一般相対論を超える重力理論と重力波

1. Introduction
2. EFT approach
3. Massive gravity
4. Summary

向山信治（京都大学基礎物理学研究所）

1. Introduction
2. EFT approach
3. Massive gravity
4. Summary

INTRODUCTION

Why gravity beyond GR?

(GR : general relativity)

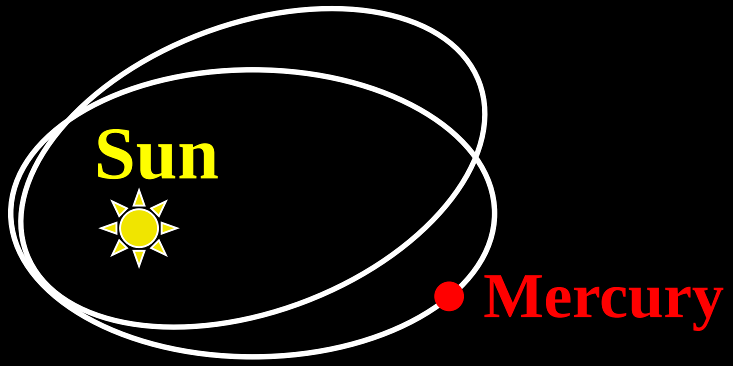
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

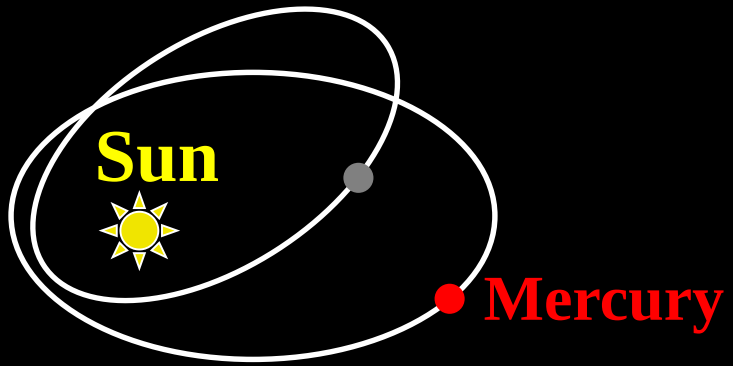
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Why gravity beyond GR?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.

Why gravity beyond GR?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.

Why gravity beyond GR?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

1. Introduction
2. EFT approach
3. Massive gravity
4. Summary

EFT APPROACH

Many gravity theories

- 3 check points
  “What are the physical d.o.f. ?”
  “How do they interact ?”
  “What is the regime of validity ?”
- If two (or more) theories give the same answers to the 3 questions above then they are the same even if they look different.
  → Effective Field Theory (EFT)
  as universal description

Scalar-tensor theories

- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), …
- Most general scalar-tensor theory with 2nd order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- All of them (and more) are universally described by an effective field theory (EFT)

EFT of inflation / dark energy = EFT of scalar-tensor theories

- **Time diffeo is broken by the background but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT of inflation / dark energy

- **Time diffeo is broken by the background but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing Nambu-Goldstone boson

Simplest : ghost condensation

ref. Arkani-Hamed, Cheng, Luty, Mukohyama 2004

Systematic construction of EFT of ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

Backgrounds characterized by

✧ $\langle \partial_\mu \phi \rangle \neq 0$ and timelike

✧Background metric is maximally symmetric, either Minkowski or dS.

⇨

$$\boxed{\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 \\ & - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \Big\} \end{aligned}}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

⇨ Write down most general action invariant under this residual symmetry.

(⇨ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\left\{ \begin{array}{l} \left(h_{00}\right)^2 \quad \textbf{OK} \\ \cancel{\left(h_{0i}\right)^2} \\ K^2, K^{ij}K_{ij} \quad \textbf{OK} \end{array} \right.$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00}\right)^2 - \frac{\alpha_1}{M^2} K^2 - \frac{\alpha_2}{M^2} K^{ij}K_{ij} + \cdots \right\}$$

Action invariant under ξ^i

$$\begin{cases} \left(h_{00}\right)^2 & \textbf{OK} \\ \xcancel{\left(h_{0i}\right)^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00}\right)^2 - \frac{\alpha_1}{M^2} K^2 - \frac{\alpha_2}{M^2} K^{ij}K_{ij} + \cdots \right\}$$

Action for π

$$\xi^0 = \pi \quad \begin{cases} h_{00} \to h_{00} - 2\partial_0 \pi \\ K_{ij} \to K_{ij} + \partial_i \partial_j \pi \end{cases}$$

$$\Rightarrow \quad \boxed{\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 \\ & - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \Big\} \end{aligned}}$$

$$E \to rE$$

$$dt \to r^{-1}dt$$

$$dx \to r^{-1/2}dx$$

$$\pi \to r^{1/4}\pi$$

Make invariant $\Rightarrow$ $$\int dt\,d^3x\left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots\right]$$

Leading nonlinear operator in infrared $\int dt\,d^3x\,\frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

$\Rightarrow$ Good low-E effective theory

Robust prediction

e.g. Ghost inflation [Arkani-hamed, Creminelli, Mukohyama, Zaldarriaga 2004]

“The Effective Field Theory of Vector-Tensor Theories”

Katsuki Aoki, Mohammad Ali Gorji, Shinji Mukohyama, Kazufumi Takahashi, JCAP01(2022)059 [arXiv: 2111.08119].

Residual symmetry in the unitary gauge

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

$$t \to t - g_M \chi(x) \,, \quad A_\mu \to A_\mu + \partial_\mu \chi(x)$$

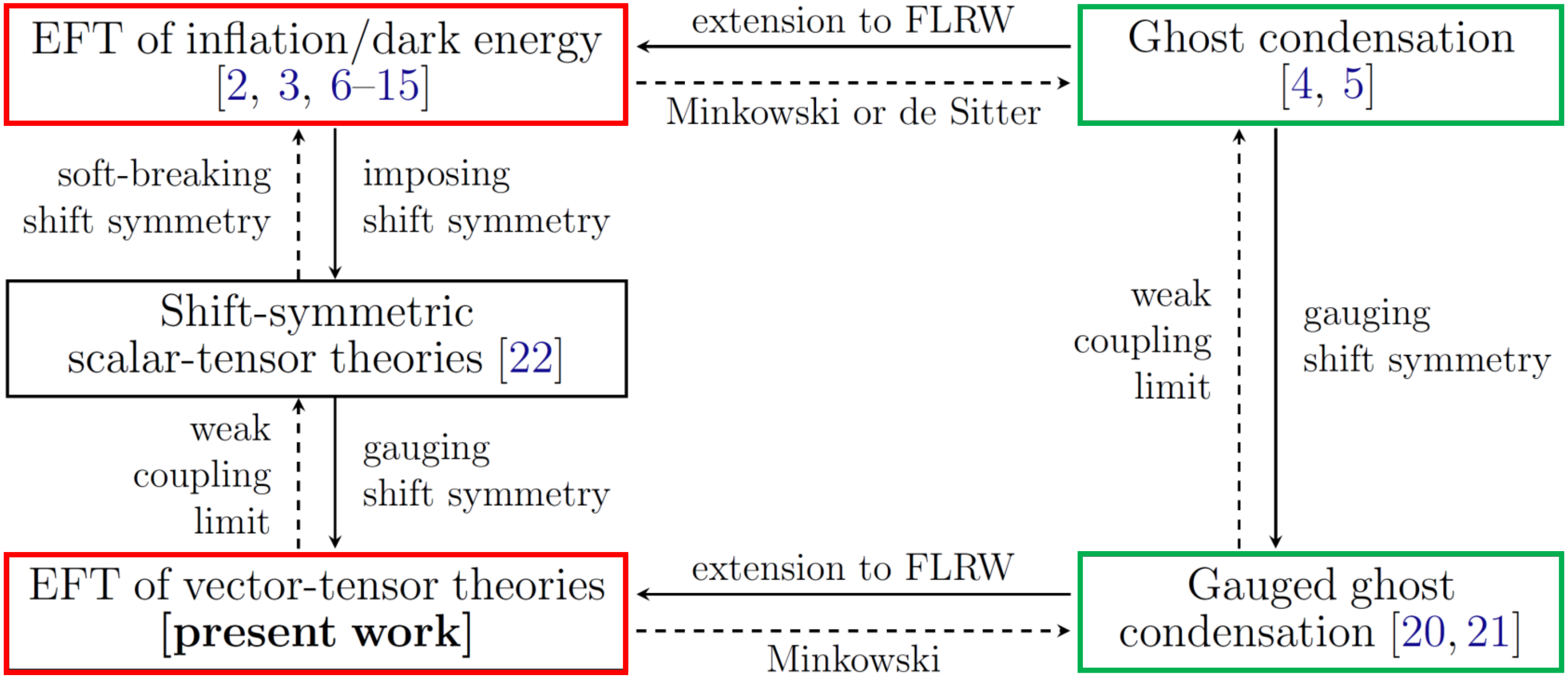
leaving $\tilde{\delta}^0{}_\mu = \delta^0{}_\mu + g_M A_\mu$ invariant

c.f. Residual symmetry in unitary gauge for scalar-tensor theories

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

The web of EFTs

Each EFT is useful for inflation/dark energy & GW phenomenology!



EFT of inflation/dark energy [2, 3, 6–15]

extension to FLRW

Minkowski or de Sitter

Ghost condensation [4, 5]

soft-breaking shift symmetry

imposing shift symmetry

Shift-symmetric scalar-tensor theories [22]

weak coupling limit

gauging shift symmetry

weak coupling limit

gauging shift symmetry

EFT of vector-tensor theories **[present work]**

extension to FLRW

Minkowski

Gauged ghost condensation [20, 21]

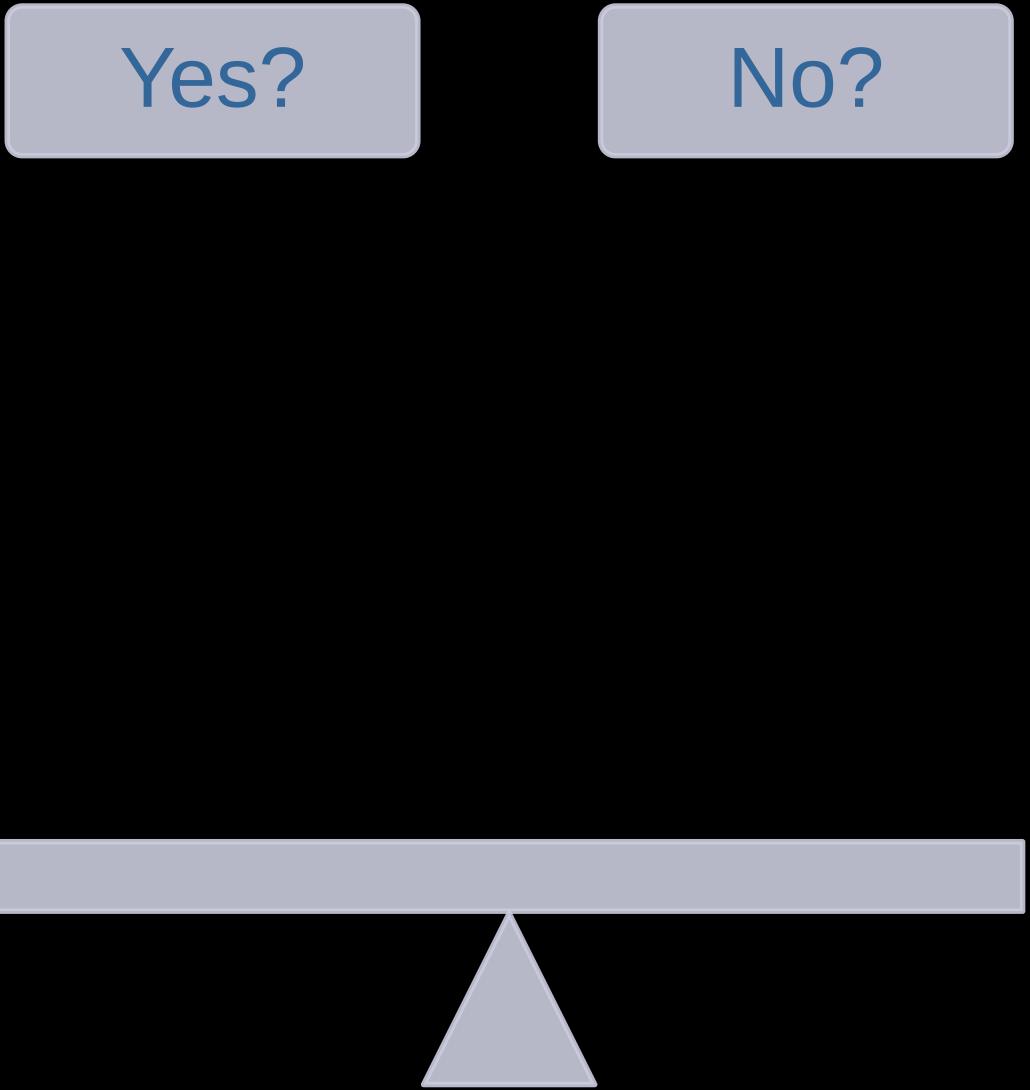
1. Introduction
2. EFT approach
3. Massive gravity
4. Summary

MASSIVE GRAVITY

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



Yes?

No?

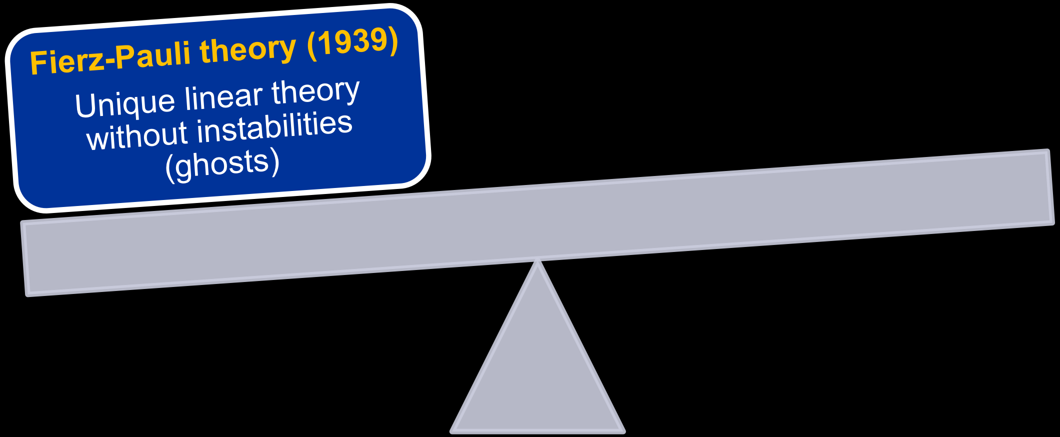
Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

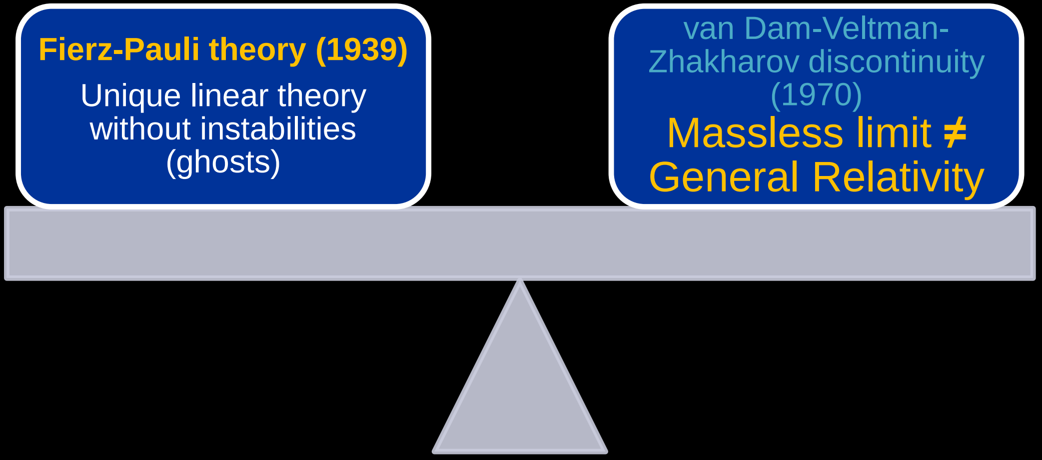
Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

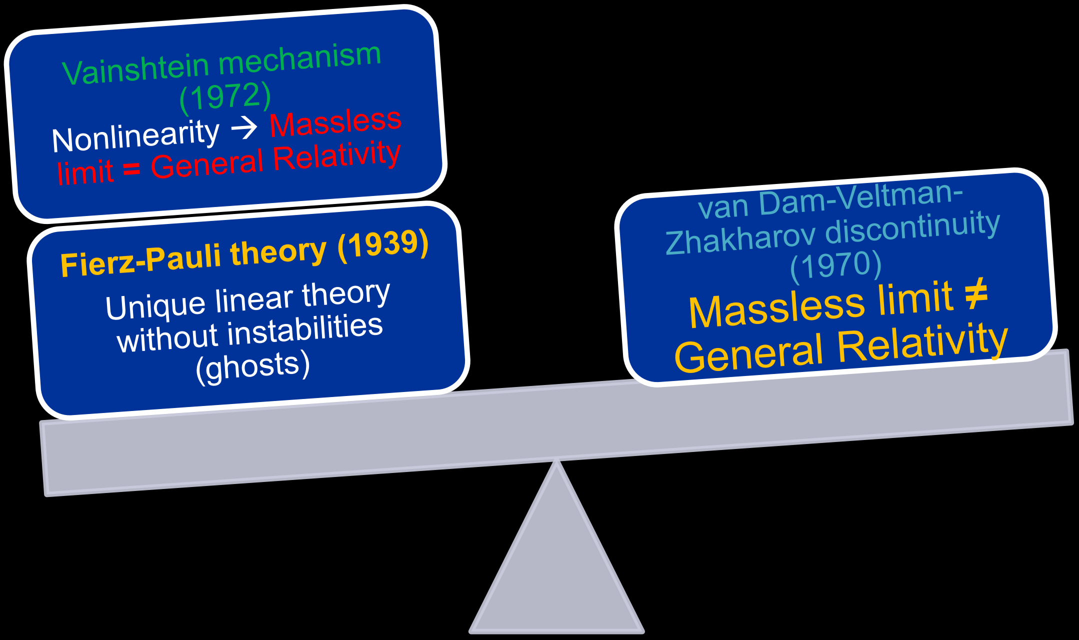
Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Vainshtein mechanism (1972)

Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)

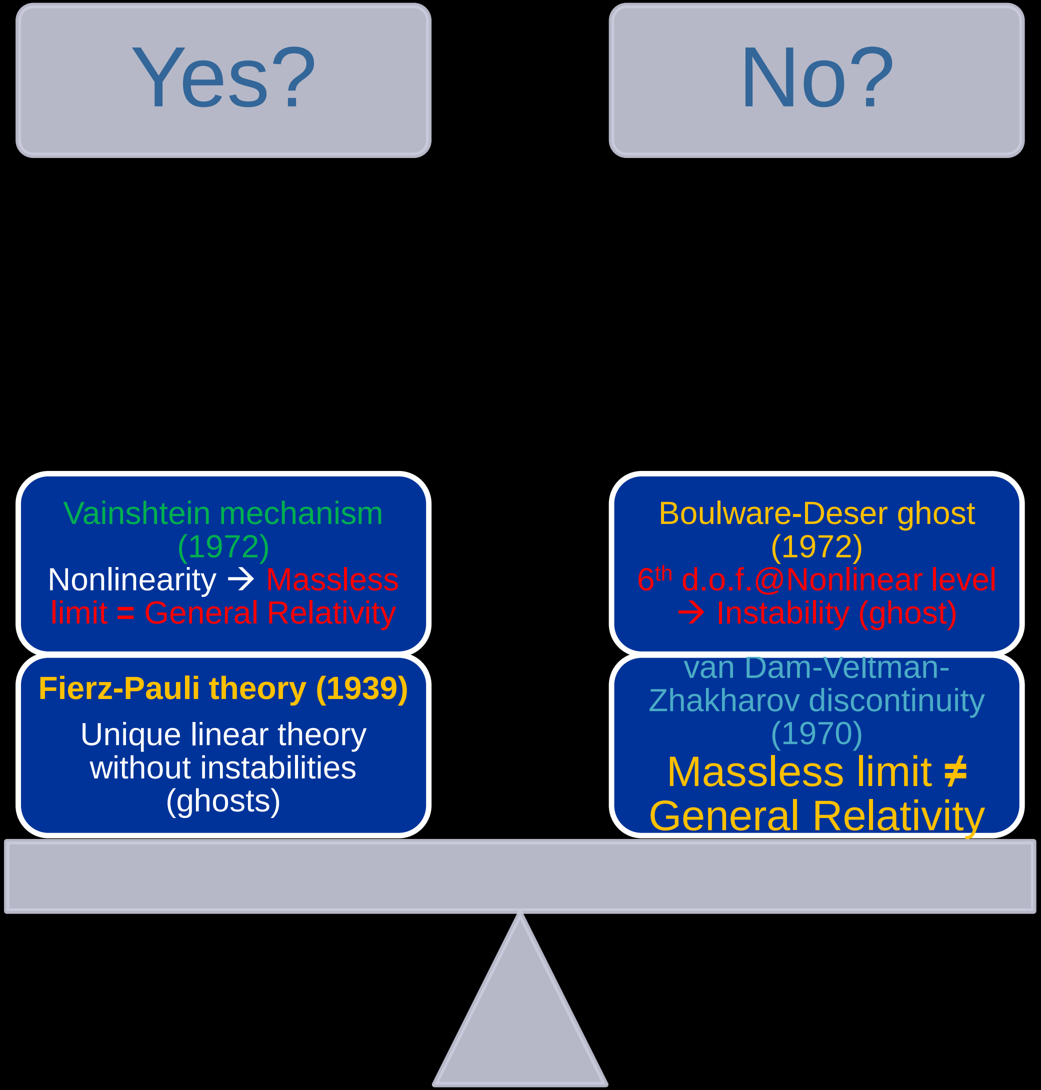
Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Massive gravity in a nutshell

Simple question: Can graviton have mass?
May lead to acceleration without dark energy



Yes?

No?

Vainshtein mechanism (1972)
Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)
Unique linear theory without instabilities (ghosts)

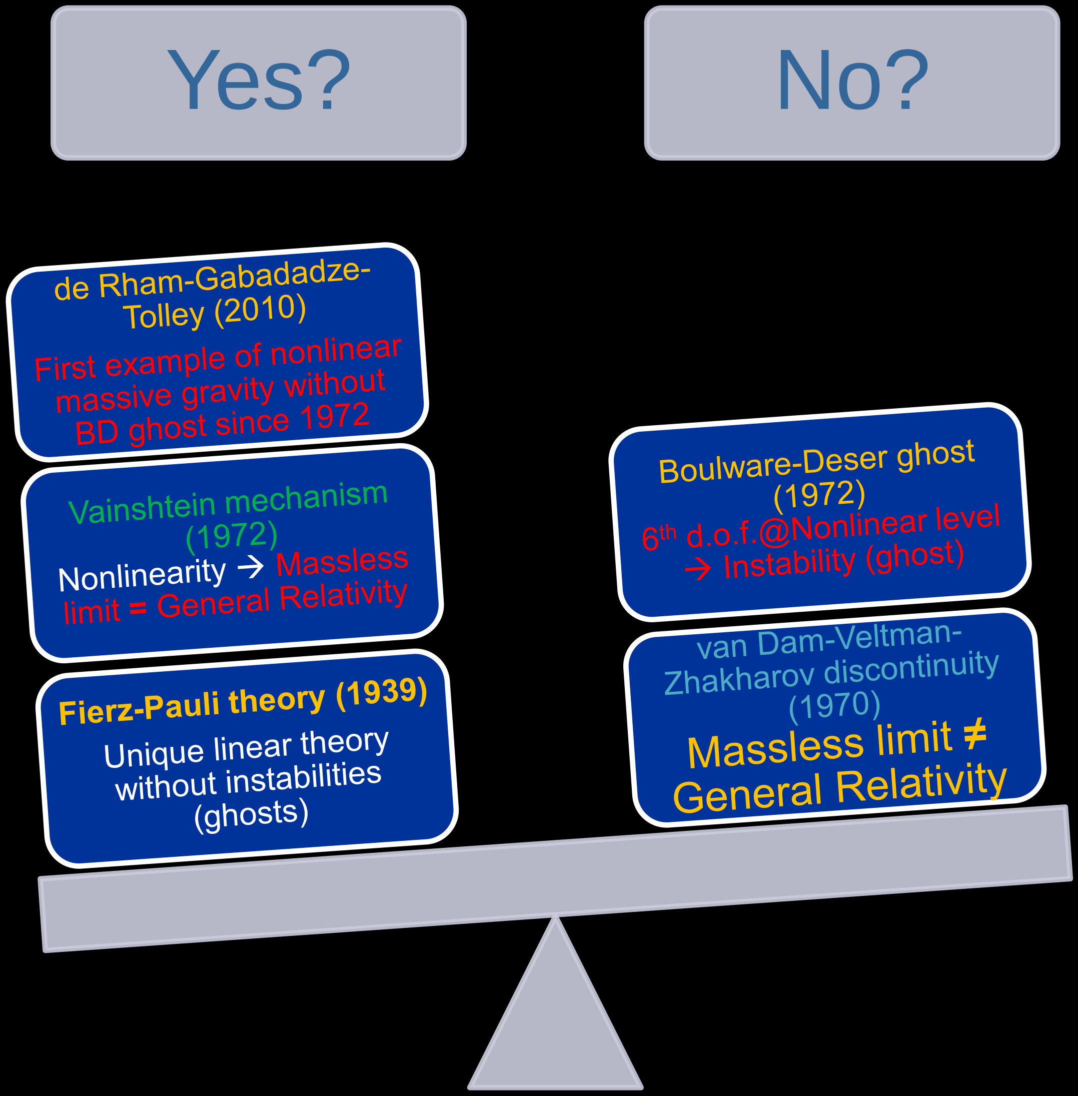
Boulware-Deser ghost (1972)
6th d.o.f.@Nonlinear level → Instability (ghost)

van Dam-Veltman-Zhakharov discontinuity (1970)
Massless limit ≠ General Relativity

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



Yes?

No?

de Rham-Gabadadze-Tolley (2010)

First example of nonlinear massive gravity without BD ghost since 1972

Vainshtein mechanism (1972)

Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)

6th d.o.f.@Nonlinear level → Instability (ghost)

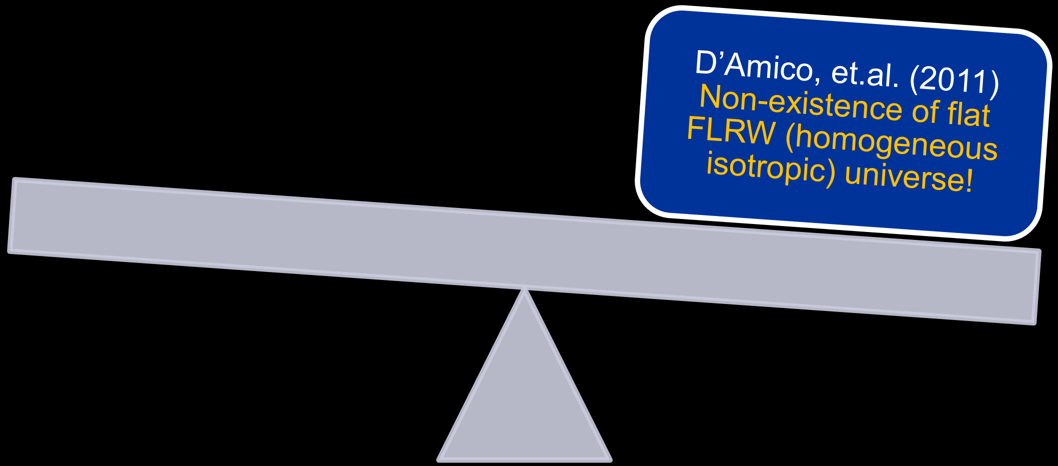
van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



D’Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

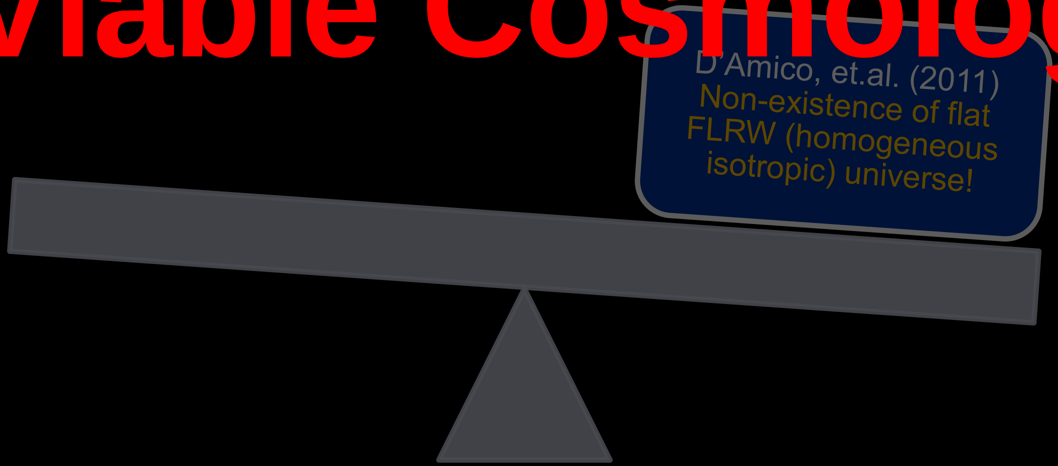
Cosmological solutions in nonlinear massive gravity

Good?

Bad?

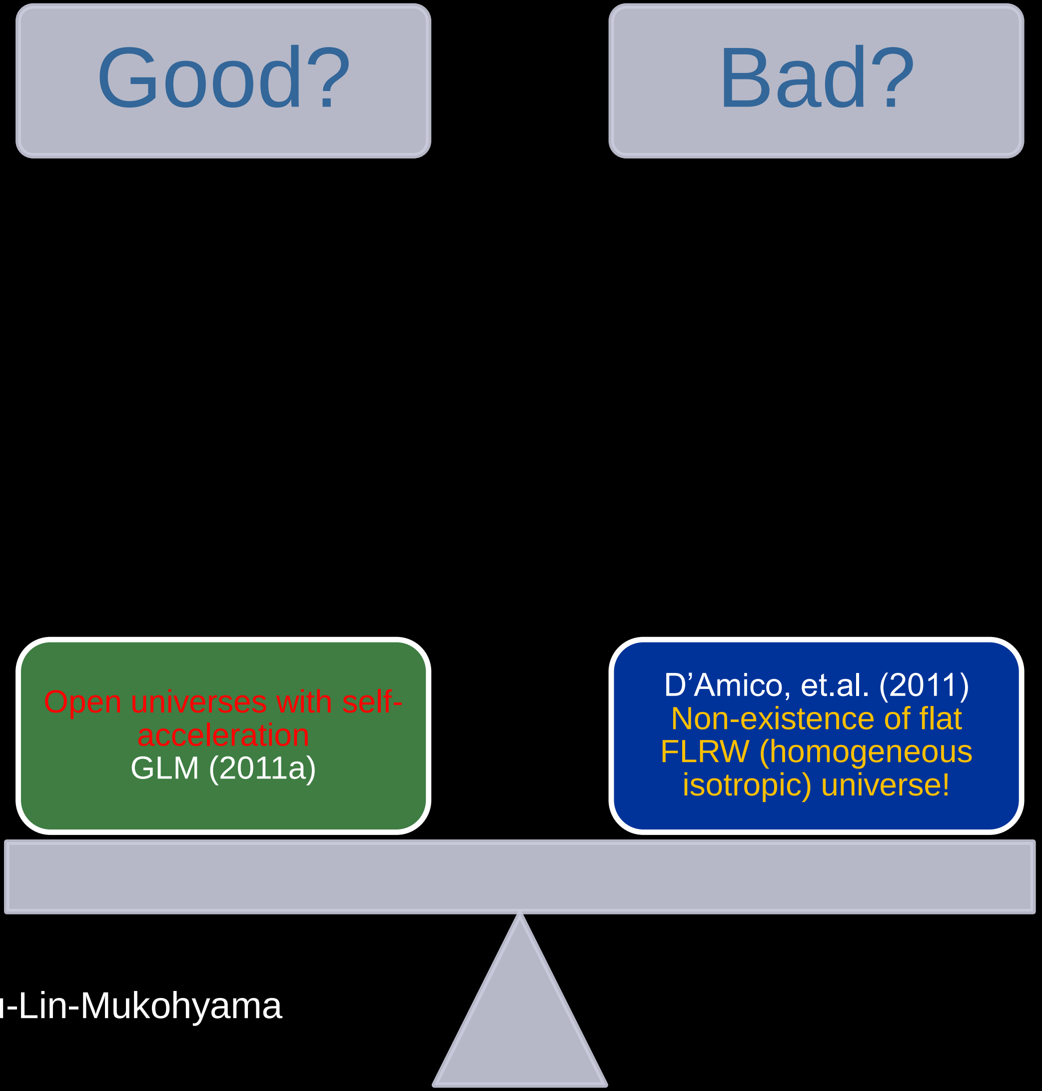
Consistent Theory found in 2010 but

No Viable Cosmology?



D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

Cosmological solutions in nonlinear massive gravity



Good?

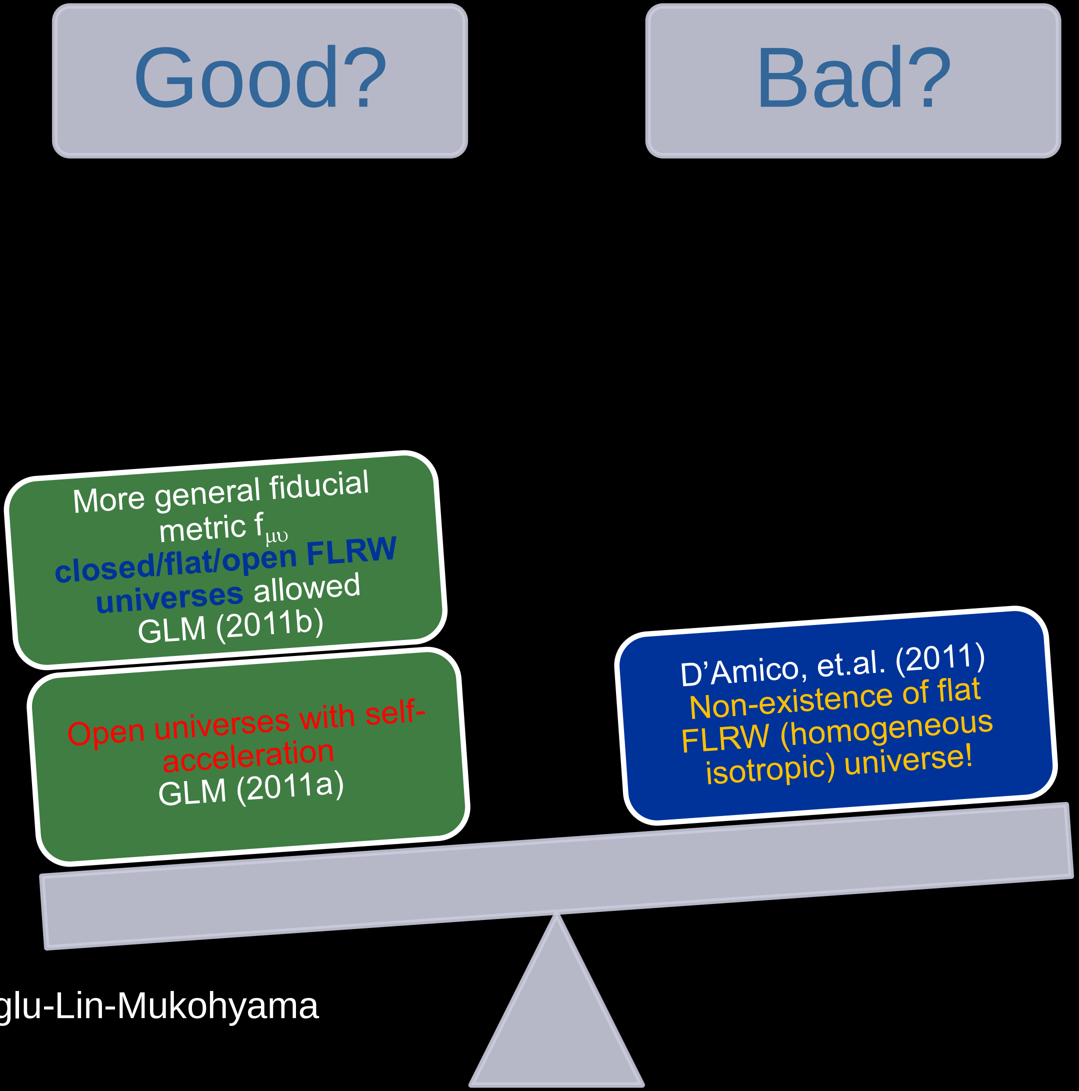
Bad?

Open universes with self-acceleration
GLM (2011a)

D’Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity



Good?

Bad?

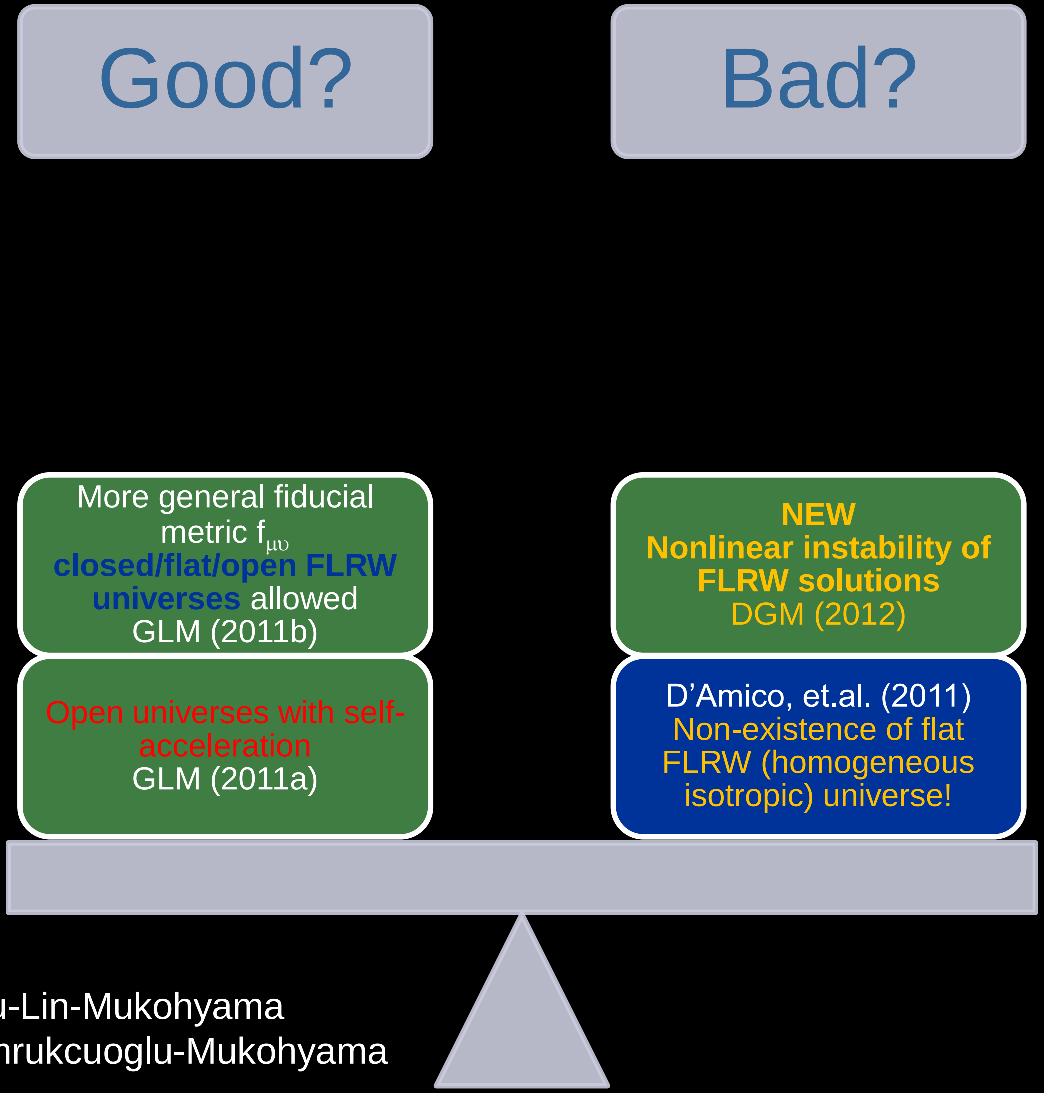
More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

D’Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity



Good?

Bad?

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

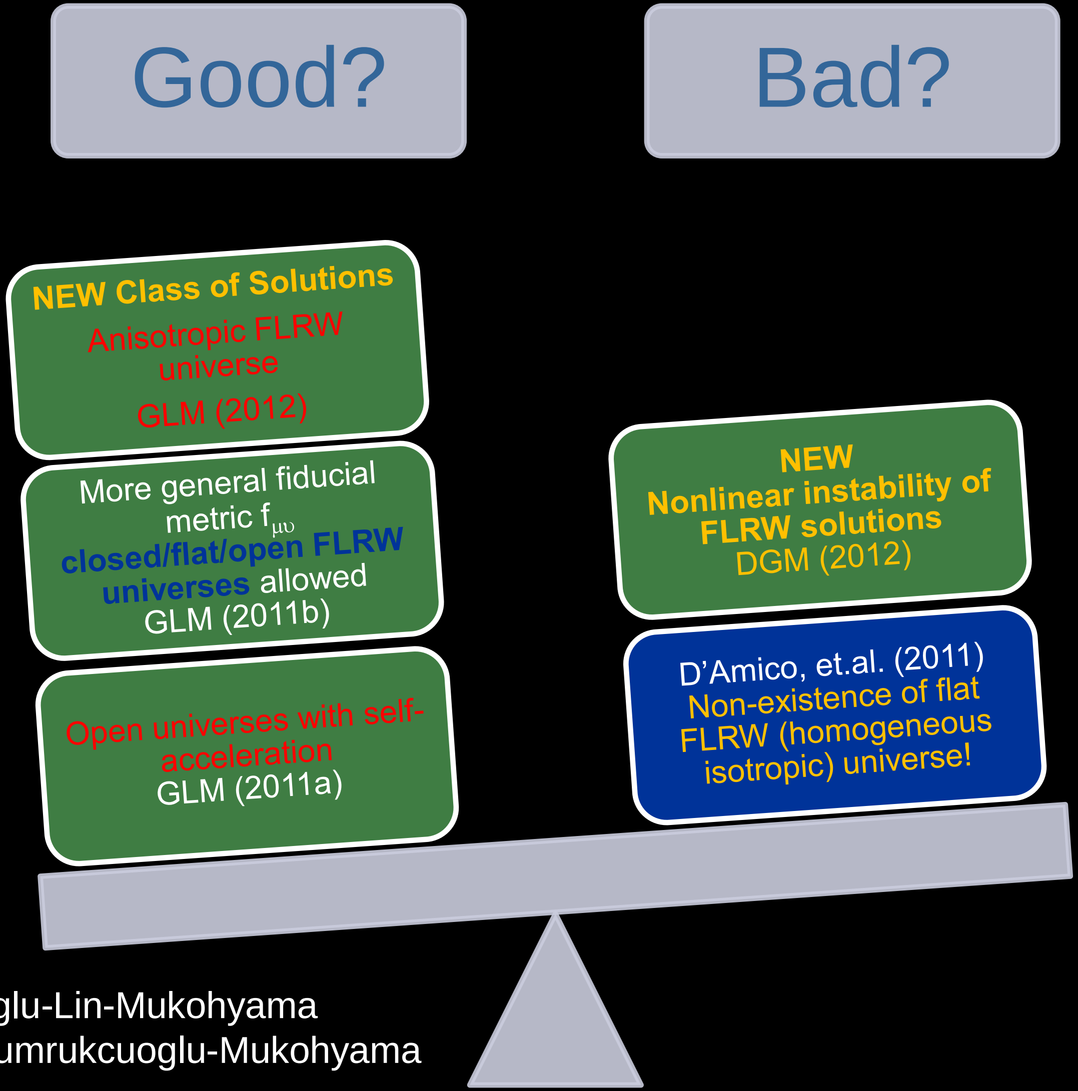
Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Cosmological solutions in nonlinear massive gravity



Good?

Bad?

NEW Class of Solutions
Anisotropic FLRW universe
GLM (2012)

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Minimal theory of massive gravity (MTMG)

De Felice & Mukohyama, PLB752 (2016) 302;
JCAP1604 (2016) 028

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Lorentz-violation due to graviton loops is suppressed by m^2/M_{Pl}^2 and thus consistent with all constraints for $m = O(H_0)$

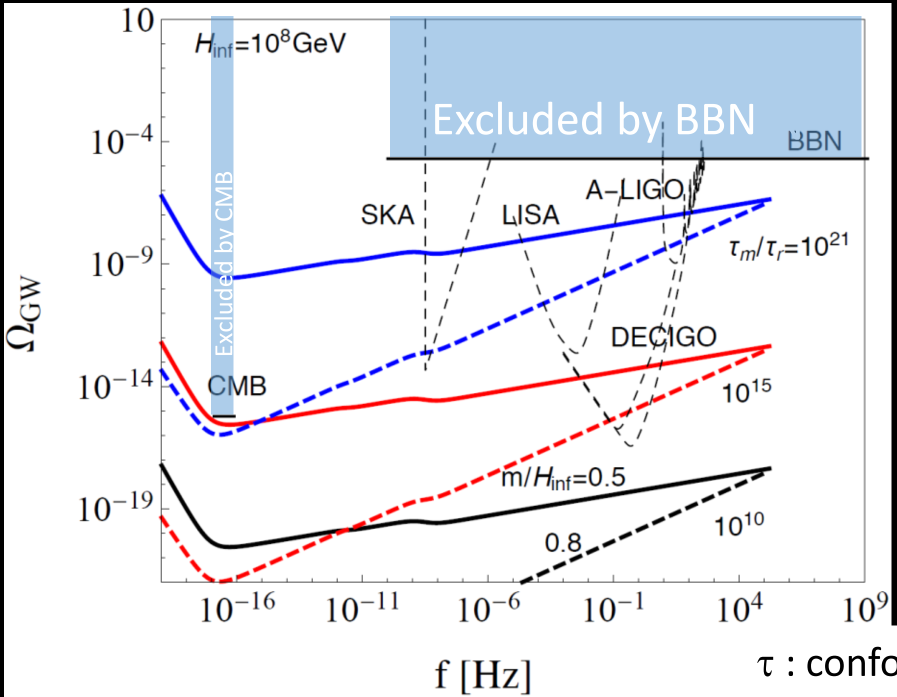
Blue-tilted & amplified primordial GW from MTMG

Fujita, Kuroyanagi, Mizuno, Mukohyama,
PLB789 (2019) 215

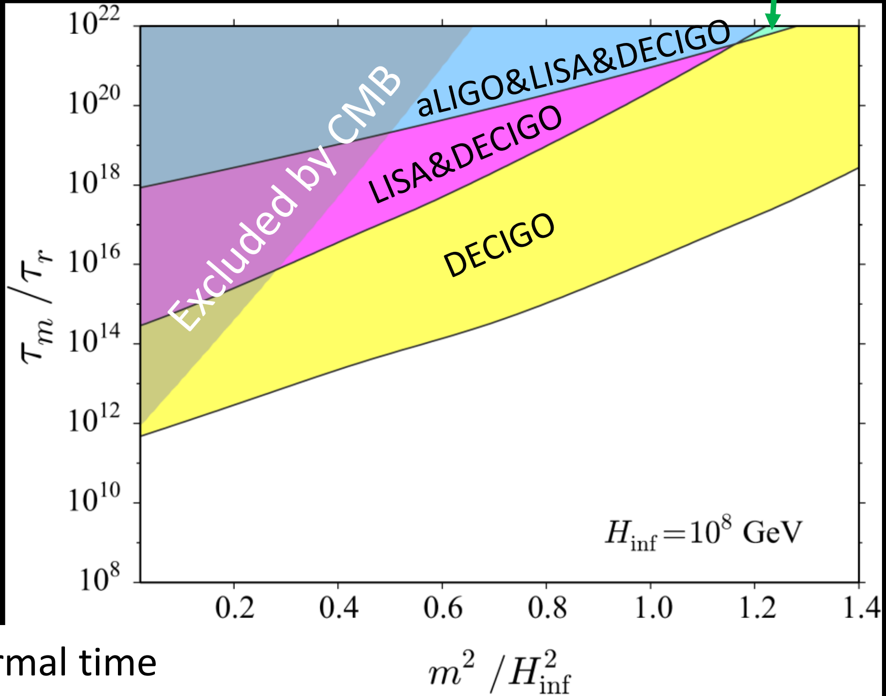
Fujita, Mizuno, Mukohyama,
JCAP 01 (2020) 023

- Simple extension: $c_i \rightarrow c_i(\phi)$ with $\phi = \phi(t)$
- m large until t_m ($t_{reh} < t_m < t_{BBN}$) but small after t_m
  cf. no Higuchi bound in MTMG
- Suppression of GW in IR due to large m → **blue spectrum**
- $\rho_{GW} \propto a^{-3}$ for $t_{reh} < t < t_m$ → **amplification relative to GR**

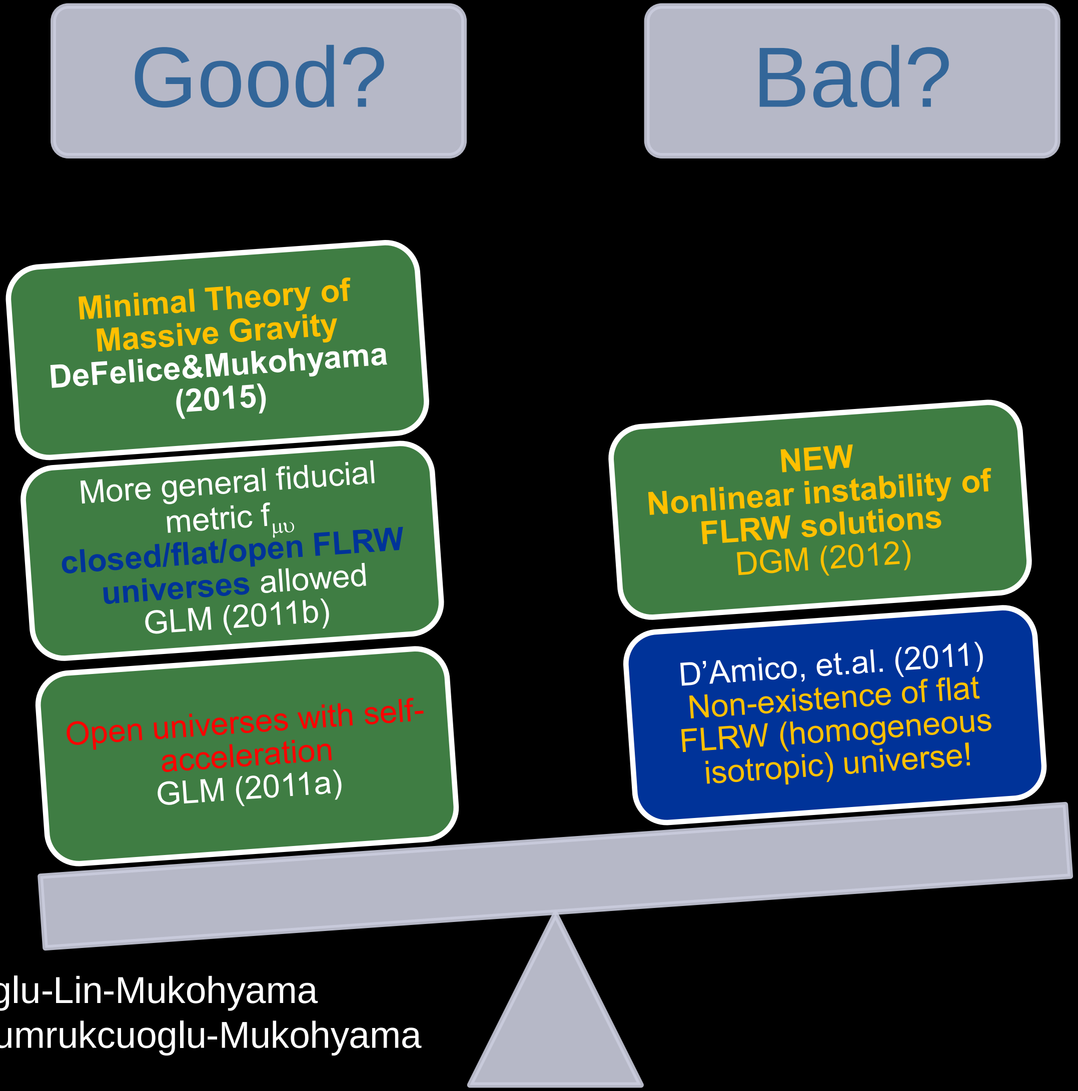
aLIGO&
DECIGO



τ : conformal time



Cosmological solutions in nonlinear massive gravity



Good?

Bad?

Minimal Theory of Massive Gravity
DeFelice&Mukohyama (2015)

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Minimal theory of bigravity (MTBG)

De Felice, Larrouturou, Mukohyama, Oliosi, arXiv:2012.01073.

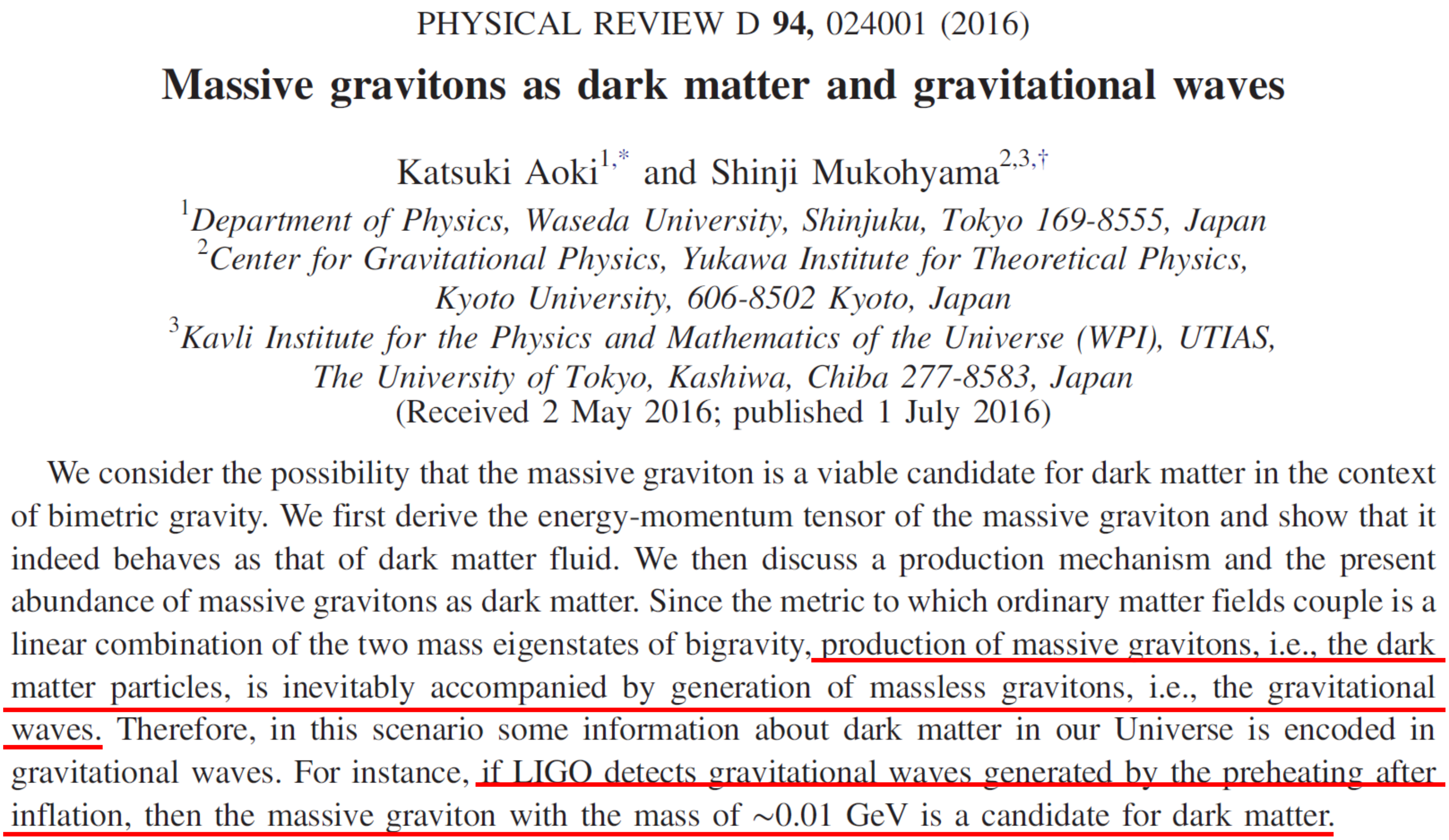
- 4 physical dof only = massless & massive GWs
- exactly same FLRW backgrounds as in HRBG
- no BD ghost, no Higuchi ghost, no strong coupling

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbeins in HRBG
2. Switch to Hamiltonian
3. Add 4 (= 5-1) additional constraints carefully

(It is easy to go back to Lagrangian after 3.)

The very first example of completely stable & cosmologically viable theory of nonlinear bigravity. A testing ground for gravitational phenomena, e.g. graviton oscillation, that can be probed by GWs.

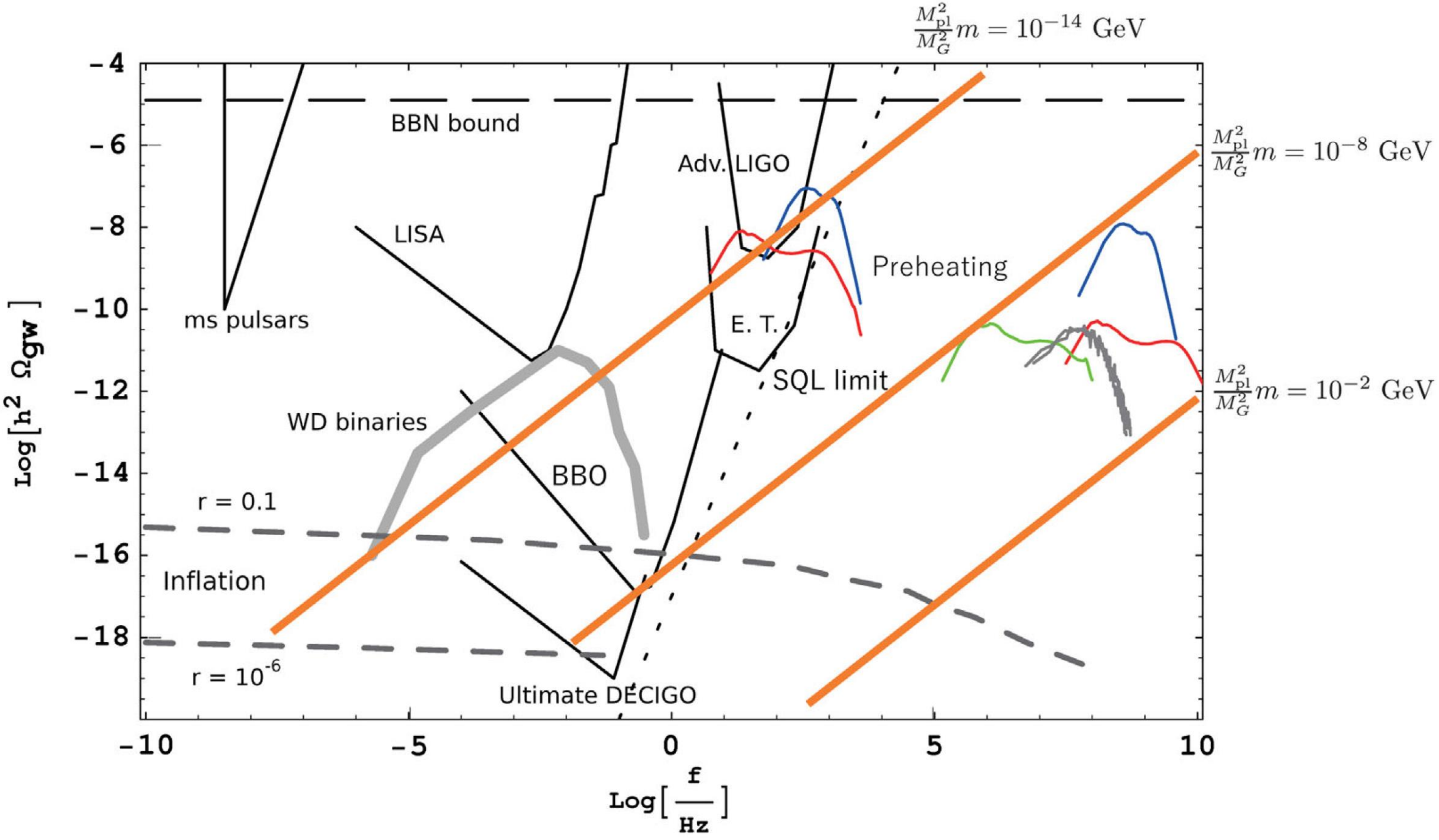


PHYSICAL REVIEW D **94,** 024001 (2016)

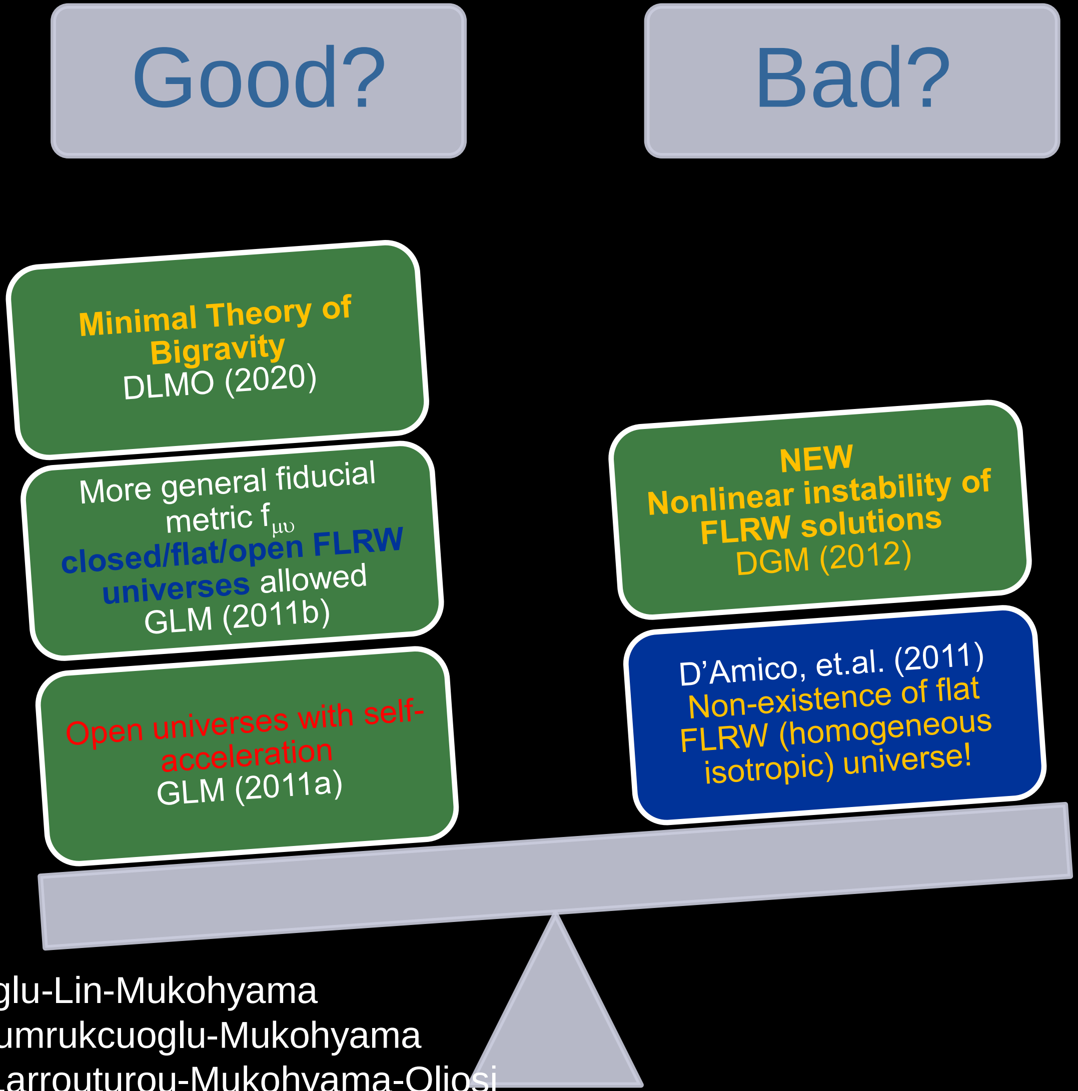
Massive gravitons as dark matter and gravitational waves

Katsuki Aoki[1,*] and Shinji Mukohyama[2,3,†]
[1]*Department of Physics, Waseda University, Shinjuku, Tokyo 169-8555, Japan*
[2]*Center for Gravitational Physics, Yukawa Institute for Theoretical Physics,*
Kyoto University, 606-8502 Kyoto, Japan
[3]*Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS,*
The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
(Received 2 May 2016; published 1 July 2016)

We consider the possibility that the massive graviton is a viable candidate for dark matter in the context of bimetric gravity. We first derive the energy-momentum tensor of the massive graviton and show that it indeed behaves as that of dark matter fluid. We then discuss a production mechanism and the present abundance of massive gravitons as dark matter. Since the metric to which ordinary matter fields couple is a linear combination of the two mass eigenstates of bigravity, production of massive gravitons, i.e., the dark matter particles, is inevitably accompanied by generation of massless gravitons, i.e., the gravitational waves. Therefore, in this scenario some information about dark matter in our Universe is encoded in gravitational waves. For instance, if LIGO detects gravitational waves generated by the preheating after inflation, then the massive graviton with the mass of ~0.01 GeV is a candidate for dark matter.


$\frac{M_{\rm pl}^2}{M_G^2}m = 10^{-14}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2}m = 10^{-8}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2}m = 10^{-2}$ GeV
-4
-6
-8
-10
-12
-14
-16
-18
BBN bound
Adv. LIGO
LISA
Preheating
ms pulsars
E. T.
SQL limit
WD binaries
BBO
r = 0.1
Inflation
r = 10^{-6}
Ultimate DECIGO
-10
-5
0
5
10
$\mathrm{Log}\left[\frac{f}{\mathrm{Hz}}\right]$
$\mathrm{Log}\left[h^2\ \Omega_{gw}\right]$

Cosmological solutions in nonlinear massive gravity



Good?

Bad?

Minimal Theory of Bigravity
DLMO (2020)

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama
DLMO = DeFelice-Larrouturou-Mukohyama-Oliosi

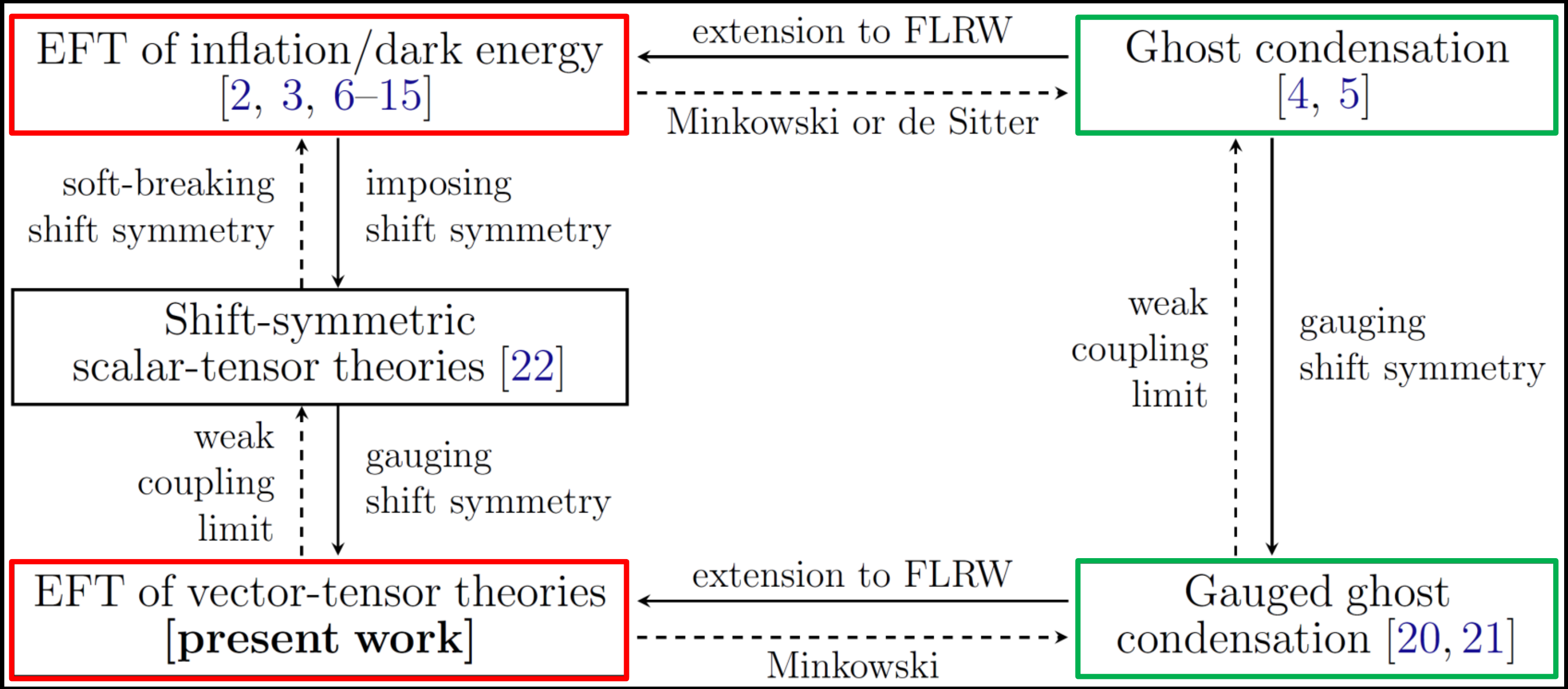
SUMMARY

Why gravity beyond GR?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

Summary of EFT approach

- Ghost condensation is a universal description of scalar-tensor theories around Minkowski and de Sitter backgrounds.
- Ghost condensation generates a web of EFTs describing scalar-tensor and vector-tensor theories.



EFT of inflation/dark energy [2, 3, 6–15]

extension to FLRW

Minkowski or de Sitter

Ghost condensation [4, 5]

soft-breaking shift symmetry

imposing shift symmetry

Shift-symmetric scalar-tensor theories [22]

weak coupling limit

gauging shift symmetry

weak coupling limit

gauging shift symmetry

EFT of vector-tensor theories [**present work**]

extension to FLRW

Minkowski

Gauged ghost condensation [20, 21]

Summary of massive gravity

- Massive gravity has a long history as a fundamental question in classical field theory.
- dRGT theory is free from BD ghost but its cosmology suffers from strong coupling and ghost instability.
  Stable cosmology requires either (i) new class of cosmological solutions or (ii) extended theories.
- MTMG and MTBG provide nonlinear completion of dRGT self-accelerating cosmology.
  → Testing grounds for gravitational phenomena that can be probed by GWs.
  \> Blue-tilted & amplified primordial GW
  \> Graviton Oscillation
  \> Massive graviton DM and GW
  and more …

Implication of GW170817 on gravity theories @ late time

$$X = -\partial^\mu \phi \partial_\mu \phi$$

- $|(c_{gw} - c_\gamma)/ c_\gamma| < 10^{-15}$
- Horndeski theoy (scalar-tensor theory with 2nd-order eom):
  Among 4 free functions, $G_4(\phi,X)$ & $G_5(\phi,X)$ are strongly constrained. Still $G_2(\phi,X)$ & $G_3(\phi,X)$ are free.
  $G_3(\phi,X)$ may be constrained due to GW-DE interactions [Creminelli, Tambalo, Vernizzi, Yingcharoenrat 2019]
- Generalized Proca theory (vector-tensor theory):
  Among 6 (or more) free functions, $G_4(X)$ & $G_5(X)$ are strongly constrained. Still $G_2(X,F,Y,U)$, $G_3(X)$, $G_6(X)$, $g_5(X)$ are free.

$$X = -A^\mu A_\mu$$

- Horava-Lifshitz theory (renormalizable quantum gravity):
  The coefficient of $R^{(3)}$ is strongly constrained
  → IR fixed point with $c_{gw} = c_\gamma$? How to speed up the RG flow?
- Ghost condensation (EFT of scalar-tensor theory in Minkowski/de Sitter):
  No additional constraint
- Massive gravity (simplest modification of GR):
  Upper bound on graviton mass ≈ 10^{-22}eV
  Much weaker than the requirement from acceleration

c.f. "All" gravity theories (including general relativity):
The cosmological constant is strongly constrained ≈ 10^{-120}.

Thank you!

Backup slides

Extension to FLRW background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under x^i → $x^i(t,x)$
- Ingredients

  $g_{\mu\nu}, g^{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu,$ $\boxed{\text{t \& its derivatives}}$

- 1st derivative of t

$$\partial_\mu t = \delta^0_\mu \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}$$

$$g^{00} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2nd derivative of t

$$K_{\mu\nu} \equiv h^\rho_\mu \nabla_\rho n_\nu$$

Unitary gauge action

$$\boxed{I = \int d^4x \sqrt{-g}\, L(t, \delta^0_\mu, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})}$$

derivative & perturbative expansions

$$\begin{aligned} I &= M_{Pl}^2 \int dx^4 \sqrt{-g} \left[\frac{1}{2} R + c_1(t) + c_2(t) g^{00} \right. \\ &\quad \left. + L^{(2)}(\tilde{\delta} g^{00}, \tilde{\delta} K_{\mu\nu}, \tilde{\delta} R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu) \right] \end{aligned}$$

$$\begin{aligned} L^{(2)} &= \lambda_1(t)(\tilde{\delta} g^{00})^2 + \lambda_2(t)(\tilde{\delta} g^{00})^3 + \lambda_3(t)\tilde{\delta} g^{00} \tilde{\delta} K^\mu_\mu \\ &\quad + \lambda_4(t)(\tilde{\delta} K^\mu_\mu)^2 + \lambda_5(t)\tilde{\delta} K^\mu_\nu \tilde{\delta} K^\nu_\mu + \cdots \end{aligned}$$

NG boson

- Undo unitary gauge $t \rightarrow \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$
\begin{aligned}
H(t) &\rightarrow H(t+\pi), \quad \dot{H}(t) \rightarrow \dot{H}(t+\pi), \\
\lambda_i(t) &\rightarrow \lambda_i(t+\pi), \quad a(t) \rightarrow a(t+\pi), \\
\delta^0_\mu &\rightarrow (1+\dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi,
\end{aligned}
$$

- NG boson in decoupling (subhorizon) limit

$$
\boxed{
\begin{aligned}
I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \Bigg\{ & -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \\
& -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta} K, \tilde{\delta} R} \Bigg\}
\end{aligned}
}
$$

$$
\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8 c_s^2 \lambda_2}{-\dot{H}}\left(\frac{1}{c_s^2} - 1\right)^{-1}
$$

- Sound speed

c_s : speed of propagation for modes with $\omega \gg H$

$\omega^2 \simeq c_s^2 \frac{k^2}{a^2}$ for $\pi \sim A(t) \exp\left(-i \int \omega dt + i\vec{k} \cdot \vec{x}\right)$

Application: non-Gaussinity of inflationary perturbation $\zeta = -H\pi$

$$I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \left\{ -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \right.$$

$$\left. -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\boxed{\dot{\pi}^3} - \dot{\pi}\boxed{\frac{(\partial_i\pi)^2}{a^2}}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\delta K, \delta R} \right\}$$

→ power spectrum

$$P_\zeta(\vec{k}) = \frac{\Delta}{k^3}, \quad \Delta = \left.\frac{H^4}{-4M_{Pl}^2 \dot{H} c_s}\right|_{c_s k \simeq aH}$$

→ non-Gaussianity

$$\langle \zeta_{\vec{k}_1}(t)\, \zeta_{\vec{k}_2}(t)\, \zeta_{\vec{k}_3}(t) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta$$

2 types of 3-point interactions

c_s^2 → size of non-Gaussianity

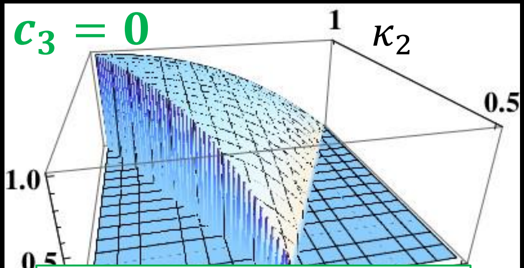
$$k^6 B_\zeta|_{k_1=k_2=k_3=k} = \frac{18}{5}\Delta^2 \left(f_{NL}^{\dot{\pi}(\partial_i\pi)^2} + f_{NL}^{\dot{\pi}^3}\right)$$

$$f_{NL}^{\dot{\pi}(\partial_i\pi)^2} = \frac{85}{324}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \qquad f_{NL}^{\dot{\pi}^3} = \frac{5\boxed{c_3}}{81}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \quad \propto \frac{1}{c_s^2}$$

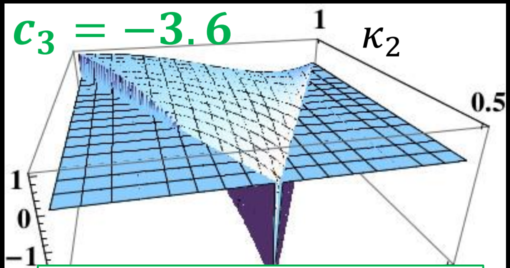
for small c_s^2

c_3 → shape of non-Gaussianity

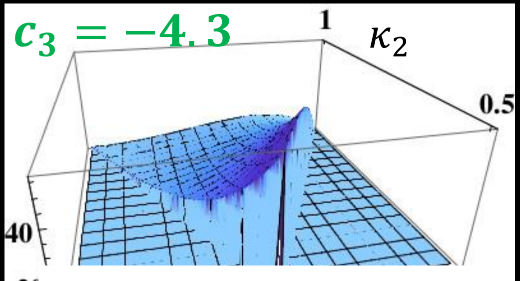
plots of $B_\zeta(k, \kappa_2 k, \kappa_3 k)/B_\zeta(k,k,k)$



Prototype of the equilateral shape



Prototype of the orthogonal shape



Linear combination of the two shapes