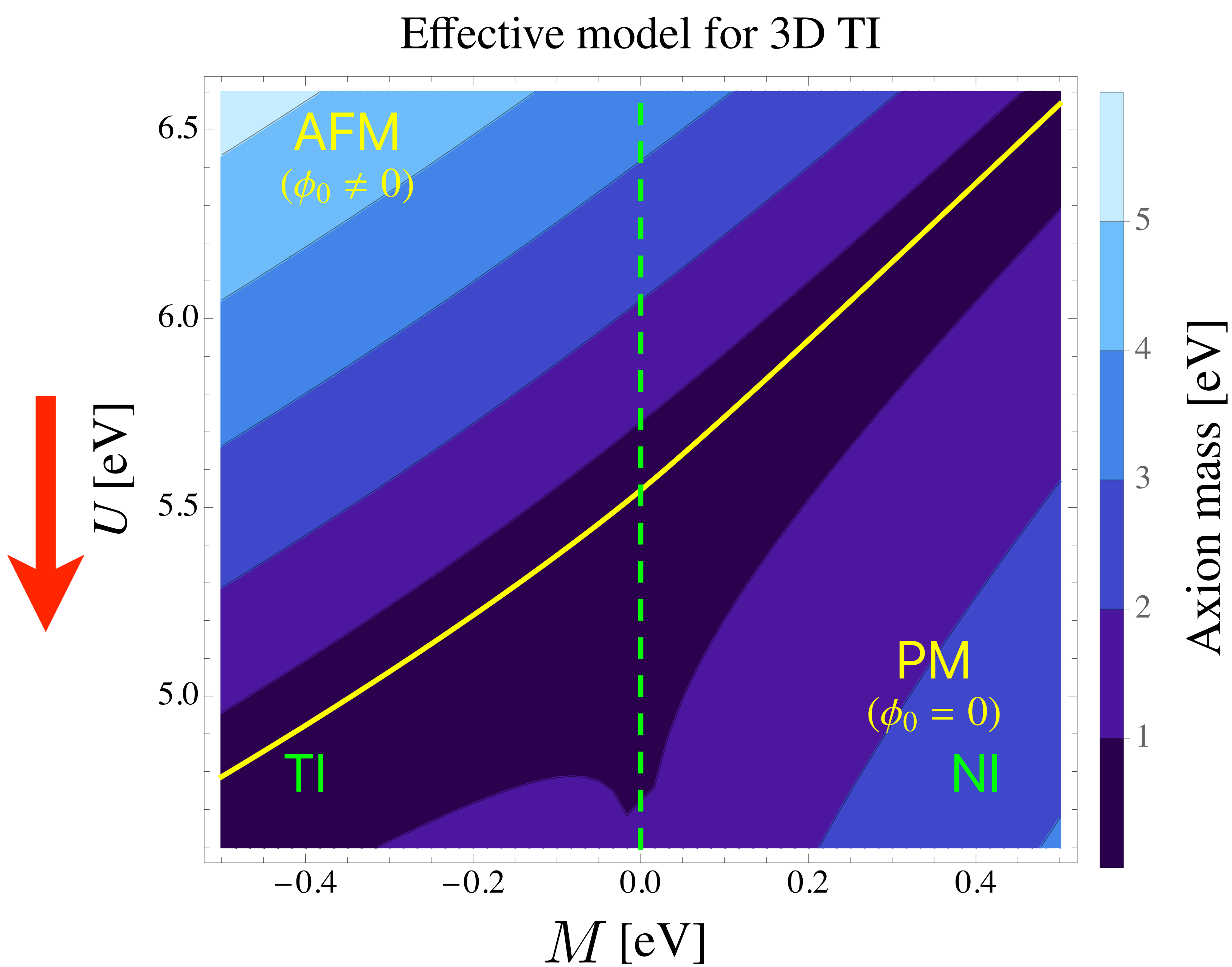
  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because\ m_a^2 \propto m_5^2)$

But this is not naively possible since

- AFM order is *assumed*

$m_5 \sim U \sim \mathrm{eV}$ (in AFM order)

Axion mass



Suppressed U → No AFM

The thing would be …

R. Li et al. '10

- $\langle\phi\rangle \ (= m_5) \ = 1 \ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $\quad (\because \ m_a^2 \propto m_5^2)$

- AFM order is *assumed*

The thing would be …

R. Li et al. '10

- $\langle\phi\rangle\ (= m_5)\ = 1\ \mathrm{meV}$ is taken

  (i.e., $\langle\phi\rangle$ is considered to be a free parameter)

  ⟶ Axion mass ~ $\mathcal{O}(\mathrm{meV})$ $(\because\ m_a^2 \propto m_5^2)$

- AFM order is *assumed*

  No AFM in TI in the first place

  ⟶ Fe-doped $\mathrm{Bi_2Se_3}$ is considered

- Fe -doped $\mathrm{Bi_2Se_3}$, $\mathrm{Bi_2Te_3}$

  "likely to be AFM" J.M. Zhang et al. '13

  (by first-principles calculation)

  → It looks unlikely to be realized …

- $\mathrm{Mn_2Bi_2Te_5}$ J. Zhang et al. '19

  "rich magnetic topological quantum states" Y. Li et al. '20

  (by first-principles calculation)

4. Conclusions and discussion

We have formulated static and dynamical axions in AFM TI consistently by using path integral

- Nonzero $\langle \phi \rangle$ is obtained from the effective potential, which gives rise to AFM order and breaks $\mathcal{T}$

- Dynamical axion appears both in TI and NI

- Axion mass is $\lesssim \mathcal{O}(\mathrm{eV})$

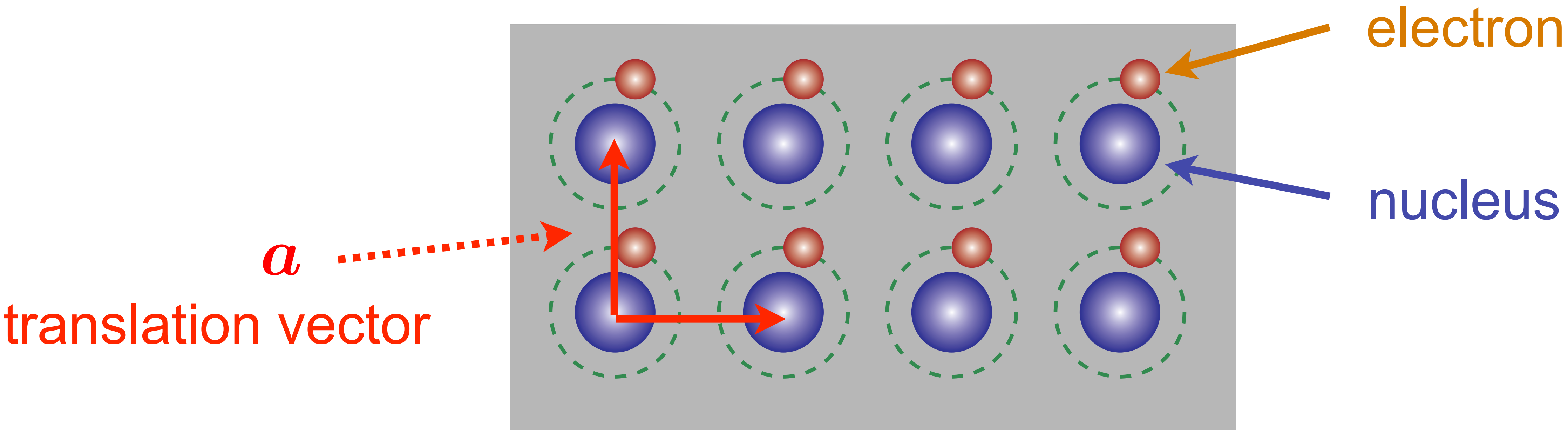
Discussion

- How do we describe axion in $\mathrm{Mn_2Bi_2Te_5}$?
- What about axion in NI ?
- Dynamical axion in ferromagnetic state or other magnetic states?

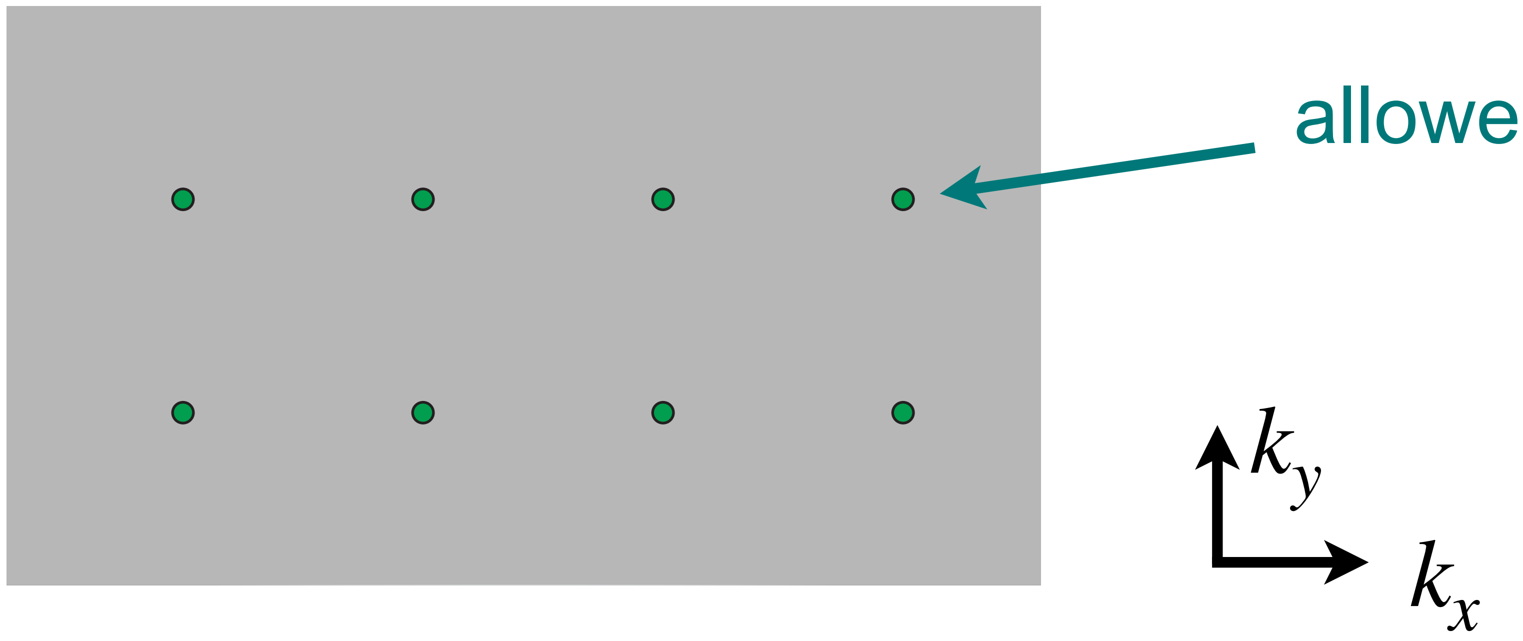
Backups

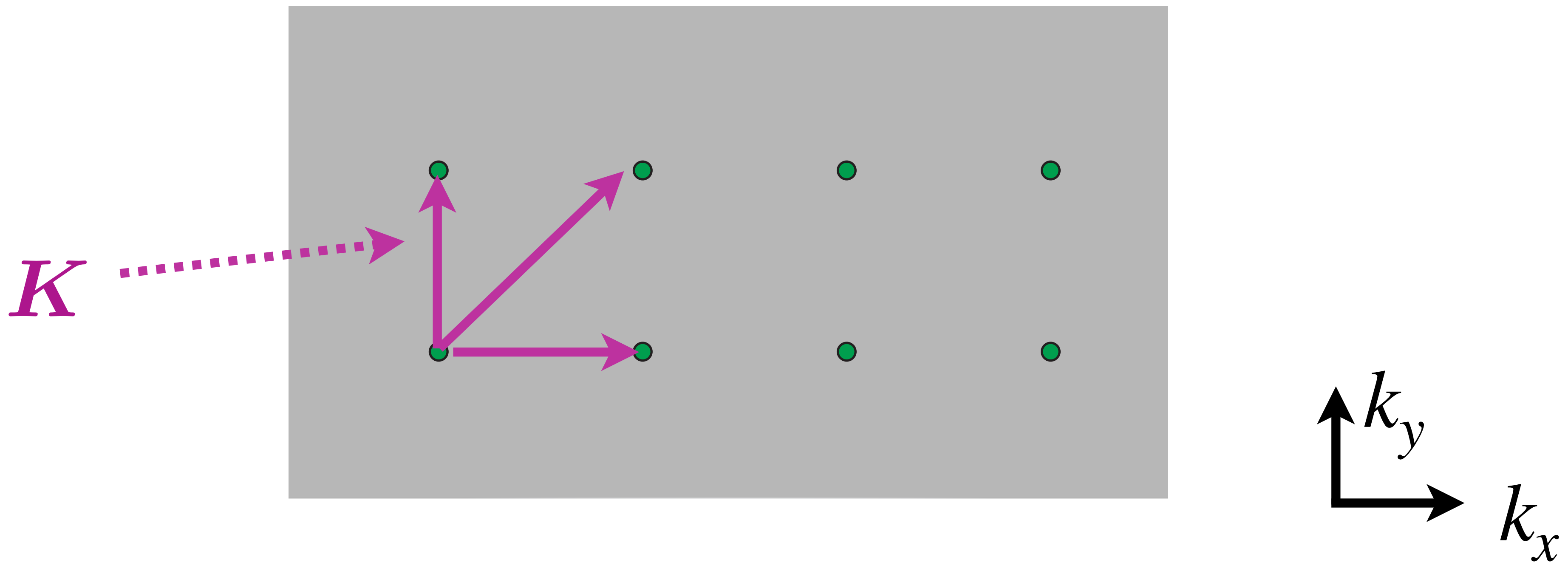
Basics

- Wavefunction of electrons is periodic (due to the crystal structure of the material)



- Consequently there is periodicity in the wavenumber space





- Periodicity $\boldsymbol{x} \to \boldsymbol{x} + \boldsymbol{a}$ corresponds to $\boldsymbol{k} \to \boldsymbol{k} + \boldsymbol{K}$

  ($\boldsymbol{K}$: reciprocal lattice vector)

- It is enough to consider region, $|\boldsymbol{k}| \lesssim |\boldsymbol{K}/2|$ (1st Brillouin zone)

- The wavefunction of the electrons is given by

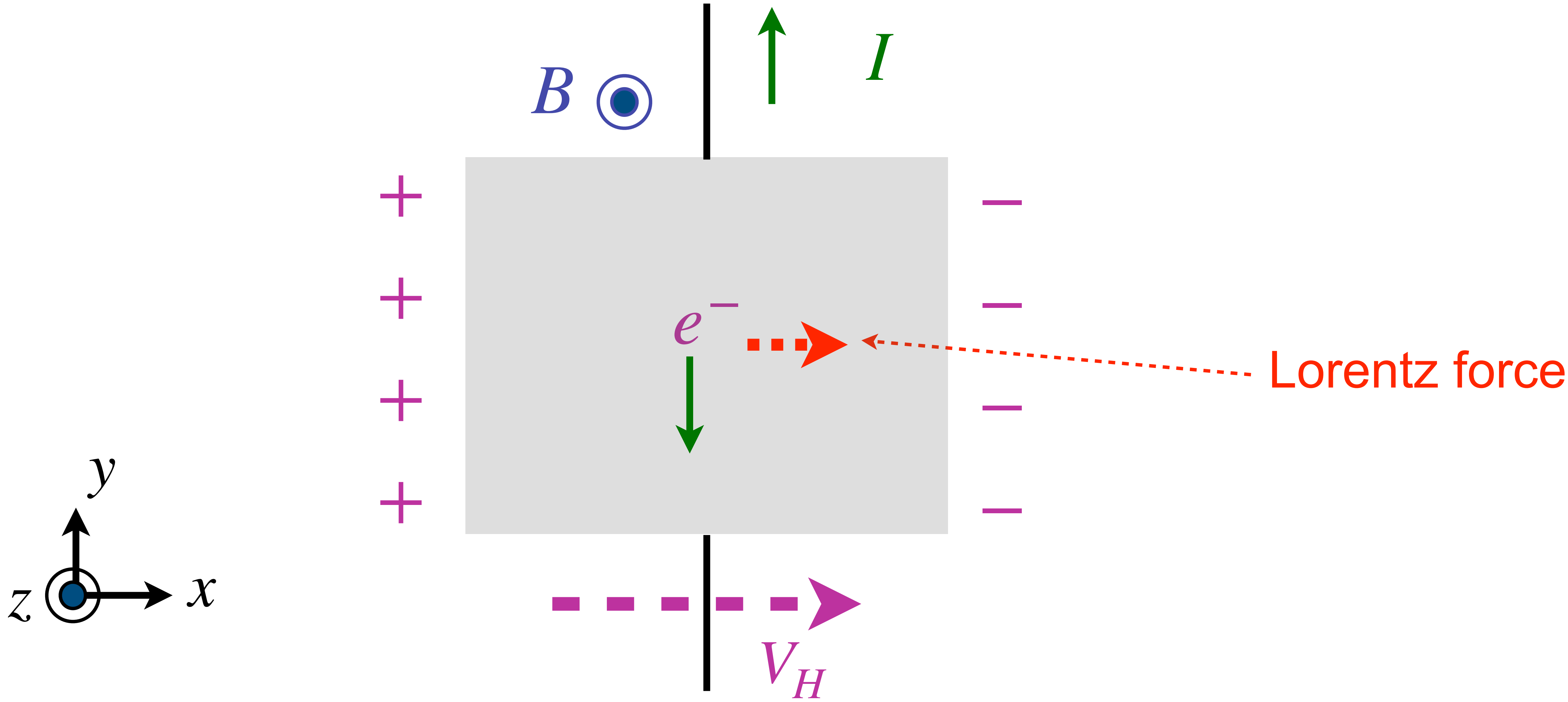
  $\psi(\boldsymbol{x}) = u_{\boldsymbol{k}}(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$ where $u_{\boldsymbol{k}}(\boldsymbol{x} + \boldsymbol{a}) = u_{\boldsymbol{k}}(\boldsymbol{x})$

  (Bloch's theorem)

  Bloch function (state)

Hall effect

e.g., semiconductor



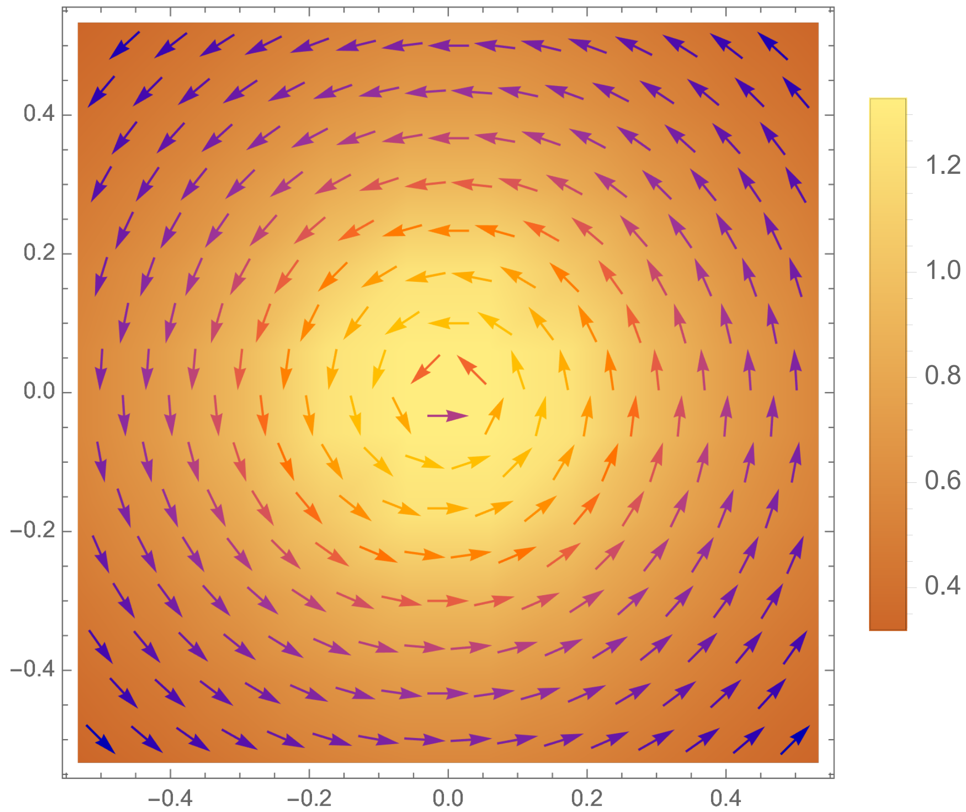
Electromotive force V_H is induced in the direction perpendicular to both the electric current and magnetic field

(In metal, V_H is too small to observe)

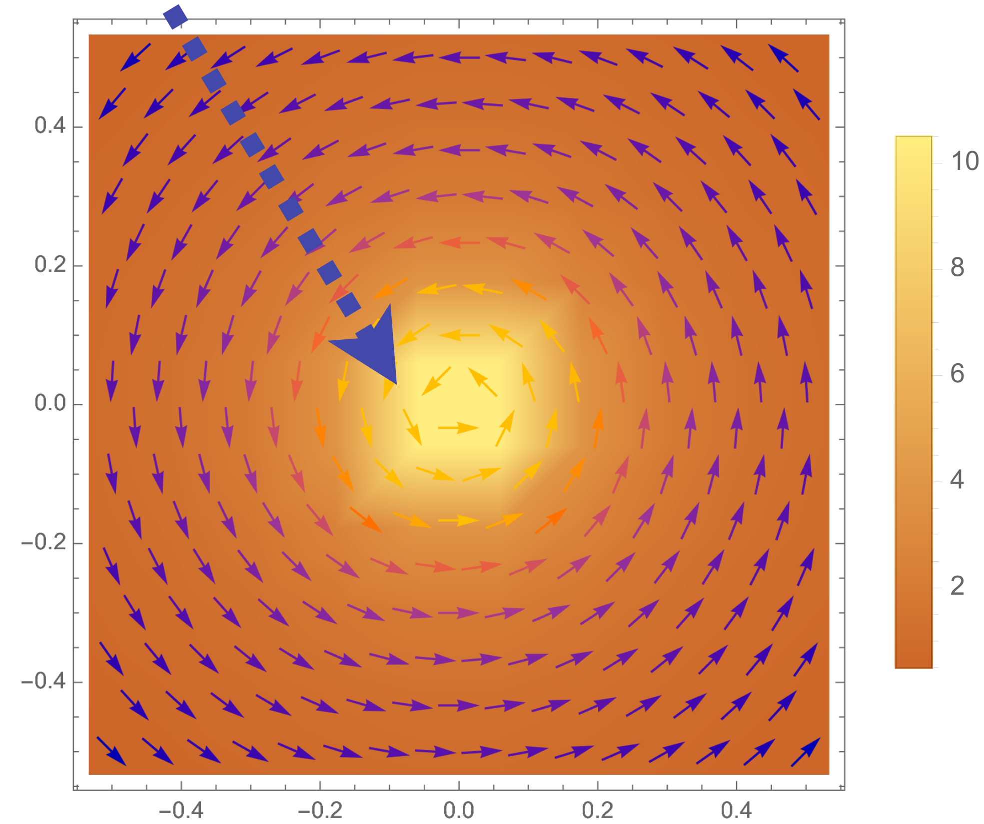
$\boldsymbol{a}(\boldsymbol{k})$

Singular at $\boldsymbol{k} = 0$

$m = 0.1$



$m = -0.1$

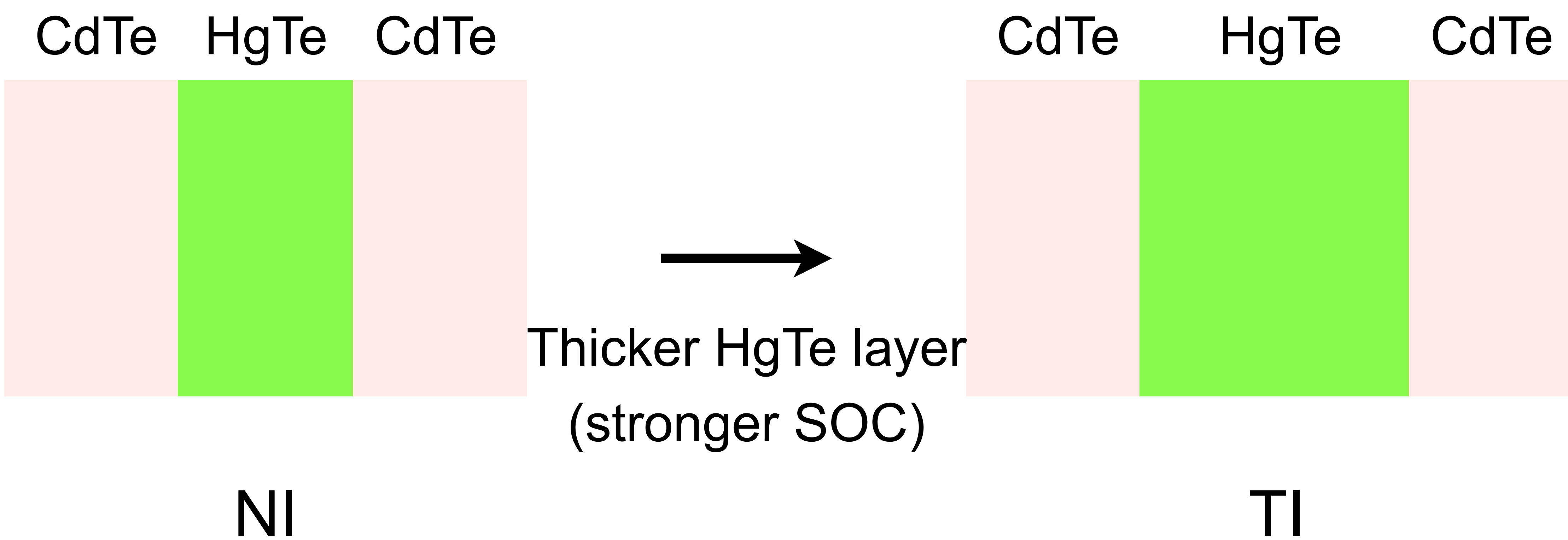


→ Integer ν

Note: ν is half-integer if the dispersion relation is Dirac type

Example of 2D TI: HgTe/(Hg , Cd)

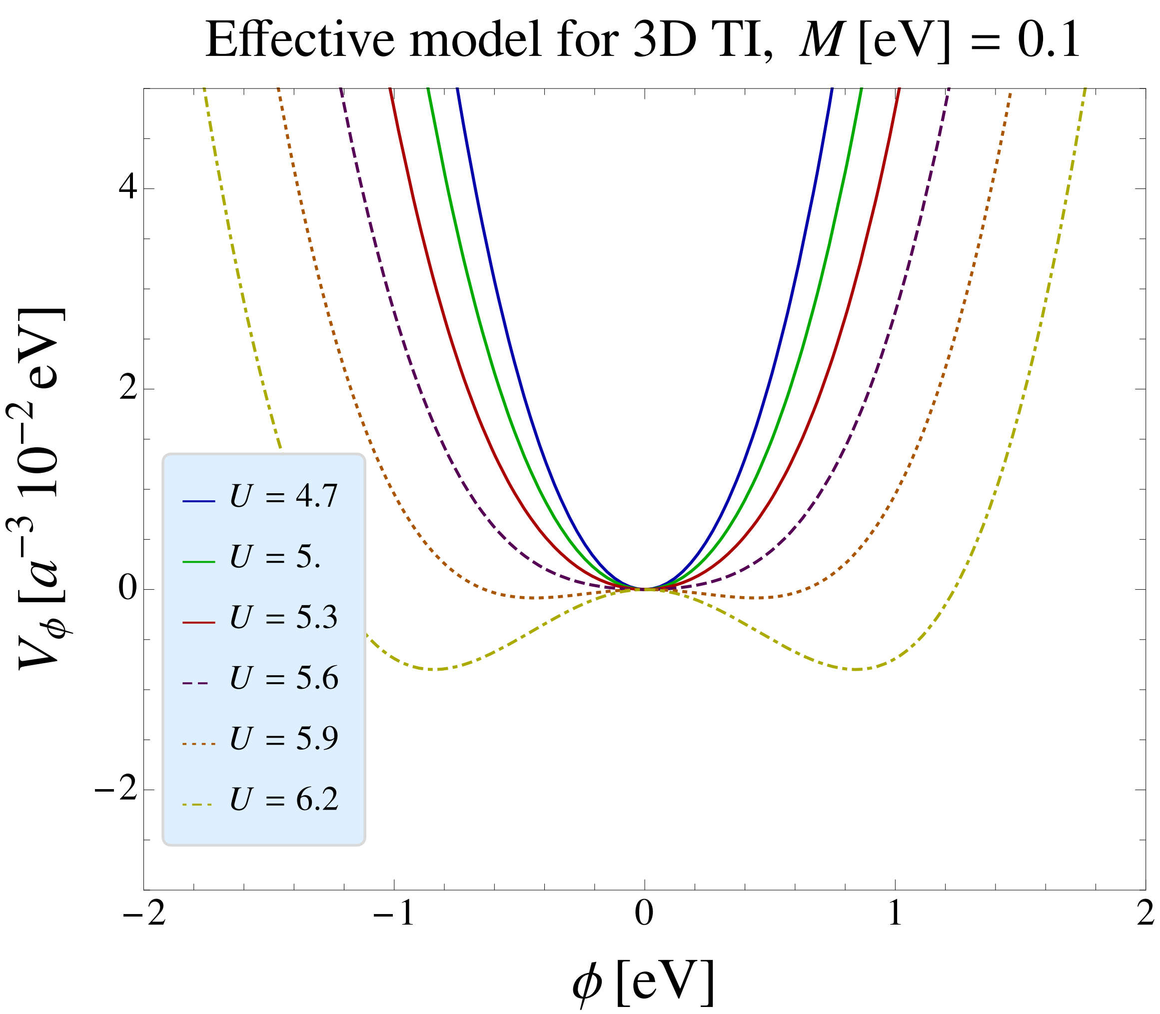
König et al. ’07

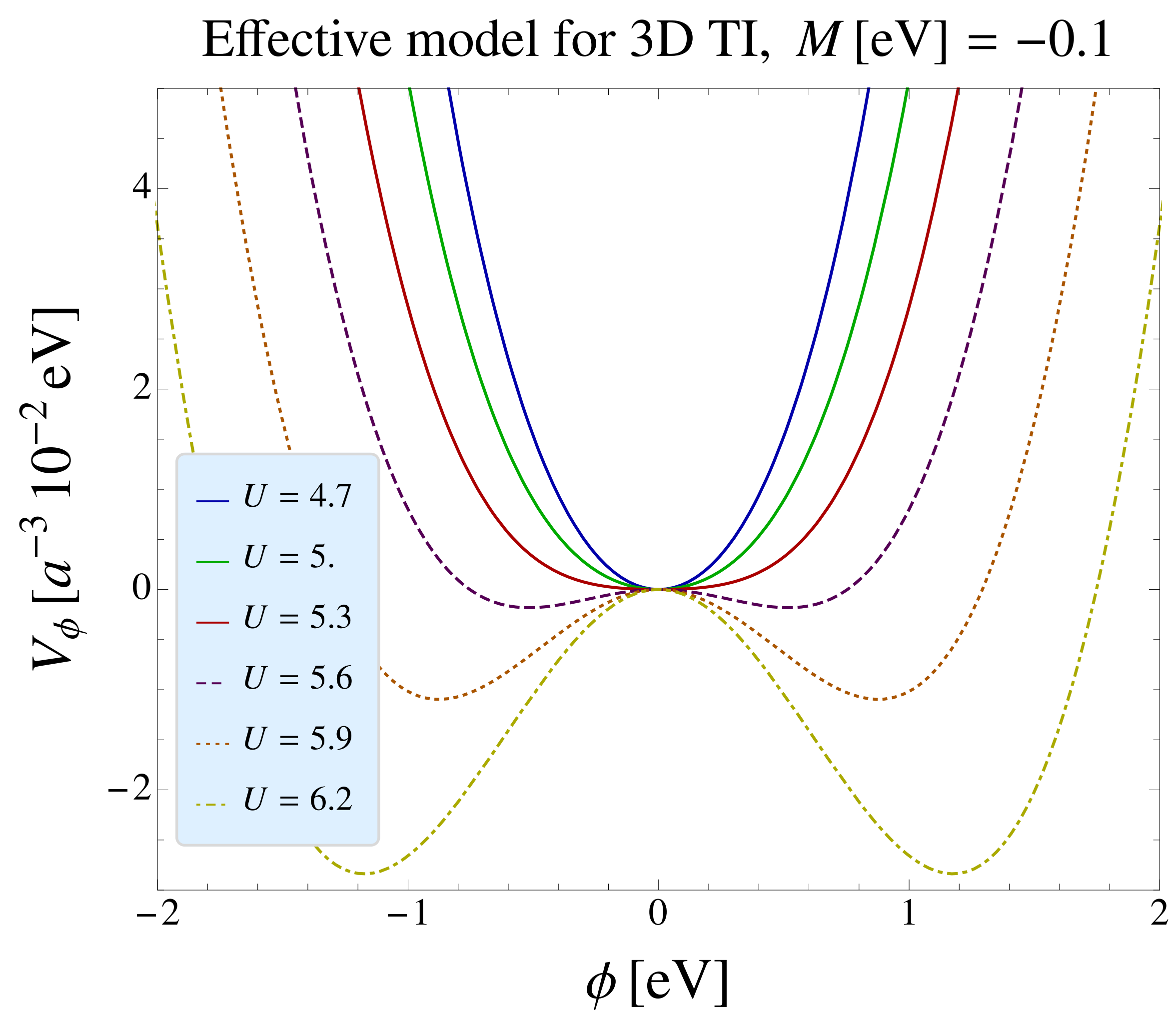


Band inversion happens in the energy band of HgTe

M dependence

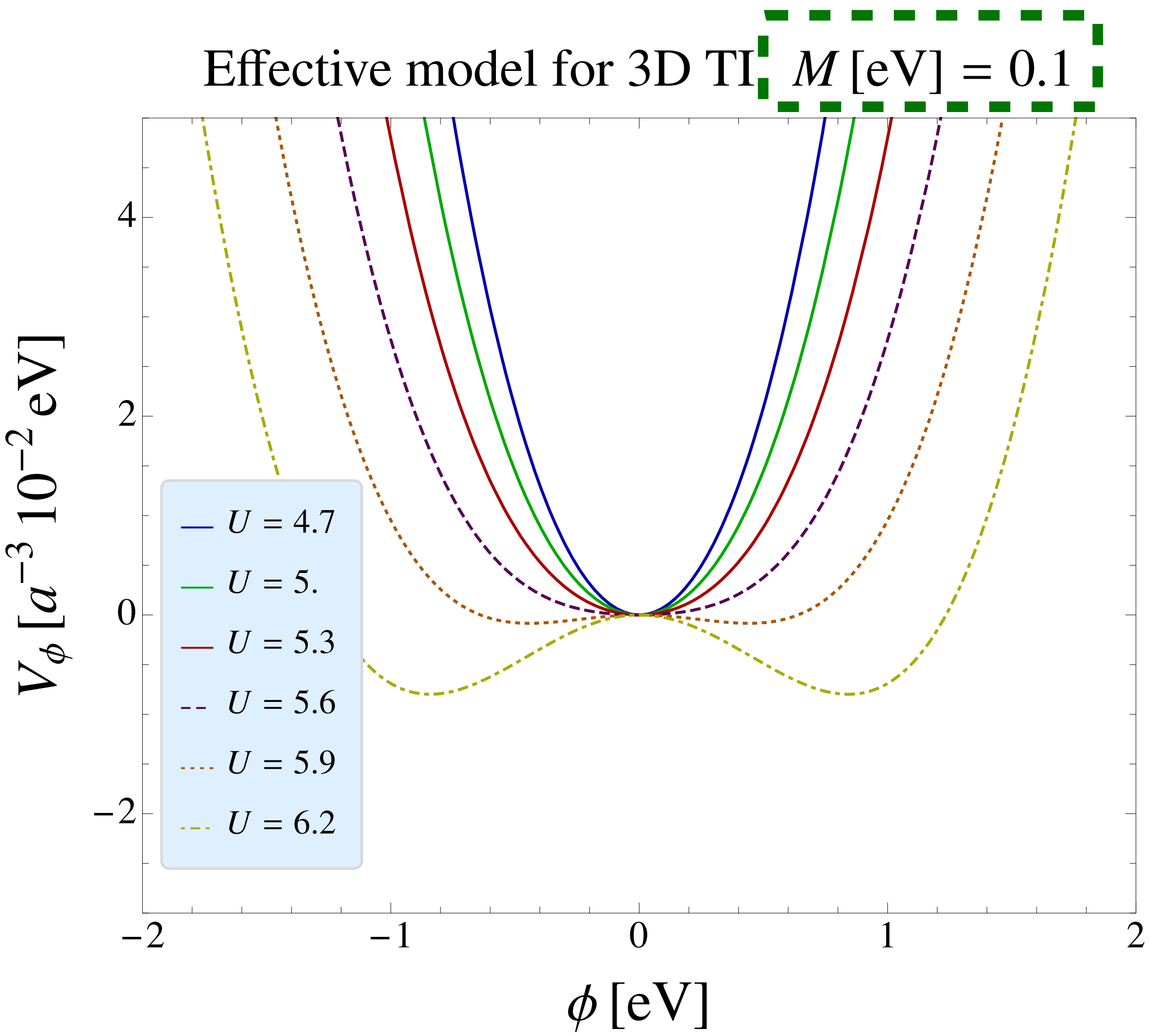
KI '21

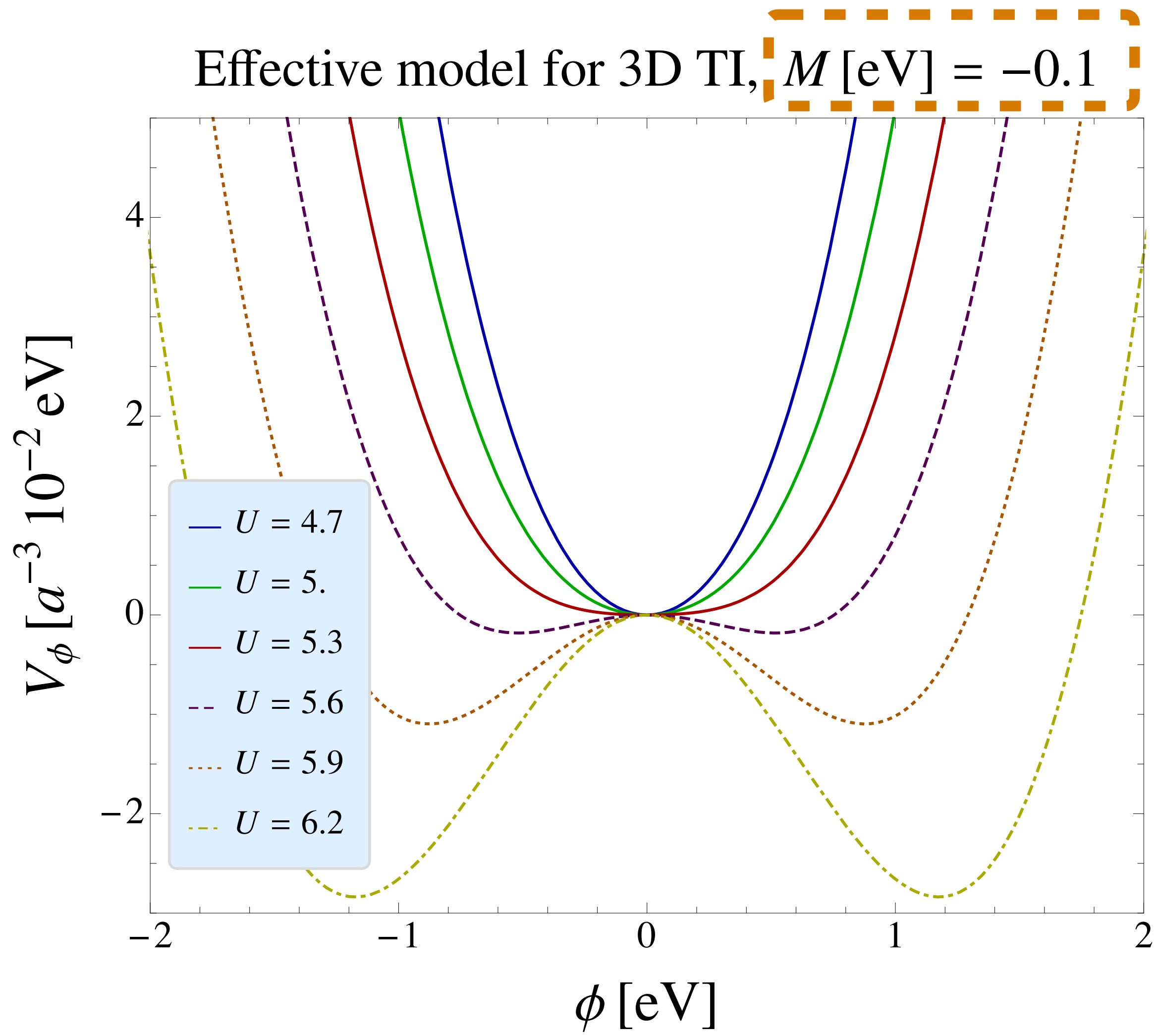




M dependence

KI '21





NI

TI

The difference between TI and NI is not clear

Recall ME effect in TI is described by

$$\mathcal{L}_\theta = -\frac{\alpha}{4\pi}\int d^4x\ \theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$ with $$\theta = \pm\pi$$

- $\theta = \pm\pi$ for TI
- $\theta = 0$ for NI

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- $\theta = \pm\pi$ for TI
- $\theta = 0$ for NI

→ θ is the order parameter of phase transition between TI and NI

We need potential for θ

θ can be computed from Hamiltonian:

- $$\theta = \frac{1}{4\pi}\int d^3k \frac{2|d| + d^4}{(|d| + d^4)^2|d|^3}\epsilon^{ijkl} d^i \partial_{k_x} d^j \partial_{k_y} d^k \partial_{k_z} d^l$$

R. Li et al. '10

- Approximately given by chiral anomaly (Fujikawa method)

Derivation as chiral anomaly

$$H(\boldsymbol{k}) = \sum_{a=1}^{5} d^a(\boldsymbol{k})\Gamma^a$$

$$(d^1,\, d^2,\, d^3,\, d^4,\, d^5) = (A_2 \sin k_x,\, A_2 \sin k_y,\, A_1 \sin k_z,\, \mathcal{M}(\boldsymbol{k}),\, \phi)$$

$$\mathcal{M}(\boldsymbol{k}) = M - 2B_1 - 4B_2 + 2B_1 \cos k_z + 2B_2(\cos k_x + \cos k_y)$$

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- expand around $\boldsymbol{k} = 0$
- redefine $\boldsymbol{k}$

$$H(\boldsymbol{k}) = k_x\Gamma^1 + k_y\Gamma^2 + k_y\Gamma^3 + M\Gamma^4 + \phi\Gamma^5$$

"Dirac model"

$$H(\boldsymbol{k}) = k_x\Gamma^1 + k_y\Gamma^2 + k_y\Gamma^3 + M\Gamma^4 + \phi\Gamma^5$$

Unitary transformation of the basis

$$\tilde{U}H(\boldsymbol{k})\tilde{U}^\dagger = \beta(\boldsymbol{\gamma}\cdot\boldsymbol{k} + M + \phi\gamma_5)$$

$$S = \int d^4x\ \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - M - i\phi\gamma_5]\psi$$

$\Gamma^5\phi$ reduces to $i\gamma^5\phi$

$i\gamma^5\phi$ term can be rotated away, which gives rise to θ term:

$$S_\Theta = -\frac{\alpha}{4\pi}\int \mathrm{d}^4x\,\Theta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Theta = \frac{\pi}{2}[1-\mathrm{sgn}(M)]\mathrm{sgn}(\phi) + \tan^{-1}\frac{\phi}{M}$$

it is consistent with

$$\theta = \frac{1}{4\pi}\int d^3k \frac{2|d|+d^4}{(|d|+d^4)^2|d|^3}\epsilon^{ijkl}d^i\partial_{k_x}d^j\partial_{k_y}d^k\partial_{k_z}d^l$$

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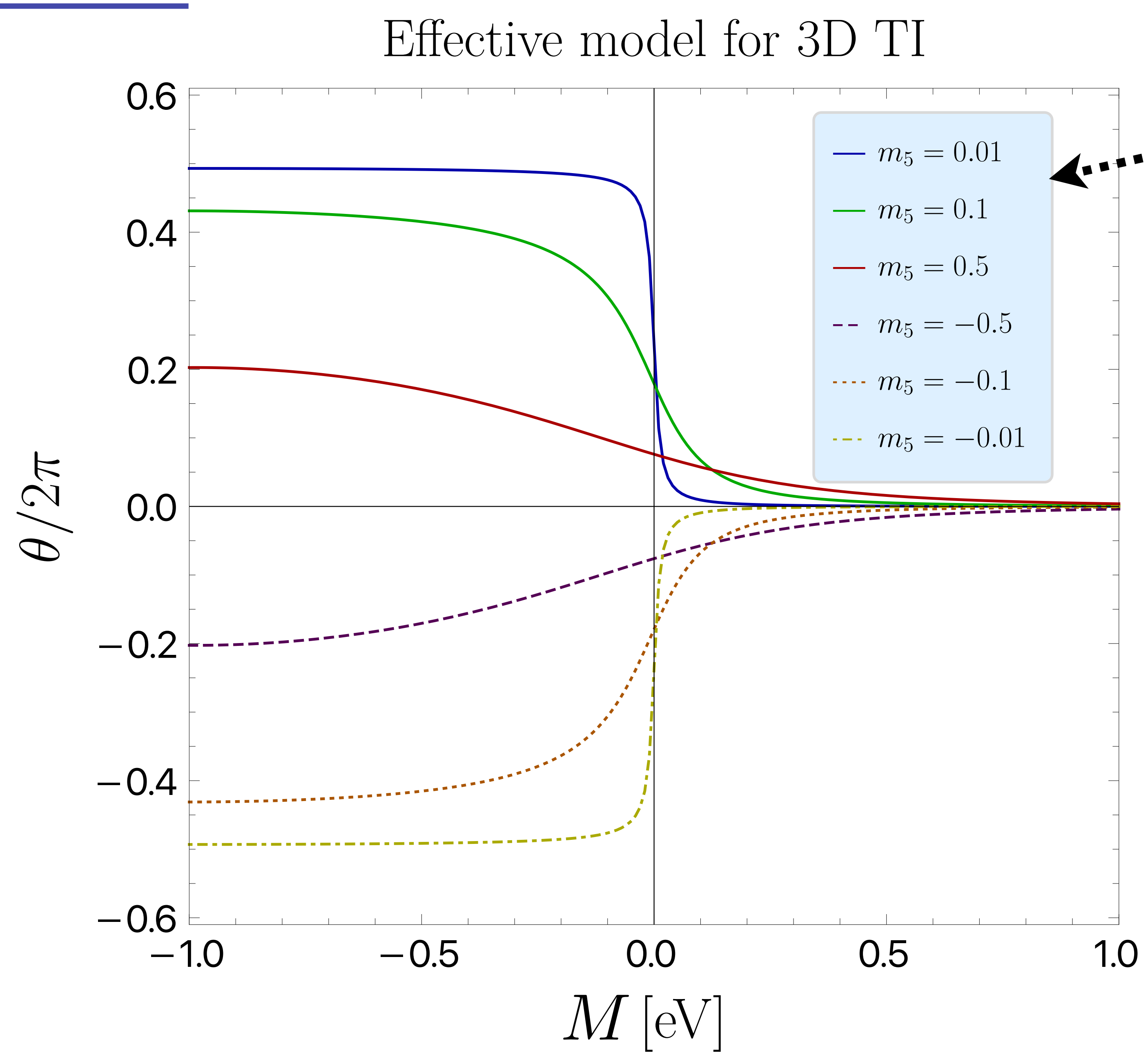
$$\phi = \phi(\theta)$$

$$V_\phi(\phi) \quad \longrightarrow \quad V_\theta(\theta)$$

θ as function of M

KI '21

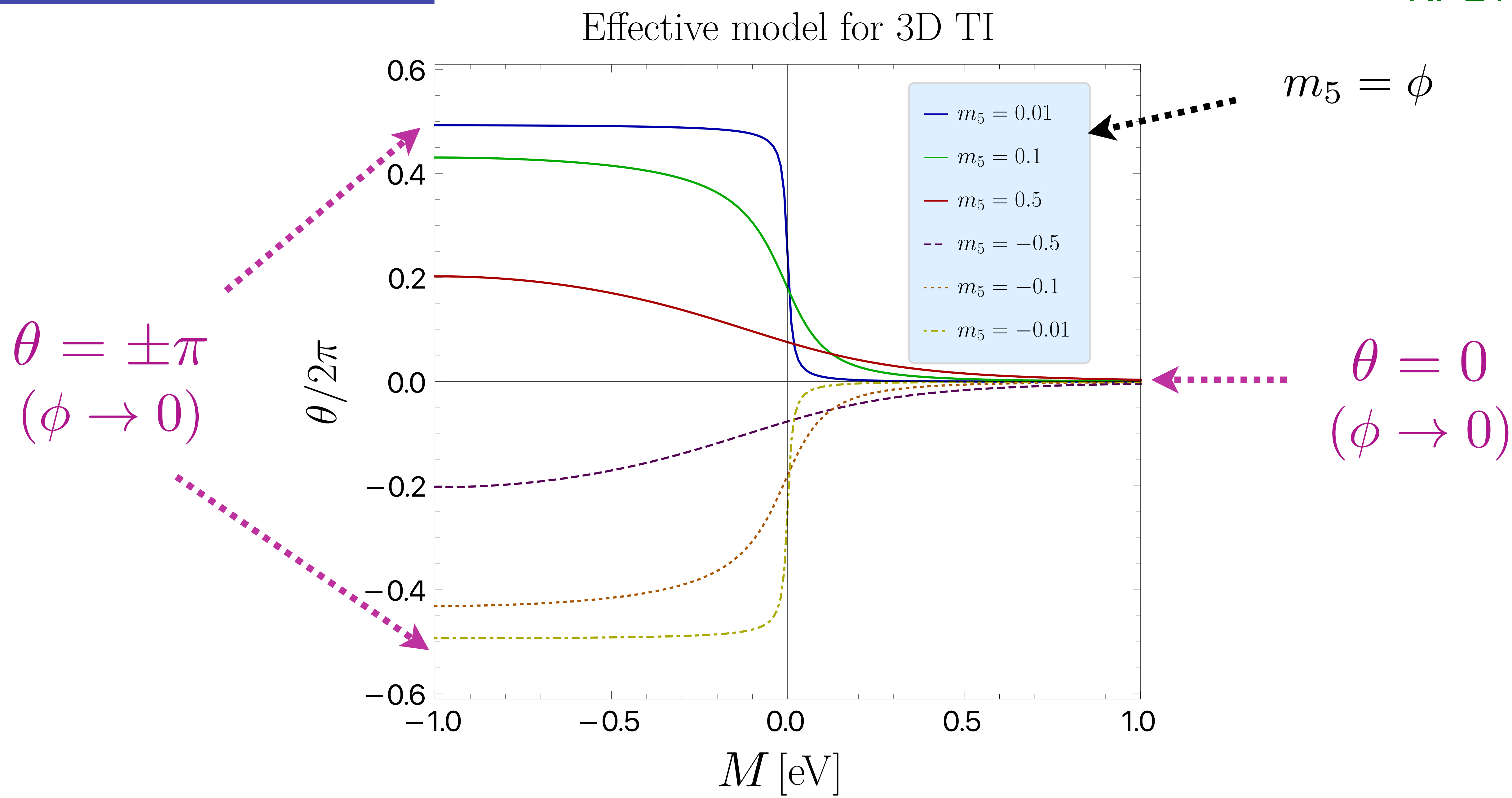


$m_5 = \phi$

calculation for Dirac model is done by Zhang '19

θ as function of M

KI ’21

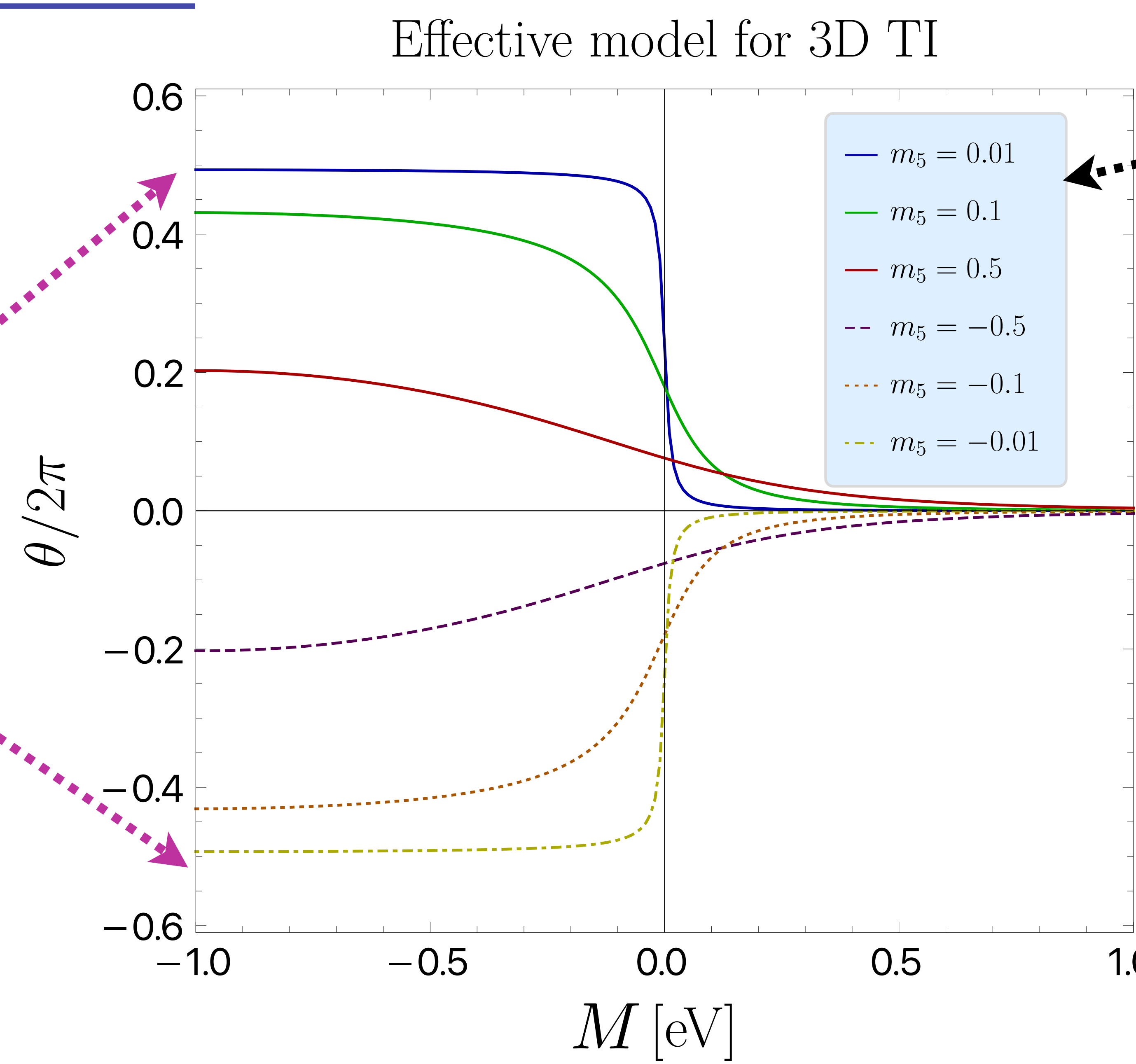


θ as function of M

KI '21

$m_5 = \phi$

$\theta = \pm\pi$
$(\phi \to 0)$



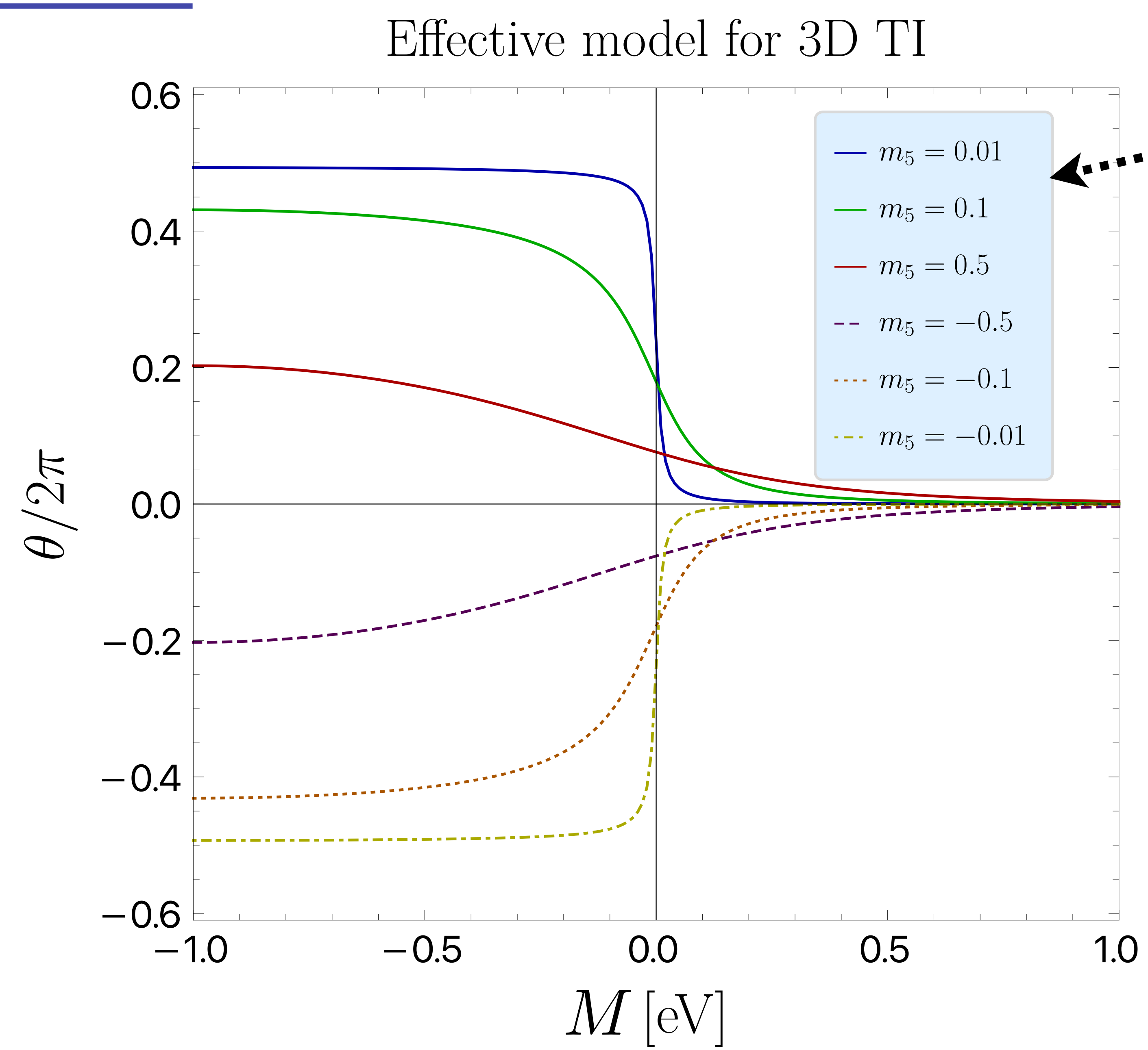
$\theta = 0$
$(\phi \to 0)$

$\phi = 0$
($\mathcal{T}$) ↔ PM order →

$\theta = \pm\pi$	$M < 0$	(TI)
$\theta = 0$	$M > 0$	(NI)

θ as function of M

KI '21



$m_5 = \phi$

$\phi \neq 0$ ($\not{\mathcal{T}}$) $\leftrightarrow$ AFM order $\rightarrow$ θ takes continuum variable

θ as function of ϕ