(宇宙論的枠組みにおける)
重力下での素粒子相互作用

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本日の問題設定

本日の議題：重力の効果は素粒子（場）の相互作用にどう影響を与えるか

- 宇宙論的背景時空は一様・等方で、時間変化する
- 背景時空・背景場は時間の並進対称性・Lorentz boost を破る
- 時空の曲率 ∼ 膨張率は Hubble rate H で与えられる

本日の相互作用：

$$\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- ゲージ場の分散は H^{-1} 程度の時間内で大きく宇宙膨張からの変更を受ける
- 初期宇宙においてこの変更は観測量の予言を修正する（し得る）
- φ の背景場（の変化）はパリティを破るようにゲージ場を生成する

本日の観測量：重力波

- 一般相対論（アインシュタイン方程式）からの不可避な帰結
- 幾何学量（重力波）は必ず物質場の振る舞いを反映する
- ゲージ場の生成は重力波検出に特徴的な痕跡を残す

Outline

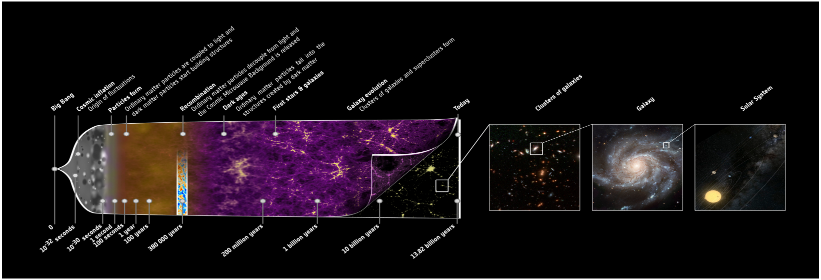
1. Introduction

2. Gauge field production during inflation
   - Generation of gravitational waves

3. Gauge field production during radiation/matter domination
   - Generation of gravitational waves

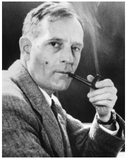
4. Summary & Conclusion

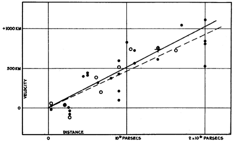
Our Expanding Universe



- Our Universe is, and has been (and will be?), expanding
- First discovered by **Edwin P. Hubble** in 1929

  The farther away a galaxy is, the faster it moves away from us.
  This means, our universe is expanding.





Hubble-Lemaître Law

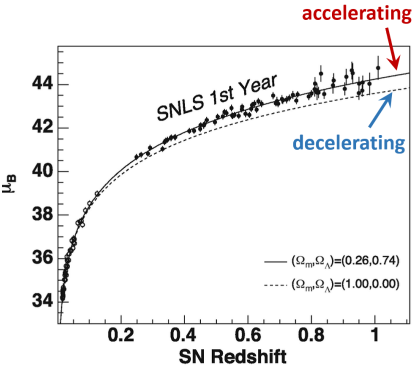
$$v = H\,L$$

v: recession speed

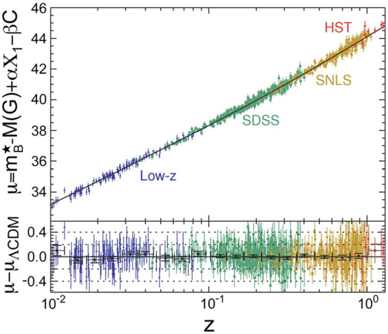
H: Hubble rate $\sim 70\,\mathrm{km/s/Mpc}$

L: proper distance

Our *Acceleratedly* Expanding Universe



R.T. Génova-Santos, 2020



Cosmic Expansion vs. Gravity

Cosmic evolution is governed by gravity

General Relativity

(GR)

$$\underset{\Downarrow}{G_{\mu\nu}} = 8\pi G_N \underset{\Downarrow}{T_{\mu\nu}}$$

Purely geometrical

Matter/energy content

- Gravity

  $\Longleftrightarrow$ structure/evolution of spacetime

- Presence of matter

  $\Longleftrightarrow$ non-trivial spacetime

- **Cosmic expansion is a natural consequence of GR**

What causes the expansion?

Cosmic expansion is a natural consequence of General Relativity

$$g_{\mu\nu} = \begin{pmatrix} -\bar{N}^2(t) & & & \\ & a^2(t) & & \\ & & a^2(t) & \\ & & & a^2(t) \end{pmatrix} , \qquad T^\mu{}_\nu = \begin{pmatrix} -\rho(t) & & & \\ & p(t) & & \\ & & p(t) & \\ & & & p(t) \end{pmatrix}$$

Friedmann equation(s)

(GR + cosmological principles)

$$3M_{\rm Pl}^2 H^2 = \rho$$
$$2M_{\rm Pl}^2 \dot{H} = -(\rho + p)$$

Hubble parameter $H = \frac{\partial_t a}{a\bar{N}}$: expansion rate

Energy density ρ, **Pressure** p,

The (reduced) Planck mass $M_{\rm Pl}^2 = (8\pi G_N)^{-1}$

- Presence of matter $\rho > 0$
  - Expansion $\Longrightarrow$ $H > 0$
- Fluids of non-relativistic particles:

  $p = 0 \quad \Longrightarrow \quad \dot{H} < 0$
- Fluids of relativistic particles:

  $p = \frac{\rho}{3} \quad \Longrightarrow \quad \dot{H} < 0$

Ordinary matter cannot drive *accelerated* expansion !

What causes the ***accelerated*** expansion?

Friedmann equation(s)

$$3M_{\rm Pl}^2 \boldsymbol{H}^2 = \boldsymbol{\rho}$$
$$2M_{\rm Pl}^2 \dot{\boldsymbol{H}} = -(\boldsymbol{\rho} + \boldsymbol{p})$$

Hubble parameter $\boldsymbol{H}$: expansion rate

Energy density ρ, **Pressure** $\boldsymbol{p}$

- The simplest solution is the cosmological constant

$$\rho_{CC} = \Lambda = \text{const.}\,, \qquad p_{CC} = -\Lambda$$

- $\dot{\boldsymbol{H}} = 0$ means ***accelerated*** expansion

Scale factor a: $\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0$

- **BUT** the current expansion is $H_0 \sim 10^{-42}\,\text{GeV} \Leftrightarrow \Lambda_{\rm obs} \sim (10^{-12}\,\text{GeV})^4$
  - **MUCH** smaller than theoretically expected vacuum energy: $\Lambda_{\rm theory} \sim M_{\rm Pl}^4 \sim (10^{18}\,\text{GeV})^4$, or at best $\Lambda_{\rm theory} \sim m_e^4 \sim (10^{-3}\,\text{GeV})^4$

What causes the ***accelerated*** expansion?

Friedmann equation(s)

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Hubble parameter H : expansion rate

Energy density ρ, **Pressure** p

- **Scalar fields** are the first non-trivial candidate
  - E.g. spin-0 condensate, dilaton, radion, Higgs, etc.
- For a canonical scalar field φ

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

$$\Longrightarrow \quad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V \,, \qquad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V$$

- Scalar fields **in the slow roll regime** are similar to cosmological const.

$$\rho_\varphi \simeq V \,, \qquad p_\varphi \simeq -V \qquad \left(\frac{1}{2}\dot{\varphi}^2 \ll V\right)$$

  - This is the only condition required for accelerated expansion

Scalar field cosmology

- In cosmology, the Poincaré invariance is broken (in temporal direction)
- Thus there is a preferred slicing, characterized by an evolving scalar $\phi(t)$

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  along time constant hypersurface, $\delta\varphi(t, \boldsymbol{x}) = 0$
- One can write down general Lagrangian with broken time diffeo.
  (& spatial diffeo. is preserved)

$$\mathcal{L}_{\text{EFT}} = \frac{M_{\text{Pl}}^2}{2} R - \Lambda(t) - c(t)\, g^{00} + M_2(t) \left(g^{00}\right)^2 + M_3(t) \left(g^{00}\right)^3 + \ldots$$

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- Recovering the action for a scalar field φ with all diffeomorphism

$$t \to \tilde{t} = \phi(t) + \delta\varphi(t, \boldsymbol{x}) \equiv \varphi(t, \boldsymbol{x}) \,, \quad x^i \to \tilde{x}^i = x^i$$

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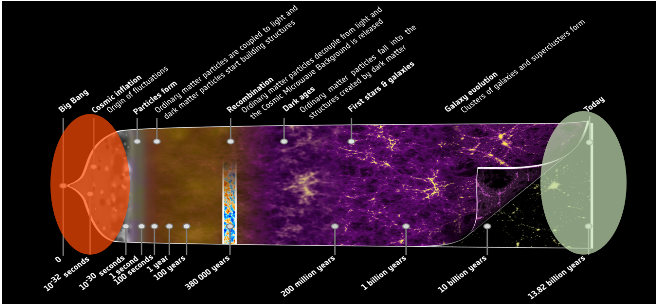
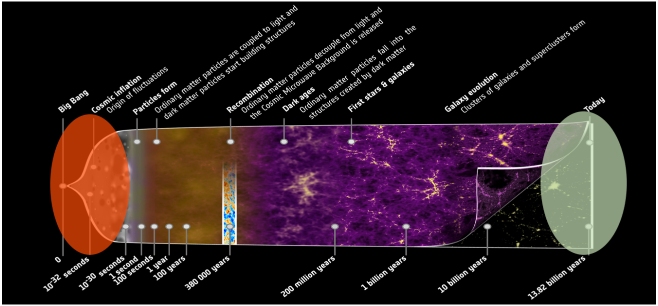
$$\boxed{\begin{aligned} \mathcal{L}_{\rm EFT} &= \frac{M_{\rm Pl}^2}{2} R - \tilde{\Lambda}(\varphi) - \tilde{c}(\varphi) X + \tilde{M}_2(\varphi) X^2 + \tilde{M}_3(\varphi) X^3 + \ldots \\ X &\equiv -\frac{1}{2}\, g^{\mu\nu} \partial_\mu \varphi\, \partial_\nu \varphi \end{aligned}}$$

Our Accelerated Cosmos



- ***Accelerated*** *expansion* at present: **dark energy**
- ***Accelerated*** *expansion* at the earliest time: **inflation**

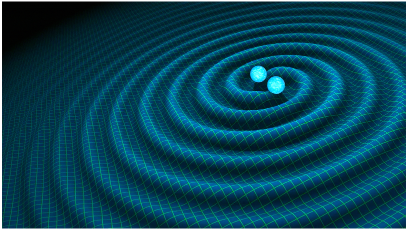
Our Accelerated Cosmos

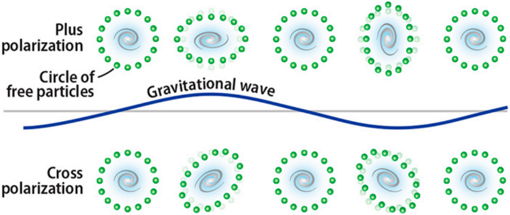


- ***Accelerated*** *expansion* at present: **dark energy**
- ***Accelerated*** *expansion* at the earliest time: **inflation**
- The source of these eras of expansion is still unknown
  - **Scalar field cosmology**
  - Interaction with other fields
  - **Particle production**

What are the observables?

1. **(Stochastic) Gravitational Wave (GW)**

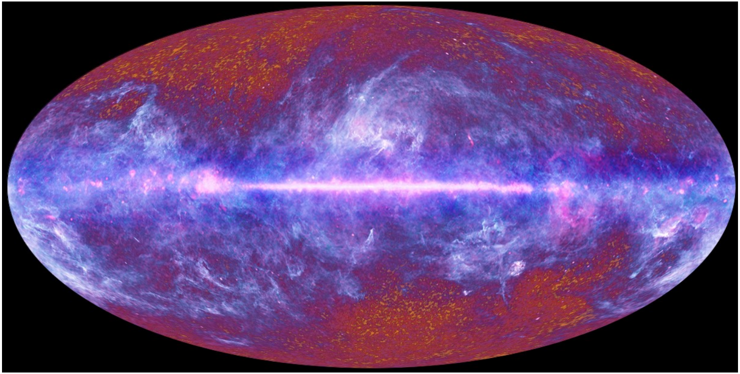




What are the observables?

2. **Cosmic Microwave Background (CMB)**

Planck/COBE collaboration



What are the observables?

❷ Cosmic Microwave Background (CMB)

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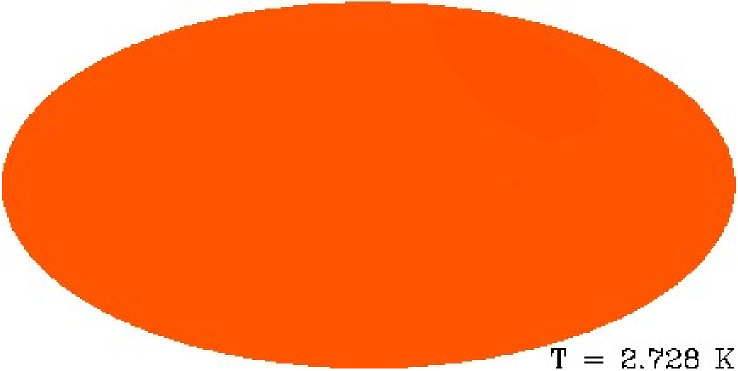

What are the observables?

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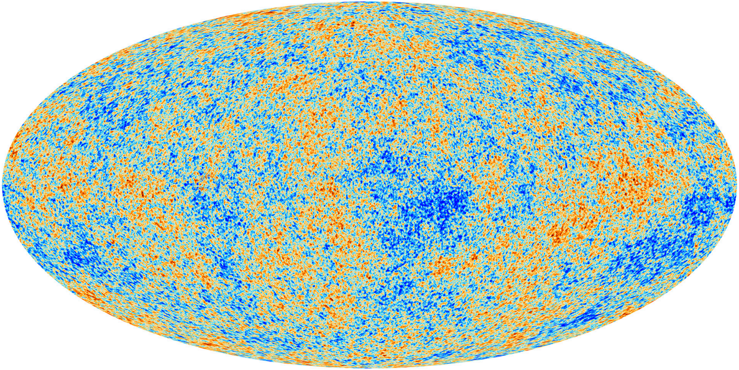
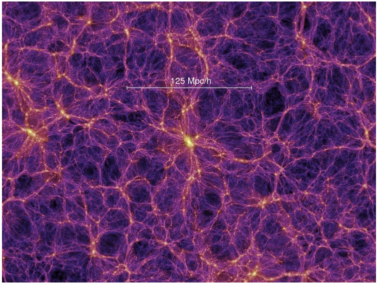


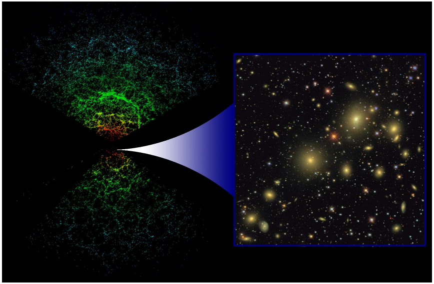
Planck/COBE collaboration

What are the observables?

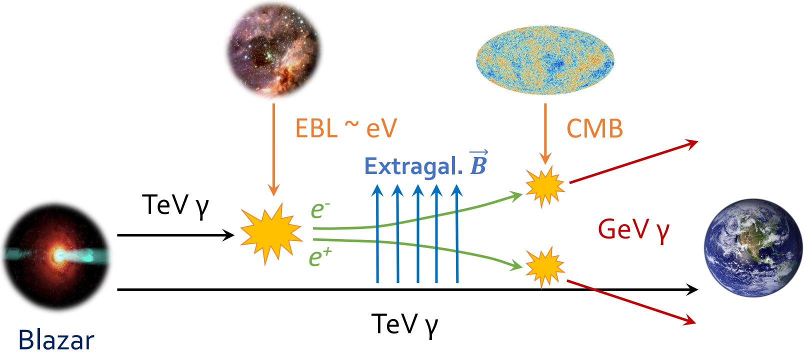
3. **Large-Scale Structure (LSS)**



SDSS



❹ Cosmic Magnetic Fields



Observe TeV γ rays

Lack of GeV γ rays

GOAL

Want to compute:

1. Gravitational wave (tensor mode) h_{ij}
2. Curvature perturbation (scalar mode) ζ
3. Magnetic fields (vector mode) $\vec{B}$

From what:

A. **Vacuum fluctuations** $\sim$ linear perturbations

B. **Particle production** $\sim$ nonlinear effects

- $\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\tilde{F}^{\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2} F_{\rho\sigma}$
- $\mathcal{L}_{\rm int} = -\frac{I(\varphi)}{4} F_{\mu\nu} F^{\mu\nu}$
- $\mathcal{L}_{\rm int} = -g^2 \varphi^2 \chi^2$
- $\mathcal{L}_{\rm int} = -|\left(\partial_\mu - ieA_\mu\right)\Phi|^2$
- $\mathcal{L}_{\rm int} = -g\varphi\,\bar{\psi}\psi$
- $\mathcal{L}_{\rm int} = -\alpha\,\frac{\partial_\mu\varphi}{f}\,\bar{\psi}\gamma^\mu\gamma_5\psi$
- ......

Some issues/differences from flat space QFT

Cosmological perturbation theory

- Time translation & Lorentz boost are usually broken by background
- Hamiltonian (for perturbations) has explicit time dependence

$$H_{\rm free} = \int \mathrm{d}^3x \left[\pi^a\, T_{ab}(t)\, \pi^b + \pi^a\, X_{ab}(t)\, \phi^b + \phi^a\, \Omega^2_{ab}(t)\, \phi^b\right]$$

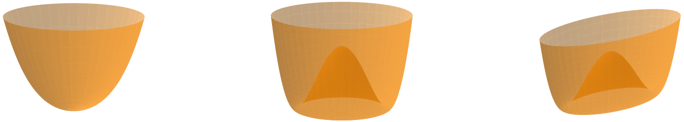
$$H_{\rm int} = \int \mathrm{d}^3x \left[c_1(t)\, \phi^3 + c_2(t)\, \phi^2 \pi + \dots\right]$$

Today's interaction

Axion-gauge field coupling

$$\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\, \varphi\, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Axion-like fields



Pseudo Nambu-Goldstone bosons

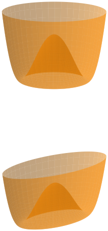
- Arise from spontaneous breaking of global symmetry [Goldstone theorem]
- Ubiquitous in UV particle theories beyond SM
  - ▷ Solution for the strong CP problem Peccei & Quinn ’77
  - ▷ Grand unified theories
  - ▷ String theory
- **Natural inflation**: Good candidate of inflaton field
- **Axion dark matter**: Possible candidate for dark matter

Axion-like fields

- **Shift symmetry**: Invariance under $\varphi \to \varphi + c$
- Slight breaking of **shift symmetry**

$$V \to V_{\text{shift}}(\rho) + V_{\text{break}}(\varphi)$$

  - In QCD, V_{break} from QCD instanton



Unique interaction

- gauge invariance
- shift symmetry
- parity

Axion-gauge coupling

$$\mathcal{L}_{\text{int}} = -\frac{\alpha}{4f}\,\varphi\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Axion-gauge coupling

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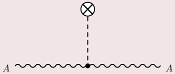
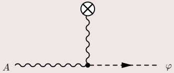
Interaction Effects

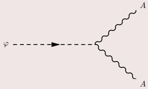
1. Non-perturbative depletion of $\dot{\varphi}^{(0)}$
   - modifies dispersion of A_μ



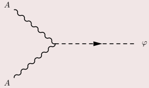
2. Photon conversion
   - External photon (magnetic) fields



3. $\delta\varphi \to A + A$, perturbative decay



4. $A + A \to \delta\varphi$, inverse decay



Axion-gauge coupling

$$\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\,\varphi\,F_{\mu\nu}\tilde{F}^{\mu\nu}$$

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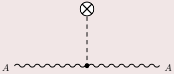
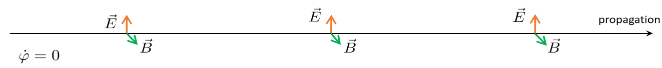


$\vec{E}\cdot\vec{B}$ in vacuum



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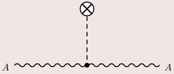
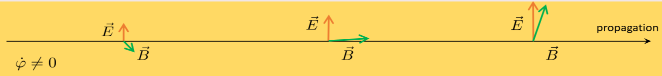


1. **Non-perturbative depletion by $\dot{\varphi}^{(0)}$**
   - ❇ modifies dispersion of A_μ

$\vec{E}\cdot\vec{B}$ in axion background



Some earlier studies

- Extra friction for inflaton motion

  Anber & Sorbo ’10
- Non-gaussianity in curvature perturbations

  Barnaby & Peloso ’11, Barnaby, Peloso & RN ’11
- Primordial gravitational waves from inflation

  Barnaby & RN et al. ’12, RN et al. ’16
- Production of primordial black holes

  Linde et al. ’12
- Generation of primordial magnetic fields

  Anber & Sorbo ’06, Durrer et al. ’10, Caprini & Sorbo ’14, Fujita & RN et al. ’15, Adshead et al. ’16
- Baryogenesis

  Bamba ’06, Anber & Sabancilar ’15, Jiménez et al. ’17, Domcke et al. ’19
- $SU(N)$ extensions

  Adshead & Wyman ’12 & ’13, Dimastrogiovanni & Peloso ’13, Dimastrogiovanni et al. ’16, Fujita & RN & Tada ’18, Fujita & RN & Obata ’18, Caldwell & Devulder ’18, Fujita et al. ’21, . . .

$\vdots$

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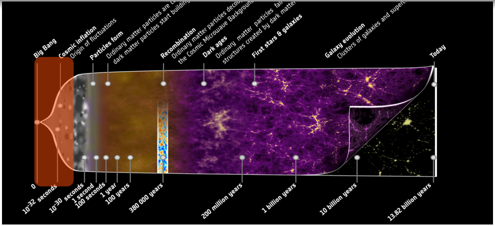
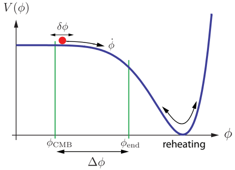
Primordial inflation



- **Extremely rapid, accelerated expansion in the earliest universe**
  - ▷ Solves the conceptual problems in the Hot Big Bang cosmology
  - ▷ Horizon, flatness, monopole problems & seeds of inhomogeneities
- **Simple realization by a single scalar field: inflaton φ**

$$\mathcal{L}_{\rm infl} = -\frac{1}{2}\left(\partial\varphi\right)^2 - V(\varphi)$$

Slow roll: $\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 , \; \eta \equiv M_{\rm Pl}^2 \frac{V''}{V} \ll 1$



Baumann, TASI Lecture, '09

Symmetries of inflation

Soda '12

- **Homogeneity** ⇔ spatial translation ⇔ momentum conservation
  - ▷ $\langle\zeta(\vec{k})\,\zeta(\vec{k}')\rangle = \delta^{(3)}(\vec{k}+\vec{k}')\,k^{-3}P_\zeta(\vec{k})$
- **Spatial isotropy** ⇔ rotational symmetry
  - ▷ $P_\zeta(\vec{k}) = P_\zeta(|\vec{k}|)$
- **Temporal de Sitter** $t \to t + c, \vec{x} \to \mathrm{e}^{-H\,c}\vec{x}$ for de Sitter expansion
  - ▷ $P_\zeta(|\vec{k}|) = \mathrm{const.}$
- **Shift symmetry** $\varphi \to \varphi + \mathrm{c}$ ⇔ small interactions ⇔ Gaussian
  - ▷ Statistical information is contained only in 2-point function

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- **Temporal de Sitter** $t \to t$ de Sitter expansion
  - ▷ $P_\zeta(|\vec{k}|) =$ const.

Not an exact symmetry

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Not an exact symmetry

Axion-gauge field coupling during inflation

Model to consider

$$\mathcal{L} = \underset{\swarrow \atop \text{Standard}}{\mathcal{L}_{\text{inflaton}}} - \frac{1}{2}(\partial\sigma)^2 - \underset{\downarrow \atop \text{Subdominant}}{V(\sigma)} - \frac{1}{4}F^2 - \frac{\alpha}{4f}\underset{\searrow \atop \text{Leads production}}{\sigma F \tilde{F}}$$

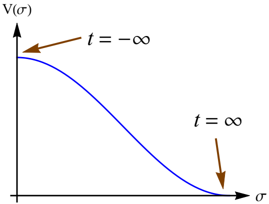
Discrete symmetry: $V(\sigma) = \Lambda^4 \left(1 + \cos\frac{\sigma}{f}\right)$

Homogeneous mode $\sigma = \sigma(t)$

$$\ddot{\sigma} + 3H\dot{\sigma} + V' = 0$$

Slow roll: $|\ddot{\sigma}| \ll 3H|\dot{\sigma}|$

$$\delta \equiv \frac{\Lambda^4}{3H^2 f^2} \ll 3$$



Gauge field production during inflation

Coupling: $\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\,\sigma F \tilde{F}$

▷ **Parity violation**

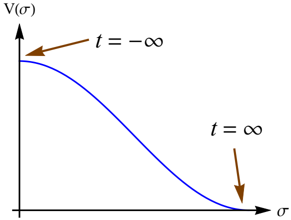
$$\frac{\partial^2}{\partial\tau^2}A_\pm + \left(k^2 \mp ak\,\frac{\alpha\dot\sigma}{f}\right)A_\pm = 0$$

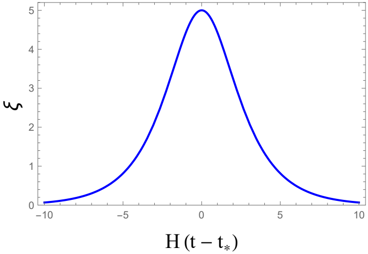
Coupling strength

$$\xi \equiv \frac{\alpha\dot\sigma}{2fH} \cong \frac{\xi_*}{\cosh\left[H\delta\left(t-t_*\right)\right]}$$

δ : controls signal **width**

ξ_* : controls signal **height**

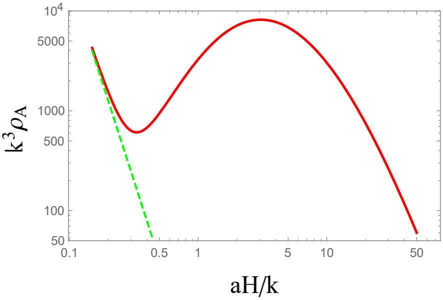




Gauge field production during inflation

$$\frac{\partial^2}{\partial\tau^2}A_\pm + \left(k^2 \pm \frac{2k\xi}{\tau}\right)A_\pm = 0\,, \qquad \xi \cong \frac{\xi_*}{\cosh\left[H\delta\left(t-t_*\right)\right]}$$

$$\boldsymbol{A}_+(\boldsymbol{\tau},\boldsymbol{k}) \simeq N[\xi_*,\tau_*,\delta]\left[\frac{-\tau}{8k\xi(\tau)}\right]^{1/4}\exp\left[-\frac{4\sqrt{-k\tau\xi_*}}{1+\delta}\left(\frac{\tau}{\tau_*}\right)^{\delta/2}\right]$$



- **Only one of the polarization states is enhanced**

Outline

1. Introduction

2. Gauge field production during inflation
   - Generation of gravitational waves

3. Gauge field production during radiation/matter domination
   - Generation of gravitational waves

4. Summary & Conclusion

Sourcing gravitational waves

Einstein equation

$$\underset{\Downarrow}{G_{\mu\nu}} = 8\pi G_N \underset{\Downarrow}{T_{\mu\nu}}$$

GW

Produced particle/field

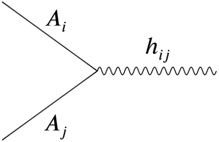
- Spacetime geometry $\Longleftrightarrow$ Matter content
- Produced fields inevitably source GW

GW ⇔ tensor mode of metric

$$\delta g_{ij} = a^2 \left(\delta_{ij} + h_{ij} \right)$$

$$\boxed{\left(\frac{\partial^2}{\partial \tau^2} - \nabla^2 - \frac{\partial_\tau^2 a}{a} \right) \left(a\, h_{ij} \right) = - \frac{2\, a^3}{M_p^2} \left(E_i E_j + B_i B_j \right)}$$

$$E_i \equiv \frac{-1}{a^2} \partial_\tau A_i \; , \quad B_i \equiv \frac{1}{a^2} \epsilon_{ijk} \partial_j A_k$$



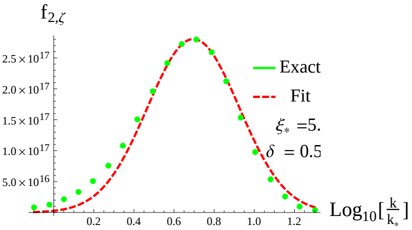
GW power spectra

GW 2-point correlator

Tensor: $\boldsymbol{\mathcal{P}}_\lambda \, \delta_{\lambda\lambda'} \, \delta^{(3)}\left(\vec{k} + \vec{k}'\right) \equiv \frac{k^3}{2\pi^2} \left\langle h_\lambda\left(\vec{k}\right) h_{\lambda'}\left(\vec{k}'\right)\right\rangle$

$$\mathcal{P}_\lambda = \underbrace{\mathcal{P}_\lambda^{(0)}}_{\text{vacuum}} + \underbrace{\boldsymbol{\mathcal{P}_\lambda^{(1)}}}_{\textbf{sourced}}$$

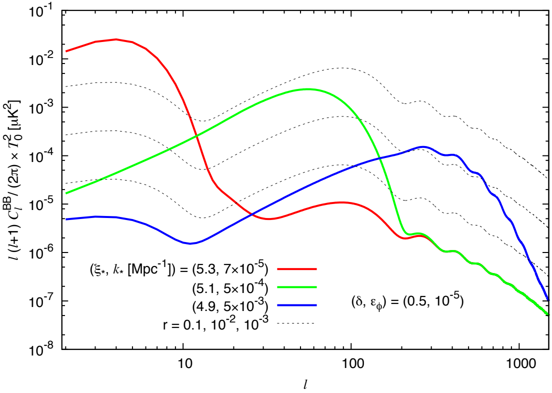
Parameterize: $\boldsymbol{\mathcal{P}^{(1)}_{\zeta/\lambda}} = \left[\epsilon_\phi \mathcal{P}_\zeta^{(0)}\right]^2 \boldsymbol{f_{2,\zeta/\lambda}\left(k_*, \delta, \xi_*; k\right)} \;, \quad k_* \equiv aH|_{t=t_*}$



Contribution to CMB B-mode polarization

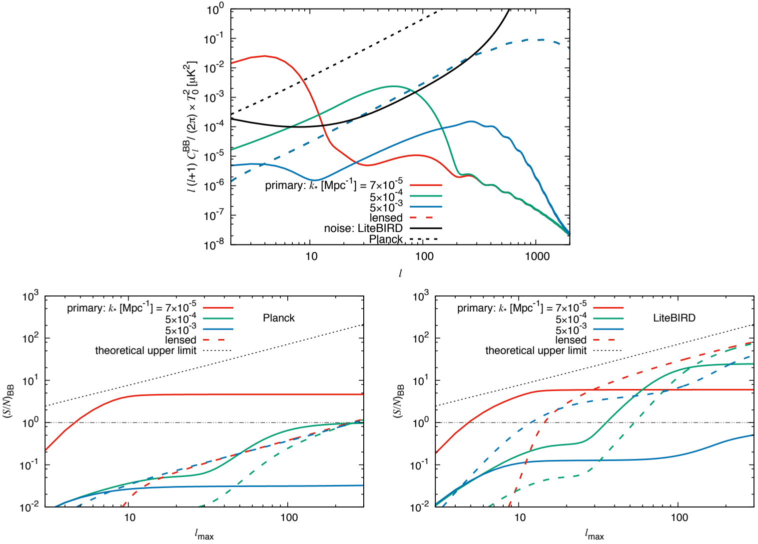
Tensor (GW) correlation functions $\mathcal{P}_\lambda$

$\Longrightarrow$ **CMB B-mode correlations C_ℓ^{BB}**



RN+ ’15

Detectability of BB Correlations



Shiraishi, Hikage, RN, Namikawa & Hazumi ’16

GW Signals from Axion + Abelian Gauge Field Model

- **Scale-dependent GW spectrum**
- **Parity-violating GW spectrum**
- **Signal-to-noise ratio (SNR) > 1**
- **Detectable BBB bispectra**

	Bispectrum SNR
Noiseless (perfect delensing)	10
Noiseless	4.6
LiteBIRD	2.5

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Axion-like field + gauge field system

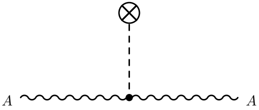
Axion–gauge field system

$$\mathcal{L}_{\text{axion+gauge}} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$-\frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Axion can typically have an oscillating VEV

$$\langle\varphi\rangle \neq 0$$

  - Long wavelength modes
  - Obeys classical equation of motion



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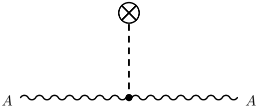
  - **Oscillation** in Minkowski



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- Axion can typically have an oscillating VEV

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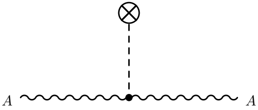
  - Long wavelength modes
  - Obeys classical equation of motion
  - **Oscillation** in Minkowski
  - **Damped oscillation** in expanding universe

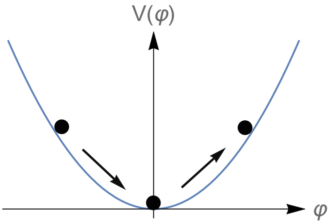


Gauge-field production in Minkowski spacetime

Equation of motion for A_μ in Minkowski

$$\left[\frac{\partial^2}{\partial t^2} + k^2 \mp k\,\frac{\alpha}{f}\,\dot{\phi}(t)\right] A_\pm(t,k) = 0$$

- $A_\pm$ ⇔ circular polarization modes for given wavenumber k
- $\phi(t) \equiv \langle\varphi\rangle$ ⇔ coherent VEV of axion
- $\phi(t)$ oscillates coherently over space



Gauge-field production in Minkowski spacetime

Axion coherent oscillation

$$\phi(t) = \phi_{\mathrm{osc}} \cos\left[m_\varphi \left(t - t_{\mathrm{osc}}\right)\right]$$

- m_φ: axion’s mass
- $\phi_{\mathrm{osc}}, t_{\mathrm{osc}}$: integration constants

Gauge-field E.o.M. in Minkowski

$$\boldsymbol{\left[\frac{\partial^2}{\partial z^2} + \kappa_k^2 \pm 2\kappa_k Q \sin 2\left(z - z_{\mathrm{osc}}\right)\right] A_\pm = 0}$$

- Dimensionless variables & parameters
  - $z \equiv \frac{m_\varphi t}{2}$ ⇔ time
  - $\kappa_k \equiv \frac{2k}{m_\varphi}$ ⇔ momentum
  - $Q \equiv \frac{\alpha \phi_{\mathrm{osc}}}{f}$ ⇔ coupling strength

Gauge-field production as parametric resonance

Gauge-field E.o.M. in Minkowski

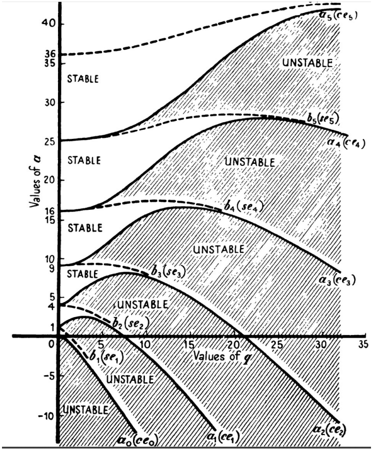
$$\left[\frac{\partial^2}{\partial z^2} + \kappa_k^2 \pm 2\kappa_k Q \sin 2\left(z - z_*\right)\right] A_\pm = 0$$

- **Mathieu equation**
- **Parametric resonance**

Dolgov&Kirilova, Traschen&Brandenberger ’90
Kofman et al., Greene et al. ’97

  - ❊ Most studied in the context of reheating after inflation
  - ❊ Extremely efficient production for some values of modes k
  - ❊ **Instability bands** $\Longrightarrow$

`McLachlan, Oxford U. Press, 1947`



Gauge-field production in **expanding background**

How do we implement background expansion?

- Flat FLRW metric: $\mathrm{d}s^2 = a^2(\tau)\left(-\mathrm{d}\tau^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right)$
- $Q \to Q(\tau)$, $\sin z \to \sin\left(z^\alpha\right)$

Axion oscillation with expansion

$$\phi(t) \approx \phi_{\rm osc}\left(\frac{a_{\rm osc}}{a}\right)^{3/2}\cos m_\varphi\left(t - t_{\rm osc}\right)$$

- $t \propto \tau^\alpha$, $\quad \alpha = \frac{3(1+w)}{(1+3w)} = 1$ (Minkowski), 2 (RD), 3 (MD)

Gauge-field production in **expanding background**

Gauge-field E.o.M. in FLRW

$$\left[\frac{\partial^2}{\partial z^2} + \kappa_k^2 \pm 2\kappa_k Q(z) \sin 2\left(z^\alpha - z_{\rm osc}^\alpha\right) \right] A_\pm = 0$$

$$z_{\rm osc} \sim \left(\frac{m_\varphi}{H_{\rm osc}} \right)^{1/\alpha} > 1 \,, \quad \kappa_k = \frac{\tau_{\rm osc}}{z_{\rm osc}} k \,, \quad Q(z) \sim \frac{m_\varphi}{H_{\rm osc}} \frac{\alpha \phi_{\rm osc}}{f} \left(\frac{\tau_{\rm osc}}{\tau} \right)^{(\alpha-1)/2}$$

- $\alpha = \frac{3(1+w)}{(1+3w)} = 1$ (Minkowski), 2 (RD), 3 (MD)
- Instability bands move over time
- **No narrow resonance around $k \approx m_\varphi/2$ for $Q < 1$**

Gauge-field E.o.M. in FLRW

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\kappa_k^2 \pm 2\kappa_k Q(z)\sin 2\left(z^\alpha - z^\alpha_{\rm osc}\right)}_{\omega_\pm^2}\right] A_\pm = 0$$

Broad instability band: $Q > 1$

Instability band: $\frac{1}{2Q(z)} < \kappa_k < 2Q(z)$

- Most of the studies have been numerical
- **Analytical approach?**

Broad instability band: $Q > 1$

Instability band: $\frac{1}{2Q(z)} < \kappa_k < 2Q(z)$

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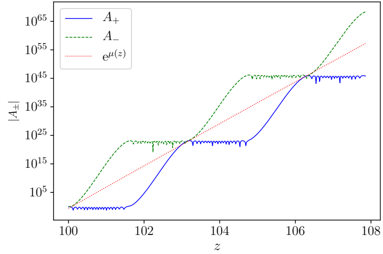
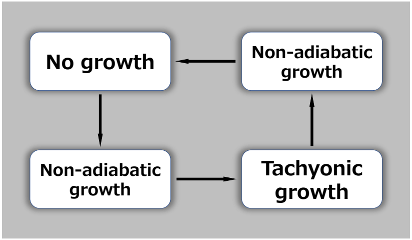


n-th cycle of growth



No growth

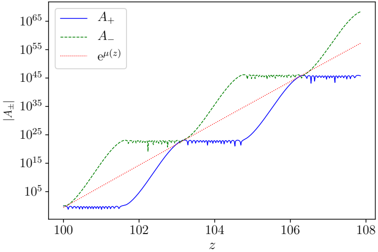
Non-adiabatic growth

Non-adiabatic growth

Tachyonic growth

Broad instability band: $Q > 1$

Instability band: $\frac{1}{2Q(z)} < \kappa_k < 2Q(z)$



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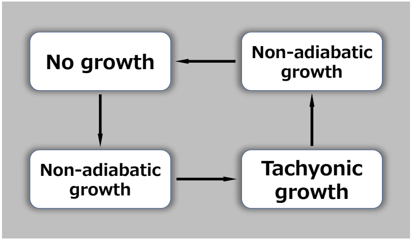

No growth

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Tachyonic growth

Non-adiabatic growth

- Adiabaticity $\frac{\partial_z \omega_\pm}{\omega_\pm^2}$

Broad instability band: $Q > 1$

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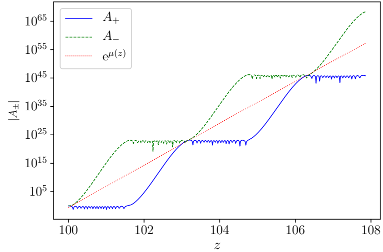

Bogolyubov transformation

$$A_\pm = \frac{\alpha_\pm \, e^{-i\int^\tau d\tau' \omega_\pm} + \beta_\pm \, e^{i\int^\tau d\tau' \omega_\pm}}{\sqrt{2\omega_\pm}}$$

$$\begin{bmatrix} \alpha_\pm(z) \\ \beta_\pm(z) \end{bmatrix} \simeq e^{\mu(z)} \, \mathcal{U} \begin{bmatrix} \alpha_\pm(z_0) \\ \beta_\pm(z_0) \end{bmatrix} ,$$

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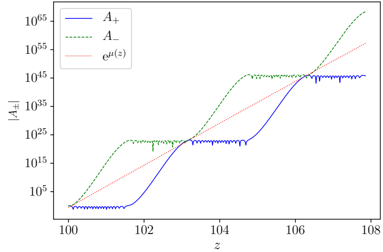

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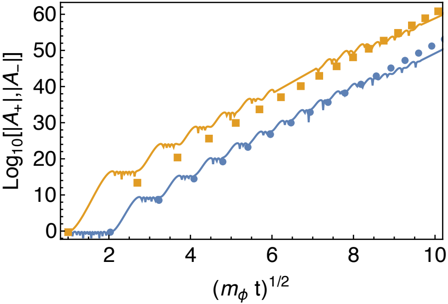
- Adiabaticity $\frac{\partial_z \omega_\pm}{\omega_\pm^2}$

Growth factor $\mu(z)$ — see our paper 2009.13909

Termination of resonant growth



Growth does not continue forever because

1. Coupling $Q(z)$ decreases with time due to expansion

$$Q(z) \propto z^{-1/2}\ \text{(RD)}\,,\ \ z^{-1}\ \text{(MD)}$$

2. Growth must not disrupt coherent motion of the axion $\phi(t)$

$$\ddot{\phi} + 3H\dot{\phi} + V_\varphi(\phi) = \frac{\boldsymbol{\alpha}}{\boldsymbol{f}}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$$

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Sourcing gravitational waves

Einstein equation

$$\underset{\underset{\text{GW}}{\Downarrow}}{G_{\mu\nu}} = 8\pi G_N \underset{\underset{\text{Produced particle/field}}{\Downarrow}}{T_{\mu\nu}}$$

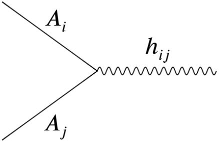
- ❄ Spacetime geometry $\Longleftrightarrow$ Matter content
- ❄ Produced fields inevitably source GW

GW ⇔ tensor mode of metric

$$\delta g_{ij} = a^2 \left(\delta_{ij} + h_{ij} \right)$$

$$\boxed{\left(\frac{\partial^2}{\partial \tau^2} - \nabla^2 - \frac{\partial_\tau^2 a}{a} \right) \left(a \, h_{ij} \right) = - \frac{2 \, a^3}{M_p^2} \left(E_i E_j + B_i B_j \right)}$$

$$E_i \equiv \frac{-1}{a^2} \partial_\tau A_i \; , \quad B_i \equiv \frac{1}{a^2} \epsilon_{ijk} \partial_j A_k$$



Gravitational-wave densities

GW energy density:

$$\rho_{\rm GW} = \frac{M_{\rm Pl}^2}{8a^2} \left\langle \partial_\tau h_{ij}\, \partial_\tau h_{ij} + \partial_k h_{ij}\, \partial_k h_{ij} \right\rangle$$

Fractional GW density spectrum:

$$\Omega_{\rm GW} = \frac{1}{3 M_{\rm Pl}^2 H^2} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k}$$

Relations between the present and time of generation:

- For amplitude:

$$\Omega_{\rm GW}(t_0) \approx 2 \times 10^{-5} \left(\frac{g_{s,0}}{g_{s,{\rm gen}}} \right)^{4/3} \frac{g_{*,{\rm gen}}}{g_{*,0}} \, \Omega_{\rm GW}(t_{\rm gen})$$

- For peak momentum/frequency:

$$f_{\rm obs} \approx 10^{-9}\,{\rm Hz}\, \frac{p(t_{\rm gen})}{10^{-20}\, g_{s,{\rm gen}}\, T(t_{\rm gen})}$$

NANOGrav observation

NANOGrav

= the **N**orth **A**merican **N**anohertz **O**bservatory for **Grav**itational Waves

- Pulsar-timing array (PTA)
  - 305-m Arecibo Observatory (Puerto Rico)
  - 100-m Green Bank Telescope (West Virginia)
- Long-term monitoring of 47 (milli-second) pulsars
- Spinning neutron stars emitting jets with **regular periods**
- **GW ⇒ change in geodesics ⇒ shift in propagation time**
- Limits on obs. GW freq. ⇔ Obs. freq. – overall duration
  - (1 week) – (10 yrs) ⇔ $1\,\mu Hz - 1\,nHz$
- Damage to the Arecibo telescope

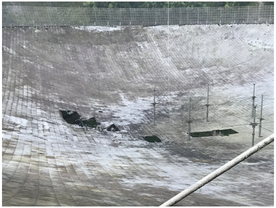


photo by Phil Perillat, Nat'l Astro and Ionosphere Ctr

NANOGrav 12.5-year result for GW

characteristic
GW strain

$$h(f) = A_{\rm GWB} \left(\frac{f}{f_{\rm obs}} \right)^{\frac{3-\gamma}{2}}$$

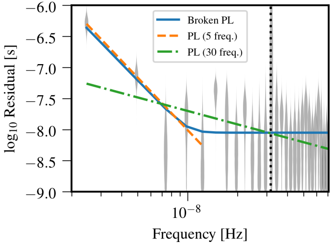
timing-residual
cross-power spectral density

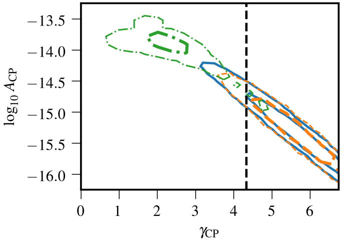
$$S_{ab}(f) = \Gamma_{ab} \frac{A_{\rm GWB}^2}{12\pi^2 f_{\rm obs}^3} \left(\frac{f}{f_{\rm obs}} \right)^{-\gamma}$$

fractional
GW density

$$\Omega_{\rm GW}(f) = \frac{2\pi^2 f_{\rm obs}^2}{3H_0^2} A_{\rm GWB}^2 \left(\frac{f}{f_{\rm obs}} \right)^{5-\gamma}$$

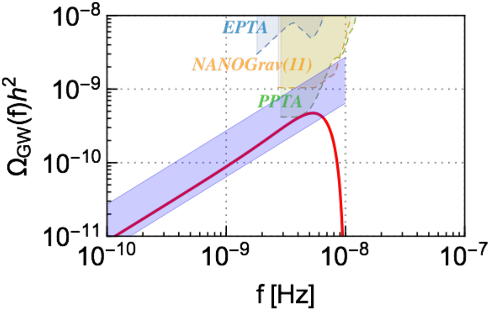
Arzoumanian et al. 2009.04496 [NANOGrav collab.]





- No significant evidence for quadrupolar spatial correlations in Γ_{ab}
- Could be spin noise, pulse profile changes, dispersion measure variations, solar system effects, clock errors, etc.
- GW signal ? . . . maybe ?

Production-driven GW interpretation



- Red curve: our GW spectrum for $\phi_{\rm osc} = 0.12\, M_{\rm Pl}$ and $m_\varphi = 10^{-12.5}\,{\rm eV}$
- Blue region: NANOGrav GW spectrum with $\gamma = 4$ within 2σ
- Parameters: $m_\varphi \sim 10^{-13}\,{\rm eV}$, $\phi_{\rm osc} \sim 0.1\, M_{\rm Pl}$, $\alpha\phi_{\rm osc}/f \sim 30$
- Contribution to $\Delta N_{\rm eff}$ from the axion abundance
  ⇒ might alleviate the Hubble tension ?

NANOGrav 12.5-year result for GW

GW interpretations

- Cosmic strings
  Blasi et al. [2009.06607], Ellis & Lewicki [2009.06555], Buchmuller et al. [2009.10649], Samanta & Datta [2009.13452], Ramberg & Visinelli [2012.06882], Blanco-Pillado et al. [2102.08194]
- Primordial black holes
  Vaskonen & Veermäe [2009.07832], De Luca et al. [2009.08268], Kohri & Terada [2009.11853], Sugiyama et al. [2010.02189], Zhou et al. [2010.03537], Domènech & Pi [2010.03976], Inomata et al. [2011.01270], Atal et al. [2012.14721], Kawasaki & Nakatsuka [2101.08012]
- Dark sector phase transition
  Nakai et al. [2009.09754], Addazi et al. [2009.10327], Ratzinger & Schwaller [2009.11875]
- Inflation
  Vagnozzi [2009.13432], Li et al. [2009.14663], Kuroyanagi et al. [2011.03323]
- Dark photon resonance by ALP
  Ratzinger & Schwaller [2009.11875], **Namba & Suzuki [2009.13909]**, Kitajima et al. [2010.10990]
- Domain walls — Liu et al. [2010.03225]
- QCD phase transition — Neronov et al. [2009.14174], Li et al. [2101.08012]
- Others — Bhattacharya et al. [2010.05071], Tahara & Kobayashi [2011.01605], Chen et al. [2101.06869]

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Summary & Conclusion

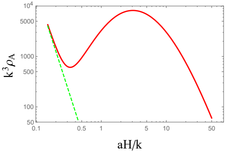
- Cosmological background breaks temporal Poincaré
- Perturbative Lagrangian/Hamiltonian explicitly depends on time

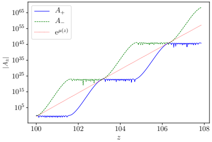
$$\mathcal{L} = \mathcal{L}[\phi, \dot{\phi}, t]\,, \qquad H = \int \mathrm{d}^3x\, \mathcal{H}[\phi, \pi, t]$$

- Background time evolution modifies dispersion of perturbative modes

$$\left[\partial_\tau^2 + k^2 \mp ak\alpha\dot{\sigma}/f\right] A_\pm = 0$$

- Copious production qualitatively different from flat spacetime (Minkowski)





- Geometrical objects (GW, spacetime curvature) are sourced by particle production as a inevitable consequence of Einstein eq.

$$\left(\frac{\partial^2}{\partial\tau^2} - \boldsymbol{\nabla}^2 - \frac{\partial_\tau^2 a}{a}\right) h_{ij} \sim \frac{E_i E_j + B_i B_j}{M_{\rm Pl}^2}$$