

# Phenomenology of superconformal subcritical hybrid inflation

Yoshihiro Gunji (Kanazawa University)

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with Koji Ishiwata (Kanazawa University)

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# 1. Introduction

# Standard Model (SM)

SM describes well the phenomena of elementary particles below 1 TeV

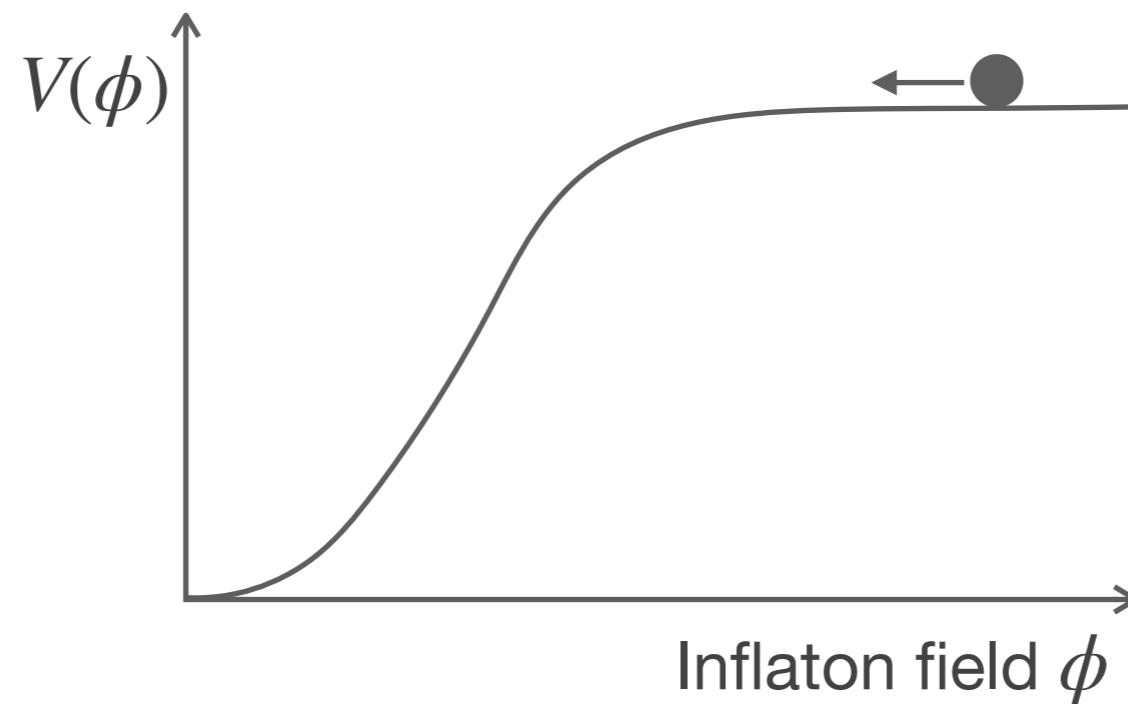
The phenomena unexplained in SM:

- Inflation
- Baryon asymmetry
- Light neutrino masses
- Dark matter

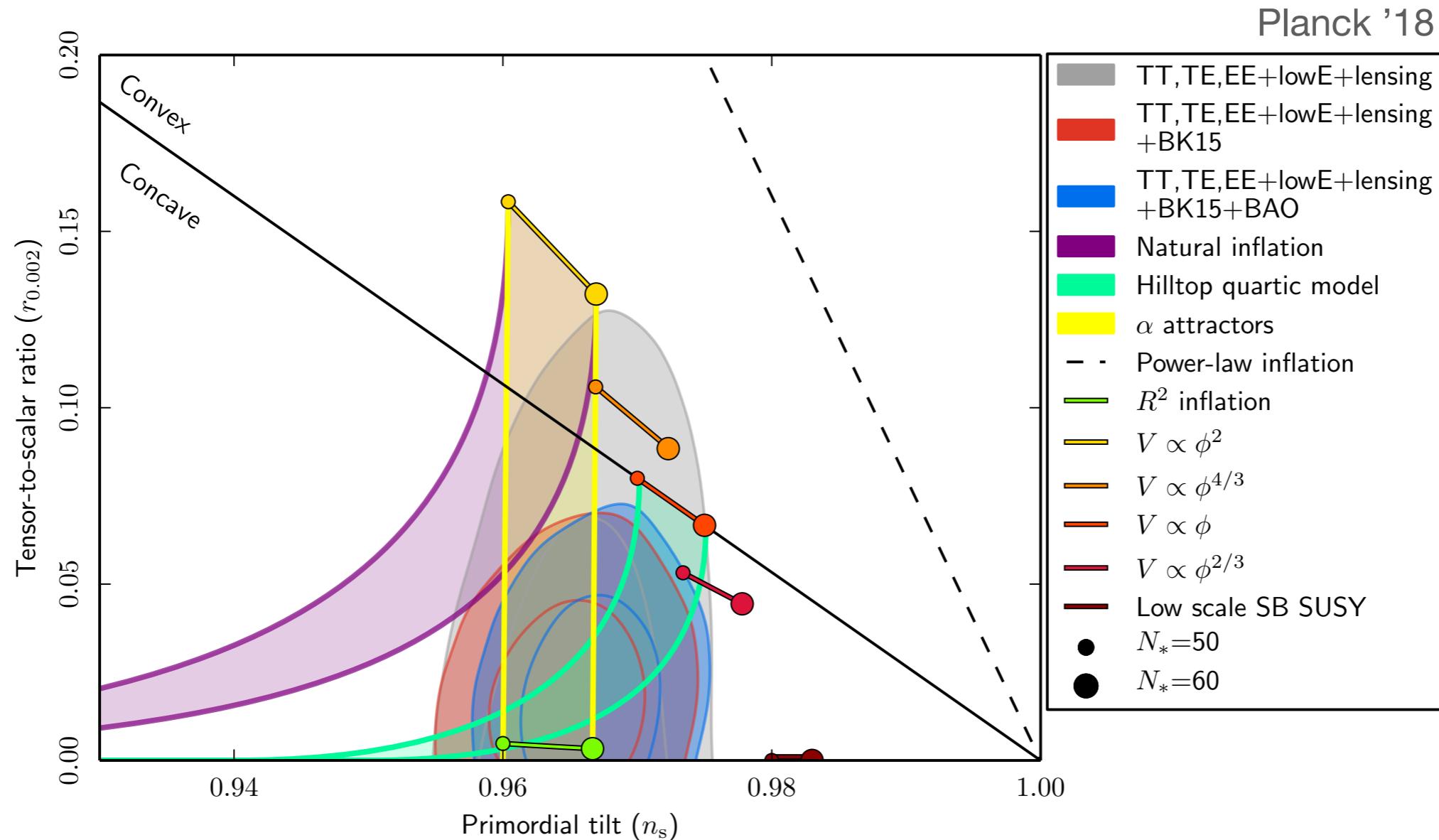
...

# Inflation

- It is a paradigm of accelerated expansion of the early universe
- It is supported by cosmic microwave background (CMB) observations
- It is realized by the potential energy of a slow-rolling scalar field (inflaton)



# Inflation

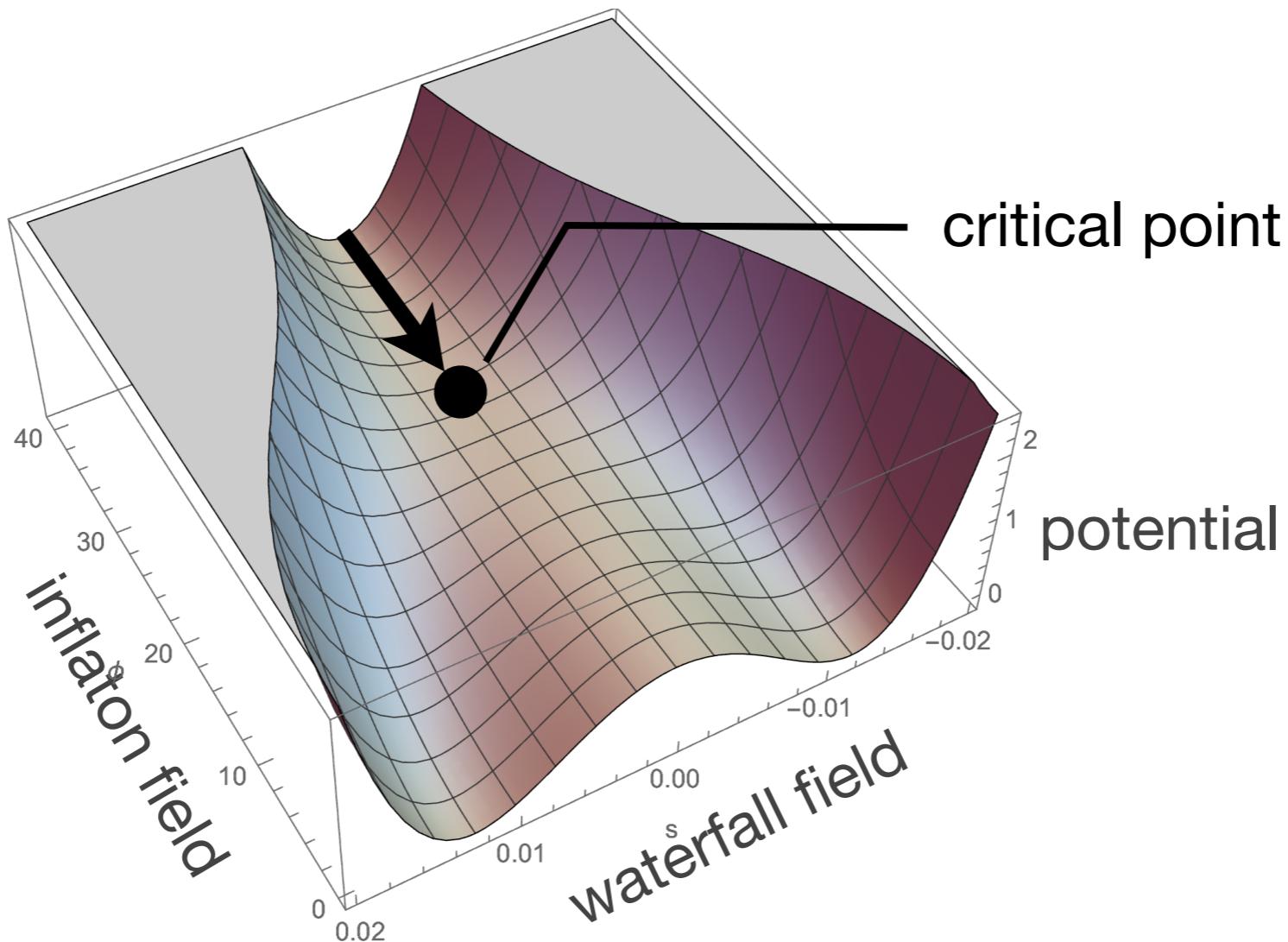


- Many inflation models have been proposed so far
- The CMB observations constrain the inflation models

# Hybrid inflation

Linde '93

- Inflation occurs until slow-rolling inflaton reaches the critical point
- It is simple model but not consistent with current CMB observations

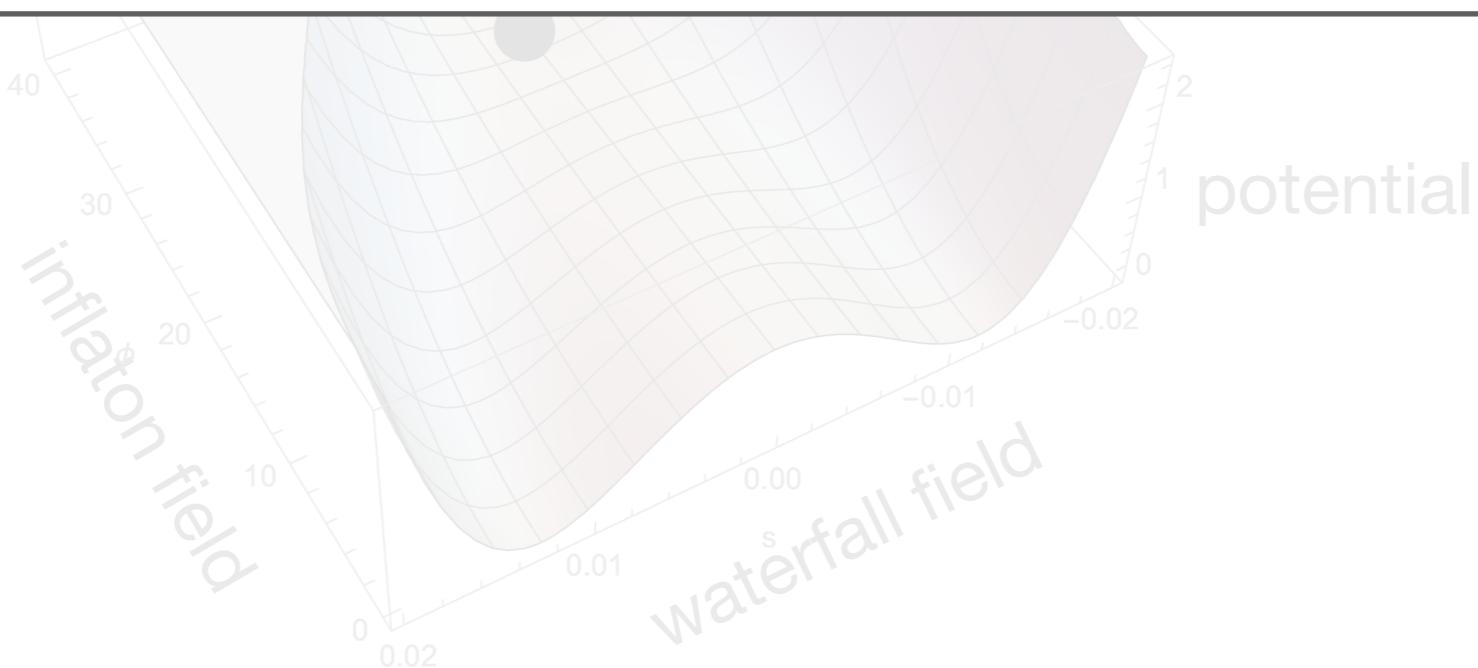


# Hybrid inflation

Linde '93

- Inflation occurs until slow-rolling inflaton reaches the critical point
- It is simple model but not consistent with current CMB observations

*D*-term hybrid inflation is revisited from new point of view



Various types of  $D$ -term hybrid inflation are realized depending on the symmetry of the Kähler potential

- Superconformal symmetry      Starobinsky type  
Buchmuller, Domcke, Schmitz '13  
Buchmuller, Domcke, Kamada '13
- Shift symmetry      Chaotic regime (**below critical point**)  
Buchmuller, Domcke, Schmitz '14  
Buchmuller, Ishiwata '13
- Superconformal  
+ approx. shift symmetry       $\alpha$ -attractor type (**below critical point**)  
Ishiwata '18

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## Subcritical hybrid inflation

- Shift symmetry

Chaotic regime (**below critical point**)

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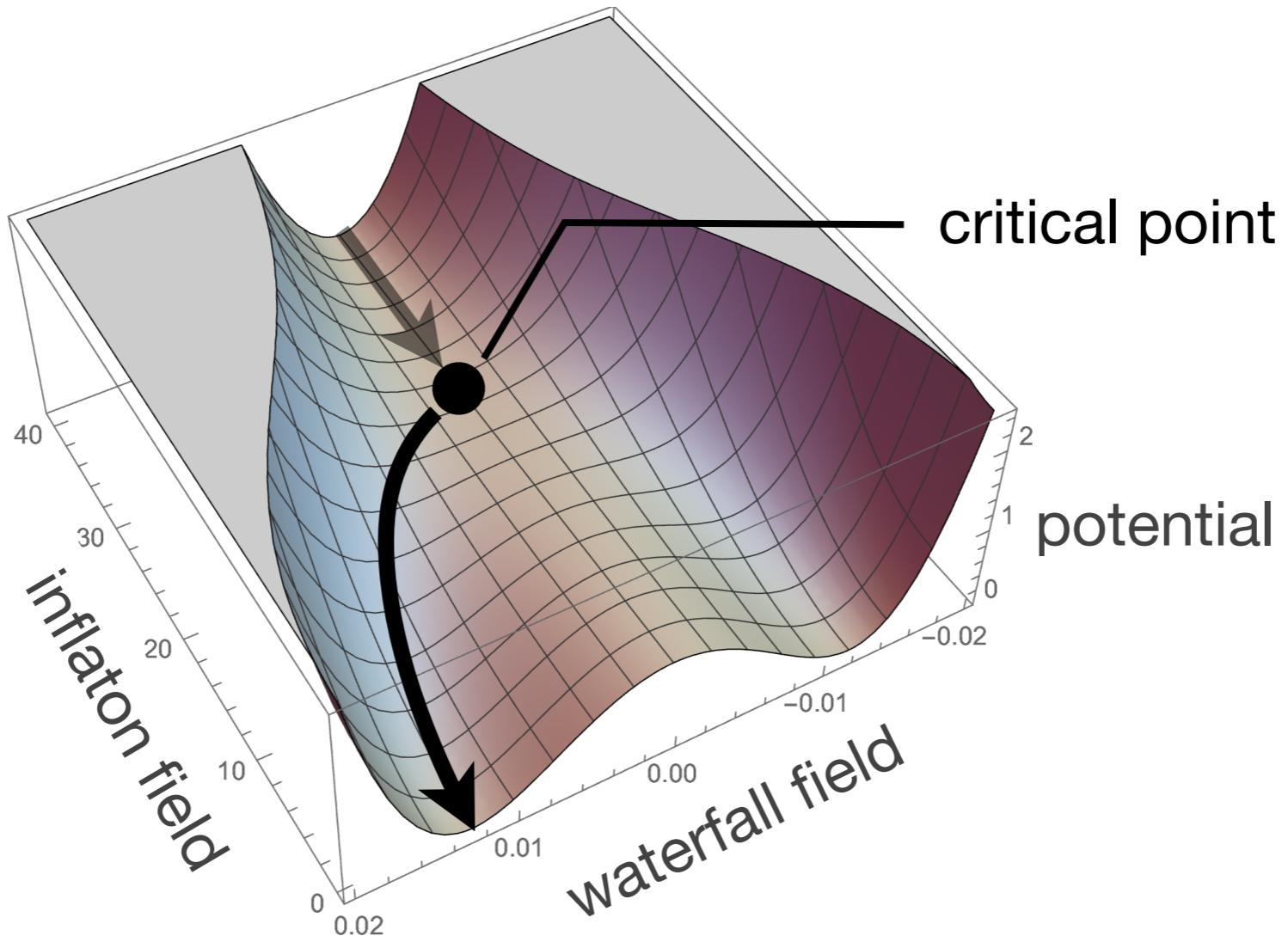
- Superconformal  
+ approx. shift symmetry

$\alpha$ -attractor type (**below critical point**)

Ishiwata '18

# Subcritical hybrid inflation

- Inflaton keeps slow-rolling after crossing the critical point
- Inflation continues in subcritical regime with growth of waterfall field



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## Superconformal subcritical hybrid inflation

- Superconformal  
+ approx. shift symmetry       $\alpha$ -attractor type (below critical point)  
Ishiwata '18

# Superconformal subcritical hybrid inflation

Ishiwata '18

- Superpotential

$$W = \lambda S_+ S_- N$$

	$S_+$	$S_-$	$N$
U(1)	$q$	$-q$	0

$q > 0$

- Kähler potential

$$K = -3 \log\left(-\frac{\Phi}{3}\right)$$

$$\text{with } \Phi = -3 + |S_+|^2 + |S_-|^2 + |N|^2 + \frac{\chi}{2}(N^2 + \bar{N}^2)$$

superconf. breaking term

$\lambda \ll 1$  &  $\chi \simeq -1$  ...  $\text{Re } N$  has an approx. shift sym.

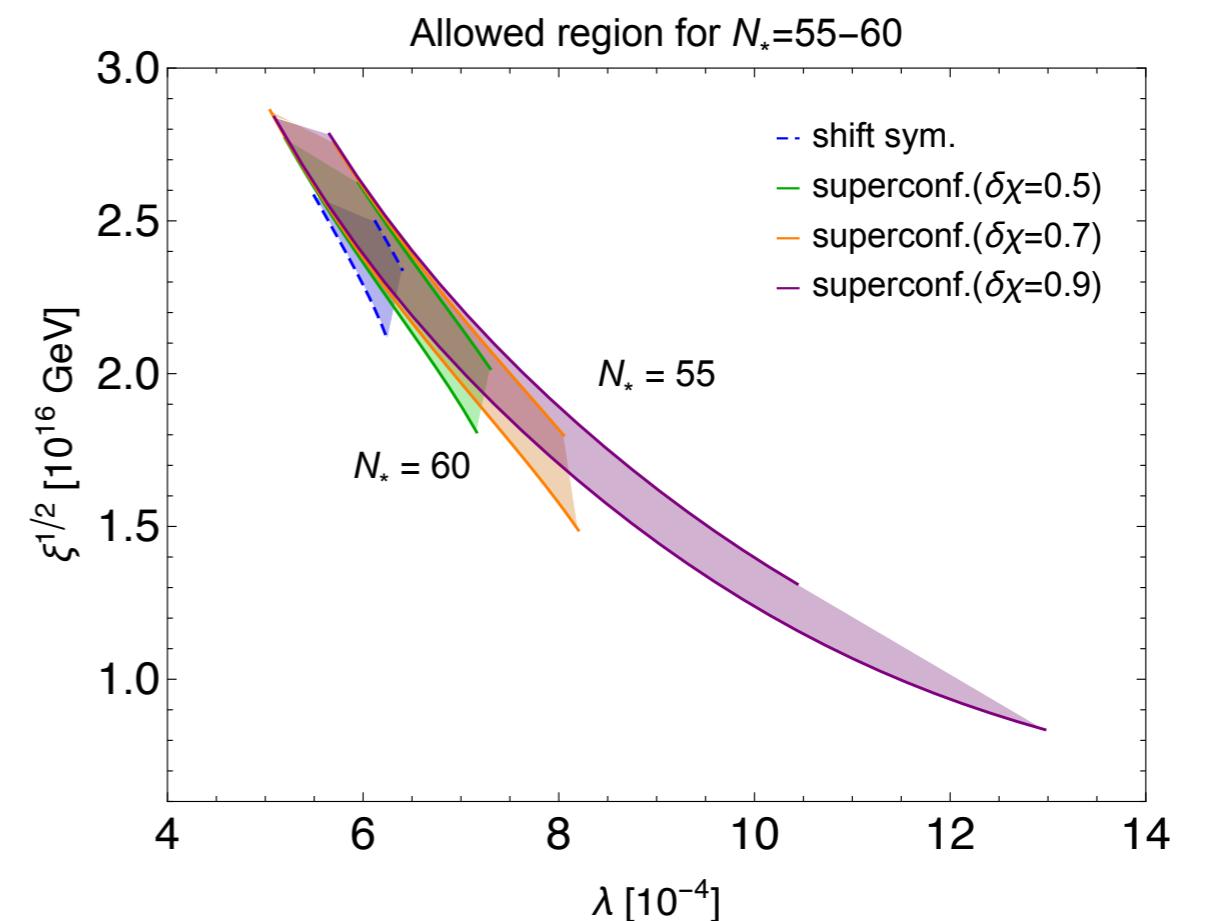
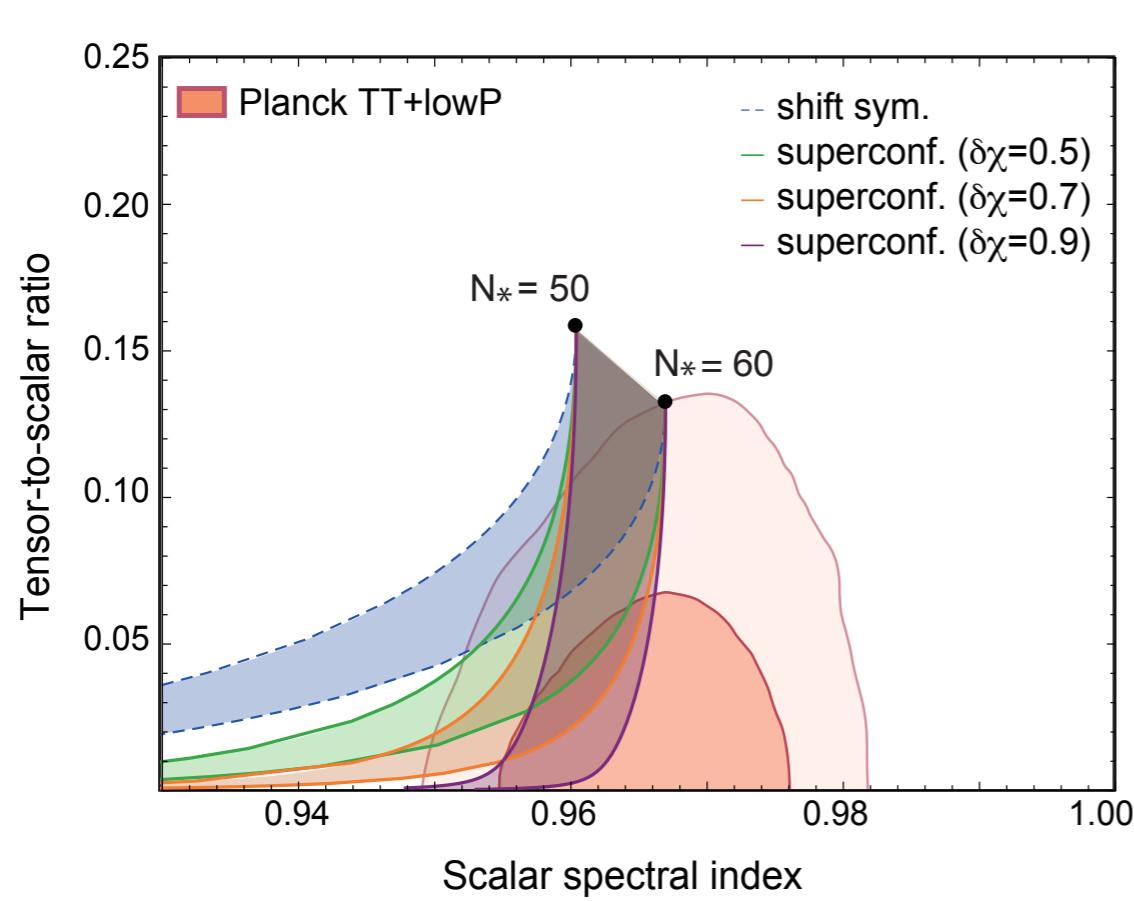
$\phi \equiv \sqrt{2} \text{Re } N$ : inflaton field

$s \equiv \sqrt{2} |S_+|$  : waterfall field

$M_{\text{pl}} = 1$

# Superconformal subcritical hybrid inflation

Ishiwata '18



This model is consistent with the CMB observations:

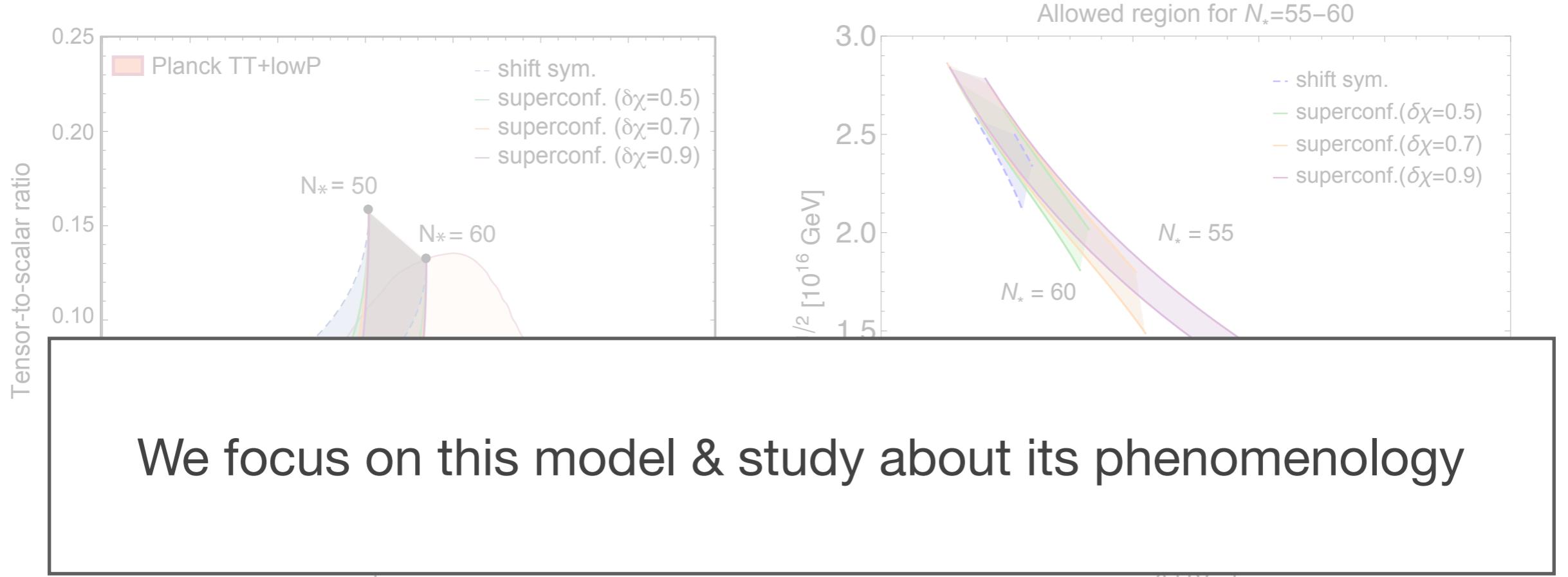
- Parameter values are  $\lambda \simeq 10^{-3}$ ,  $\sqrt{\xi} \simeq 10^{16} \text{ GeV}$
- Inflaton mass is  $m_\phi \simeq \lambda \sqrt{\xi} \simeq 10^{13} \text{ GeV}$

$\xi$ : constant Fayet-Iliopoulos term

$(\xi > 0)$

# Superconformal subcritical hybrid inflation

Ishiwata '18



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# Outline

1. Introduction
2. Leptogenesis after the inflation
3. Generalized superconformal subcritical hybrid inflation
4. Conclusions

## 2. Leptogenesis after the inflation

# The extended model

YG & Ishiwata '19

Introduce three right-handed neutrinos  $N_i^c$  with the Majorana masses

- Superpotential

$$W_{\text{neu}} = \lambda_i S_+ S_- N_i^c + \frac{1}{2} M_{ij} N_i^c N_j^c + y_{\nu ij} N_i^c L_j H_u$$

- Kähler potential

$$K = -3 \log\left(-\frac{\Phi}{3}\right)$$

$$\Phi = -3 + |S_+|^2 + |S_-|^2 + |N_i^c|^2 + \frac{\chi_i}{2} (N_i^{c2} + \bar{N}_i^{c2})$$

# The extended model

YG & Ishiwata '19

Consider a minimal extension  $\begin{cases} \lambda_3 \neq 0, \chi_3 \simeq -1, \text{ The others} = 0 \\ M_{ij} = \text{diag}(M_1, M_2, M_3) \end{cases}$

$$W_{\text{neu}} = \lambda_3 S_+ S_- N_3^c + \frac{1}{2} M_i N_i^c N_i^c + y_{\nu ij} N_i^c L_j H_u$$

$$\Phi = -3 + |S_+|^2 + |S_-|^2 + |N_i^c|^2 + \frac{\chi_3}{2} (N_3^{c2} + \bar{N}_3^{c2})$$

$\phi \equiv \sqrt{2} \operatorname{Re} \tilde{N}_3^c$  : inflaton field

$s \equiv \sqrt{2} |S_+|$  : waterfall field

We study the thermal history after the inflation in this setup

# The effect of $M_3$ on inflation

An additional term appears in the inflaton potential by the extension

$$V_{\text{inf}} = V + \Delta V(M_3) \quad W_{\text{inf}} = \lambda_3 N_3^c S_+ S_- + \frac{1}{2} M_3 N_3^c N_3^c + \dots$$

The condition to avoid the effect of  $\Delta V$  on the inflationary trajectory

$$\Delta V(M_3)/V \ll 1$$

The upper bound on  $M_3$

$$M_3 \lesssim 2 \times 10^{11} \text{ GeV} \ll m_\phi \simeq 10^{13} \text{ GeV} \quad m_\phi : \text{inflaton mass}$$

# The effect of $M_{1,2}$ on inflation

On the other hand, there are no restrictions on  $M_{1,2}$

...  $M_{1,2}$  are free parameters

We consider the thermal history in two cases:

(I) .  $M_1, M_2 < m_\phi$

(II) .  $M_1, M_2 > m_\phi$   $m_\phi \simeq 10^{13} \text{ GeV}$ : Inflaton mass

# Light neutrino masses

Seesaw mechanism:  $M_\nu = -\tilde{m}_\nu^T \tilde{M}^{-1} \tilde{m}_\nu$

$$\tilde{m}_\nu = \begin{pmatrix} m_\nu \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_{\nu ij} = y_{\nu ij} \langle H_u^0 \rangle$$

4 × 3 matrix

$$\tilde{M} = \begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & M_3 & \\ \hline & & m_\phi & 0 \end{pmatrix}$$

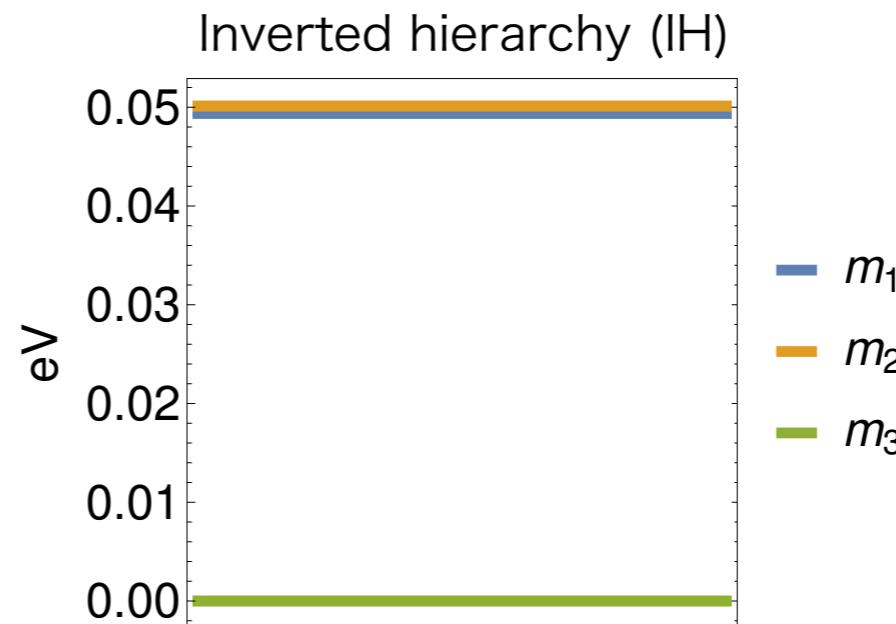
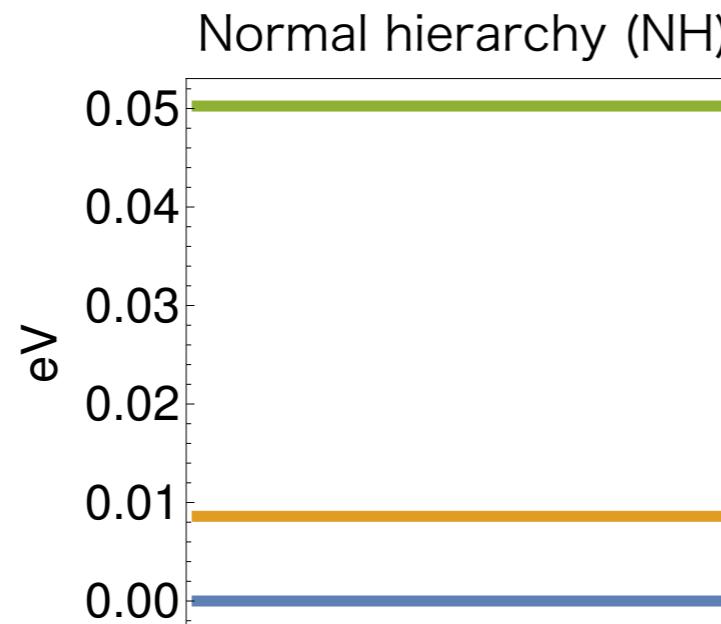
4 × 4 matrix

The mass matrix is different from a conventional one

# Light neutrino masses

$$M_{\nu ij} \simeq \langle H_u^0 \rangle^2 \sum_{k=1}^2 \frac{y_{\nu ki} y_{\nu kj}}{M_k}$$

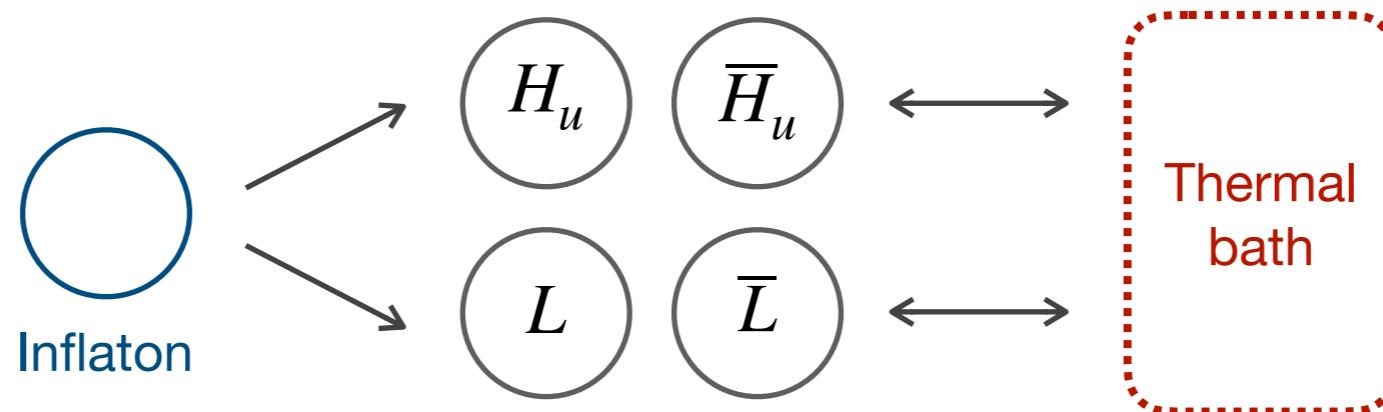
- The lightest neutrino mass is zero ( $\because \text{rank } M_\nu = 2$ )



- $m_\phi$ ,  $M_3$ , and  $y_{\nu 3i}$  are not restricted from neutrino experiments  
It determines the reheating temperature

# Reheating

The universe becomes radiation dominated by the inflaton decay



Reheating temperature

$$T_R \simeq 1.4 \times 10^{10} \text{ GeV} \left( \frac{(y_\nu y_\nu^\dagger)_{33}}{10^{-9}} \right)^{1/2} \propto |y_{\nu 3i}|$$

...  $T_R$  is a free parameter

For simplicity, we focus on a simple situation,  $T_R < m_\phi$

# Leptogenesis

Lepton asymmetry is produced, which is then converted to baryon asymmetry  
Fukugita, Yanagida '86

We have considered two representative cases:

(I) .  $M_1, M_2 < m_\phi$  ... Thermal leptogenesis

(II) .  $M_1, M_2 > m_\phi$  ... Non-thermal leptogenesis

$$m_\phi \simeq 10^{13} \text{ GeV}$$

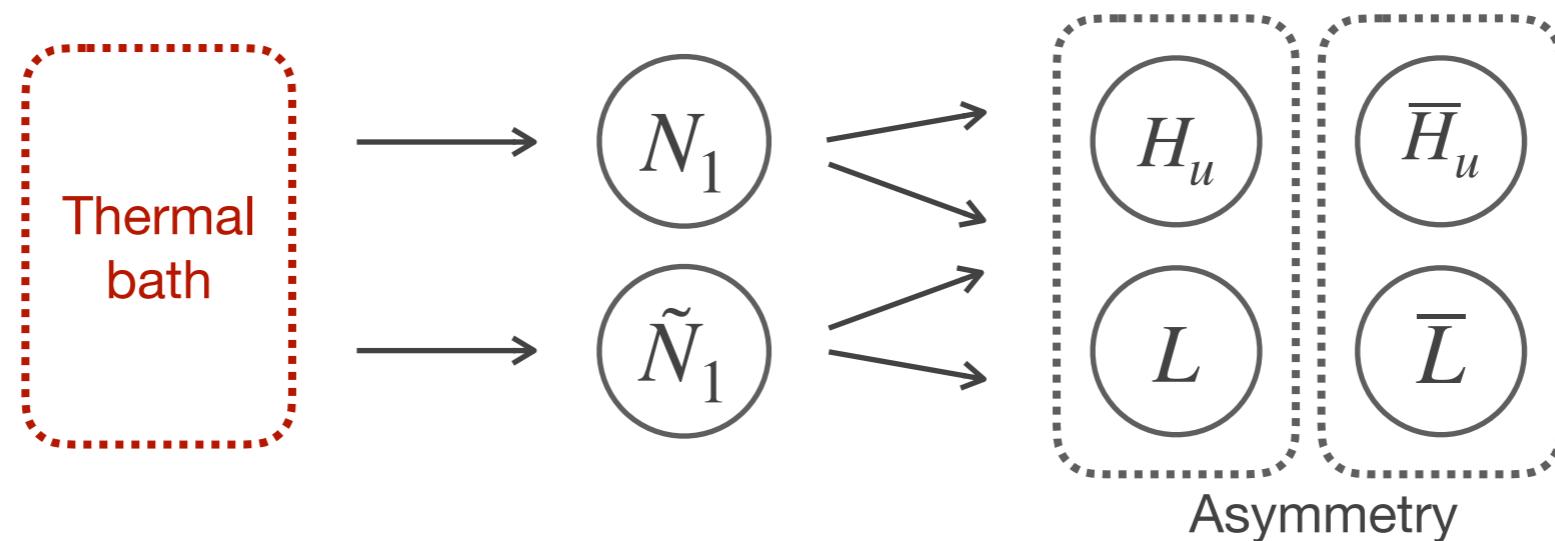
Current baryon asymmetry

$$\eta_B^{\text{obs}} \equiv \frac{n_{B0}}{n_{\gamma 0}} = (6.12 \pm 0.03) \times 10^{-10}$$

Planck '18

# Case (I) . $M_1, M_2 < m_\phi$

Leptogenesis by decay of thermally produced **right-handed (s)neutrinos**



Simple situation  $M_1 \ll M_2 \& M_1 \lesssim T_R$

**Produced baryon asymmetry**

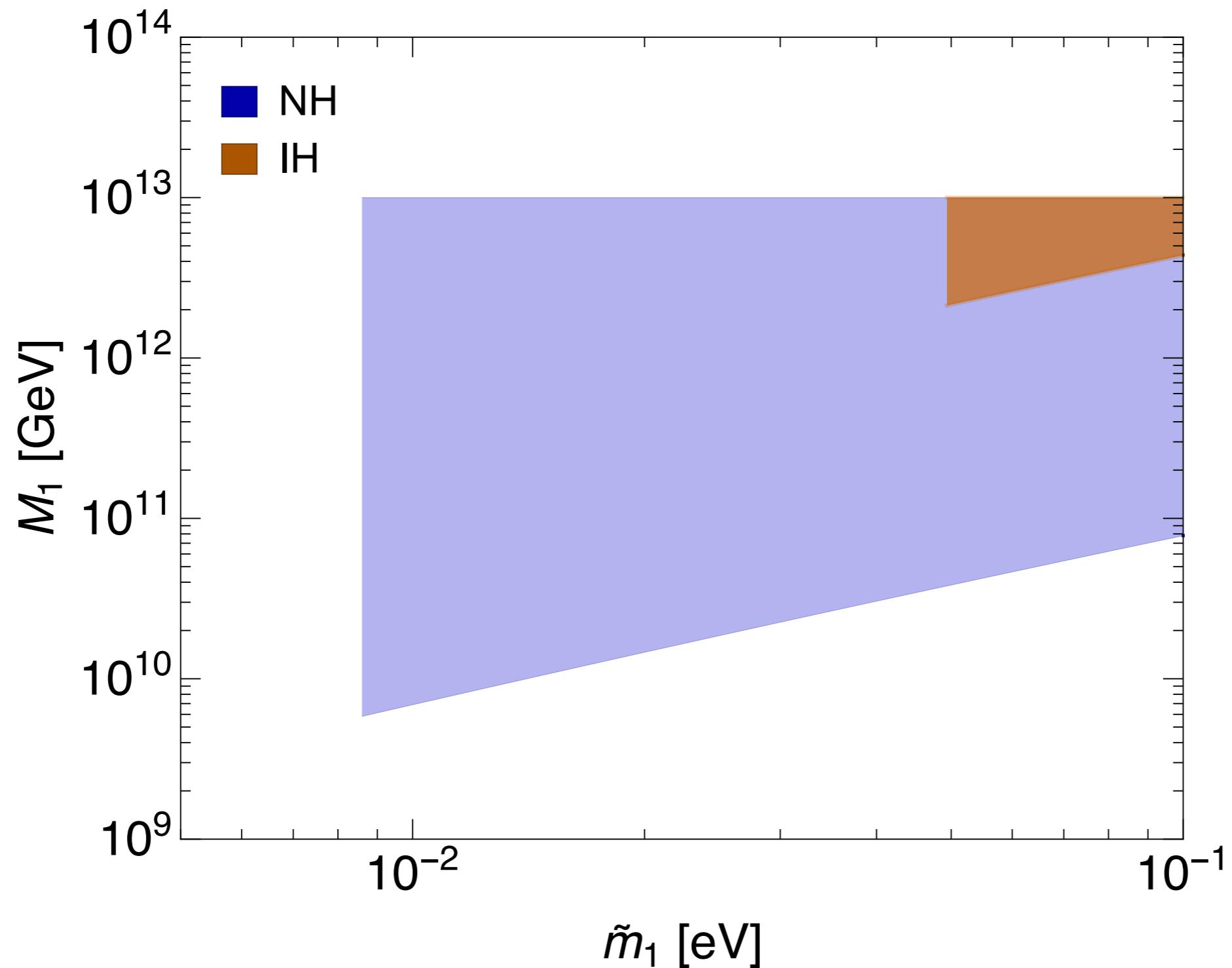
$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq 2.7 \times 10^{-10} \left( \frac{\epsilon_1}{10^{-6}} \right) \left( \frac{\kappa_f}{2 \times 10^{-2}} \right)$$

Buchmüller, Di Bari, Plümacher '05

Asymmetric parameter

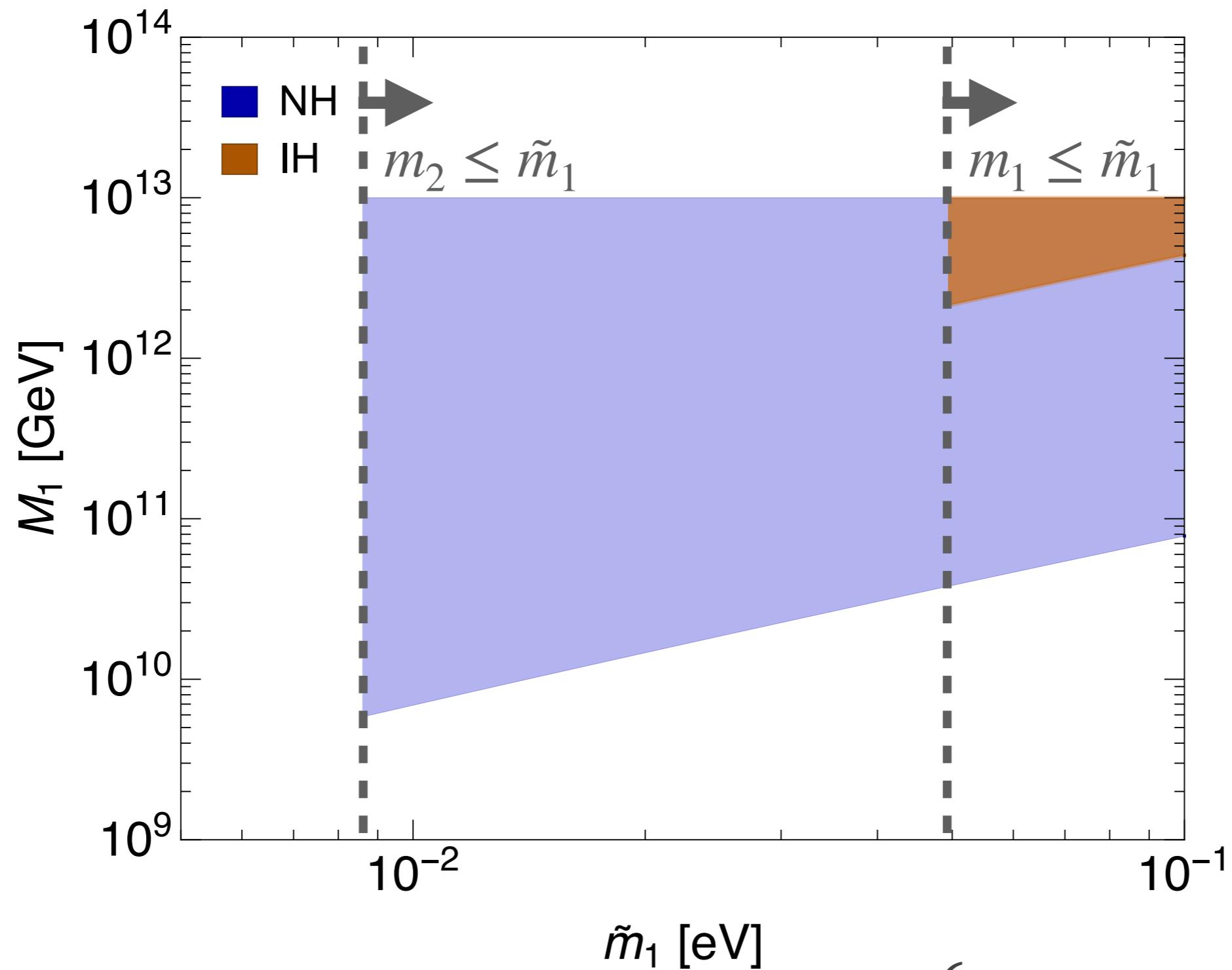
$$\epsilon_1 = \begin{cases} 8.2 \times 10^{-7} \text{ (NH)} \\ 1.5 \times 10^{-8} \text{ (IH)} \end{cases} \times \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \left( \frac{\sin \delta}{0.5} \right)$$

Case (I) .  $M_1, M_2 < m_\phi$



$$\tilde{m}_1 \equiv \frac{(m_\nu m_\nu^\dagger)_{11}}{M_1}$$

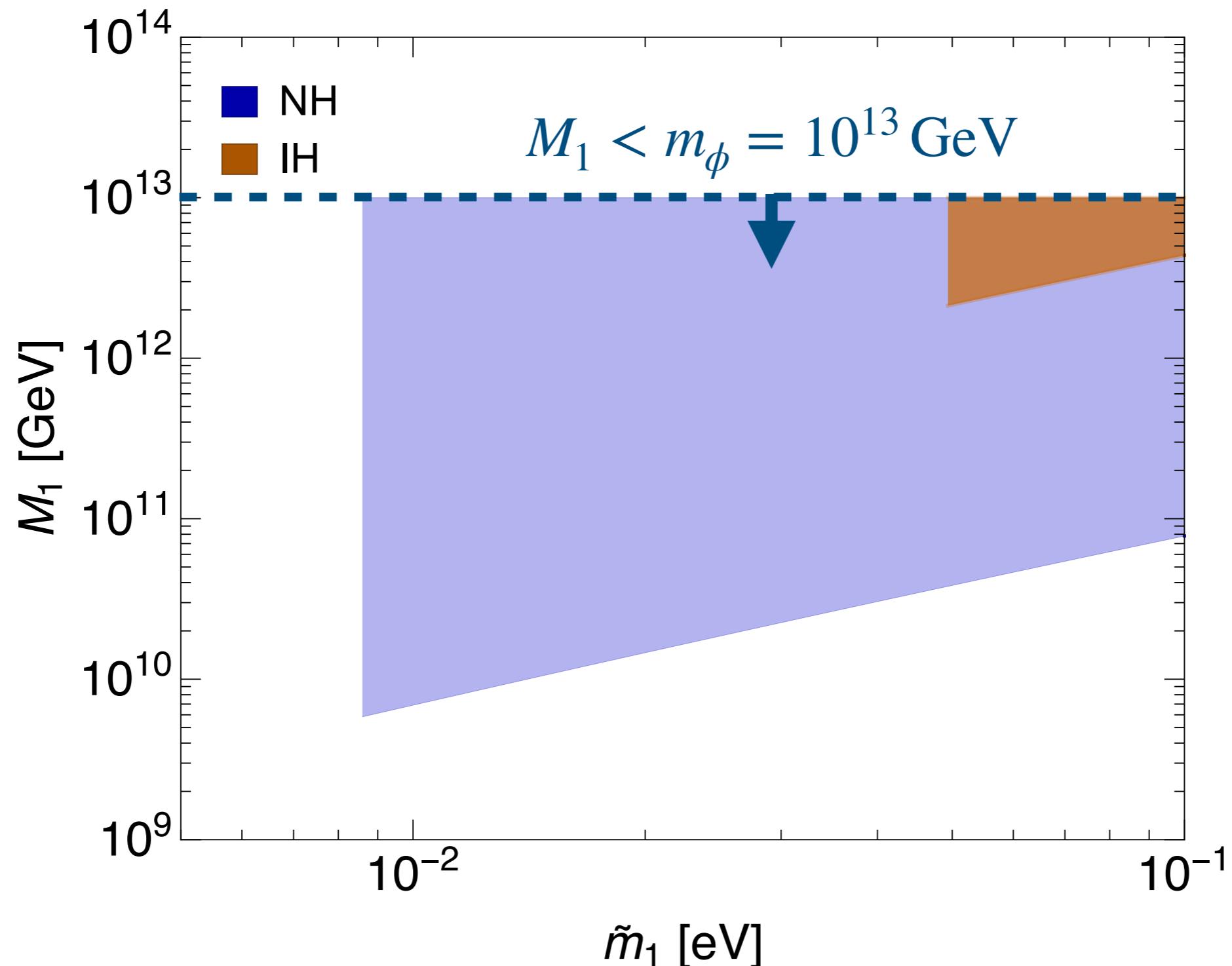
Case (I) .  $M_1, M_2 < m_\phi$



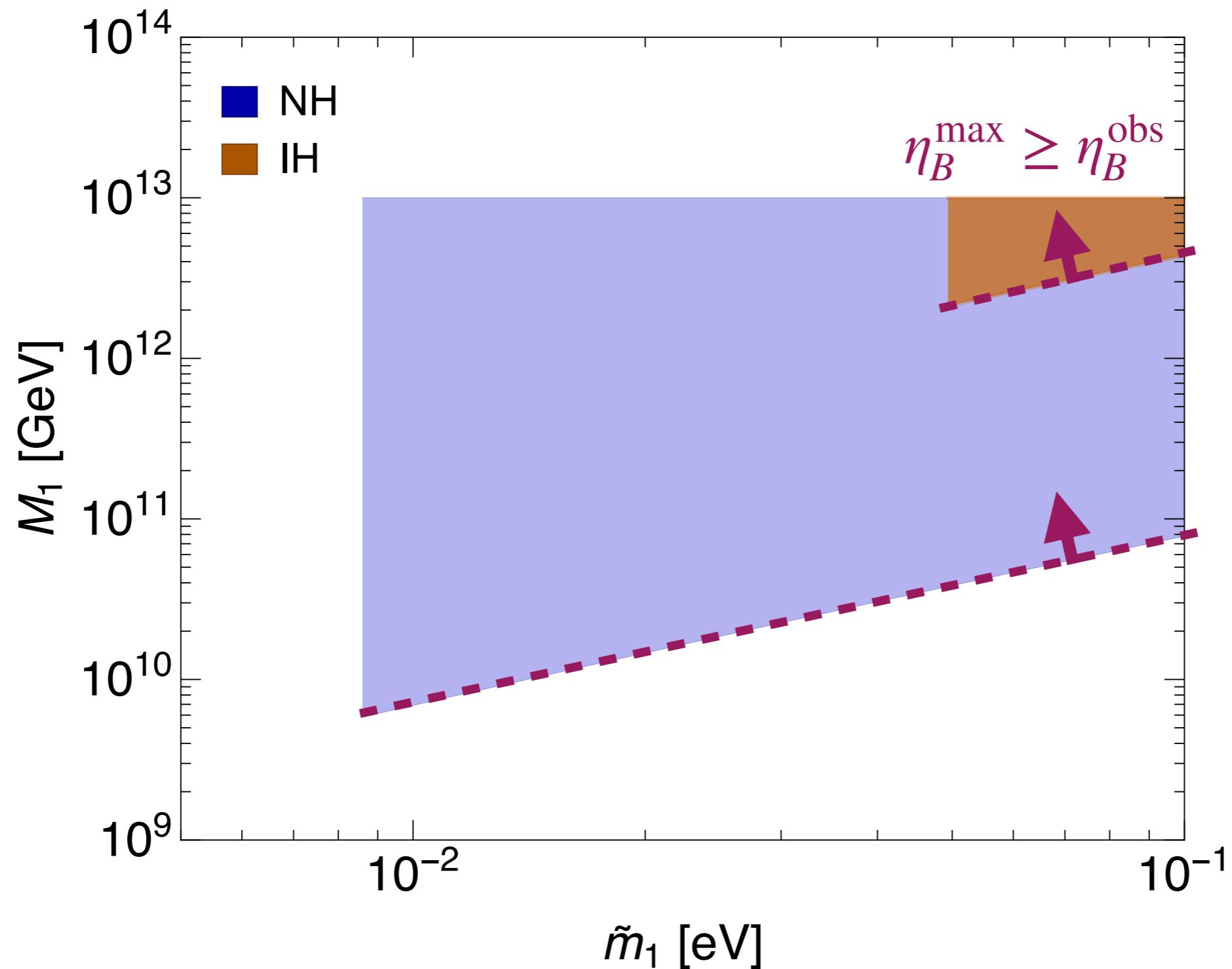
$\tilde{m}_1$  [eV]

$$\tilde{m}_1 \geq \begin{cases} m_2 \simeq 8.6 \times 10^{-3} \text{ eV (NH)} \\ m_1 \simeq 4.9 \times 10^{-2} \text{ eV (IH)} \end{cases}$$

Case (I) .  $M_1, M_2 < m_\phi$



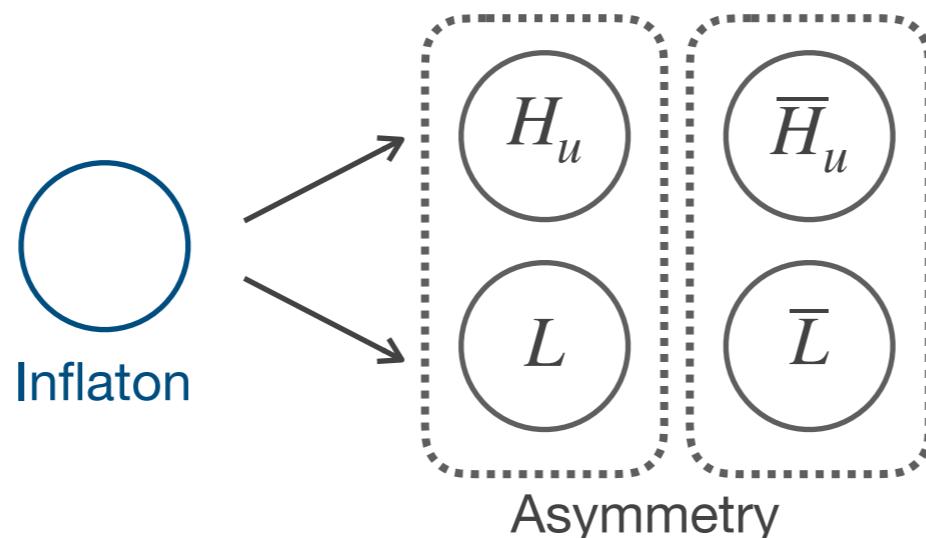
Case (I) .  $M_1, M_2 < m_\phi$



Successful leptogenesis is realized in wide range of parameter space

## Case (II) . $M_1, M_2 > m_\phi$

Leptogenesis by the **inflaton decay**



Murayama, Suzuki, Yanagida, Yokoyama '93

Hamaguchi, Murayama, Yanagida '02

Ellis, Raidal, Yanagida '04

Nakayama, Takahashi, Yanagida '16

Simple situation  $T_R/m_\phi \ll 1$

Produced baryon asymmetry

$$\eta_B = \frac{3}{4} \frac{T_R}{m_\phi} a_{\text{sph}} d \epsilon_\phi$$

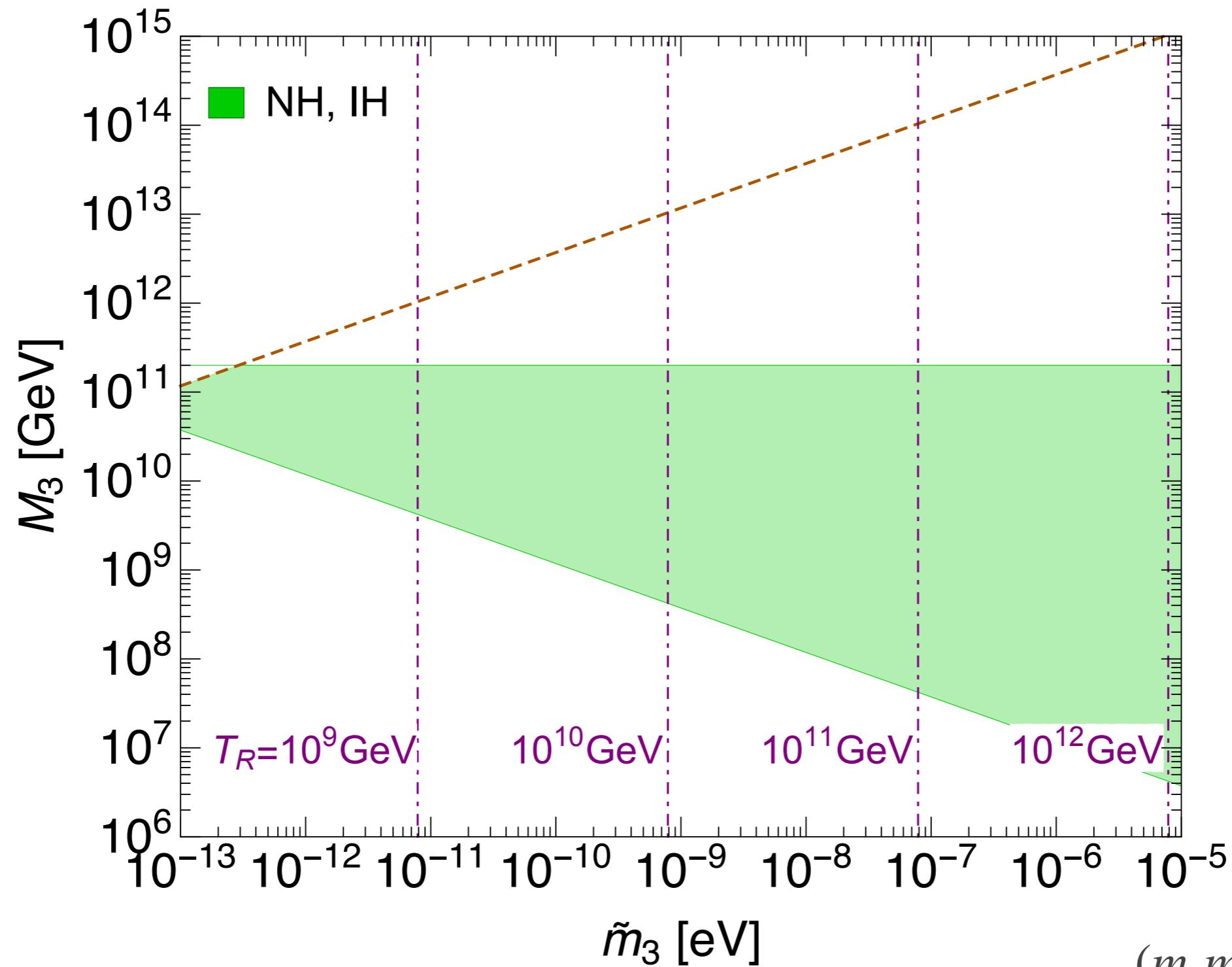
$$a_{\text{sph}} = 28/79$$

$$d = s_0 / n_{\gamma 0}$$

Asymmetric parameter

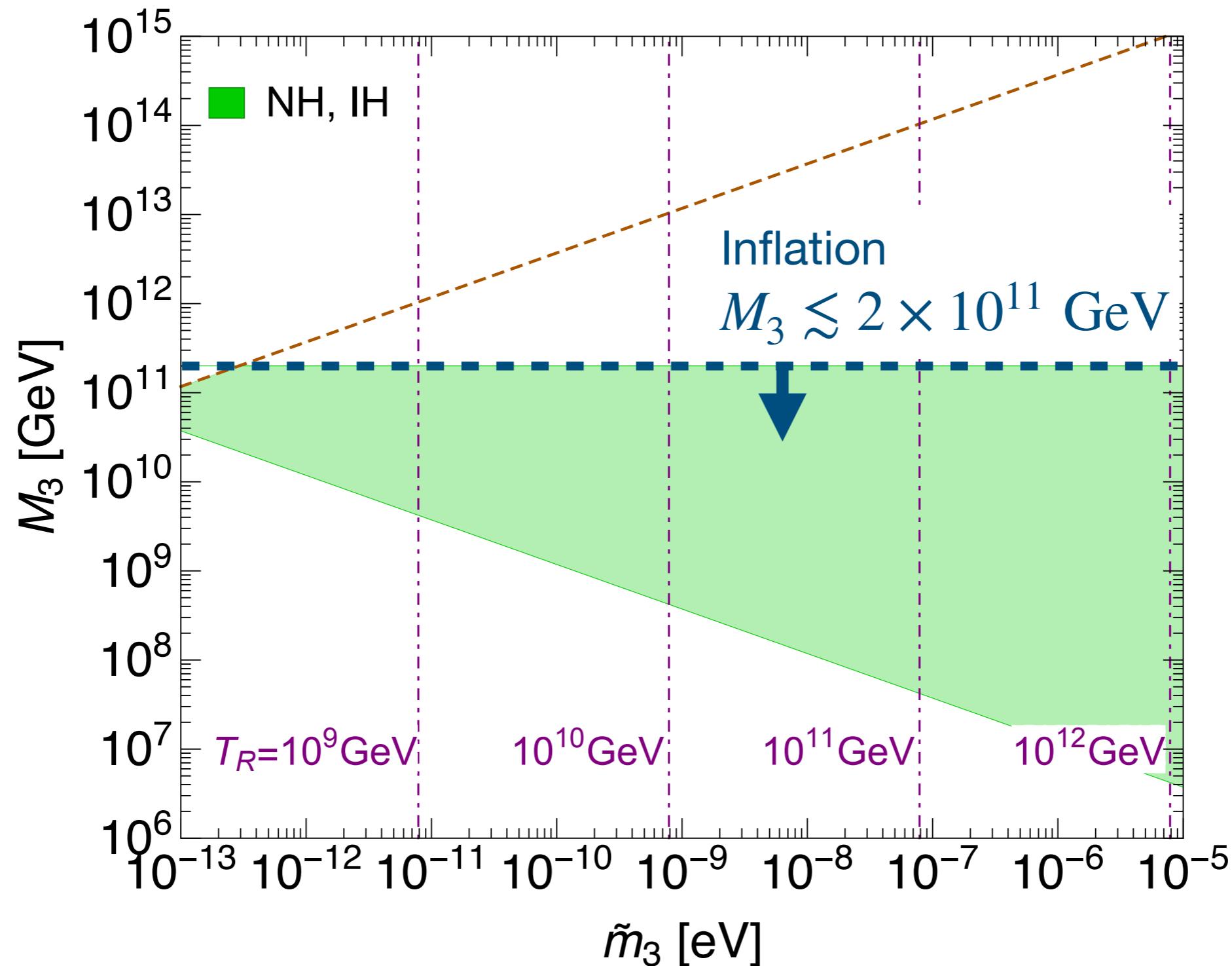
$$\epsilon_\phi \simeq 3.9 \times 10^{-9} \left( \frac{M_3}{10^7 \text{ GeV}} \right) \left( \frac{\sin \delta'}{0.5} \right)$$

Case (II) .  $M_1, M_2 > m_\phi$

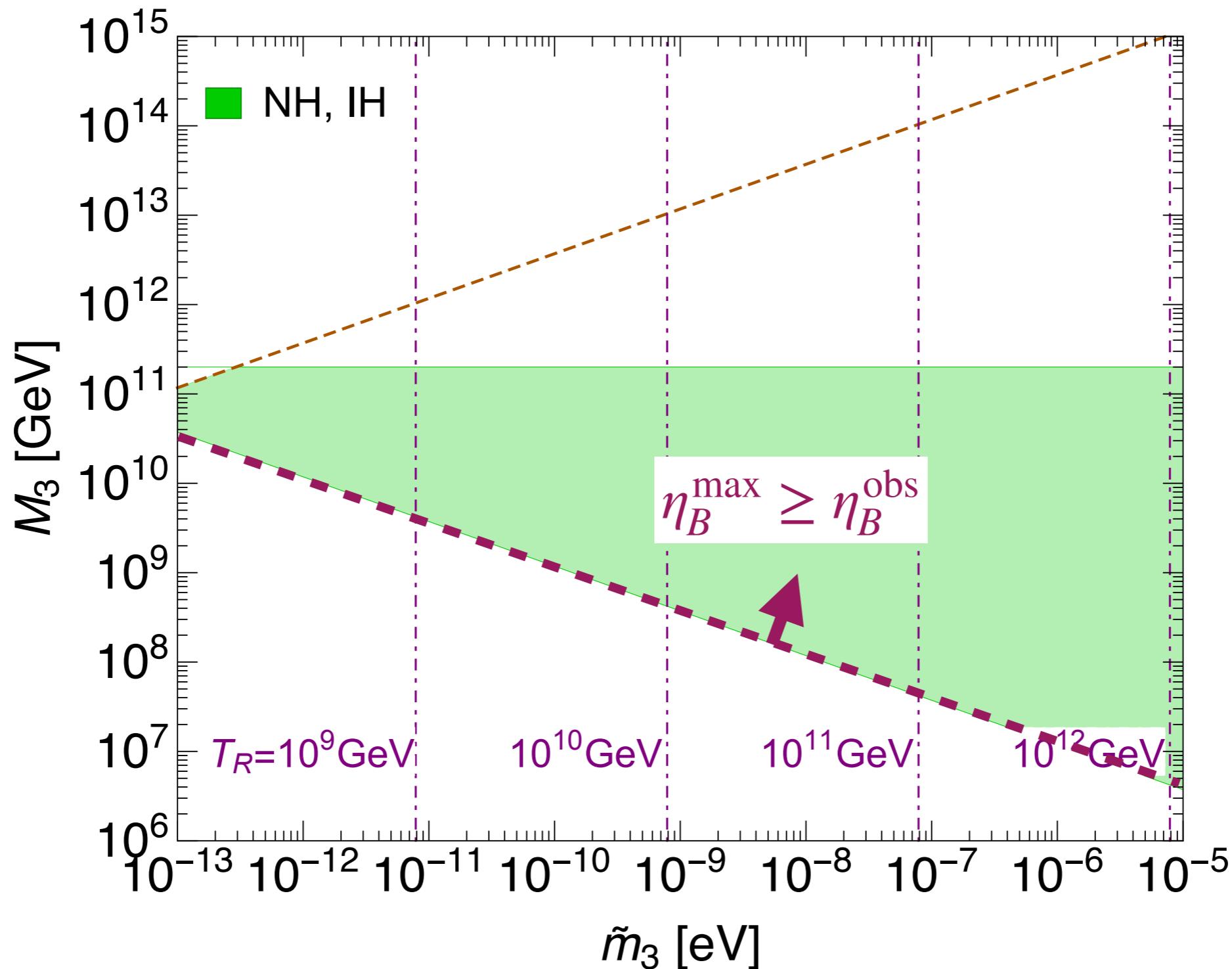


$$\tilde{m}_3 \equiv \frac{(m_\nu m_\nu^\dagger)_{33}}{m_\phi} \propto |y_{\nu 3i}|^2$$

Case (II) .  $M_1, M_2 > m_\phi$



## Case (II) . $M_1, M_2 > m_\phi$



Successful leptogenesis is realized in wide range of parameter space

# Summary

We have studied the leptogenesis after superconformal subcritical hybrid inflation in an extended model by introducing three right-handed neutrinos

- One of the right-handed sneutrinos plays a role of inflaton
- Light neutrino mass matrix given by seesaw mechanism has an unconventional structure
- Inflaton decay reheats the universe and the temperature is a free parameter
- Thermal or Non-thermal leptogenesis can be realized

### 3. Generalized superconformal subcritical hybrid inflation

# The generalized model YG & Ishiwata '21

- Superpotential

$$W = \lambda S_+ S_- N$$

	$S_+$	$S_-$	$N$
U(1)	$q$	$-q$	0

$q > 0$

- Kähler potential

$$K = -3\alpha \log\left(-\frac{\Phi}{3}\right)$$

$\alpha = 1$  in typical model  
 $\alpha > 0$  in this model

$$\text{with } \Phi = -3 + |S_+|^2 + |S_-|^2 + |N|^2 + \frac{\chi}{2}(N^2 + \bar{N}^2)$$

superconf. breaking term

$\phi \equiv \sqrt{2} \operatorname{Re} N$ : inflaton field

$s \equiv \sqrt{2} |S_+|$  : waterfall field

$M_{\text{pl}} = 1$

# The generalized model YG & Ishiwata '21

- $F$ -term potential

$$V_F = \left( -\frac{\Phi(\phi, s)}{3} \right)^{1-3\alpha} \frac{\lambda^2}{4\alpha} \phi^2 s^2$$

- $D$ -term potential

$$V_D = \frac{g^2}{8} \left[ \left( -\frac{\Phi(\phi, s)}{3} \right)^{-1} \alpha q s^2 - 2\xi \right]^2$$

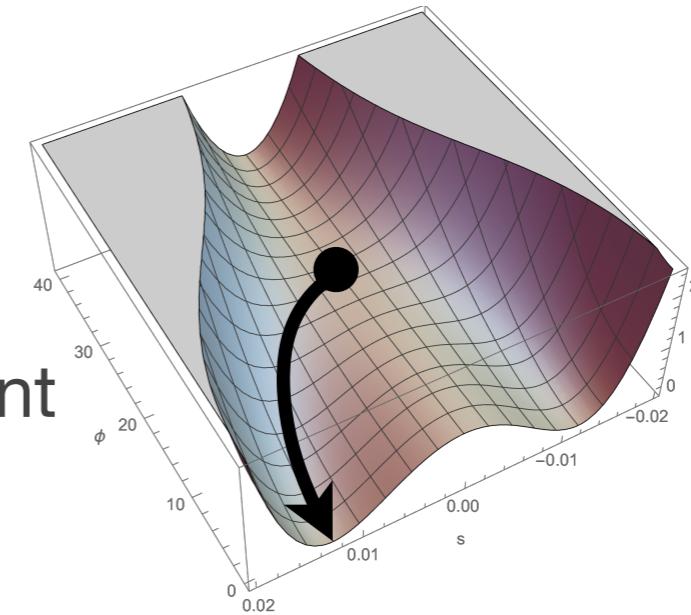
$$\Phi(\phi, s) = -3 + \frac{1}{2} (s^2 + (1 + \chi)\phi^2)$$

$g$  : gauge coupling constant

$\xi$  : constant Fayet-Iliopoulos term ( $\xi > 0$ )

# Inflaton potential

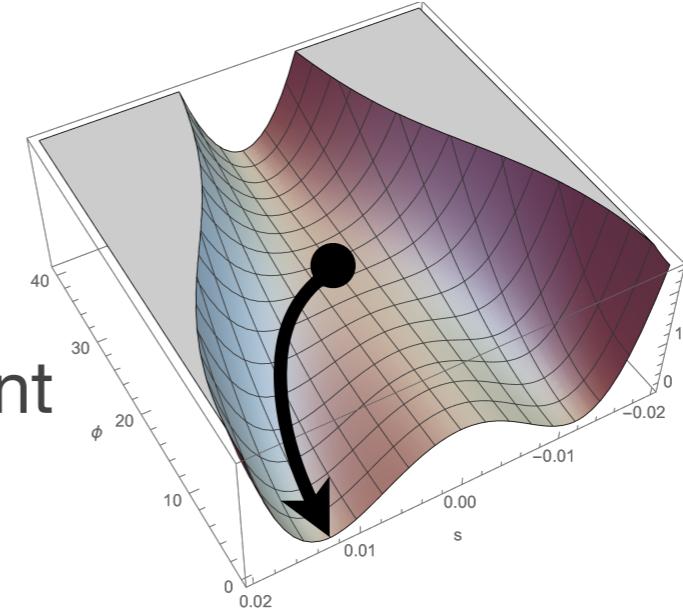
$s = s_{\min}(\phi)$  after critical point



# Inflaton potential

$$V_{\text{tot}}(\phi, s) = V_F + V_D$$

$s = s_{\min}(\phi)$  after critical point



Potential in subcritical regime

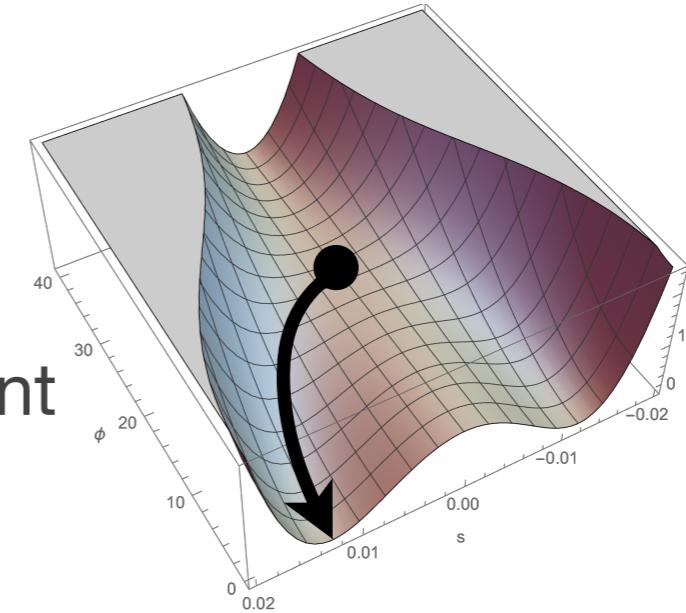
$$V(\phi) = g^2 \xi^2 \Psi(\phi) \left( 1 - \frac{1}{2} \Psi(\phi) \right)$$

$$\Psi(\phi) = \frac{k}{2\alpha^2} \left( \frac{\Phi(\phi, 0)}{3} \right)^{2-3\alpha} \phi^2$$
$$k \equiv \lambda^2 / q g^2 \xi$$

# Inflaton potential

$$V_{\text{tot}}(\phi, s) = V_F + V_D$$

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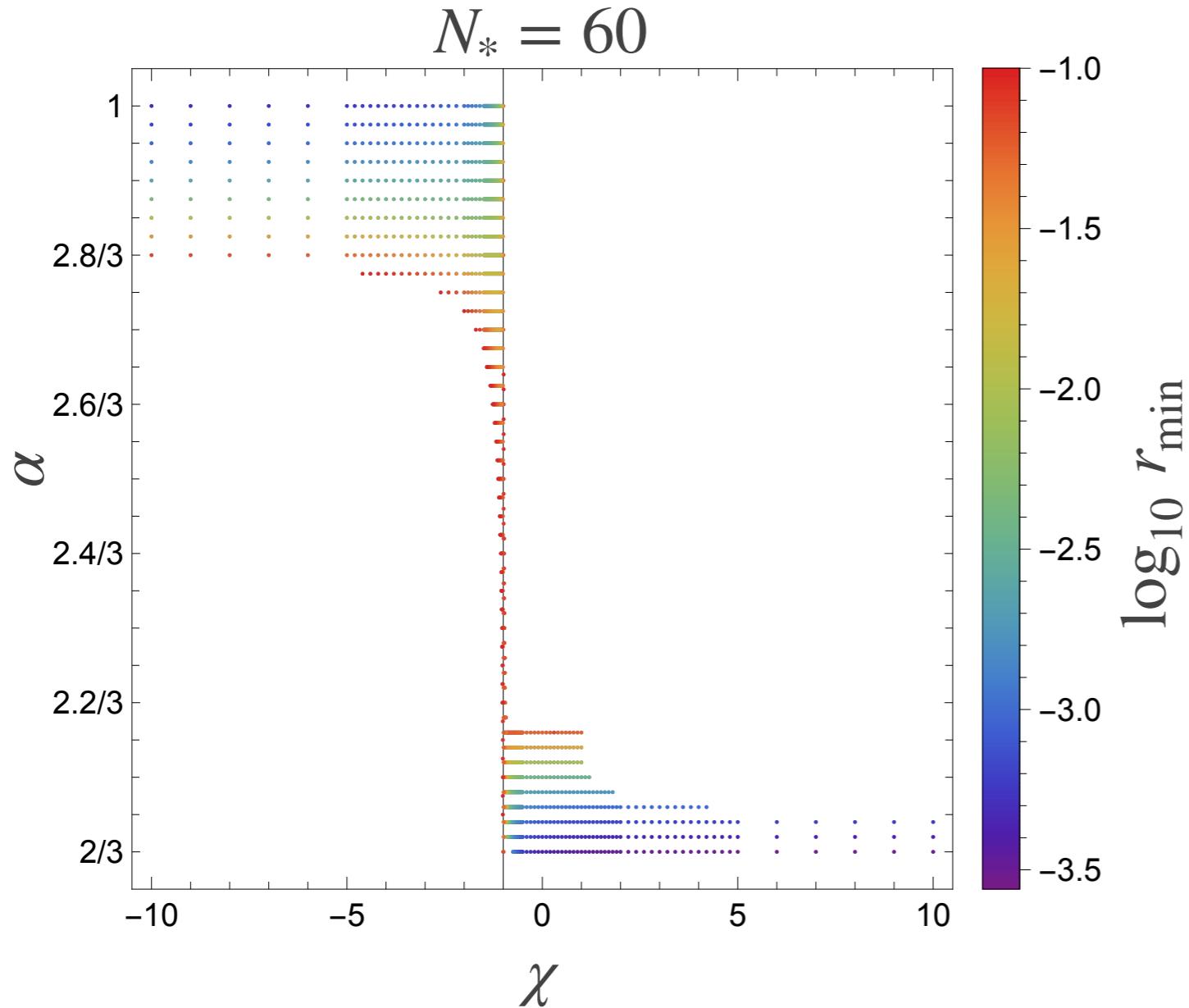
Potential in subcritical regime

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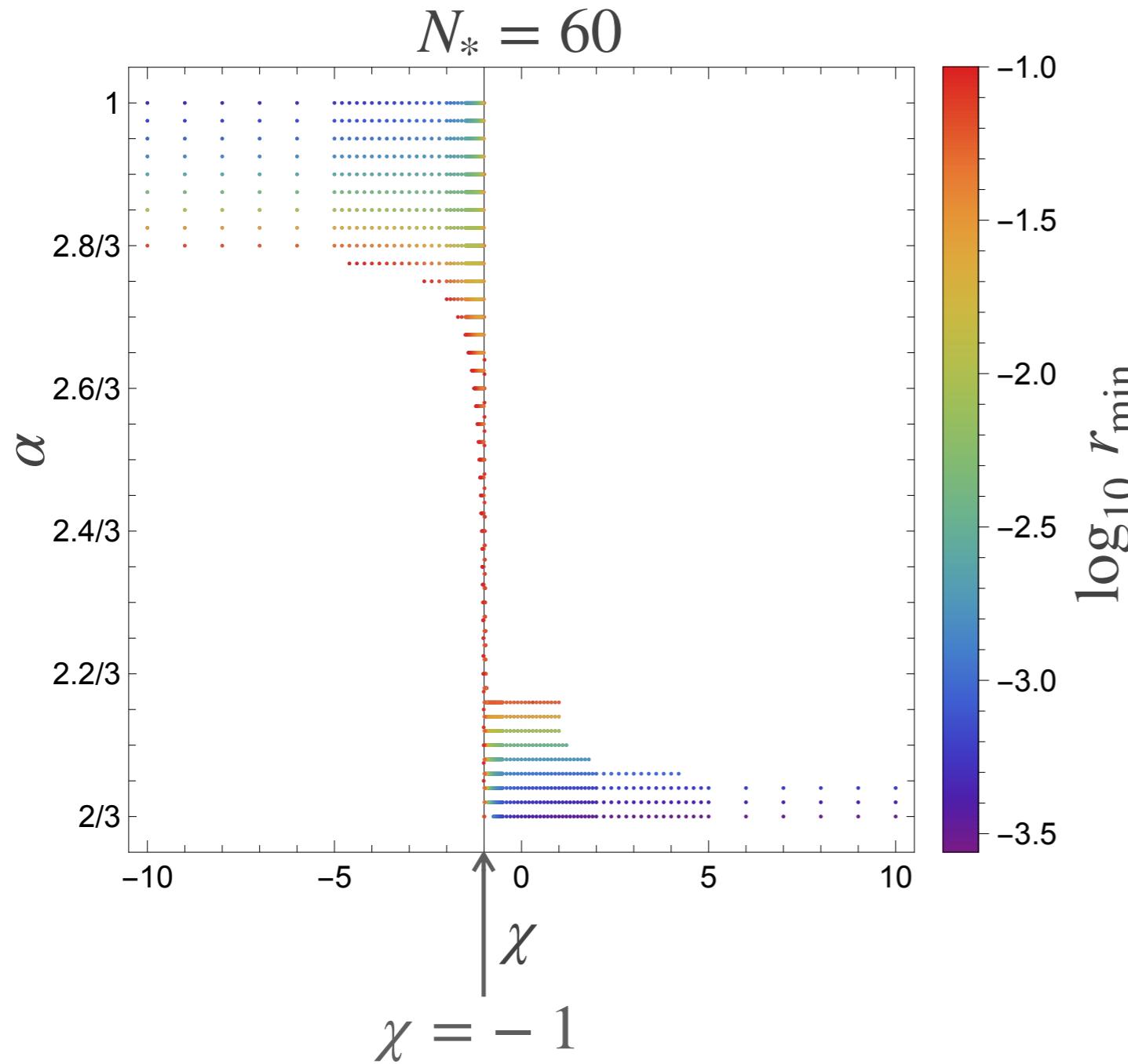
$$\begin{aligned}\Psi(\phi) &= \frac{k}{2\alpha^2} \left( \frac{\Phi(\phi, 0)}{3} \right)^{2-3\alpha} \phi^2 \\ k &\equiv \lambda^2 / qg^2 \xi\end{aligned}$$

Use  $V(\phi)$  to identify params. consistent with CMB  
& to clarify prediction of tensor-to-scalar ratio  $r$

# Allowed region for $\alpha$ & $\chi$



# Allowed region for $\alpha$ & $\chi$

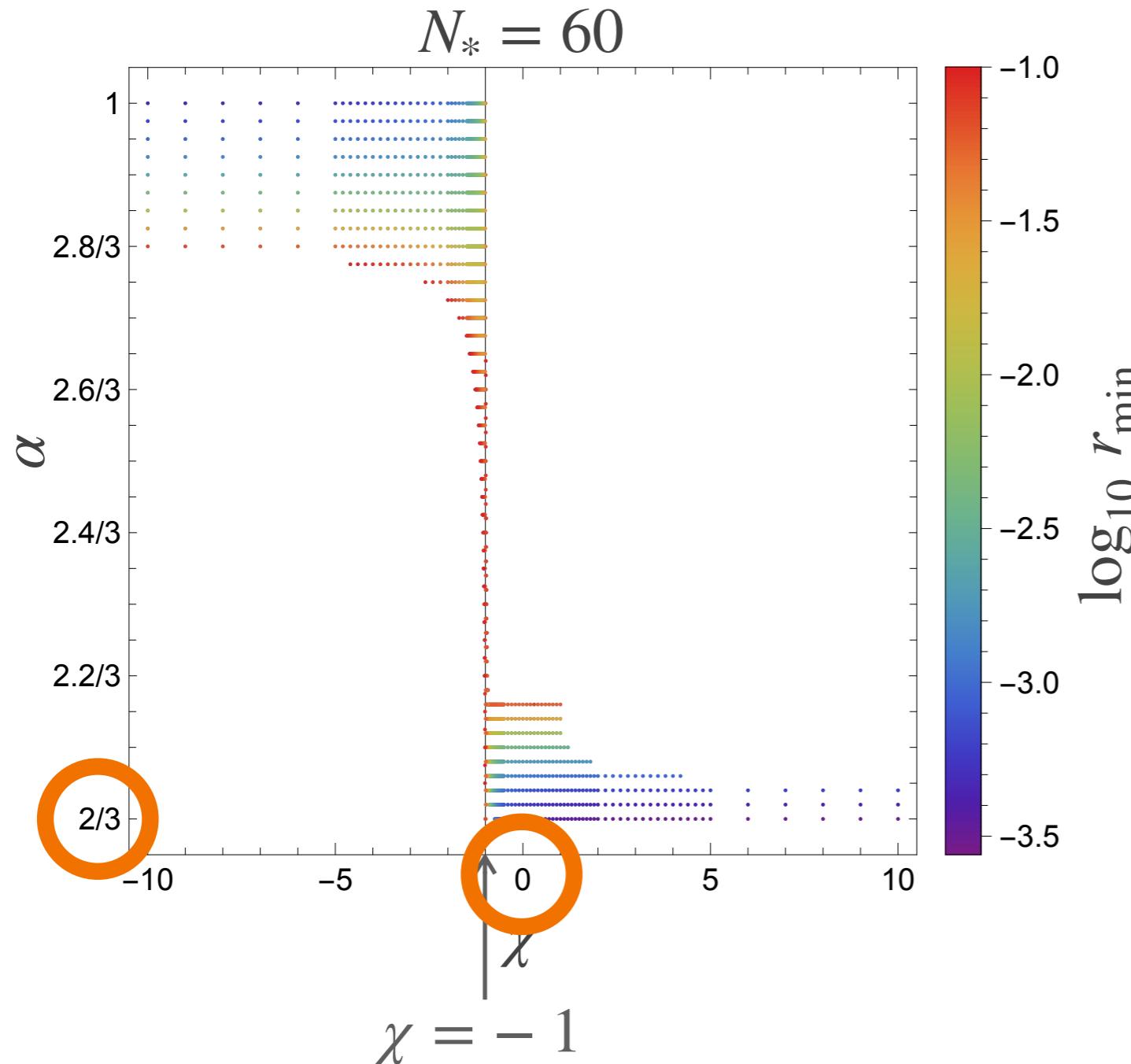


- Behavior of allowed region changes around  $\chi = -1$ 

$$\begin{cases} \alpha \approx 1 & (\chi \lesssim -5) \\ 2/3 \lesssim \alpha \lesssim 1 & (\chi \approx -1) \\ \alpha \approx 2/3 & (\chi \gtrsim 5) \end{cases}$$
- Predicted  $r$  changes depending on  $\alpha$  &  $\chi$ 

$$10^{-4} \lesssim r \lesssim 10^{-1}$$

# Allowed region for $\alpha$ & $\chi$

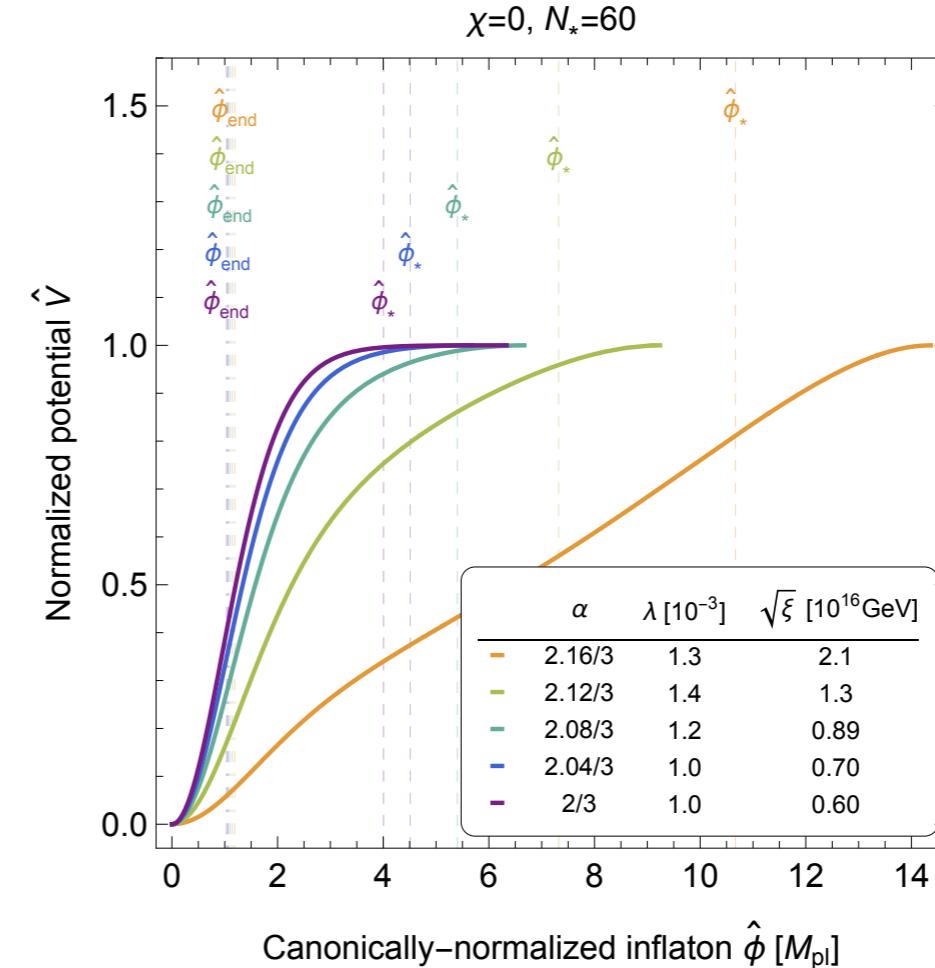
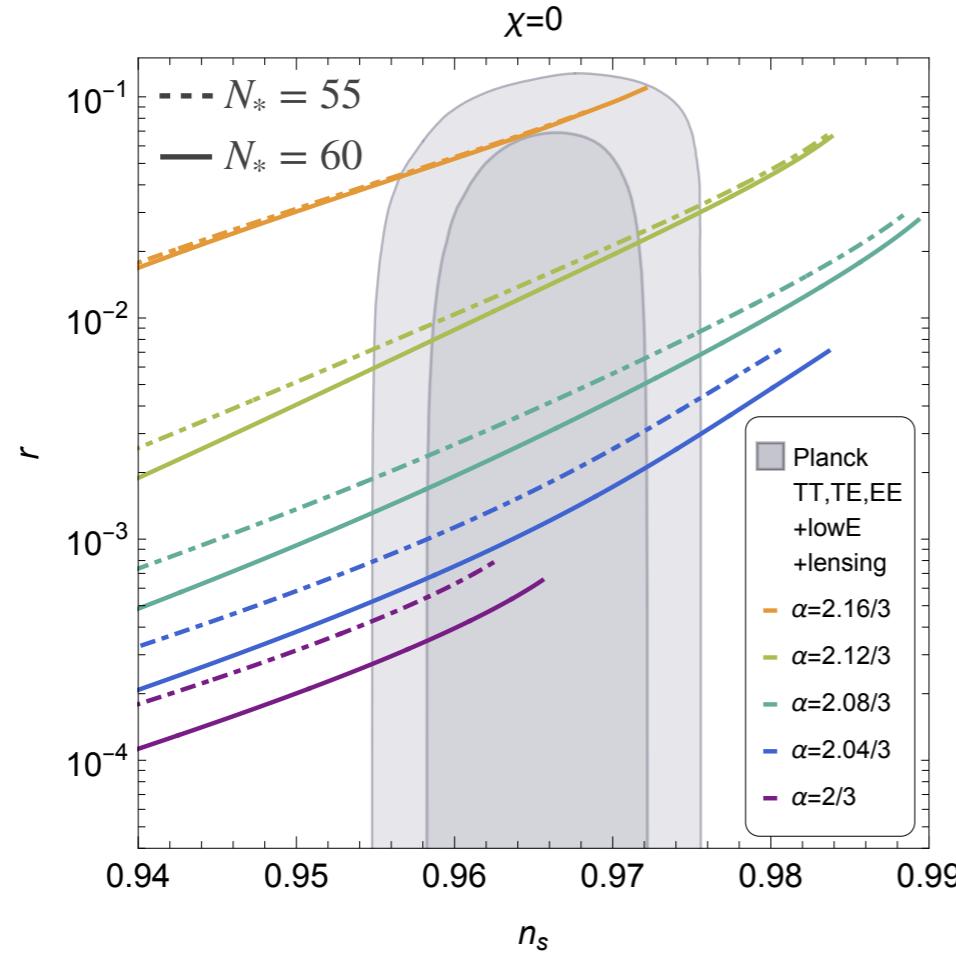


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- Predicted  $r$  changes depending on  $\alpha$  &  $\chi$ 

$$10^{-4} \lesssim r \lesssim 10^{-1}$$

# $\chi = 0$ case



As  $\alpha \rightarrow 2/3$ :

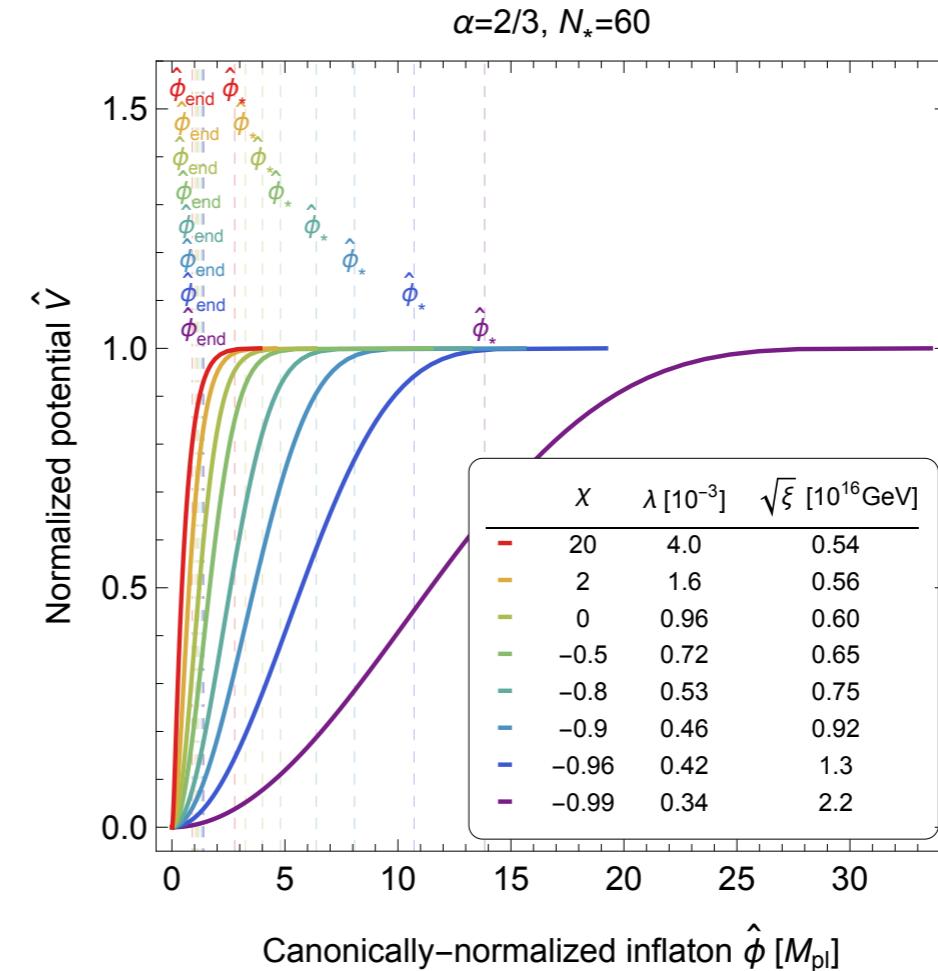
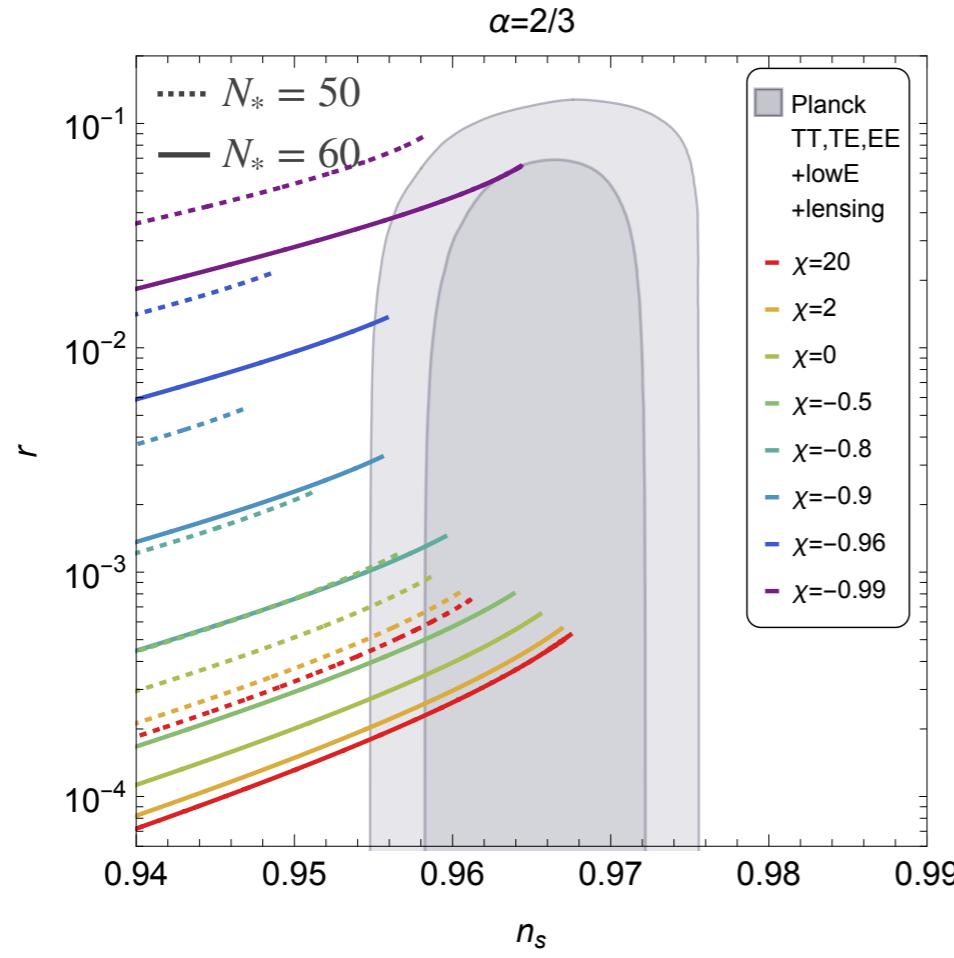
- $\hat{V}$  becomes flatter
- $\hat{\phi}_*$  becomes smaller
- $r$  becomes smaller

$$\hat{V} = V/V(\phi_c)$$

$\hat{\phi} = \hat{\phi}_{\text{end}}$  at the end of inflation

$\hat{\phi} = \hat{\phi}_*$  at 60  $e$ -folds

# $\alpha = 2/3$ case



As  $\chi$  increases:

- $\hat{V}$  becomes flatter
- $\hat{\phi}_*$  becomes smaller
- $r$  becomes smaller

$$\hat{V} = V/V(\phi_c)$$

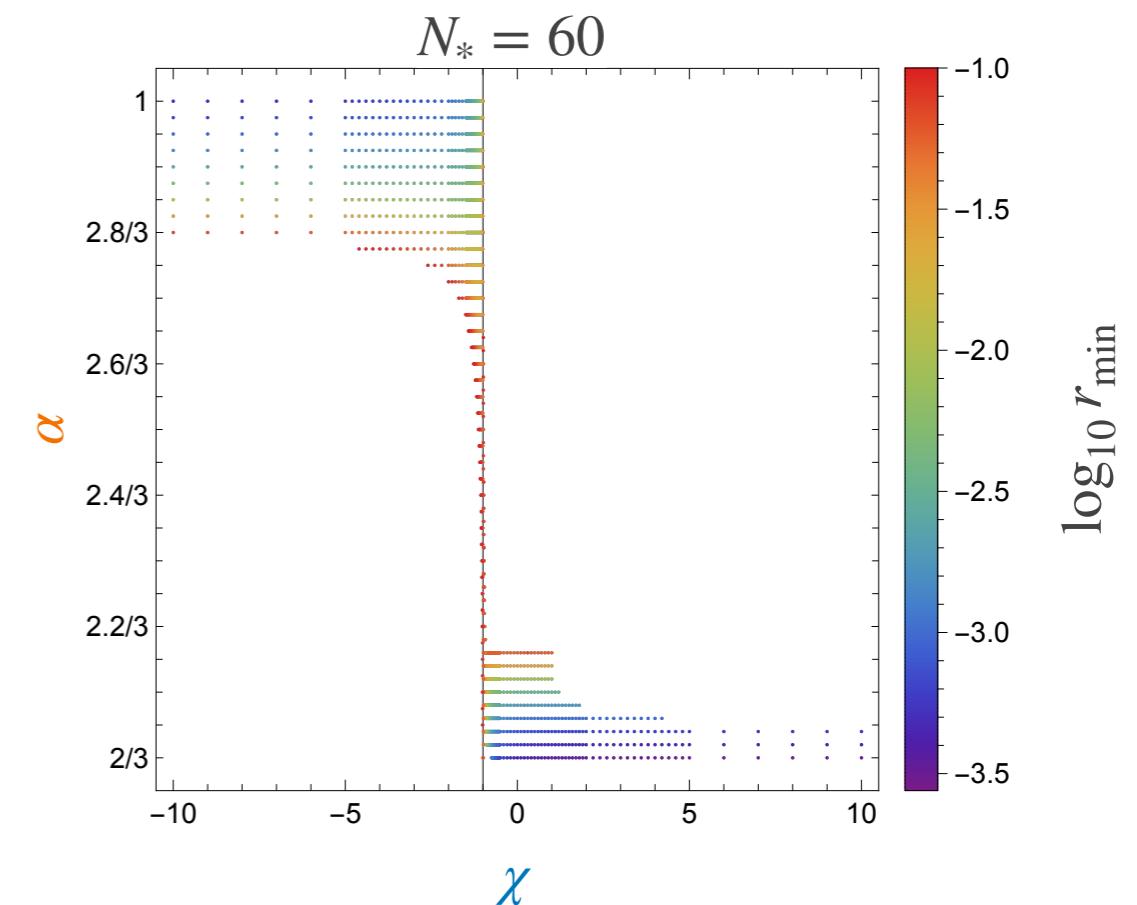
$\hat{\phi} = \hat{\phi}_{\text{end}}$  at the end of inflation

$\hat{\phi} = \hat{\phi}_*$  at 60  $e$ -folds

# Summary

We have studied subcritical hybrid inflation in a generalized superconformal model

- Successful inflation is realized in wide range of parameters
- Potential changes drastically depending on  $\alpha$  &  $\chi$
- $r$  is found to range from  $10^{-4}$  to  $10^{-1}$

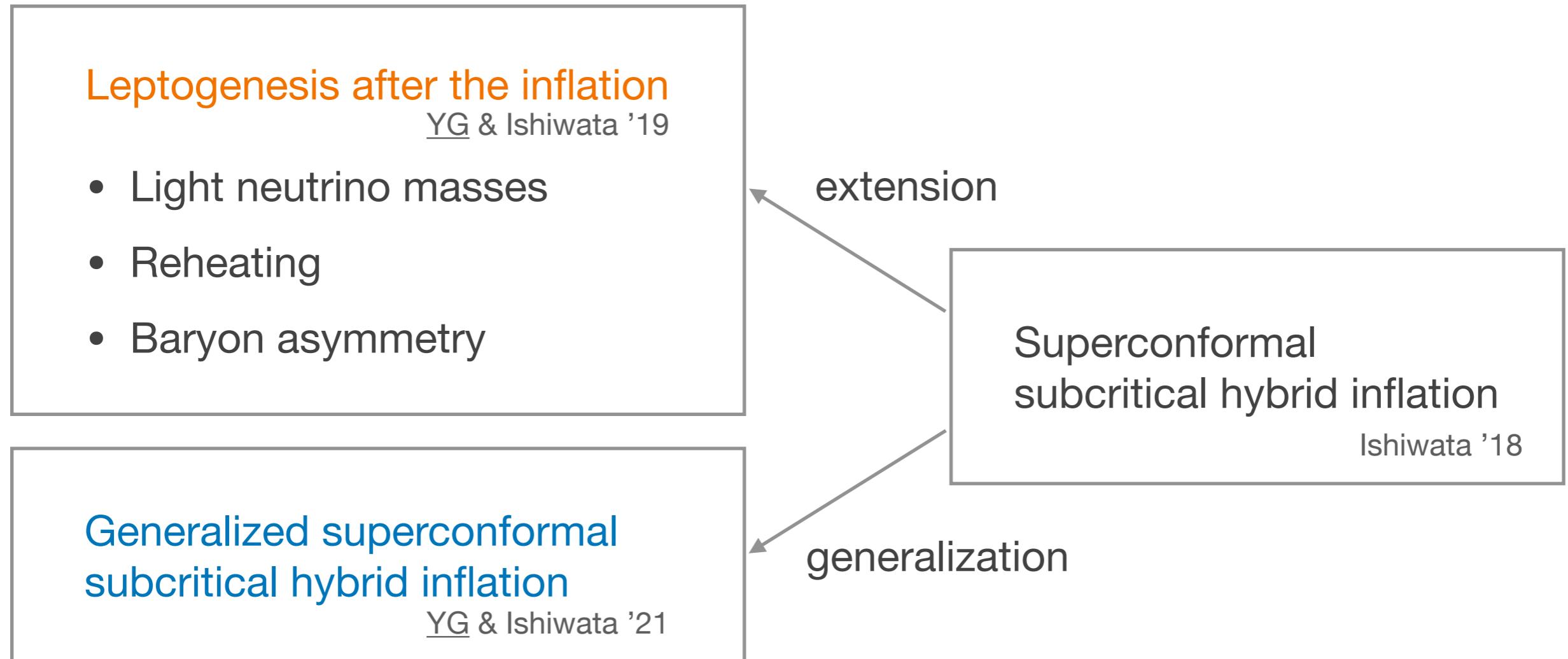


# Conclusions

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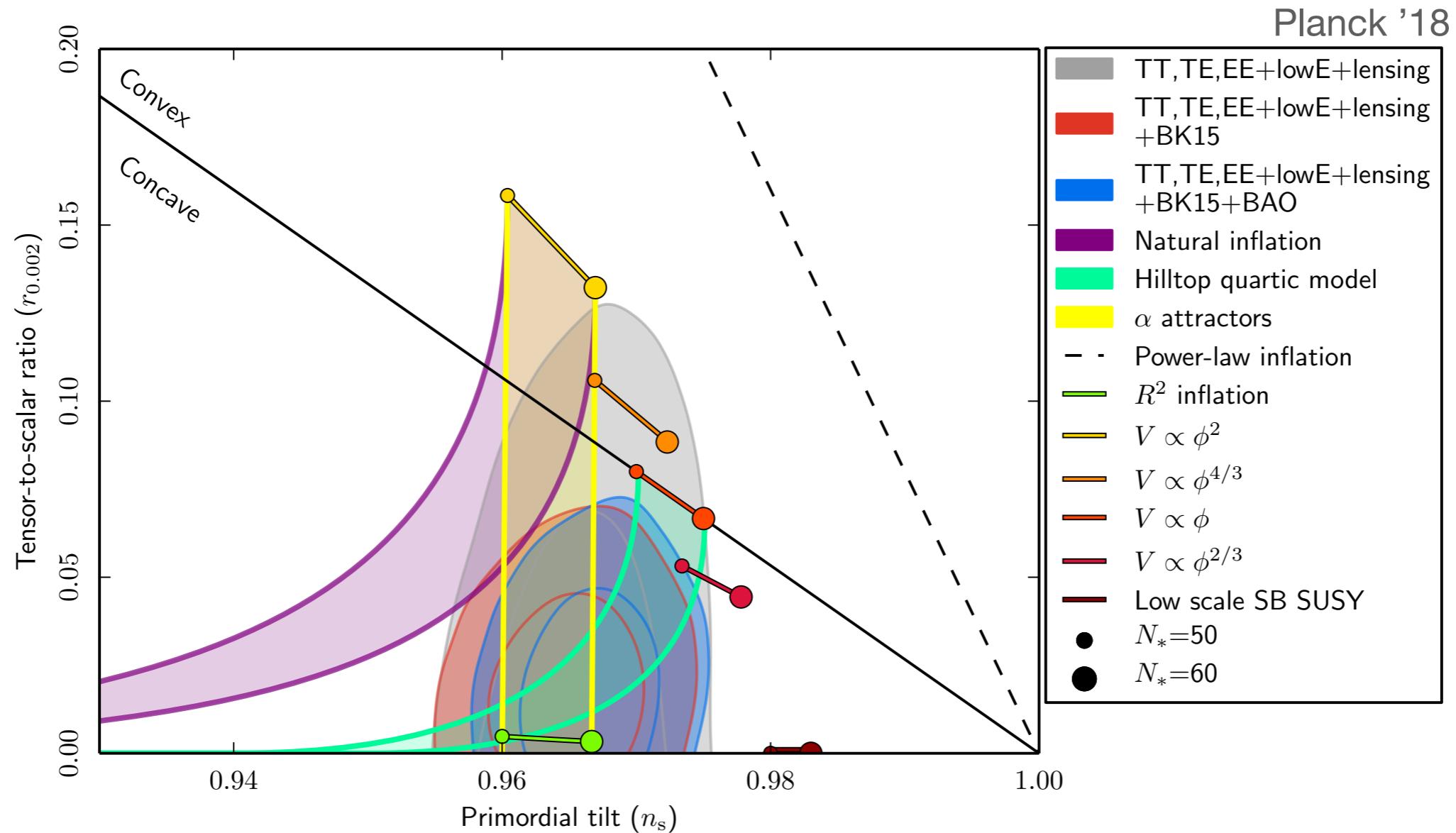
We have studied the phenomenology of superconformal subcritical hybrid inflation

- Successful leptogenesis is realized in the extended model
- Subcritical hybrid inflation is realized in a generalized model



Back up

# Inflation



CMB constraints on inflationary models  
may provide hints for new physics

# Inflation

Supersymmetry (SUSY) is one of the candidates for new physics

- Superstring theory requires SUSY
- Dark matter candidate exists
- Flat directions appear  $\longrightarrow$  Suitable for inflation

Many F, D-term hybrid inflation models have been considered

$$V_{\text{scalar}} = V_F + V_D$$

# Inflation

Supersymmetry (SUSY) is one of the candidates for new physics

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- Dark matter candidate exists
- Flat directions appear  $\longrightarrow$  Suitable for inflation

Many F, D-term hybrid inflation models have been considered

$$V_{\text{scalar}} = V_F + \boxed{V_D}$$

D-term hybrid inflation has been revisited from new point of view

# Reheating

Decay width

$$\Gamma_\phi \simeq \frac{(y_\nu y_\nu^\dagger)_{33}}{8\pi} m_\phi$$

$$\Gamma_{\phi \rightarrow L\tilde{H}_u} = \Gamma_{\phi \rightarrow \bar{L}\tilde{\bar{H}}_u} = \frac{(y_\nu y_\nu^\dagger)_{33}}{16\pi} m_\phi, \quad \Gamma_{\phi \rightarrow \tilde{L}H_u} = \Gamma_{\phi \rightarrow \tilde{L}^*H_u^*} = \frac{(y_\nu y_\nu^\dagger)_{33}}{16\pi} \frac{M_3^2}{m_\phi}$$

Reheating temperature

$$T_R \simeq \left(90/\pi^2 g_*(T_R)\right)^{1/4} \sqrt{\Gamma_\phi M_{pl}}$$
$$\simeq 1.4 \times 10^{10} \text{GeV} \left(\frac{m_\phi}{10^{13} \text{GeV}}\right)^{1/2} \left(\frac{(y_\nu y_\nu^\dagger)_{33}}{10^{-9}}\right)^{1/2} \left(\frac{g_*(T_R)}{228.75}\right)^{-1/4}$$

# Case (I) . $M_1, M_2 < m_\phi$

Produced baryon asymmetry

Buchmüller, Di Bari, Plümacher '05

$$\eta_B = \frac{3}{4} \frac{a_{\text{sph}}}{f} \epsilon_1 \kappa_f$$

Efficiency factor in strong washout regime ( $\tilde{m}_1 > m_* \sim 10^{-3}$  eV)

$$\kappa_f = (2 \pm 1) \times 10^{-2} \left( \frac{0.01 \text{ eV}}{\tilde{m}_1} \right)^{1.1 \pm 0.1}$$

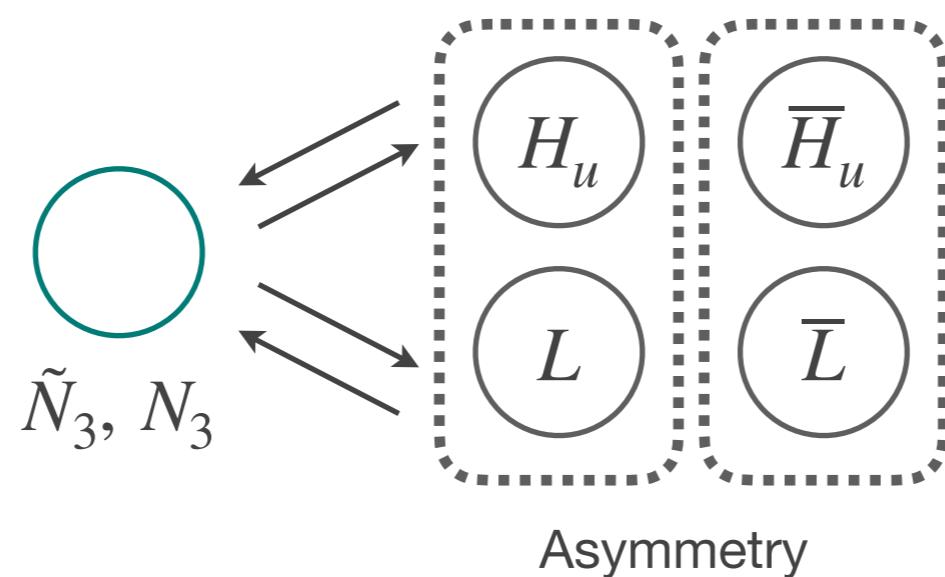
$$\tilde{m}_1 \geq \begin{cases} m_2 \simeq 8.6 \times 10^{-3} \text{ eV (NH)} \\ m_1 \simeq 4.9 \times 10^{-2} \text{ eV (IH)} \end{cases}$$

Asymmetric parameter

$$\epsilon_1 = \begin{cases} 8.2 \times 10^{-7} \text{ (NH)} \\ 1.5 \times 10^{-8} \text{ (IH)} \end{cases} \times \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \left( \frac{\sin \delta}{0.5} \right)$$

## Case (II) . $M_1, M_2 > m_\phi$

When  $T_R \gtrsim m_\phi$ ,  $\tilde{N}_3$  &  $N_3$  are thermally produced



We need to evaluate the time evolution equation of the lepton asymmetry

$$\begin{cases} \dot{n}_L + 3Hn_L = (\text{source}) - (\text{washout}) \\ \dot{n}_{\tilde{N}_3^c} + 3Hn_{\tilde{N}_3^c} = -D_{\tilde{N}_3^c} + ID_{\tilde{N}_3^c} - S_{\tilde{N}_3^c} \\ \dot{n}_{N_3} + 3Hn_{N_3} = -D_{N_3} + ID_{N_3} - S_{N_3} \end{cases}$$

Case (II) .  $M_1, M_2 > m_\phi$

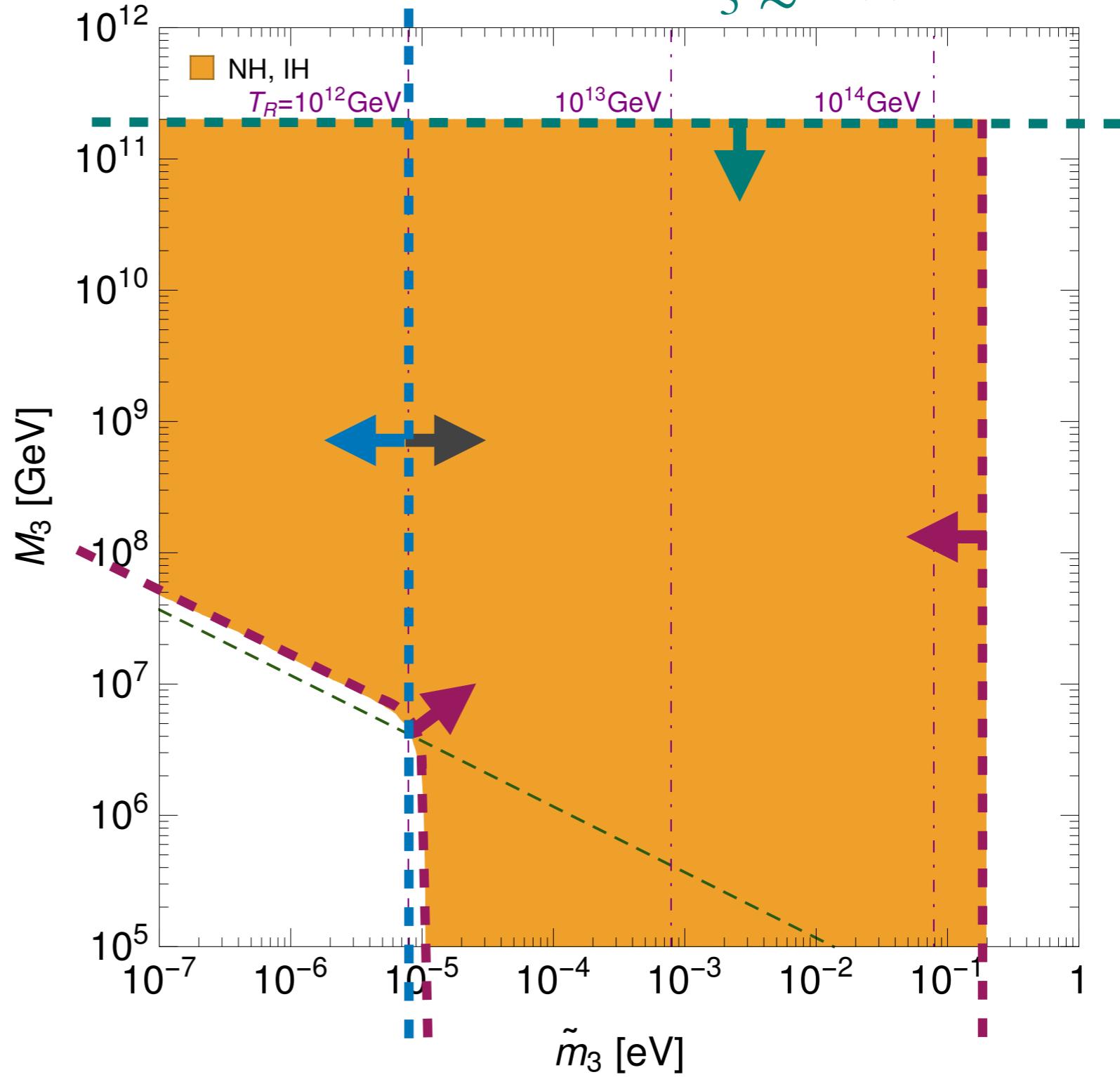
$T_R \ll m_\phi$

Non-thermal leptogenesis  
(by the inflaton decay)

$T_R \gtrsim m_\phi$

Thermal leptogenesis  
(by thermally-produced  
neutrino decay)

Inflation  
 $M_3 \lesssim 2 \times 10^{11} \text{ GeV}$



$$\eta_B^{\max} \geq \eta_B^{\text{obs}}$$

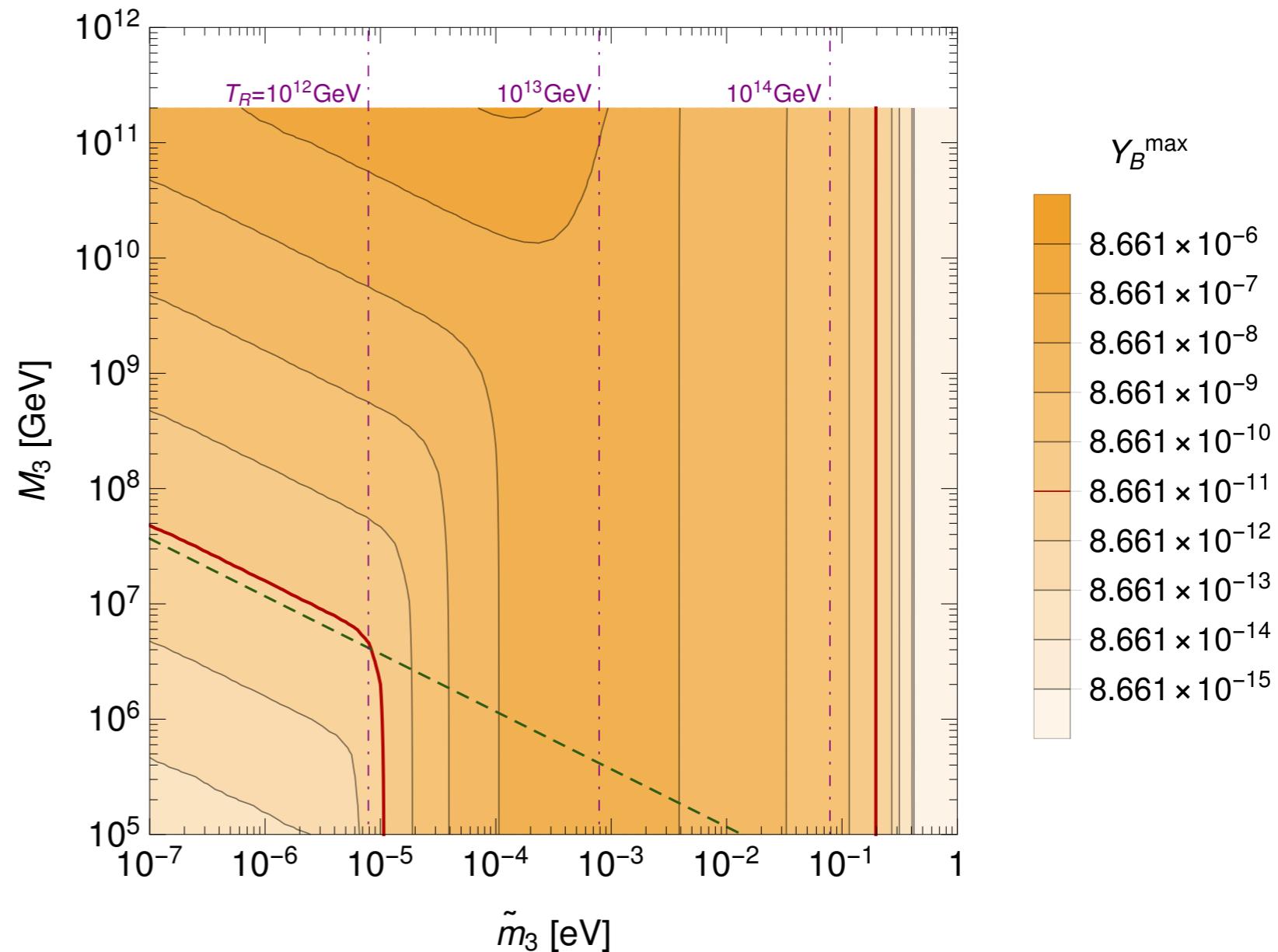
## Case (II). $M_1, M_2 > m_\phi$

Non-thermal leptogenesis

$$Y_B^{\max} \propto \tilde{m}_3^{1/2} M_3$$

Thermal leptogenesis

$$Y_B^{\max} = Y_B^{\max}(\tilde{m}_3, m_\phi)$$



$\phi$  decay

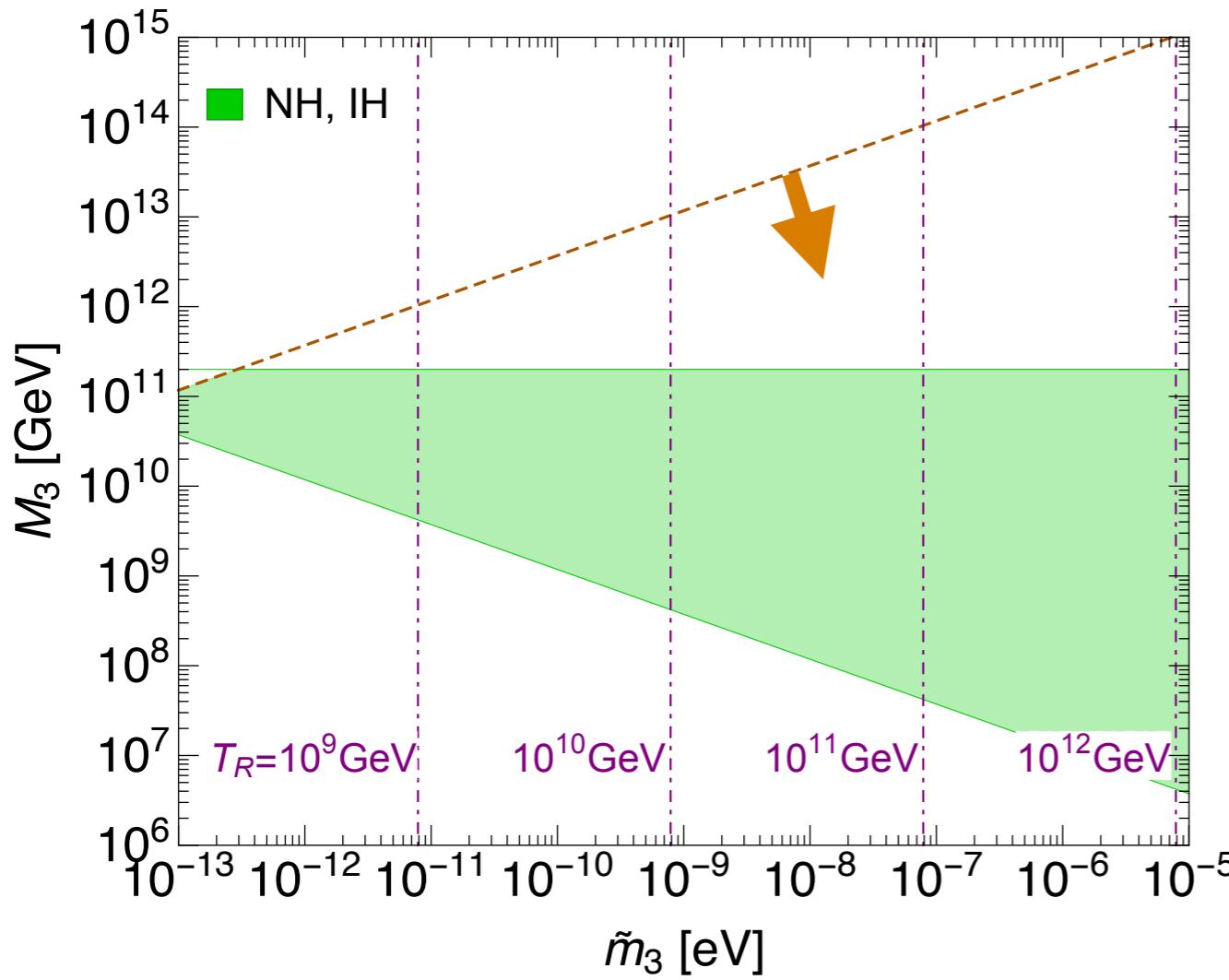
$$\epsilon_\phi \simeq 4 \times 10^{-9} \left( \frac{M_3}{10^7 \text{ GeV}} \right) \left( \frac{\sin \delta'}{0.5} \right)$$

$N_3$  decay

$$\epsilon_3 \simeq 2 \times 10^{-4} \left( \frac{m_\phi}{10^{13} \text{ GeV}} \right) \left( \frac{\sin \delta'}{0.5} \right)$$

# Stability of inflationary trajectory

$$W_{\text{neu}} = \lambda_3 N_3^c S_+ S_- + \frac{1}{2} M_i N_i^c N_i^c + \boxed{y_{\nu ij} N_i^c L_j H_u}$$



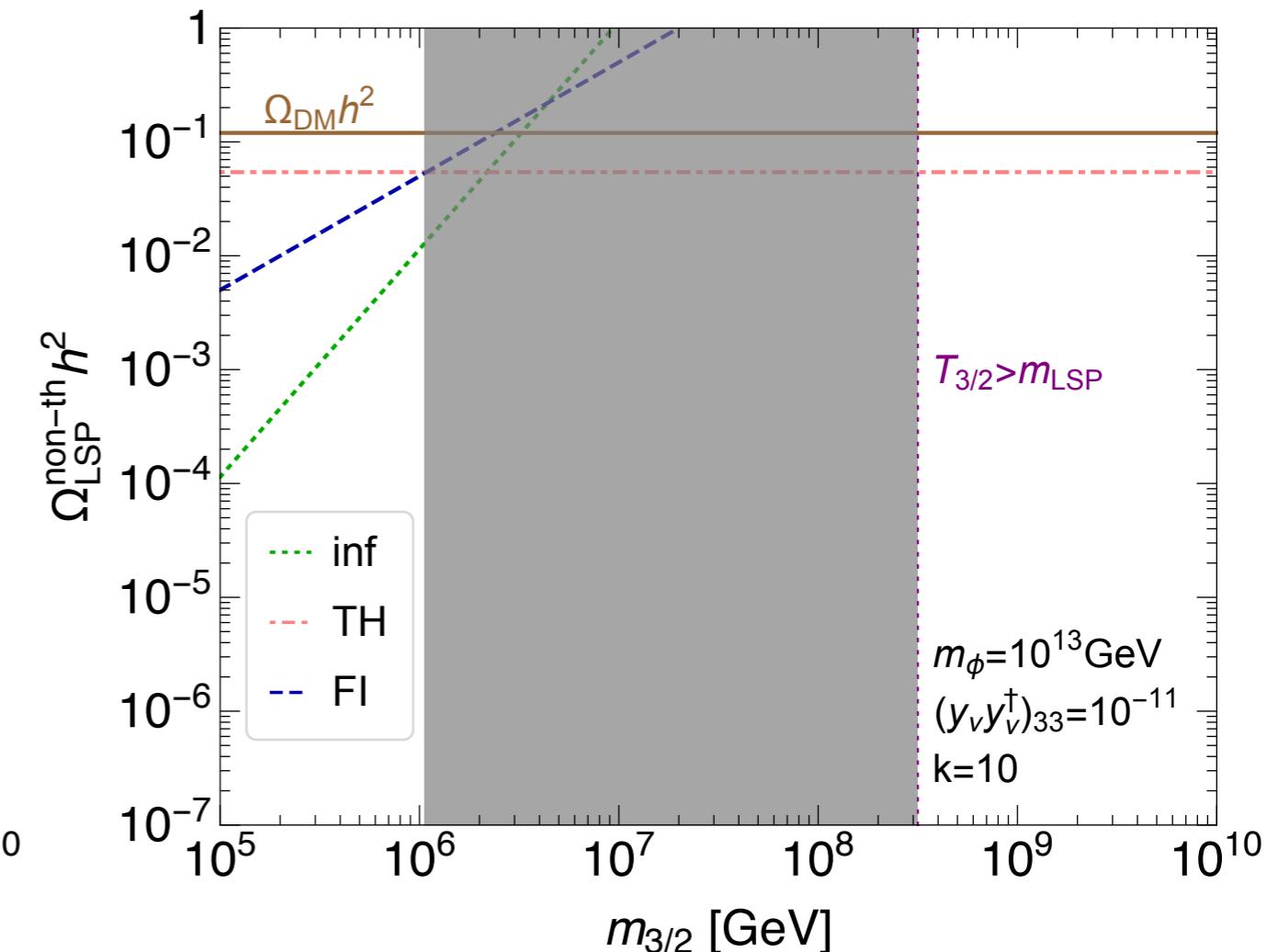
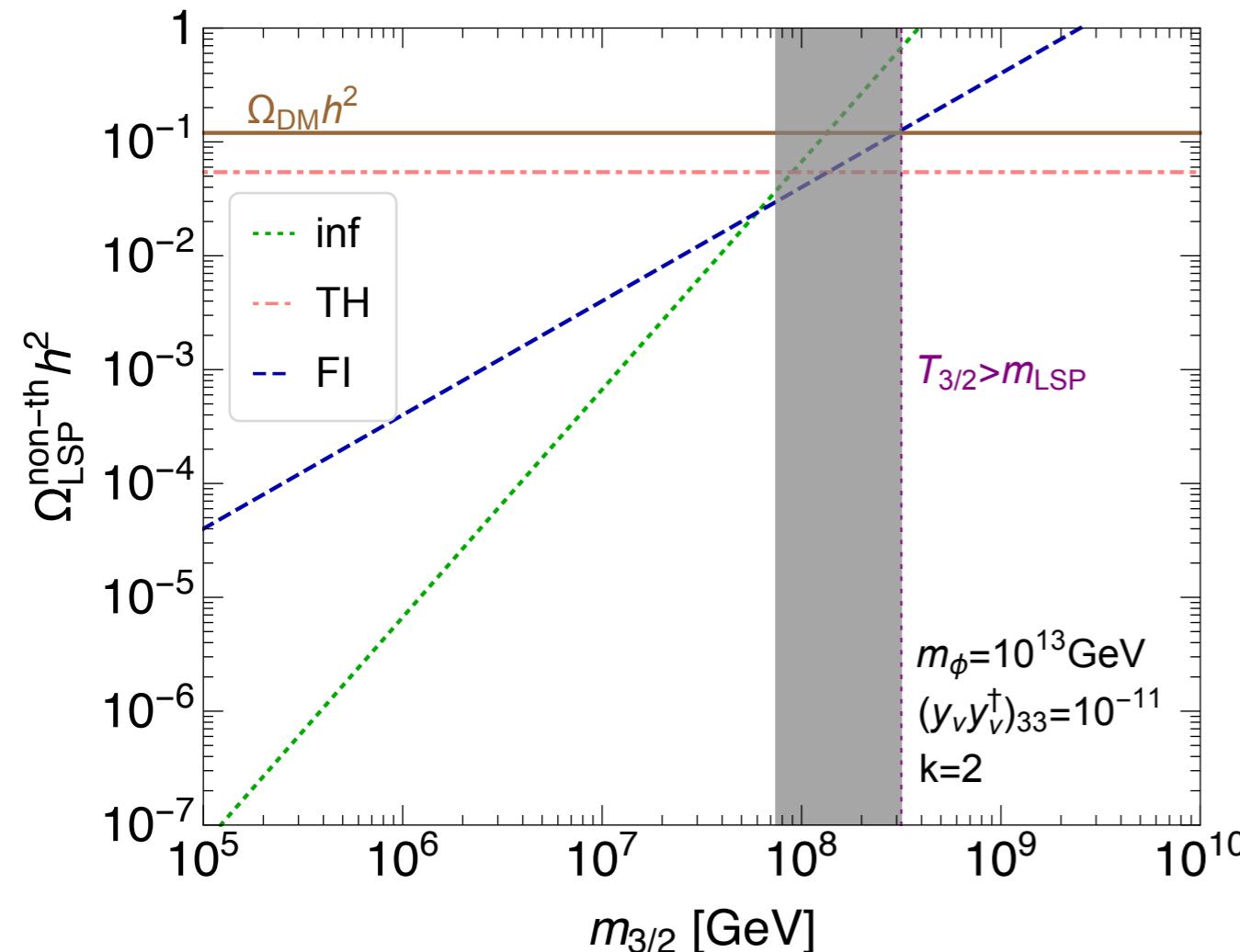
Consistency with the inflation

$$|M_3| \leq 1.4 \times 10^{14} \text{ GeV} \left( \frac{(y_\nu y_\nu^\dagger)_{33}}{10^{-7}} \right)^{1/2}$$

# LSP production by gravitino decay

Gravitino decays to the lightest supersymmetric particles (LSP)

When  $R$ -Parity is conserved, LSP becomes DM



Gravitino production processes

- (i) Inflaton decay
- (ii) Thermal production by scattering
- (iii) SUSY particle decay in thermal bath

$$k \equiv \tilde{m}/m_{3/2}$$

$m_{3/2}$  : gravitino mass

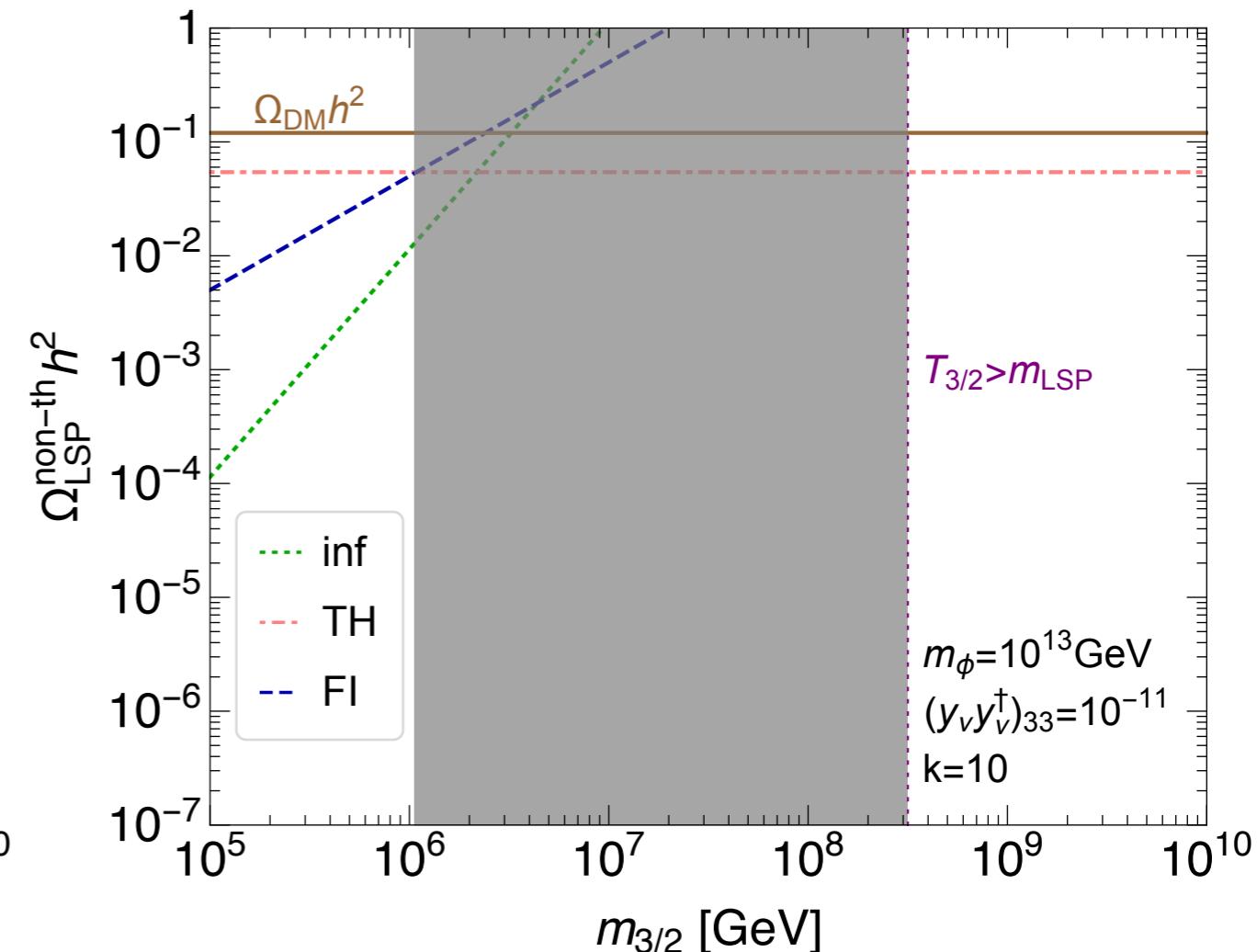
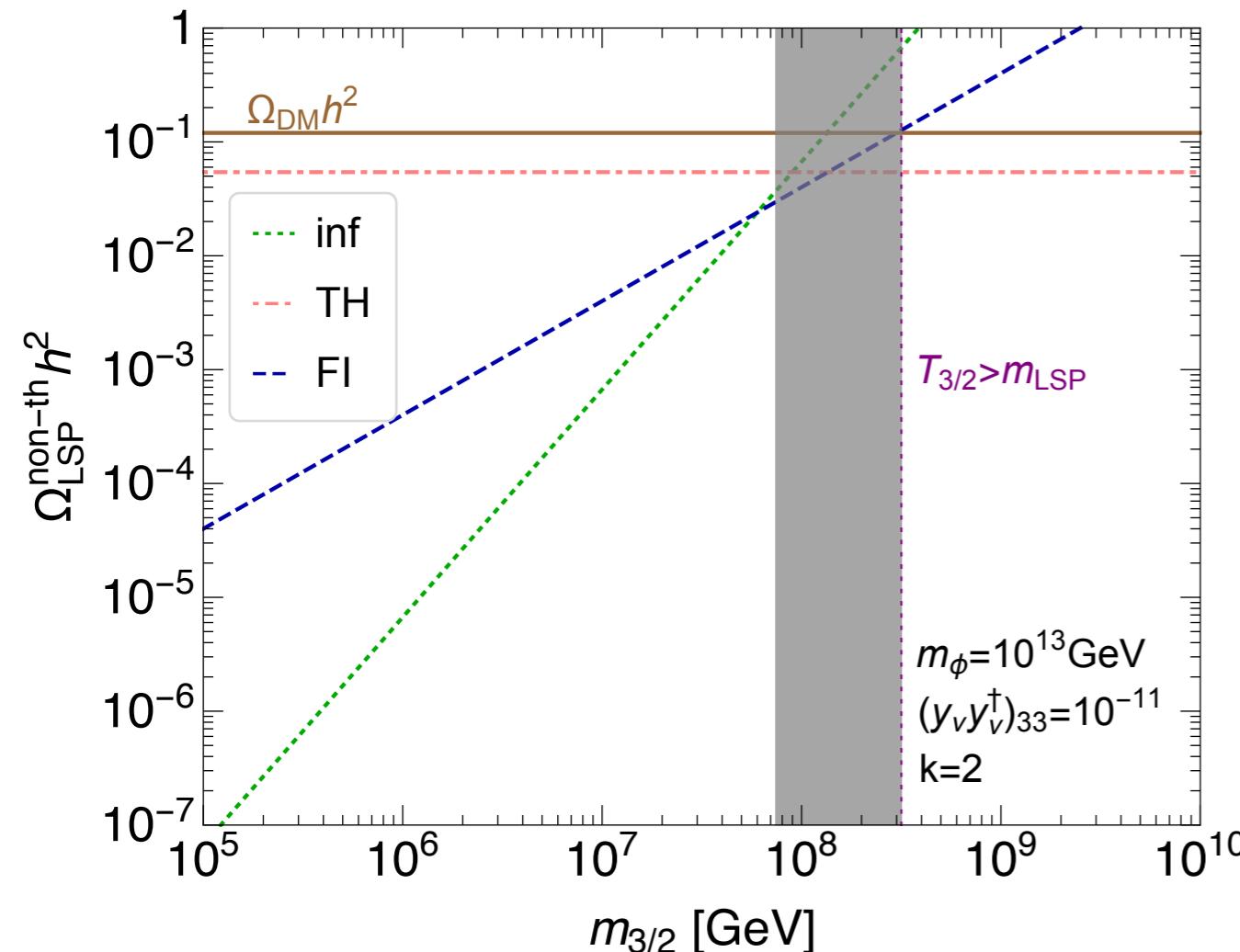
$\tilde{m} = km_{2/3}$  : soft ~~SUSY~~ mass scale ( $k > 1$ )

$m_{\text{LSP}}$  : LSP mass scale ( $\sim 1$  TeV)

# LSP production by gravitino decay

Gravitino decays to the lightest supersymmetric particles (LSP)

When  $R$ -Parity is conserved, LSP becomes DM



- Gravitino decays to LSP before thermal freeze-out of LSP when  $m_{2/3} \gtrsim 3 \times 10^8$  GeV
- $m_{2/3} \lesssim 10^{10}\text{-}10^{11}$  GeV should be satisfied to realize the reheating ( $\text{Br}_{\phi \rightarrow \psi_\mu N_3} \lesssim 0.1$ )

# Mass spectrum

	Case . (I)	Case . (II)
$m_\phi = 10^{13} \text{ GeV}$	$\frac{\phi}{N_3}$	$\frac{N_{1,2}}{M_{1,2}(>m_\phi)}$
$m_f = m_\phi - \tilde{m}$	$\tilde{Q}, \tilde{L}$	$\frac{N_{1,2}}{M_{1,2}(<m_\phi)}$
$\tilde{m} = km_{2/3} (k > 1)$	$\psi_\mu$	
$m_{2/3} (\lesssim 10^{10} \text{ GeV})$		
$m_{\text{LSP}} \simeq 1 \text{ TeV}$	LSP	

# Potential in the pre-critical regime

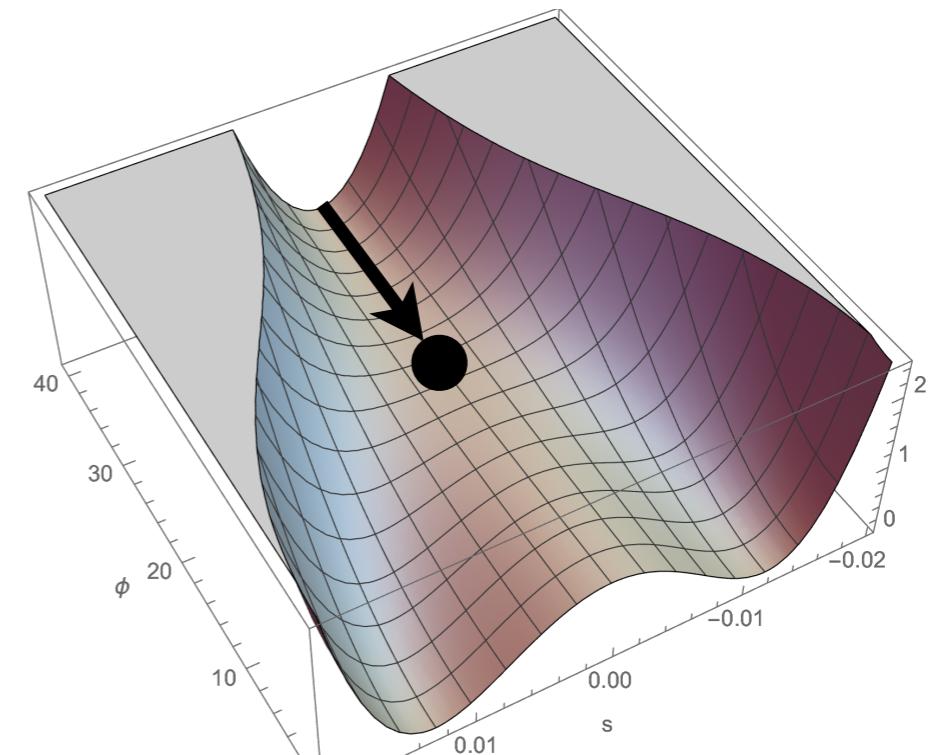
$$V(\phi) = V_{\text{tree}} + V_{\text{1 loop}}$$

$$V_{\text{tree}} = g^2 \xi^2 / 2$$

$$V_{\text{1 loop}} = \frac{g^4 q^2 \xi^2}{32\pi^2} L(\Psi)$$

$$L(\Psi) = (\Psi - 1)^2 \ln(\Psi - 1) + (\Psi + 1)^2 \ln(\Psi + 1) - 2\Psi^2 \ln \Psi - \ln 16$$

$$\Psi(\phi) = \frac{k}{2\alpha^2} \left( \frac{\Phi(\phi, 0)}{3} \right)^{2-3\alpha} \phi^2$$

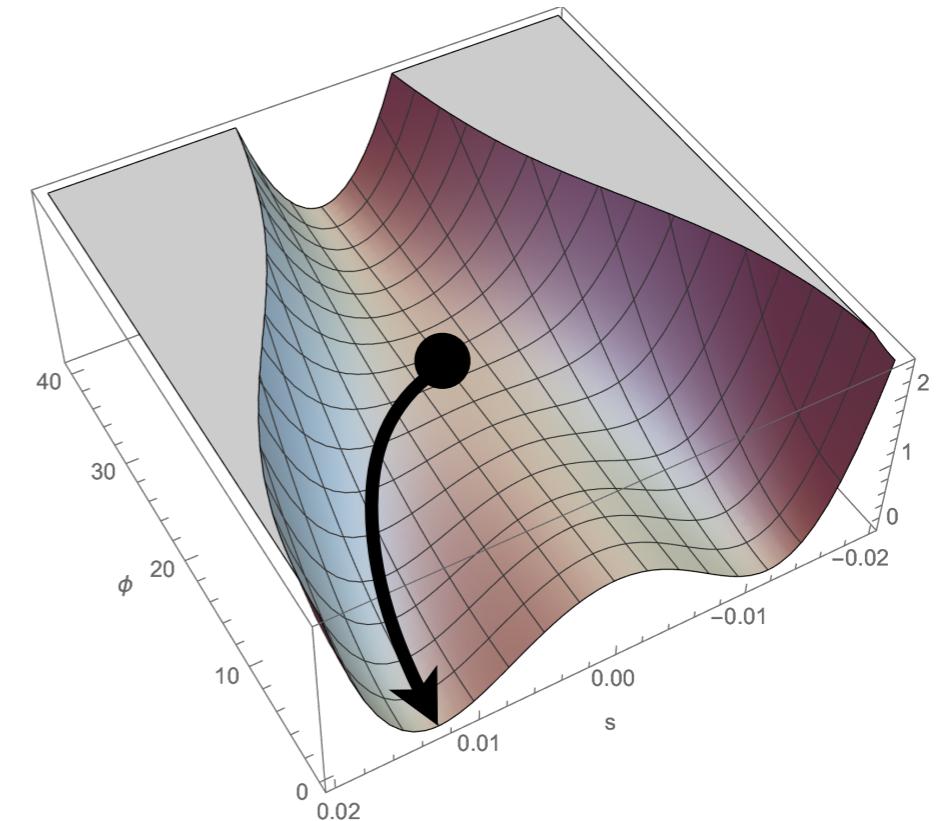


# Potential in the subcritical regime

$$V(\phi) = g^2 \xi^2 \Psi(\phi) \left( 1 - \frac{1}{2} \Psi(\phi) \right)$$

$$\Psi(\phi) = \frac{k}{2\alpha^2} \left( \frac{\Phi(\phi,0)}{3} \right)^{2-3\alpha} \phi^2$$

$$k \equiv \lambda^2 / q g^2 \xi$$



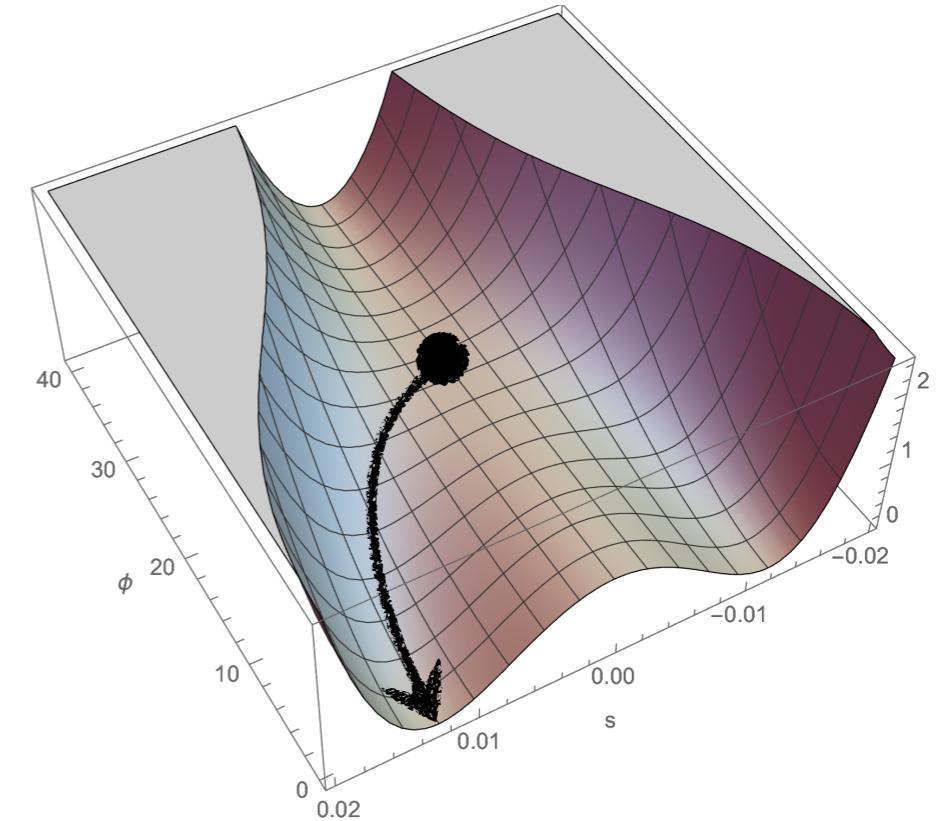
$q$  &  $g$  can be absorbed into redefining  $\lambda$  &  $\xi$

... We take  $q = g = 1$  in numerical analysis

# Masses of $S_{\pm}$

$$m_{\pm}^2 = \left(-\frac{\Phi(\phi)}{3}\right)^{2-3\alpha} \frac{\lambda^2}{2\alpha^2} \phi^2 \mp qg^2\xi$$

$$\Phi(\phi) = 1 - \frac{1}{6}(1 + \chi)\phi^2$$



The critical point value  $\phi_c$  can be determined from  $m_+^2 = 0$ , i.e.,

$$\left(-\frac{\Phi(\phi_c)}{3}\right)^{2-3\alpha} \phi_c^2 = \frac{2\alpha}{k}$$

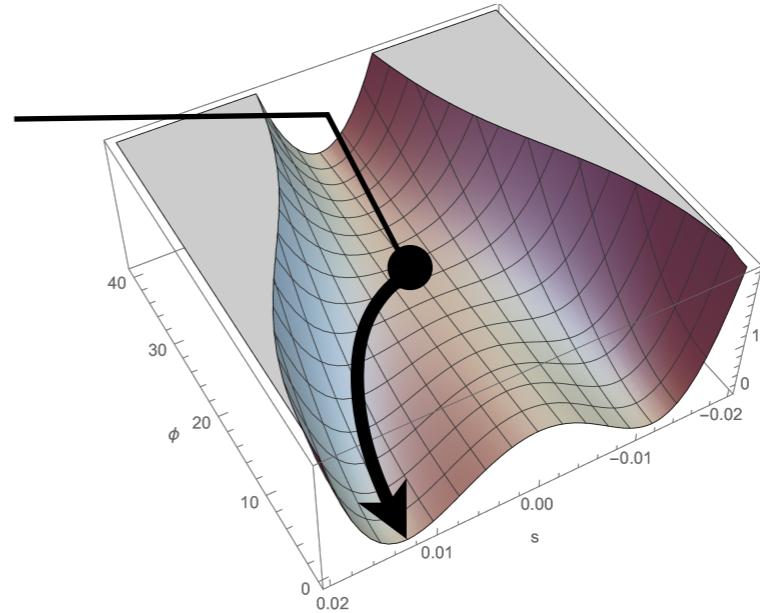
$$k \equiv \lambda^2/qg^2\xi$$

$\phi = \phi_c$  at the critical point

# Waterfall field value in subcritical regime

$\phi = \phi_c$  at the critical point

$s = s_{\min}(\phi)$  for  $\phi < \phi_c$

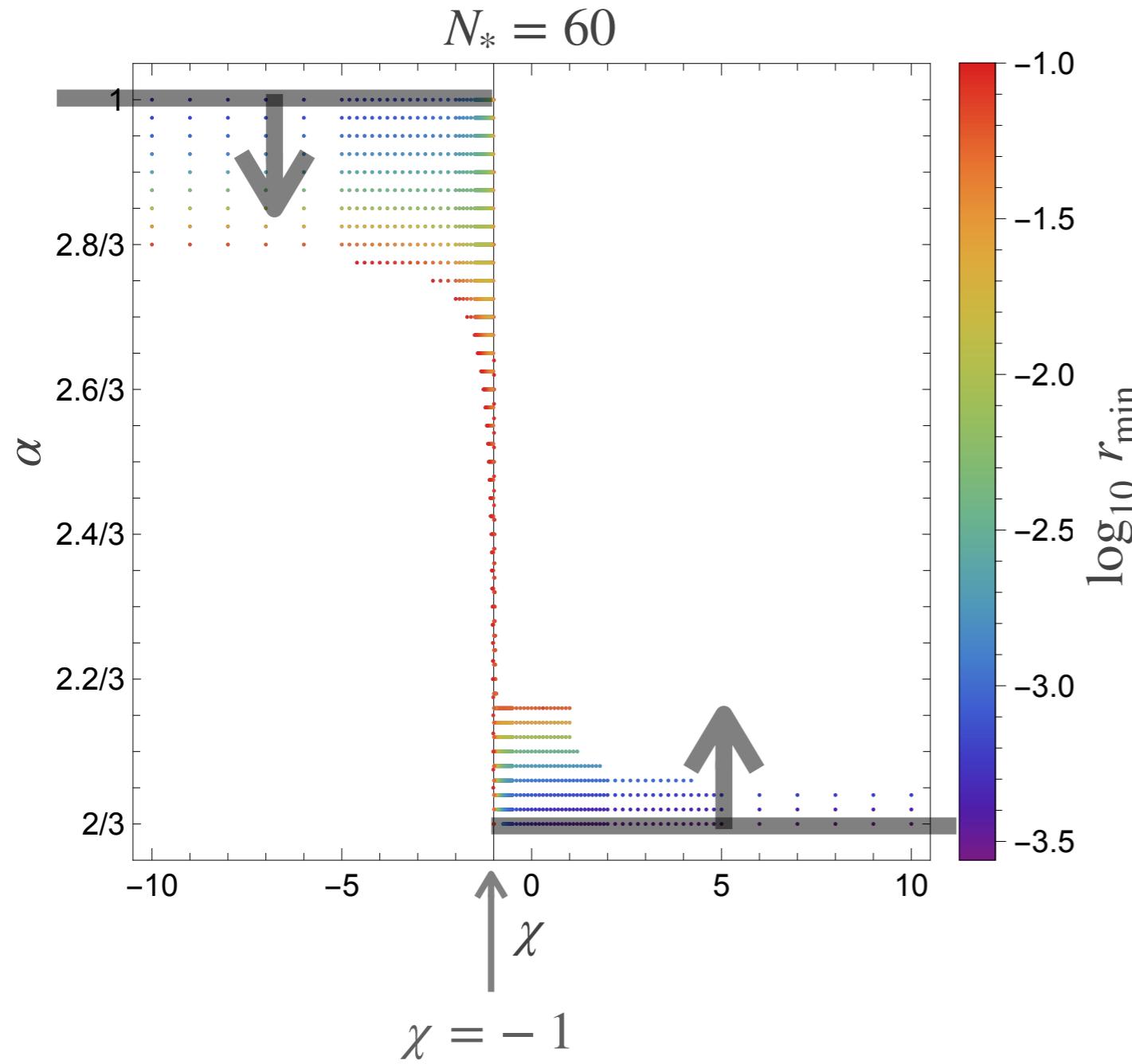


$$s_{\min}^2(\phi) = -\frac{\Phi(\phi)}{3} \frac{2\xi}{q\alpha} (1 - \Psi(\phi))$$

$$\Phi(\phi) = 1 - \frac{1}{6}(1 + \chi)\phi^2$$

$$\Psi(\phi) = \left(\frac{\Phi(\phi)}{\Phi(\phi_c)}\right)^{2-3\alpha} \frac{\phi^2}{\phi_c^2} = \frac{k}{2\alpha} \left(-\frac{\Phi(\phi)}{3}\right)^{2-3\alpha} \phi^2$$

# Allowed region for $\alpha$ & $\chi$

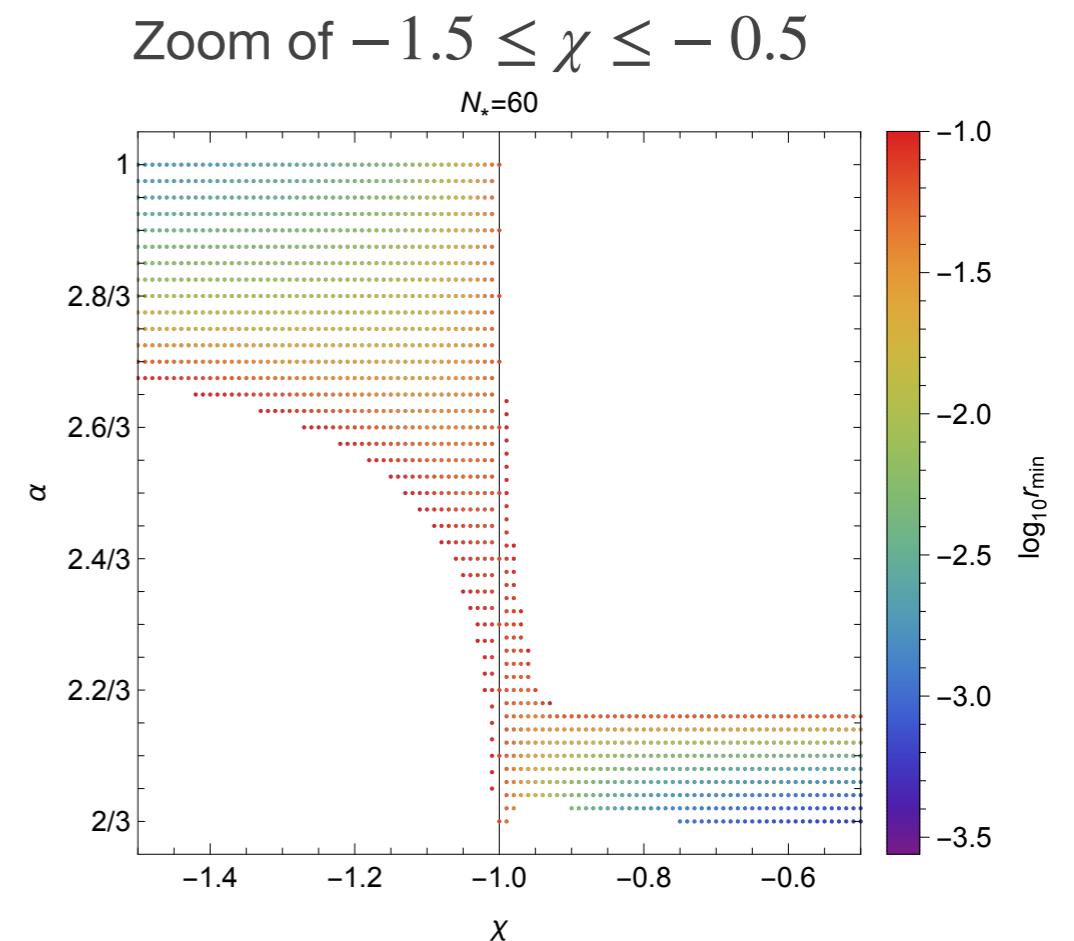
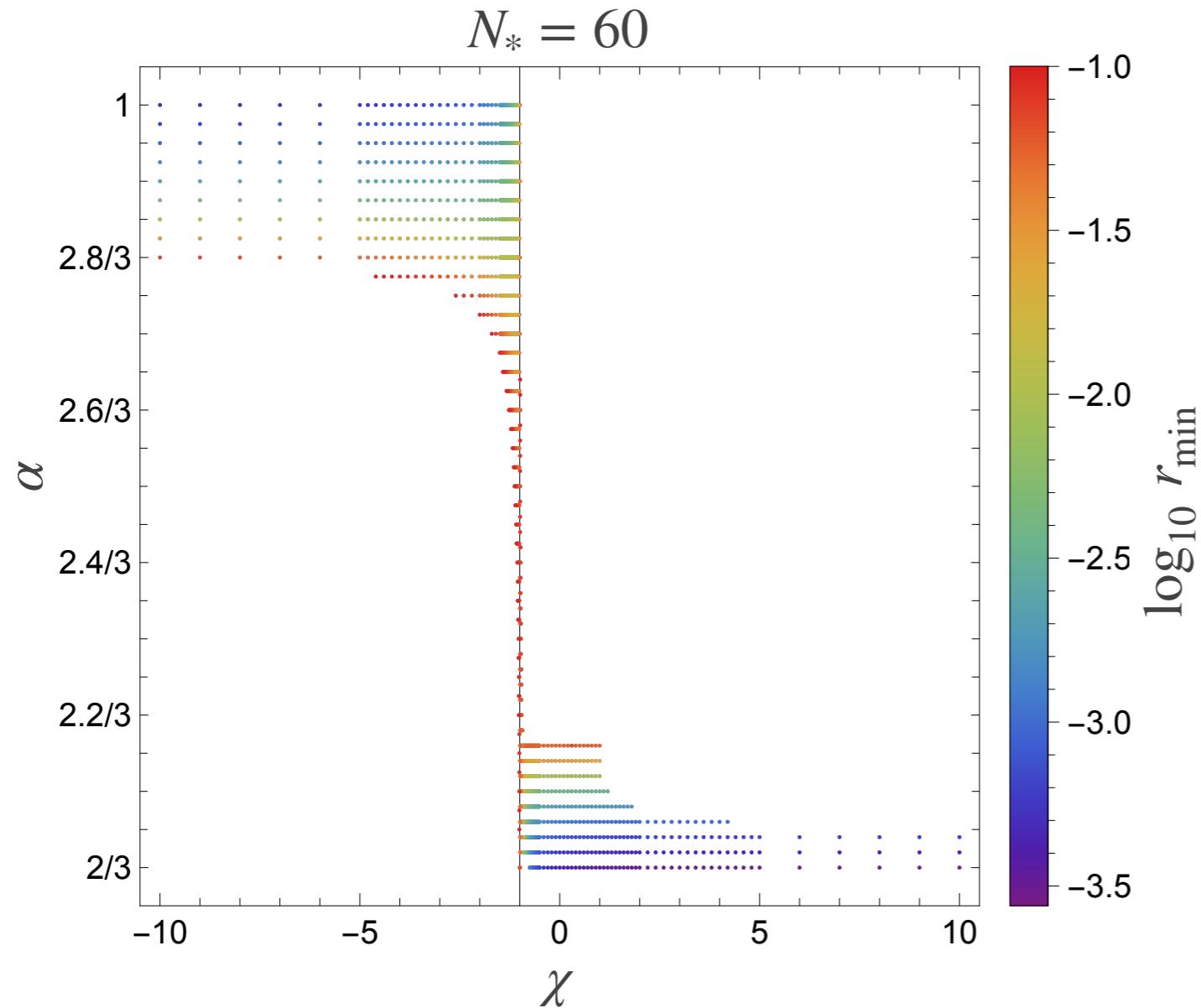


We focus on

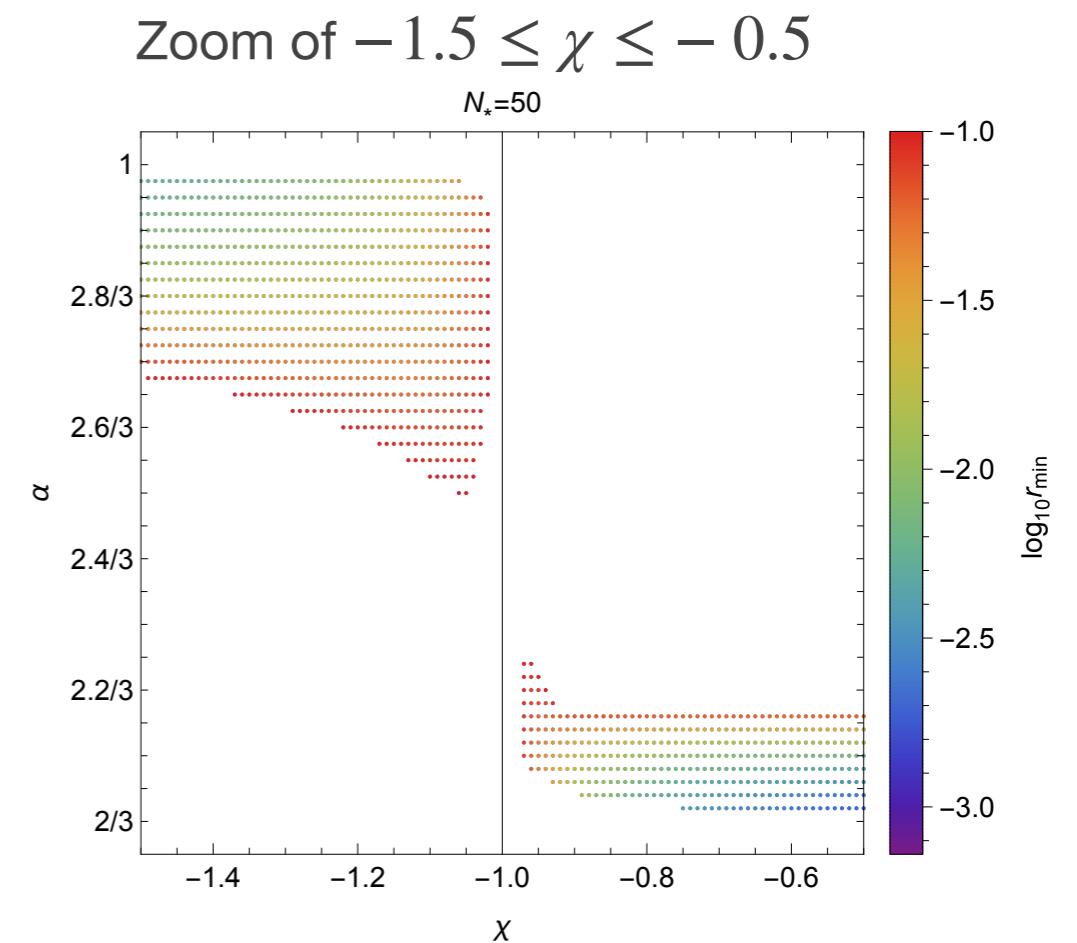
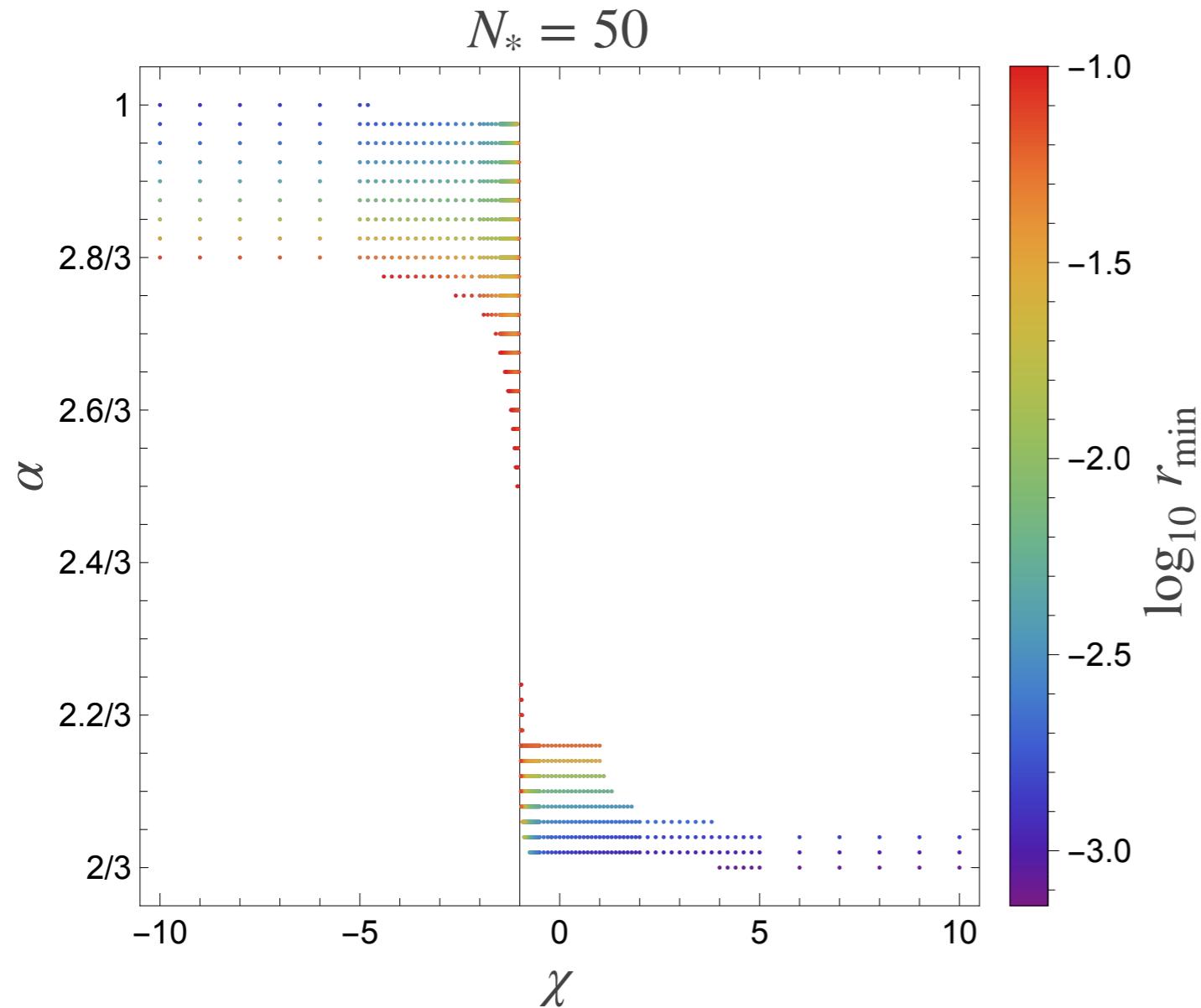
$$\begin{cases} \alpha \leq 1 & (\chi < -1) \\ \alpha \geq 2/3 & (\chi > -1) \end{cases}$$

to give single critical point

# Allowed region for $\alpha$ & $\chi$



# Allowed region for $\alpha$ & $\chi$



# The generalized Model

Lagrangian in a Jordan frame:

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{6}R_J \mathcal{N} - \mathcal{N}_{\beta\bar{\beta}} g_J^{\mu\nu} \mathcal{D}_\mu z^\beta \mathcal{D}_\nu \bar{z}^{\bar{\beta}} - V_J \quad z^\beta = \{S_+, S_-, N\}$$

$$\mathcal{N} = -|X^0|^2 \left[ 1 - \frac{|S_+|^2 + |S_-|^2 + |N|^2}{|X^0|^2} - \frac{\chi}{2} \left( \frac{N^2 \bar{X}^0}{X^0} + \frac{\bar{N}^2 X^0}{\bar{X}^0} \right) \right]^\alpha$$

↓

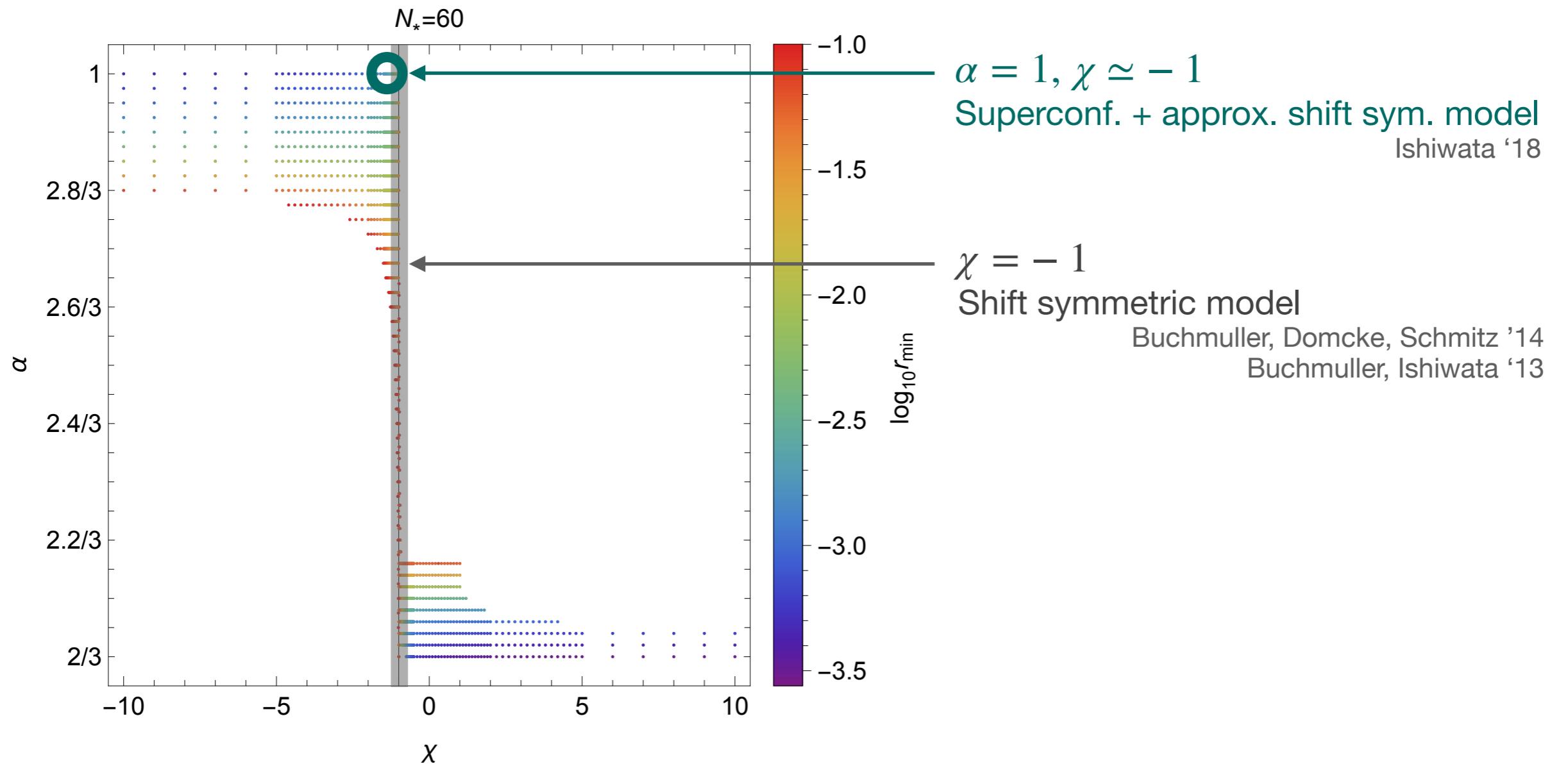
Gauge fixing  $X^0 = \bar{X}^0 = \sqrt{3}$   
 Weyl transformation  $g_{J\mu\nu} = (-\mathcal{N}/3)^{-1} g_{E\mu\nu}$

Lagrangian in the Einstein frame:

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{1}{2}R_E - K_{\beta\bar{\beta}} g_E^{\mu\nu} \mathcal{D}_\mu z^\beta \mathcal{D}_\nu \bar{z}^{\bar{\beta}} - V_E, \quad V_E = \left(\frac{\mathcal{N}}{3}\right)^{-2} V_J = V_F + V_E$$

$$K = -3\alpha \log\left(-\frac{\Phi}{3}\right), \quad \Phi = -3 + |S_+|^2 + |S_-|^2 + |N|^2 + \frac{\chi}{2}(N^2 + \bar{N}^2)$$

# Correspondence with the previous studies



# The generalized Model

Constant Fayet-Iliopoulos term

Introduce an additional term in the Lagrangian in the Jordan frame

$$\frac{\Delta \mathcal{L}_J}{\sqrt{-g_J}} = g \frac{-\mathcal{N}\xi}{3} \mathcal{P} \quad \text{Buchmuller, Domcke, Schmitz '13}$$
$$\mathcal{P} = -gQz^\beta \mathcal{N}_\beta - g\mathcal{N}\xi/3$$

Constant FI term appears in  $D$ -term potential in the Einstein frame

$$V_D = \frac{g^2}{2} (K_\beta Q z^\beta - \xi)^2$$

# Effect of $s$ to the adiabatic curvature perturbation

Trajectory of the inflation is almost straight along the inflation field

The effect on the scalar amplitude

$$A_s \rightarrow A'_s = e^\beta A_s$$

...  $e^\beta$  gives an impact on  $A_s$

The effect is sufficient small

$$e^\beta - 1 \simeq \eta_\perp^2 \xi \sim 10^{-10}$$

... The effective description in the subcritical regime is valid

# Stability of $\text{Im}N$

Mass of  $\tau \equiv \sqrt{2} \text{Im}N$  in subcritical regime

$$m_\tau = \frac{g^2 \xi^2 k}{\alpha^2} \left( -\frac{\Phi(\phi)}{3} \right)^{1-3\alpha} (1 - \Psi(\phi)) \left[ 1 - \frac{\phi^2}{6} \{3 - \chi + 3\alpha(\chi - 1)\} \right]$$

Proffered region to satisfy stability condition, i.e.,  $m_\tau^2 > 0$ :

- $\chi < -1$

$$\begin{cases} 1/3 + 2/3(1 - \chi) < \alpha \leq 1 \\ \alpha < 1/3 + 2/3(1 + \chi), \text{ depending on other parameter} \end{cases}$$

- $\chi > -1$

- $\begin{cases} \chi < 1 \\ \chi \gg 1 \text{ and small } \alpha \text{ (but } \leq 2/3) \end{cases}$

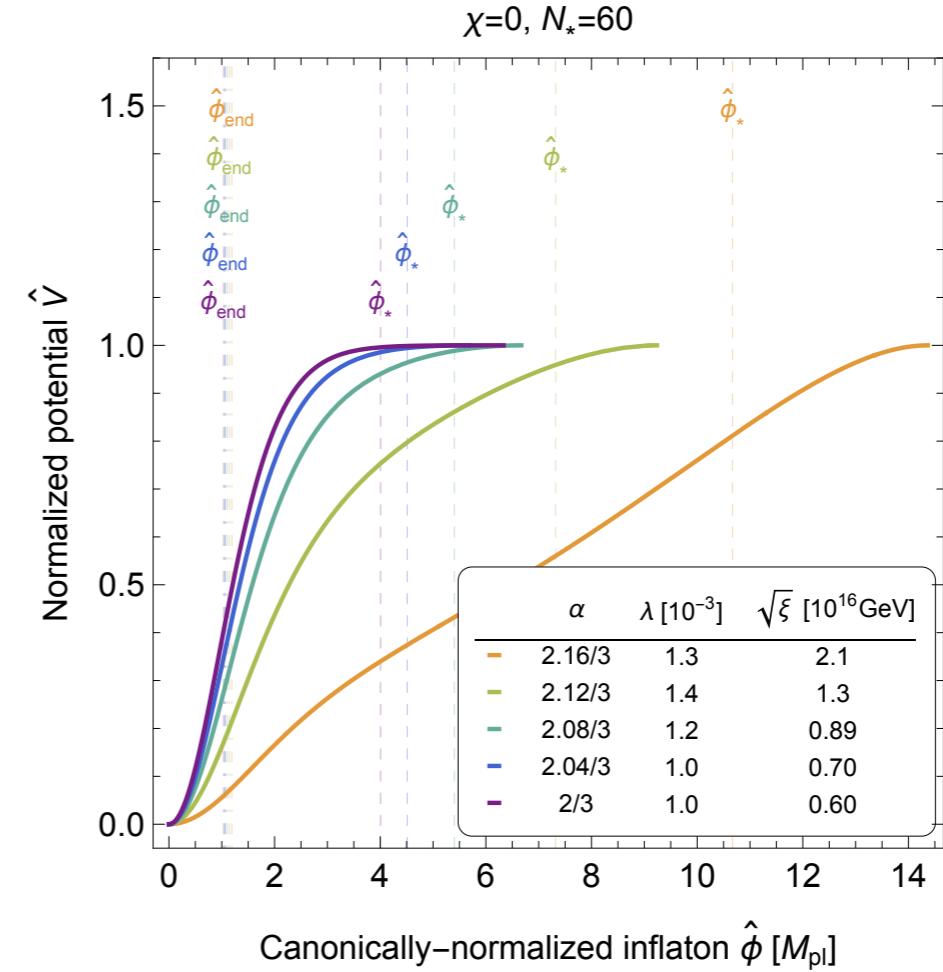
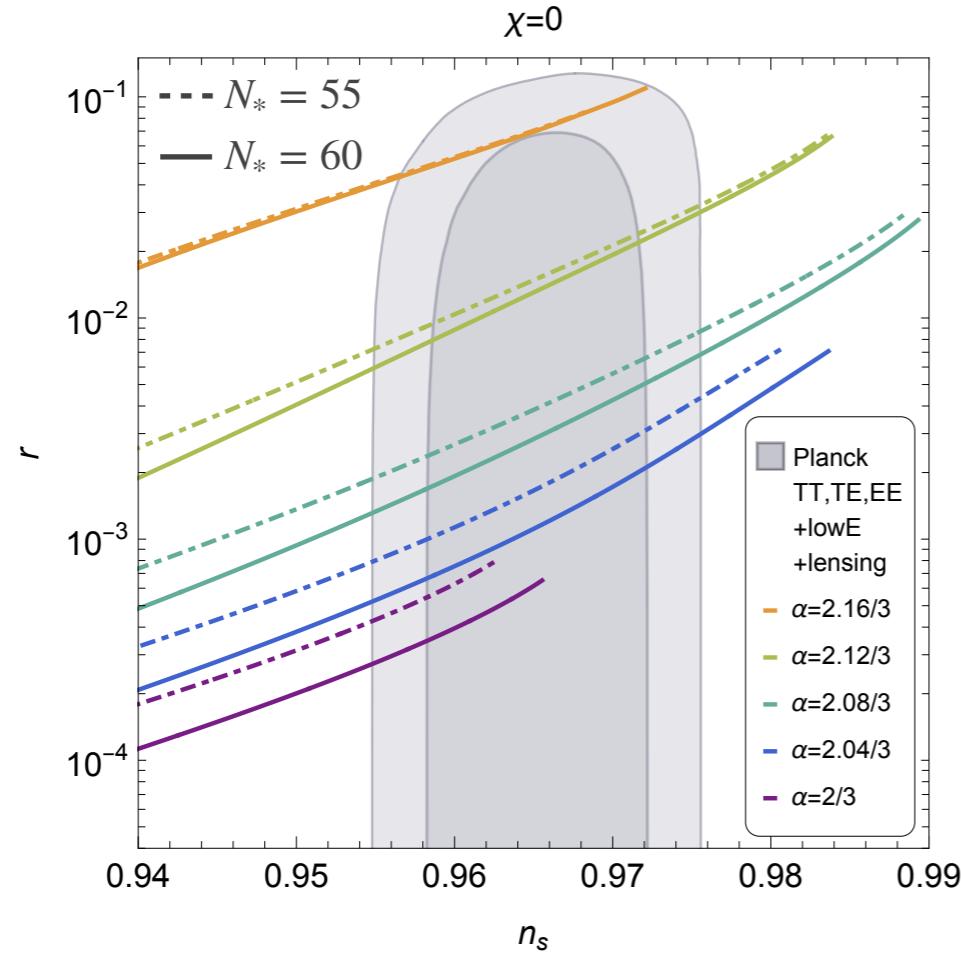
# Canonically normalized inflaton

Canonically normalized inflaton  $\hat{\phi}$  determined by solving the equation:

$$\frac{d\hat{\phi}}{d\phi} = K_{N\bar{N}}^{1/2} \Big|_{s=s_{\min}}$$

$$K_{N\bar{N}} \simeq \frac{3\alpha}{-\Phi(\phi)} \left[ 1 + \frac{(1+\chi)^2 \phi^2}{-2\Phi(\phi)} \right] \quad (\because s_{\min} \simeq 0)$$

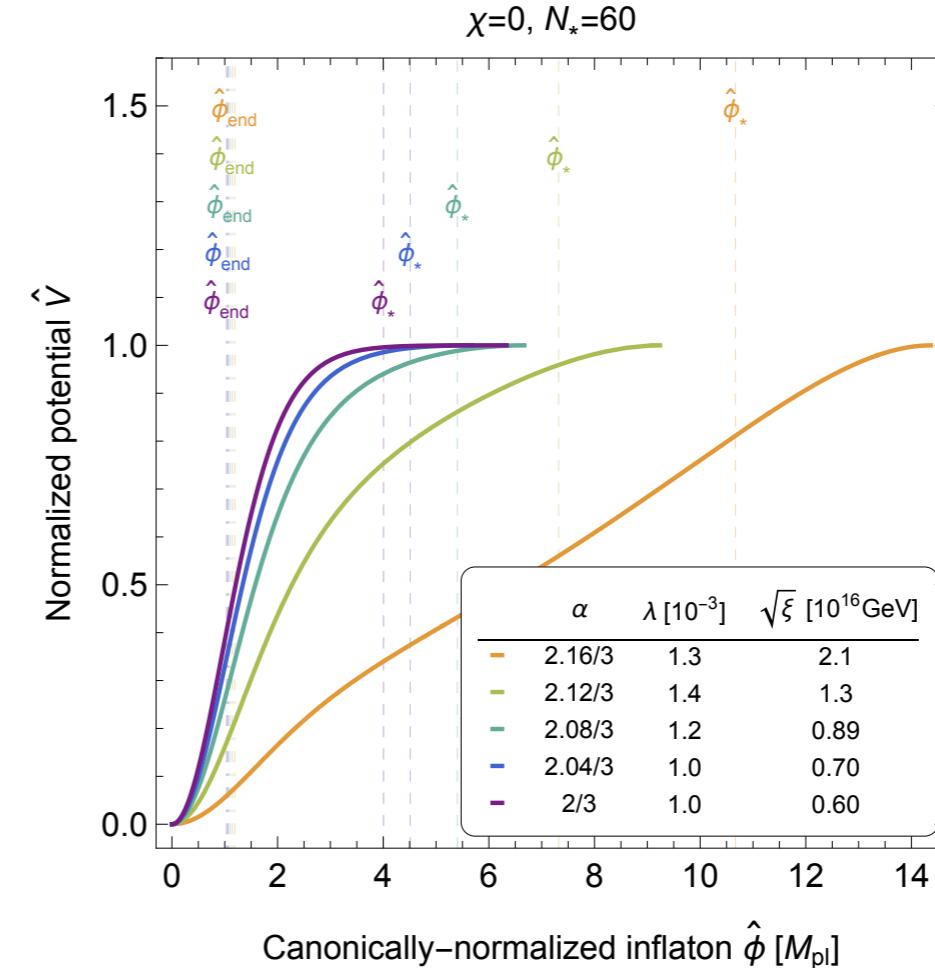
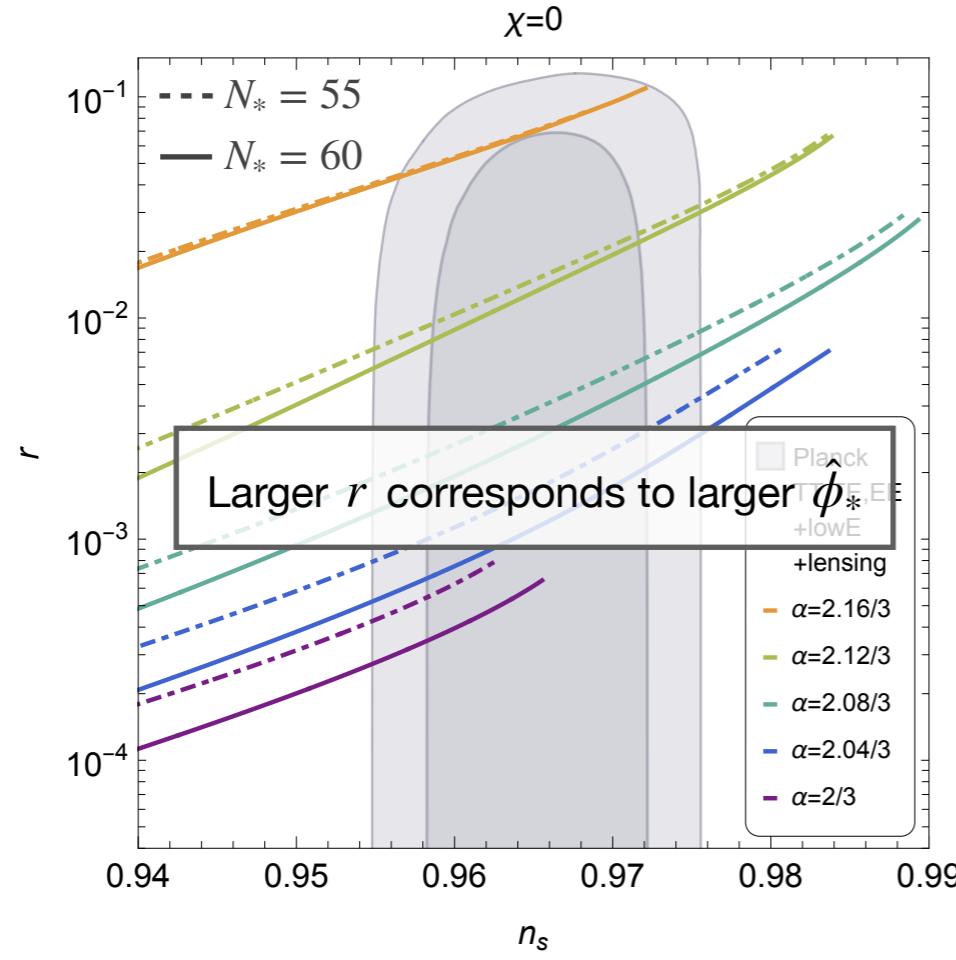
# $\chi = 0$ case



$$V = g^2 \xi^2 \Psi \left( 1 - \frac{1}{2} \Psi \right)$$

$$\begin{aligned} \phi^2 &= 6 \tanh^2 \frac{\hat{\phi}}{\sqrt{6\alpha}} \\ \Psi &= \frac{3k}{\alpha^2} \tanh^2 \frac{\hat{\phi}}{\sqrt{6\alpha}} \times \cosh^{2(3\alpha-2)} \frac{\hat{\phi}}{\sqrt{6\alpha}} \end{aligned}$$

# $\chi = 0$ case



Potential in large field value limit for  $\alpha \neq 2/3$

$$V = g^2 \xi^2 \Psi \left( 1 - \frac{1}{2} \Psi \right) \sim V_0 e^{p\hat{\phi}} \quad (\because \Psi \sim C e^{p\hat{\phi}}) \quad p = \sqrt{2/3\alpha}(3\alpha - 2)$$

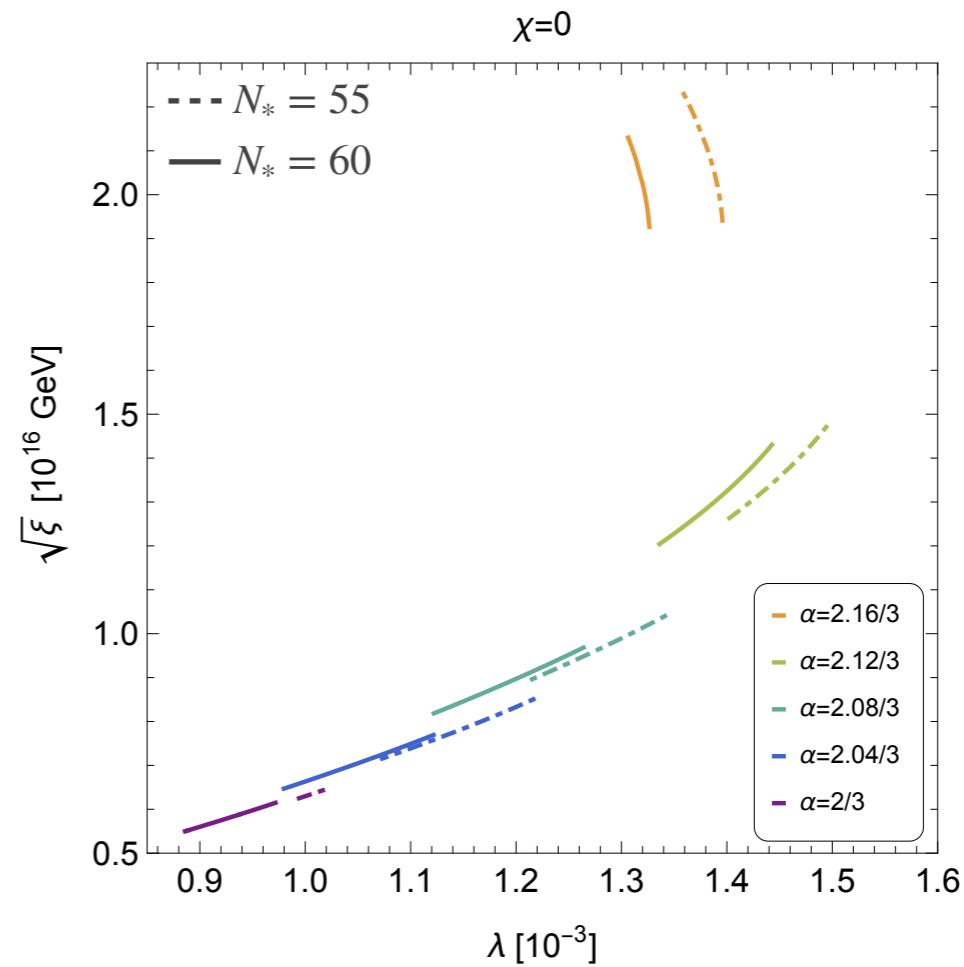
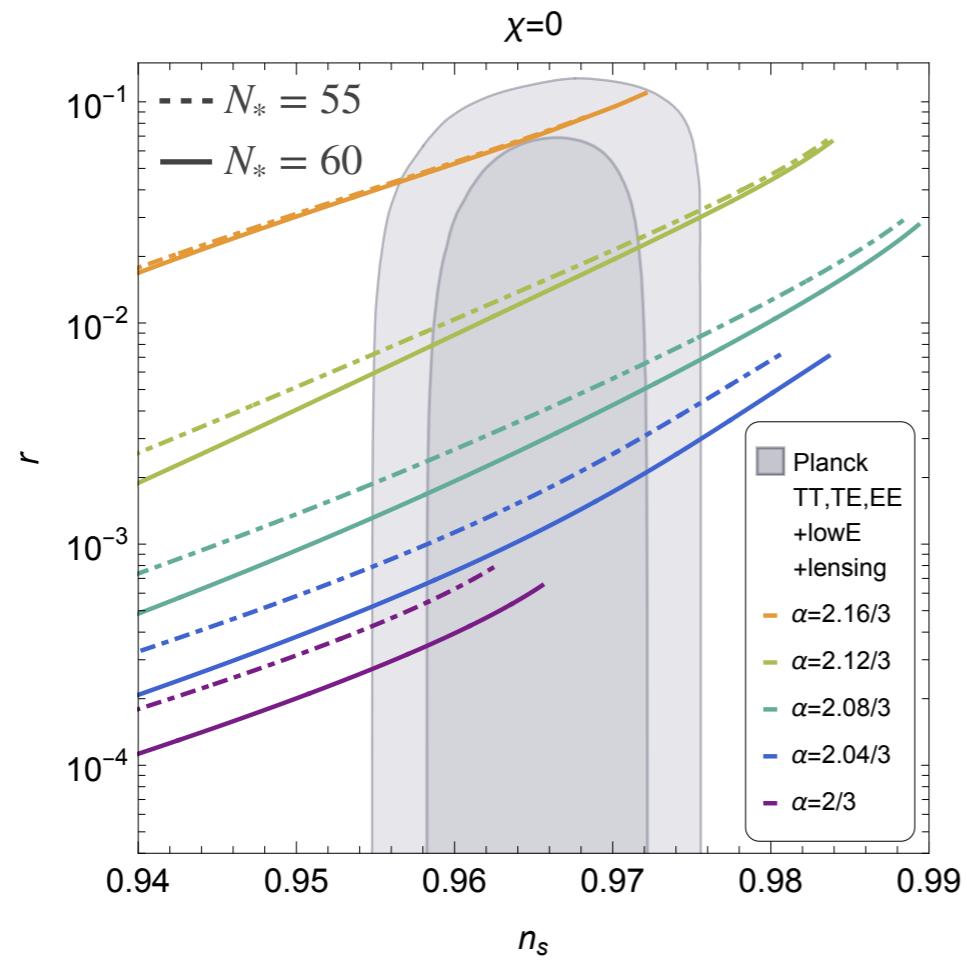
$$(n_s, r) \sim (1 - p^2, 8p^2)$$

$$C = 3k/4^{3\alpha-2}\alpha^2$$

$$k = \lambda^2/qg^2\xi$$

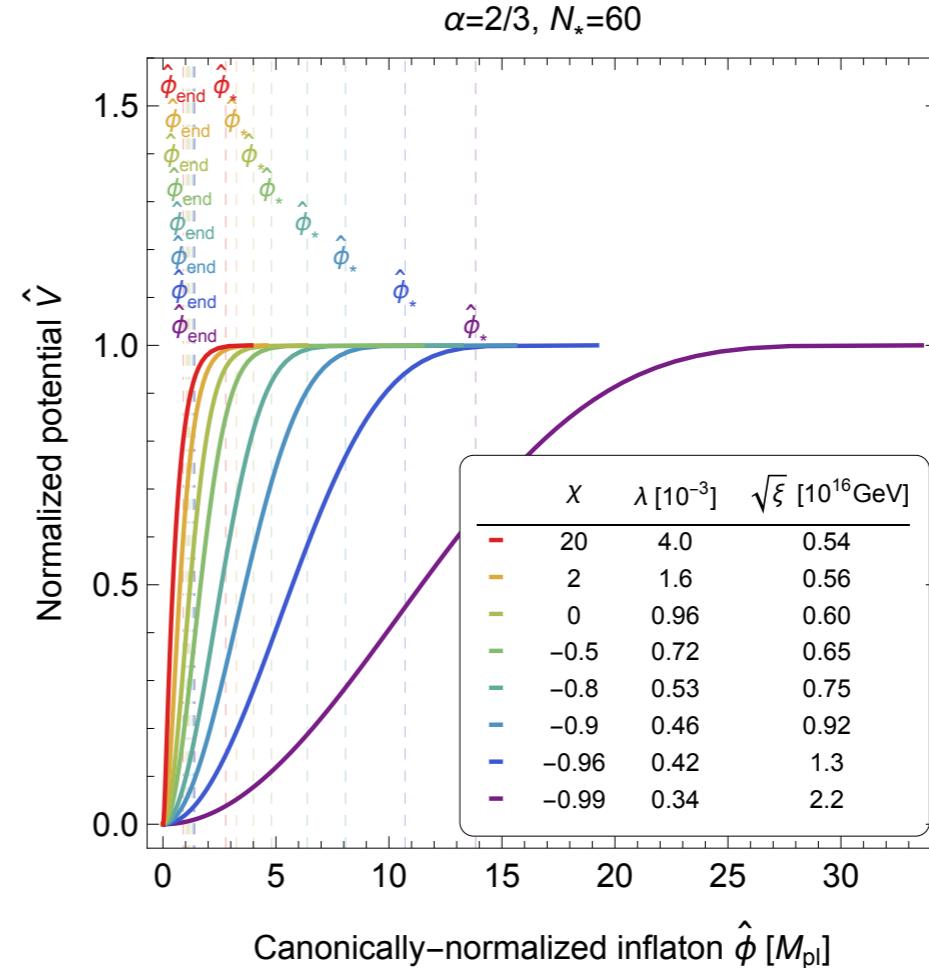
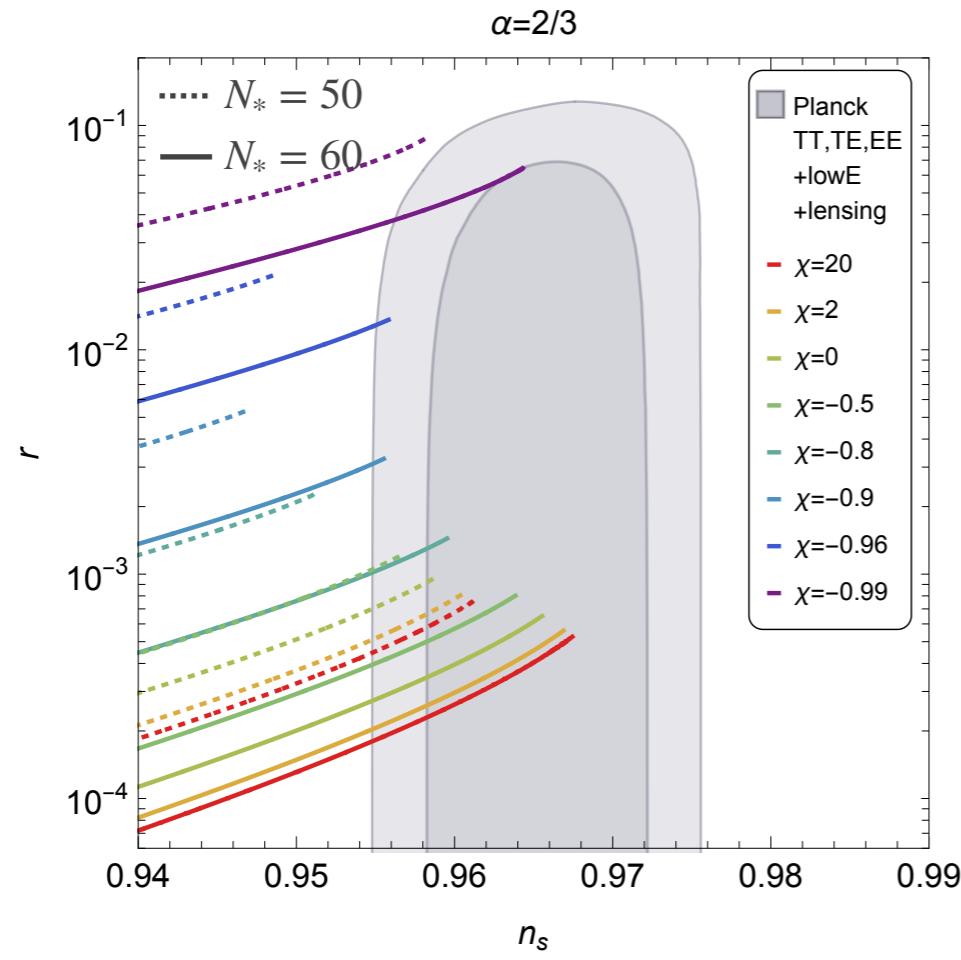
Numerical results deviate from the approximation due to  $\Psi^2$  term in the potential

# $\chi = 0$ case



Allowed  $\lambda$  &  $\sqrt{\xi}$  are  $\lambda \sim 10^{-3}$  &  $\sqrt{\xi} \sim 10^{16} \text{ GeV}$

# $\alpha = 2/3$ case

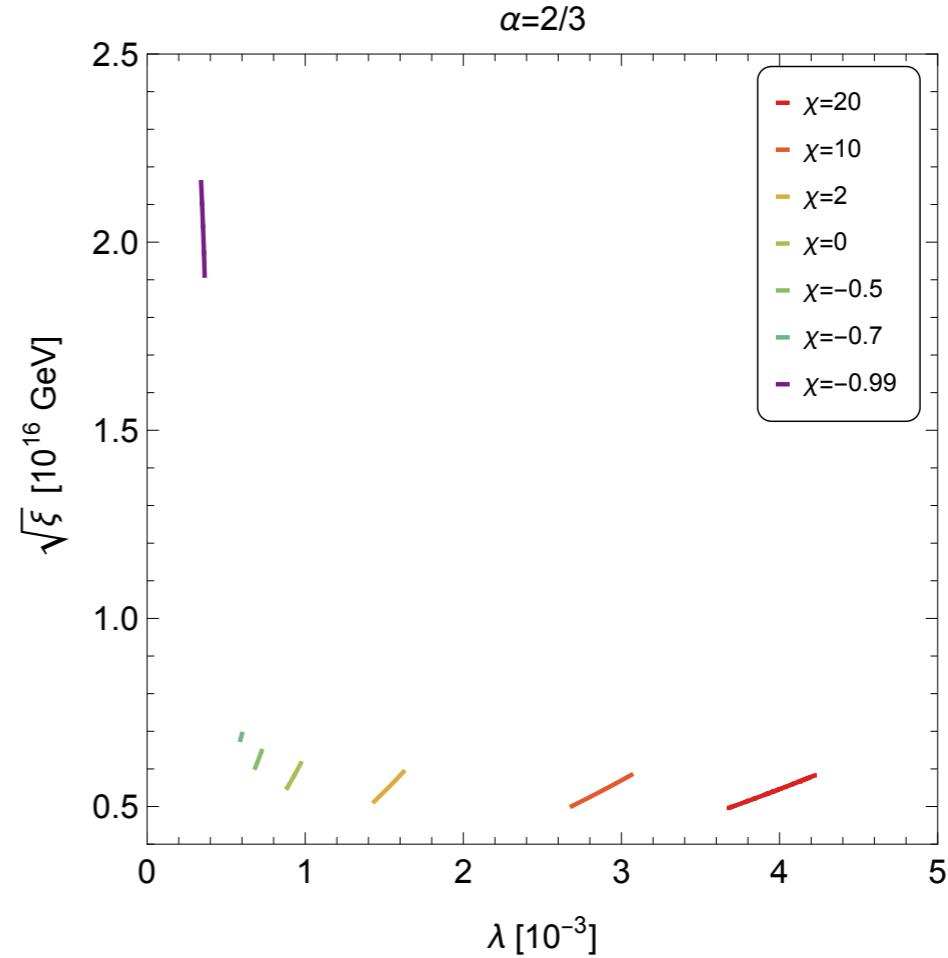
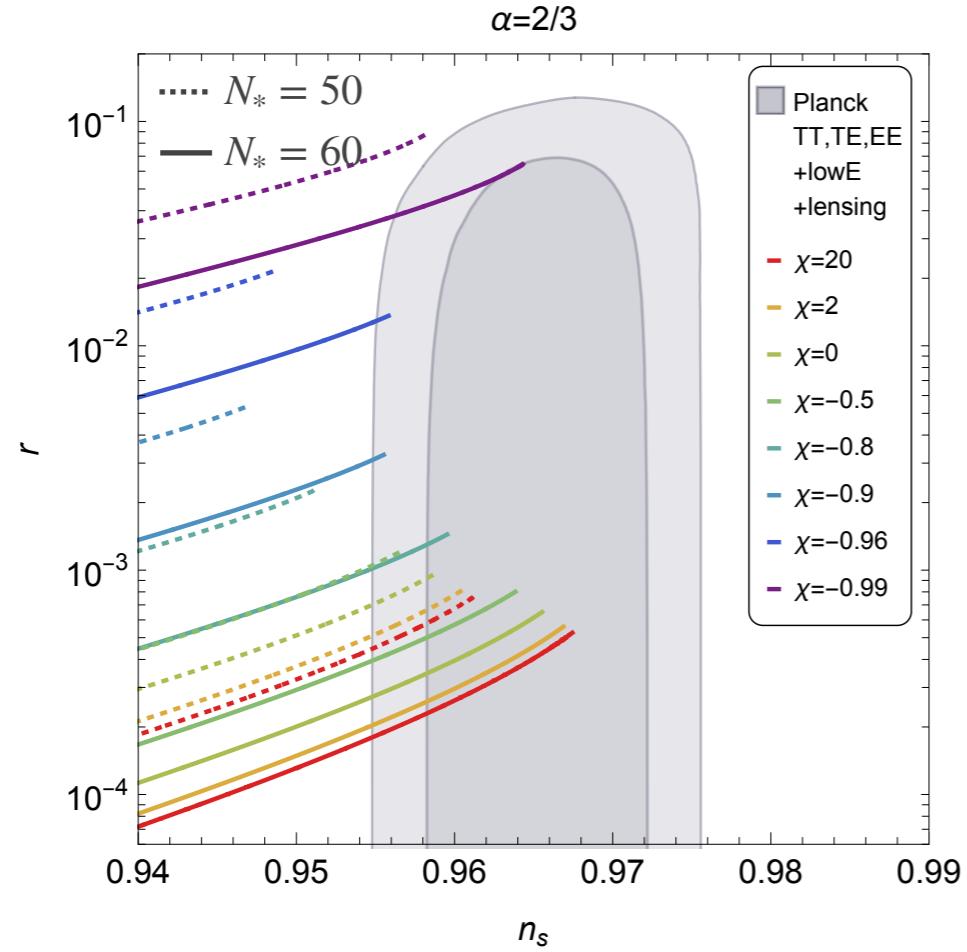


- Larger  $r$  corresponds to smaller  $k$  ( $k = \lambda^2/qg^2\xi$ )
- Largest  $r$  is determined by  $k > 4(1 + \chi)/27$

---

This is given by  $-\Phi(\phi_c)/3 > 0$ , i.e.,  $\phi_c < \sqrt{6/(1 + \chi)}$

# $\alpha = 2/3$ case



Allowed  $\lambda$  &  $\sqrt{\xi}$  are  $\lambda \sim 10^{-3}$  &  $\sqrt{\xi} \sim 10^{16} \text{ GeV}$